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by Richard Harrison

FILE	13679_RICHARD_HARRISON_1809806_179202_1384904216.DOCX (4.66M)		
TIME SUBMITTED	29-APR-2021 10:19AM (UTC+0100)	WORD COUNT	2771
SUBMISSION ID	150949732	CHARACTER COUNT	16368

Course:	HND Electrical and Electronic Engineering
Module:	SGEE5009 Control Engineering
Assignment:	Control Systems Analysis, Modelling and Simulation.
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University number:	1809806
Date submitted:	27/04/2021
Word count:	2041

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1. Executive summary

This report focuses on control systems analysis, modelling and simulation.
Firstly, using chosen values for “K” and “a” a transient response was calculated along with all the key factors of the response waveform like natural frequency, peak time and rise time.

Secondly, a control system was modelled using Laplace transforms to translate the equation into the frequency domain and then my own chosen values were inputted to produce a transfer function and develop a set of results. Use the third Person

Use the third Person

Thirdly, a block diagram was given which needed to be reduced to a single transfer function, then my chosen values were inputted to produce the final transfer function which summed up the original block diagram.

Fourthly, a frequency response was calculated with chosen values for p, q and r.

Lastly, using chosen values the minimum sampling frequency for a digital control system which ensured z-plane stability.

For each of the tasks a simulation was done to prove the calculations done were correct and the results were compared, evaluated, and explained.

2. Transient response of a second order system

2.1 Method and workings

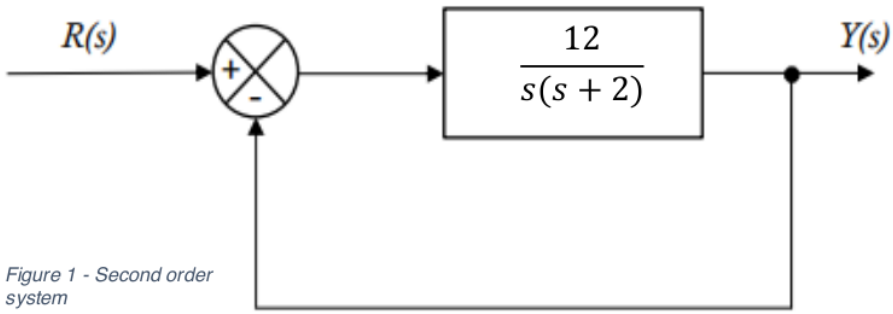


Figure 1 - Second order system

Reducing to a single transfer function by multiplying both the numerator and denominator by s(s+4):

$$T(s) = \frac{\frac{12}{s(s+2)}}{1 + \frac{12}{s(s+2)} \times 1} = \frac{12}{s(s+2) + 12} = \frac{12}{s^2 + 2s + 12}$$

Using the standard transfer function for second order circuits, the natural frequency can be solved:

$$\frac{12}{s^2 + 2s + 12} \; compares\ to \; \frac{K\omega n^2}{s^2 + 2\Sigma\omega_n^s + \omega_n^2}$$

From this,

$$\omega_n^2 = 12 \quad therefore, \quad \omega_n = \sqrt{12} = 3.46 rad/s$$

$$K\omega n^2 = 12 \; and \; \omega_n^2 = 12 \quad therefore \; K = 1$$

From the comparison of the transfer functions above, the damping factor is:

$$2\Sigma\omega_n = 2$$

$$\Sigma = \frac{2}{2\omega_n} = \frac{2}{2 \times 3.46} = 0.289$$

Which is underdamped

To work out the damped frequency:

$$\omega = \omega_n\sqrt{1 - \Sigma^2}$$

$$\omega = 3.46\sqrt{1 - 0.289^2} = 3.312 rad/s$$

Rise time:

$$t_r = \frac{\pi}{2 \times 3.46\sqrt{1 - 0.289^2}} = 0.474$$

Or

Most of these calculations should be in the appendix as stated in the brief with a summary table of results placed in the main report.

$$t_r = \frac{\pi - tan^{-1}(\frac{\sqrt{1-0.289^2}}{0.289})}{3.312} = 0.563$$

Peak time:

$$t_p = \frac{\pi}{3.46\sqrt{1-0.289^2}} = 0.949 seconds$$

Percentage overshoot:

$$Percentage\ overshoot = 100_e^{(\frac{-0.289\pi}{\sqrt{1-0.289^2}})} = 38.74\%$$

Settling time within 2%:

$$t_s(2\%) = \frac{4}{0.289 \times 3.46} = 4 seconds$$

To work out the number of oscillations during the settling time:

$$n = \frac{2}{\pi}\sqrt{\frac{1}{\Sigma^2} - 1} = \frac{2}{\pi}\sqrt{\frac{1}{0.289^2} - 1} = 2.11$$

2.2 MatLab results and comparison

2.2.1 peak time, Rise time, Settling time.

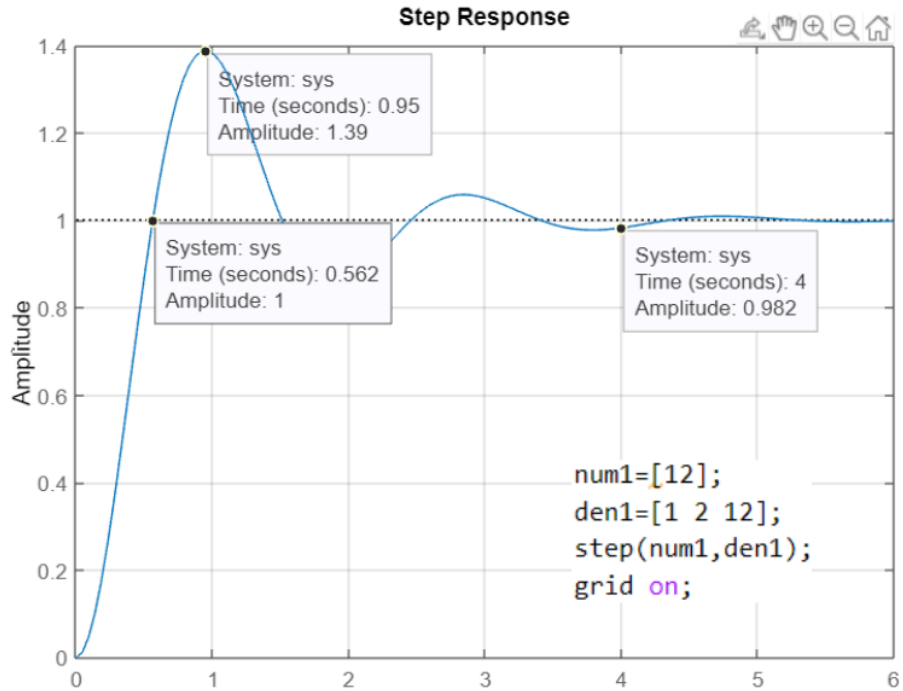


Figure 2 - Peak time, rise time, settling time, oscillation number

The peak time measured in MatLab is 0.95 seconds which matches the calculated value of 0.949.

From the MatLab simulation, the time taken to rise to its final value is 0.562 seconds. The calculations rise time value is 0.563 seconds which is acceptable.

Settling time from the calculation (Calculated as 4 seconds) is close to the MatLab simulation. The curve settles to within 2% of its final value at the 4.31 second mark.

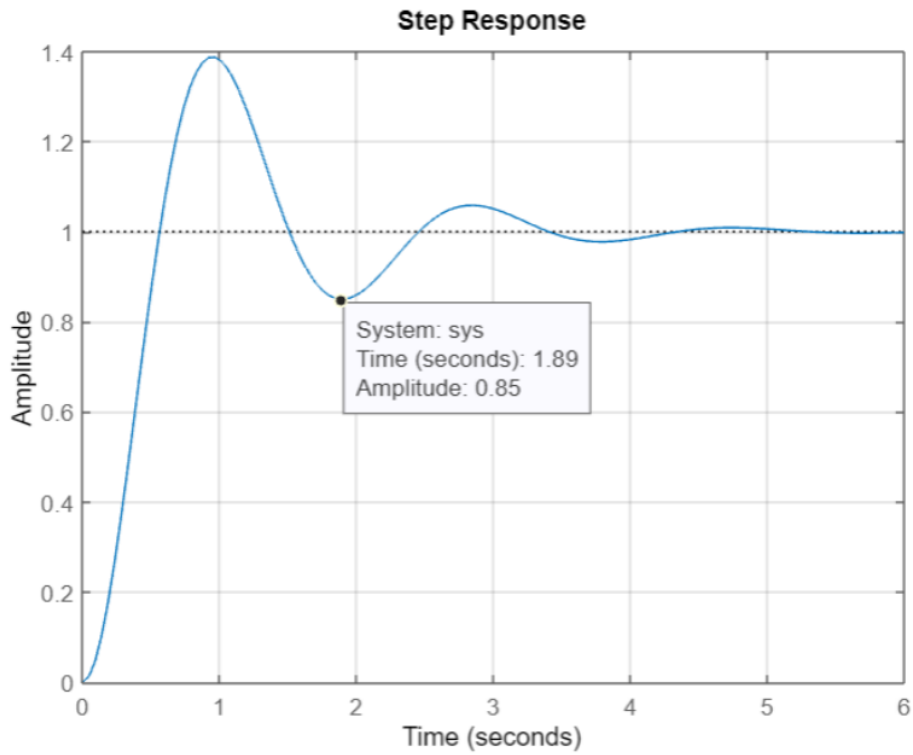


Figure 3 - percentage overshoot and number of oscillations

As shown in figure 3 the percentage overshoot in the simulation is 39%. The calculated overshoot is 38.74% which proves the simulation is accurate. Also, the number of oscillations was worked out to be 2.11. This is also proven as the number of full rotations before the waveform falls within 2% is 2.

The damped frequency from the MatLab simulation works out as:

$$\pi\left(\frac{1}{1.89} \times 2\right) = 3.324\, rad/s$$

The calculated value for the damped frequency is 3.312rad/s which proves the formulae is correct.

3. Control system modelling

Open loop control system differential equation with own values:

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y = \frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 4x$$

3.1 Producing the general solution

Assuming y(0)=0:

$$\mathcal{L}\left\{\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 2y\right\} = s^2Y(s) + 4sY(s) + 2Y(s)$$

Assuming x(0)=0:

$$\mathcal{L}\left\{\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + 4x\right\} = s^2X(s) - 2sX(s) + 4X(s)$$

By putting both equations back into their original positions:

$$s^2Y(s) + 4sY(s) + 2Y(s) = s^2X(s) - 2sX(s) + 4X(s)$$

$$Y(s)[s^2 + 4s + 2] = X(s)[s^2 - 2s + 4]$$

Therefore:

$$\frac{Y(s)}{X(s)} = \frac{s^2 - 2s + 4}{s^2 + 4s + 2} \; and \; G(s) = \frac{s^2 - 2s + 4}{s^2 + 4s + 2}$$

3.2 Poles location, response type, natural frequency, damping factor

By then taking the G(s) numerator the zero positions can be calculated:

$$s^2 - 2s + 4 = 0$$

$$s = \frac{-(-2) \pm \sqrt{-2^2 - 4 \times 1 \times 4}}{2 \times 1}$$

$$s = \frac{2 \pm \sqrt{4 - 16}}{2}$$

Since a root cannot be negative:

$$s = \frac{2 \pm \sqrt{12}}{2}$$

$$s = 1 \pm j1.732$$

To find the system poles using G(s) denominator:

$$s^2 + 4s + 2 = 0$$

$$s = \frac{-4 \pm \sqrt{4^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$s = -0.586, -3.414$$

These results are confirmed by Matlab in figure 4 below:

```
clc;
s = tf('s');
system1 = (s^2 - 2*s + 4)/(s^2 + 4*s + 2)
pole(system1)
zero(system1)

grid on
hold off
step(system1)
```

Command Window

```
system1 =

  s^2 - 2 s + 4
  -------------
  s^2 + 4 s + 2

Continuous-time transfer function.


ans =

   -3.4142
   -0.5858


ans =

   1.0000 + 1.7321i
   1.0000 - 1.7321i
```

Figure 4 - Matlab results

3.2.1 Argand diagram

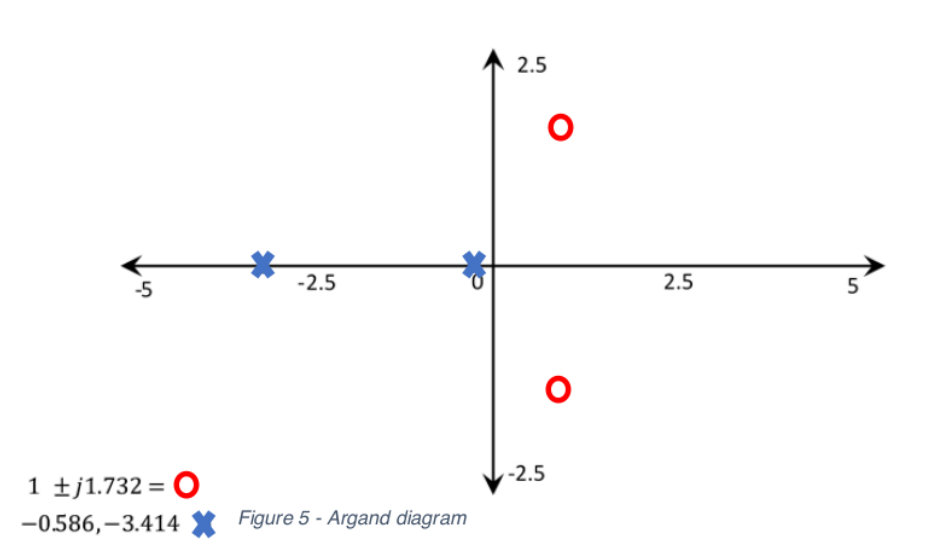


Figure 5 - Argand diagram

Figure 5 shows the pole positions on the argand diagram.
The expected response is for it to be stable and overdamped since the zero positions are in the right-hand plane and the poles are on the real axis.
Use the denominator of the general solution to compare against the characteristic equation:

$$s^2 + 2\Sigma\omega_n^s + \omega_n^2 \; compares \; with \; s^2 + 4s + 2$$

$$\omega_n^2 = 2$$

$$\omega_n = 1.414 rad/s$$

$$2\Sigma\omega_n^s = 4$$

$$\Sigma = \frac{4}{2 \times 1.414} = 1.414$$

Figure 6 shows the step response when the general solution is inputted into Matlab. Figure 7 shows the Matlab code used to create the response. The step response does not oscillate as it reaches steady state proving it is overdamped. Despite the initial drop to begin with.

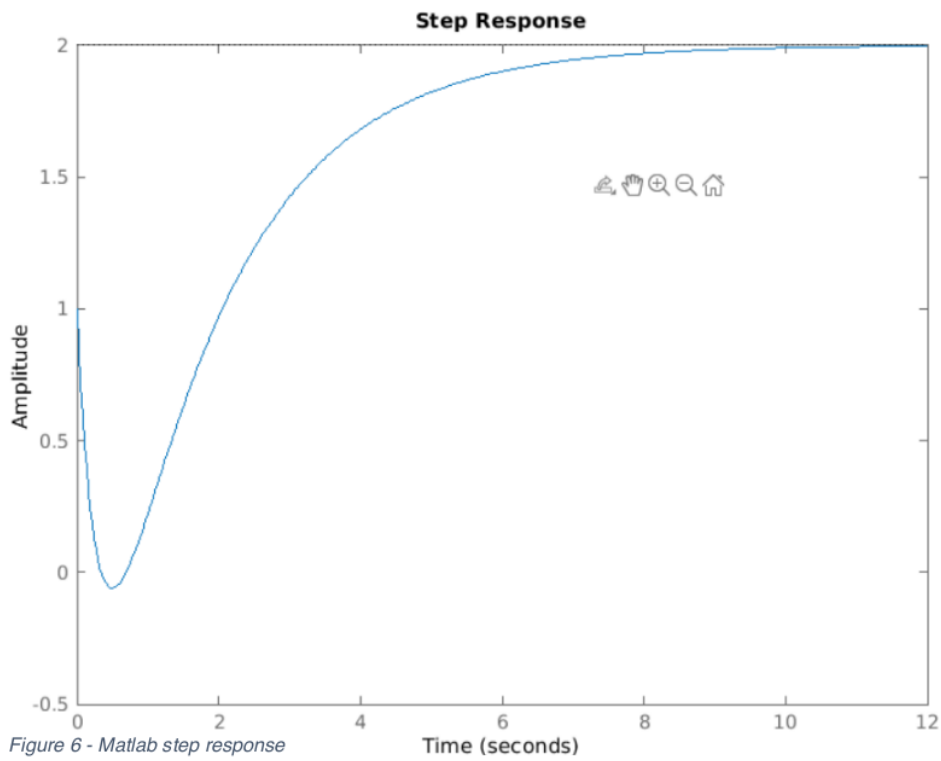


Figure 6 - Matlab step response

```
clc;
s = tf('s');
system1 = (s^2 - 2*s + 4)/(s^2 + 4*s + 2)
pole(system1)
zero(system1)

grid on
hold off
step(system1)
```

Figure 7 - Step response Matlab code

Figure 8 shows the positions on a Matlab generated root locus diagram which can be used to prove the locations of the poles and zeros. As calculated, the zero positions are within the right-hand plane at 1+1.73 and 1-1.73 while the pole positions are precisely where calculated. -0.586 and -3.41.

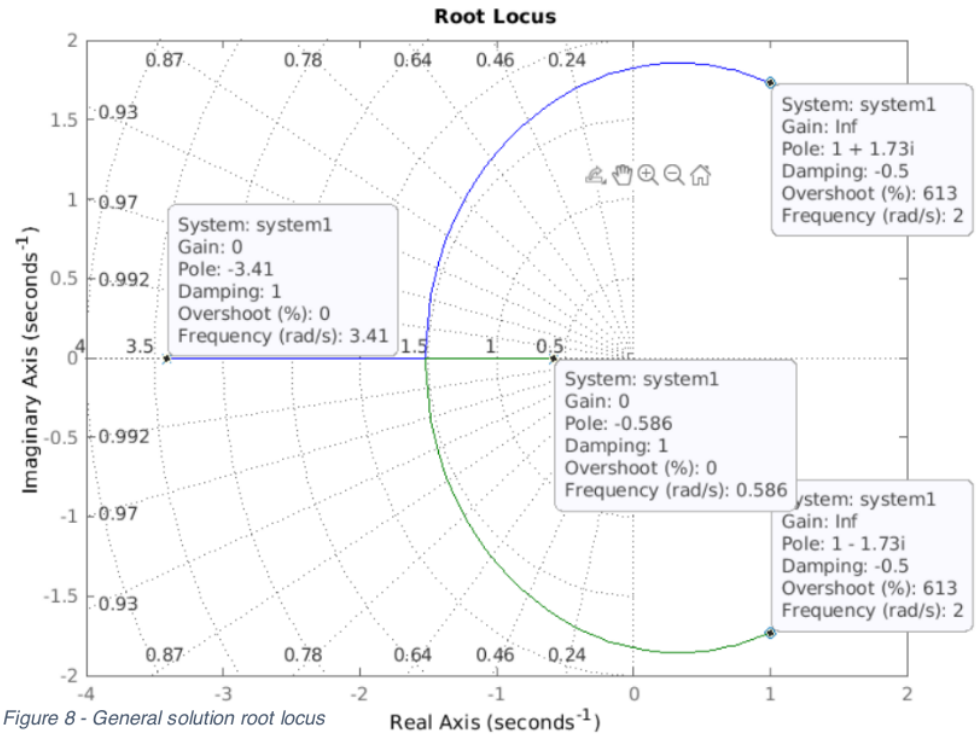


Figure 8 - General solution root locus

3.3 Effects of negative feedback and critical damping

By using the proven root locus response diagram shown in figure 9 and the pointer, the point of critical damping can be shown as 0.189 which can be used as a value of "k".

Using 0.189 as the value of "k" with the transfer function:

$$T(s) = \frac{G(s)}{1 + H(s)G(s)}$$

$$T(s) = s^2 + \frac{4 - 2(0.189)}{1 + 0.189}s + \frac{2 + 4(0.189)}{1 + 0.189}$$

$$T(s) = s^2 + 3.046s + 2.318$$

By comparing the G(s) characteristic equation:

$$\omega n^2 = 2.318$$

$$\omega n = \sqrt{2.318} = 1.523 rad/s$$

Therefore:

$$2\Sigma\omega_n^s = 3.046$$

$$\Sigma = \frac{3.046}{2\sqrt{1.523}} = 1.234$$

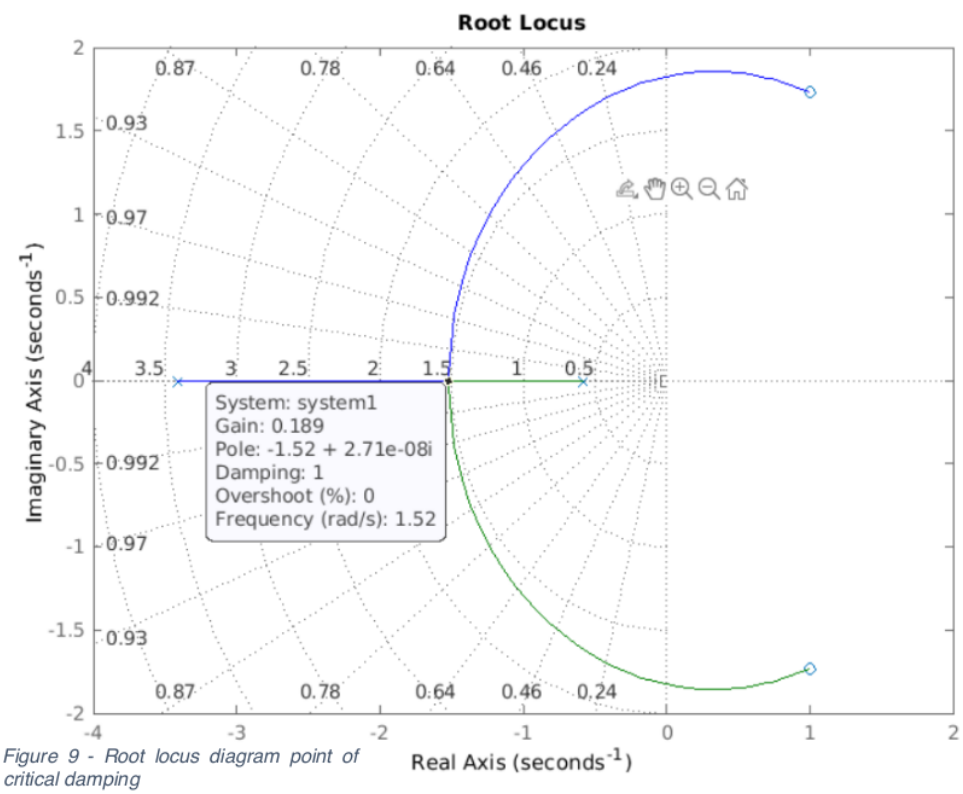


Figure 9 - Root locus diagram point of critical damping

Figure 10 shows the points of critical stability on the root locus diagram generated using Matlab and the G(s). Critical stability refers to the point were the lines intersect the real axis at zero. As shown in figure 10, the gain is 1.99 (which can be rounded to 2). This can be checked with the following calculation to add negative feedback. (closed loop system)

$$T(s) = \frac{G(s)}{1 + H(s)G(s)}$$

$$= \frac{\frac{s^2 - 2s + 4}{s^2 + 4s + 2}}{1 + K \times (\frac{s^2 - 2s + 4}{s^2 + 4s + 2})}$$

$$= \frac{s^2 - 2s + 4}{s^2 + 4s + 2 + Ks^2 - 2Ks + 4K}$$

$$= \frac{s^2 - 2s + 4}{(1 + K)s^2 + (4 - 2K)s + (2 + 4K)}$$

$$T(s) = \frac{\frac{s^2 - 2s + 4}{(1 + K)}}{s^2 + \frac{(4 - 2K)}{(1 + K)}s + \frac{(2 + 4K)}{(1 + K)}}$$

Using this equation, the value of "K" can be calculated and then checked against the value of "K" in Matlab to verify that it is correct.

Using the denominator of T(s):

$$T(s) = s^2 + \frac{4 - 2(K)}{1 + K}s + \frac{2 + 4(K)}{1 + K} = 0$$

Simplified:

$$4 - 2K = 0$$

$$-2K = -4$$

$$K = \frac{-4}{-2}$$

$$K = 2$$

Figure 10 shows the gain to be 1.99 which can be rounded to 2, to match the calculation. This could be due to I minor error in the Matlab software but the difference is negligible.

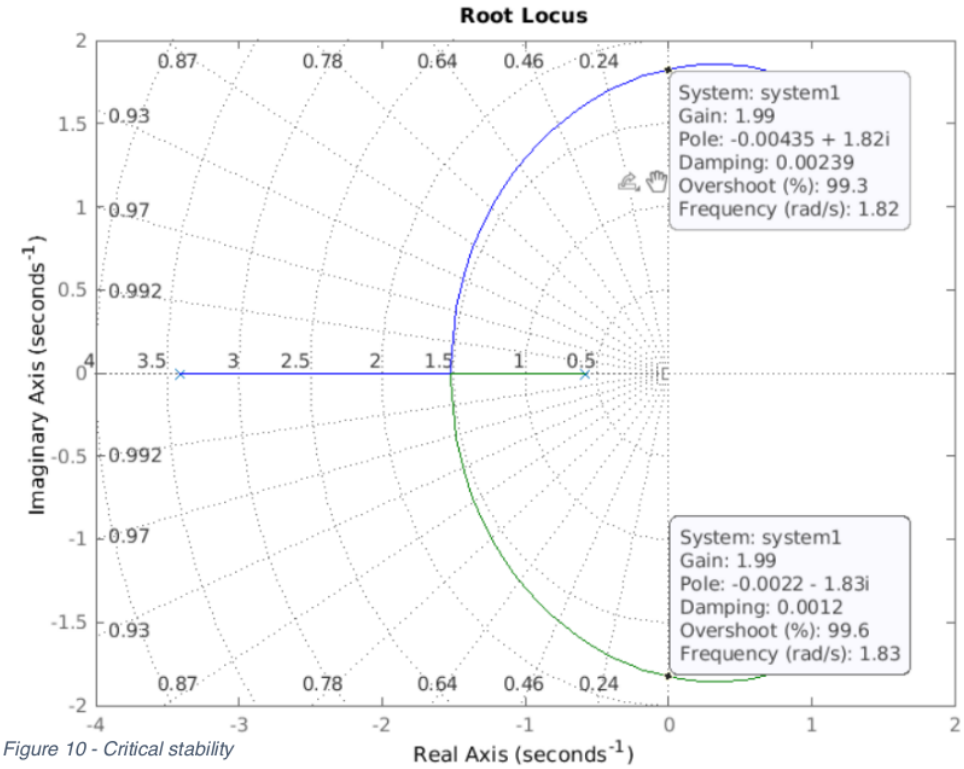


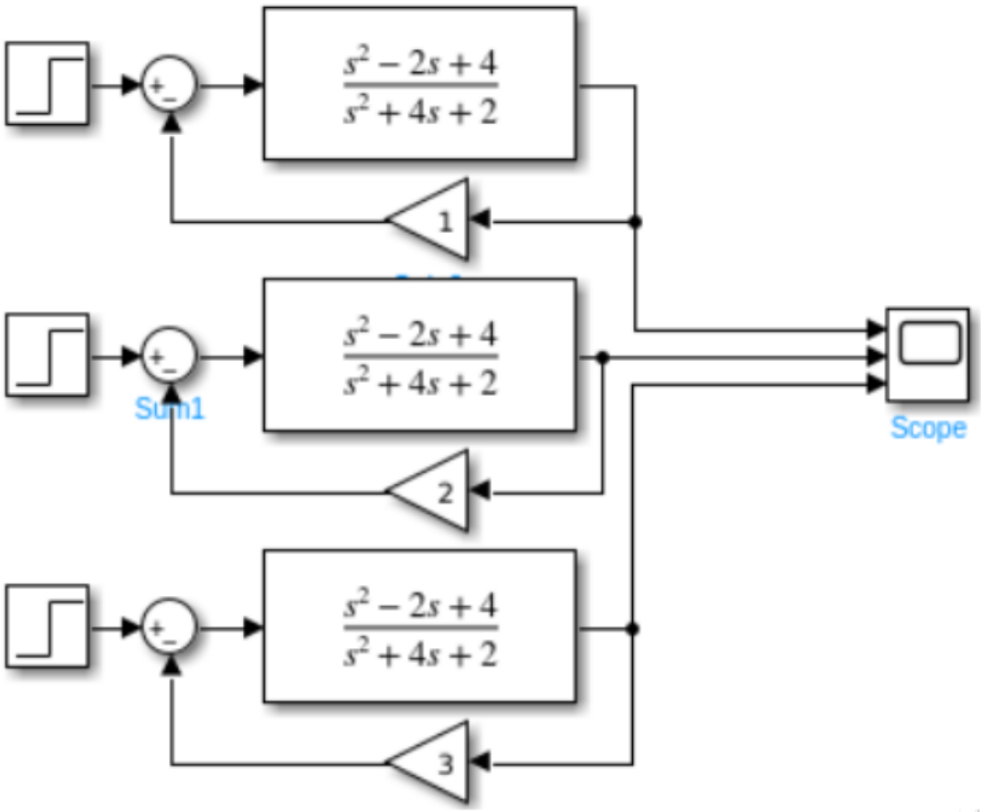
Figure 10 - Critical stability

By adding negative feedback, the zeros move along the root locus within their respective margins.

Negative feedback and critical damping are what allows the curve to smoothen out (settle) into a straight line instead of fluctuating.

Figure 11 shows the Simulink and the result on the built-in scope utility. The feedback values used are 2, for critical stability which should produce a waveform which will not settle but will oscillate continuously at the same rate (shown in brown). This also means the zeroes lie on the $j\omega$ axis. Decreasing the feedback by 50% gives an underdamped response which will peak and oscillate a little before settling (shown in red) making it stable while meaning $\Sigma < 1$. Lastly, increasing the feedback by an additional 50% causes an unstable waveform which increases in oscillation size indefinitely as time goes on (shown in yellow). Adding unity negative feedback should stabilise the unstable waveform.

When does critical damping occur?



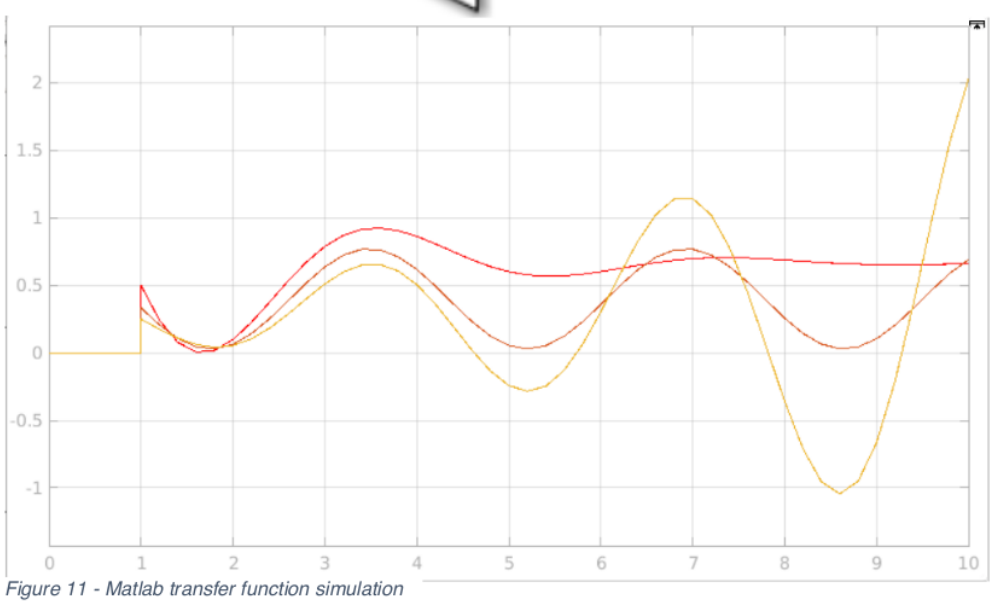


Figure 11 - Matlab transfer function simulation

a good way of proving stability

3.4 Root locus method

The root locus method allows the user to closely analyse the poles and zero points of any given transfer function. It also allows the user to find useful information like the point of critical stability/damping. Once produced, the rlocus diagram can be used to find the point of critical damping or “K”. Meaning further calculations can be done. The rlocus is vital in proving that the calculations are correct.

4. Block diagram reduction method

4.1 Reducing to single transfer function and finding T

$$G_5 = G_3G_4$$

$$G_6 = \frac{G_5}{1 - G_5H_1}$$

$$G_7 = G_6G_1 + G_2$$

H1 is positive feedback with G5 and H2 is negative feedback with G8.

$$T = \frac{G_7}{1 + G_7H_2}$$

Suitable for G7

$$T = \frac{G_6G_1 + G_2}{1 + (G_6G_1 + G_2)H_2}$$

$$T = \frac{G_6G_1 + G_2}{1 + H_2G_6G_1 + H_2G_2}$$

Suitable for G6

$$T = \frac{\left(\frac{G_5}{1 - G_5H_1}\right)G_1 + G_2}{1 + H_2\left(\frac{G_5}{1 - G_5H_1}\right)G_1 + H_2G_2}$$

Multiply numerator and denominator by 1-G_5H_1

$$T = \frac{G_5G_1 + G_2(1 - G_5H_1)}{(1 - G_5H_1) + H_2G_5G_1 + H_2G_2(1 - G_5H_1)}$$

$$T = \frac{G_5G_1 + G_2 - G_2G_5H_1}{1 - G_5H_1 + H_2G_5G_1 + H_2G_2 - H_2G_2G_5H_1}$$

Suitable for G5

$$T = \frac{(G_3G_4)G_1 + G_2 - G_2(G_3G_4)H_1}{1 - (G_3G_4)H_1 + H_2(G_3G_4)G_1 + H_2G_2 - H_2G_2(G_3G_4)H_1}$$

$$T = \frac{G_3G_4G_1 + G_2 - G_2G_3G_4H_1}{1 - G_3G_4H_1 + H_2G_3G_4G_1 + H_2G_2 - H_2G_2G_3G_4H_1}$$

Sub in Values:

G1=5 **G2=0.1** **G3=2/s+1** **G4=1/s** **H1=1/s+1** **H2=3**

$$T = \frac{\left(\frac{2}{s+1} \times \frac{1}{s} \times 5\right) + 0.1 - \left(0.1 \times \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}\right)}{1 - \left(\frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}\right) + \left(3 \times \frac{2}{s+1} \times \frac{1}{s} \times 5\right) + (3 \times 0.1) - \left(3 \times 0.1 \times \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}\right)}$$

$$T = \frac{\frac{10}{s(s+1)} + 0.1 - \frac{0.2}{s(s+1)^2}}{1 - \frac{2}{s(s+1)^2} + \frac{30}{s(s+1)} + 0.3 - \frac{0.6}{s(s+1)^2}}$$

$$T = \frac{10(s+1) + 0.1(s+1)^2 - 0.2}{s(s+1)^2 - 2 + 30(s+1) + 0.3s(s+1)^2 - 0.6}$$

$$T = \frac{10s + 10 + 0.1s^3 + 0.1s^2 + 0.1s^2 + 0.1s - 0.2}{s^3 + s^2 + s^2 + s - 2 + 30s + 30 + 0.3s^3 + 0.3s^2 + 0.3s^2 + 0.3s - 0.6}$$

$$T = \frac{0.1s^3 + 0.2s^2 + 9.9s + 9.8}{1.3s^3 + 2.6s^2 + 31.3s + 27.4}$$

Figures 12, 13 and 14 show the results of the transfer function using Matlab. Figure 14 shows it proven in Matlab code (Bode and Nyquist methods) while 12 and 13 show the pole and zero positions on a Rlocus diagram. For each of the three methods, the results are all the same. -0.5 ± 9.89 pole position and a $-0.543 \pm j4.77$ zero position.

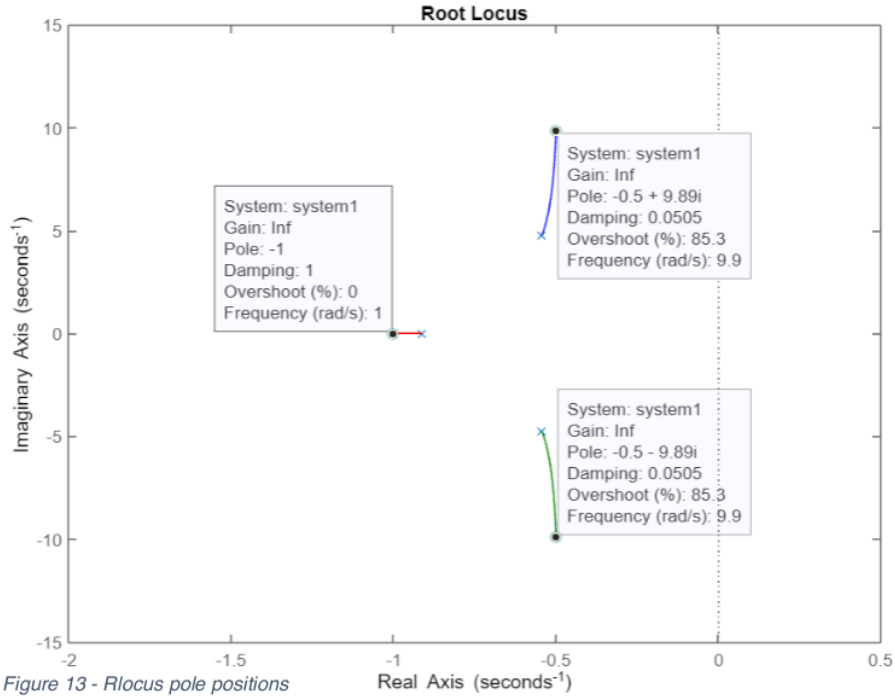


Figure 13 - Rlocus pole positions

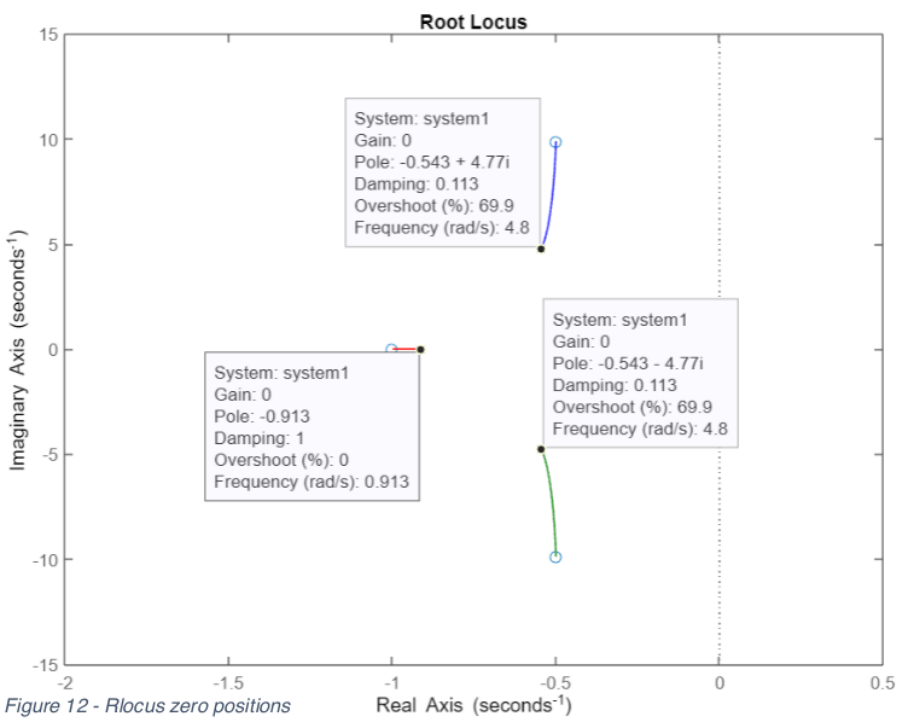


Figure 12 - Rlocus zero positions

```
clc;
s=tf('s');
system1=(0.1*s^3+0.2*s^2+9.9*s+9.8)/(1.3*s^3+2.6*s^2+31.3*s+27.4);
pole(system1)
zero(system1)

grid on
hold off
step(system1)
```

Command Window

```
ans =

  -0.5435 + 4.7738i
  -0.5435 - 4.7738i
  -0.9130 + 0.0000i


ans =

  -0.5000 + 9.8869i
  -0.5000 - 9.8869i
  -1.0000 + 0.0000i
```

```
clc;
num=[0.1 0.2 9.9 9.8];
den=[1.3 2.6 31.3 27.4];

[z,p,k]=tf2zp(num,den)
```

Command Window

```
z =

  -0.5000 + 9.8869i
  -0.5000 - 9.8869i
  -1.0000 + 0.0000i


p =

  -0.5435 + 4.7738i
  -0.5435 - 4.7738i
  -0.9130 + 0.0000i


k =

    0.0769
```

Figure 14 - MatLab Pole and zero positions proved with both proving methods

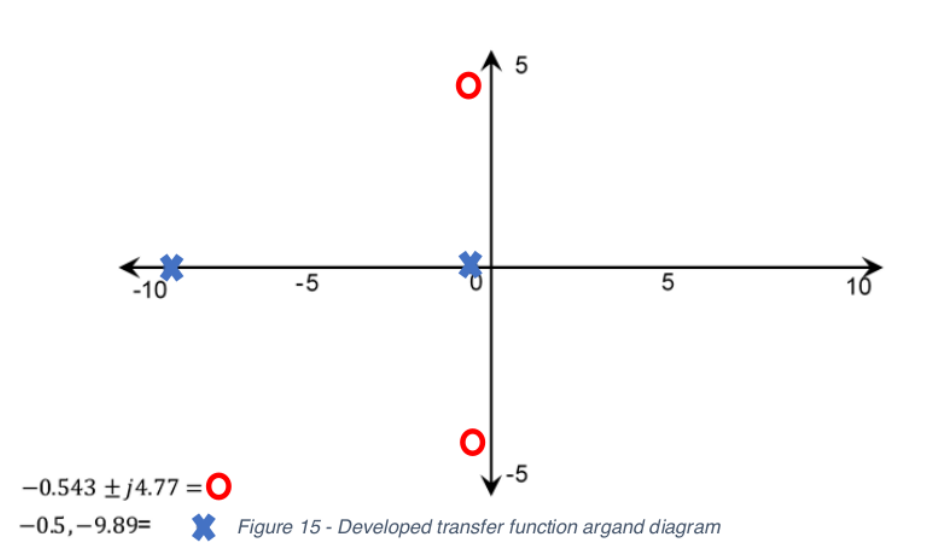
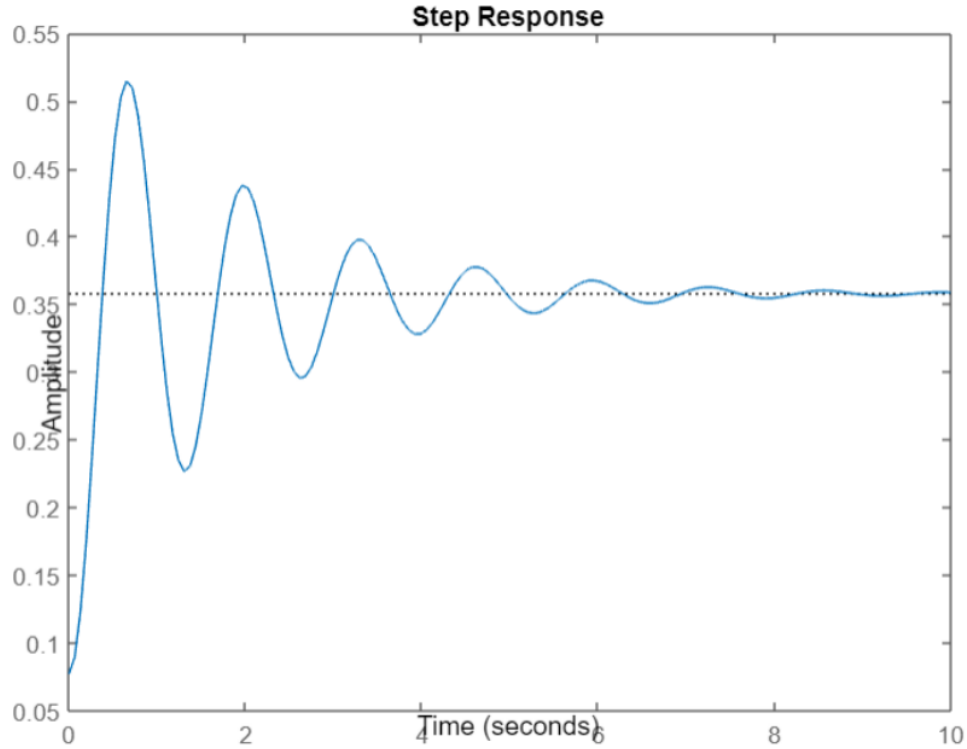


Figure 15 - Developed transfer function argand diagram

Figure 15 shows the developed transfer functions argand diagram produced from the zero and pole positions produced in figures 12-14. The pole positions on the real axis prove the resultant waveform should be stable while the zero positions mean the system will be underdamped. This will mean the waveform will overshoot and oscillate before reaching a steady state because the zeroes have imaginary parts. The oscillations will only occur and degrade to steady state as long as the imaginary parts are in the left hand plane.




```
clc;
s=tf('s');
system1=(0.1*s^3+0.2*s^2+9.9*s+9.8)/(1.3*s^3+2.6*s^2+31.3*s+27.4);
pole(system1);
zero(system1);
step(system1)
```

Figure 16 - Developed solution step response

Figure 16 shows the step response of the developed transfer function. As shown by the oscillations and overshoot of the waveform before reaching its settling point at approximately 8 seconds according to the graph, the waveform is underdamped.

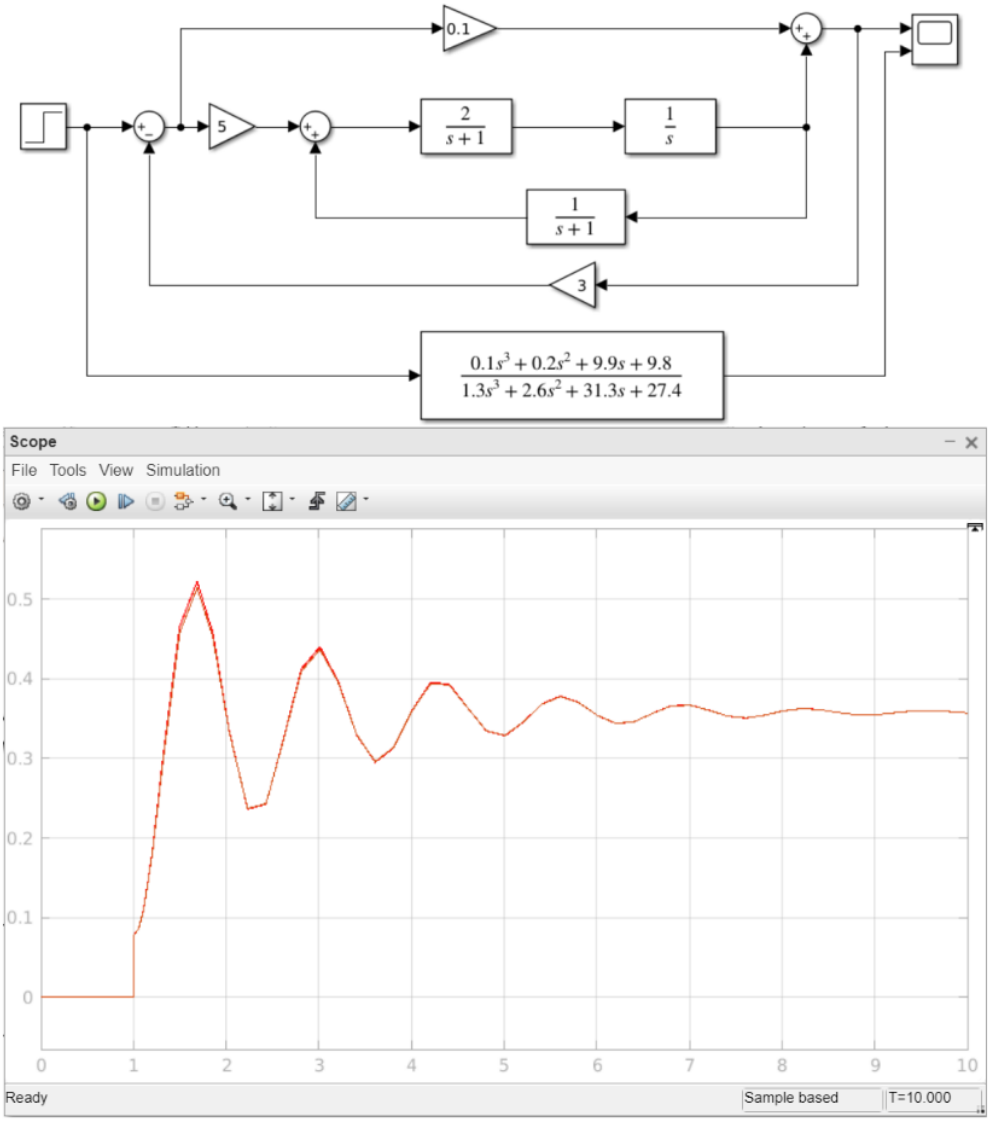


Figure 17 - Simulink results from transfer function and original block diagram

Figure 17 above shows that the results of both the developed transfer function and the and the original both matches. Proving that the developed one is correct since there are no differences in the two waveforms.

Good evidence of testing

5. Frequency response method

This method is often used to investigate how a system behaves and is best described as the steady state of a transient waveform (meaning the output is constant) responding to a range of set sinusoidal frequencies.

P=2, q=20, r=300

$$G(s) = \frac{K(s+p)}{(s+q)(s+r)}$$

P=Wc1=2 q=Wc2=20 r=Wc3=300

$$T(s) = \frac{K_1(s+\omega_{c1})}{(s+\omega_{c2})(s+\omega_{c3})}$$

ω_c1,2,3 refer to the cut-off frequencies and the times at which the response changes state. From climbing stated by P, to peaking stated by Q to falling stated by C.

$$T(j\omega) = \frac{K_1(j\omega+\omega_{c1})}{(j\omega+\omega_{c2})(j\omega+\omega_{c3})}$$

$$T(j\omega) = \frac{K_1\left(\frac{j\omega}{\omega_{c1}}+1\right)\omega_{c1}}{\left(\frac{j\omega}{\omega_{c2}}+1\right)\omega_{c2}\left(\frac{j\omega}{\omega_{c3}}+1\right)\omega_{c3}}$$

Separating out the brackets:

$$T(j\omega) = \frac{K_1\omega_{c1}}{\omega_{c2}\omega_{c3}} \times \frac{\left(1+\frac{j\omega}{\omega_{c1}}\right)}{\left(1+\frac{j\omega}{\omega_{c2}}\right)\left(1+\frac{j\omega}{\omega_{c3}}\right)}$$

The first fraction in the equation above can be used to calculate the overall gain (K). The second fraction can be used in the bode magnitude diagram.

For bode magnitude diagram:

$$|T(j\omega)|_{dB} = +20log_{10}\left|1+\frac{j\omega}{\omega_{c1}}\right| \qquad so, at\ \omega = \omega_{c1} \qquad Asymptote\ gradient = +20log|K|$$

$$|T(j\omega)|_{dB} = -20log_{10}\left|1+\frac{j\omega}{\omega_{c2}}\right| \qquad so, at\ \omega = \omega_{c2} \qquad Asymptote\ gradient = -20log|K|$$

$$|T(j\omega)|_{dB} = -20log_{10}\left|1+\frac{j\omega}{\omega_{c3}}\right| \qquad so, at\ \omega = \omega_{c3} \qquad Asymptote\ gradient = -20log|K|$$

When dBs are used to add magnitudes, the frequency levels can be measured on a logarithmic scale. This means that approximations can be made with regards to the response with an error range of 3dBs at the cut-off frequencies.

$$where\ \omega \ll \omega_{c1}\ at\ lower\ frequencies\ |T(j\omega)|_{dB} = 20log|K|$$

Bearing in mind: **P=Wc1=2, q=Wc2=20, r=Wc3=300** which are used to find the overall bode response

The overall gain (K):

$$K = \frac{K_1 P}{q \times r}$$

Where the graph rises at p, peaks at q and falls at r. Using K_1=12000:

$$K = \frac{12000 \times 2}{20 \times 300} = 4$$

$$20log_{10}|4| = 12.04$$

$$T(s) = \frac{12000(s+2)}{(s+20)(s+300)}$$

$$T(s) = \frac{12000s + 24000)}{s^2 + 320s + 6000)}$$

Using MatLab, P=Wc1=2, q=Wc2=20, r=Wc3=300 and K=6000:

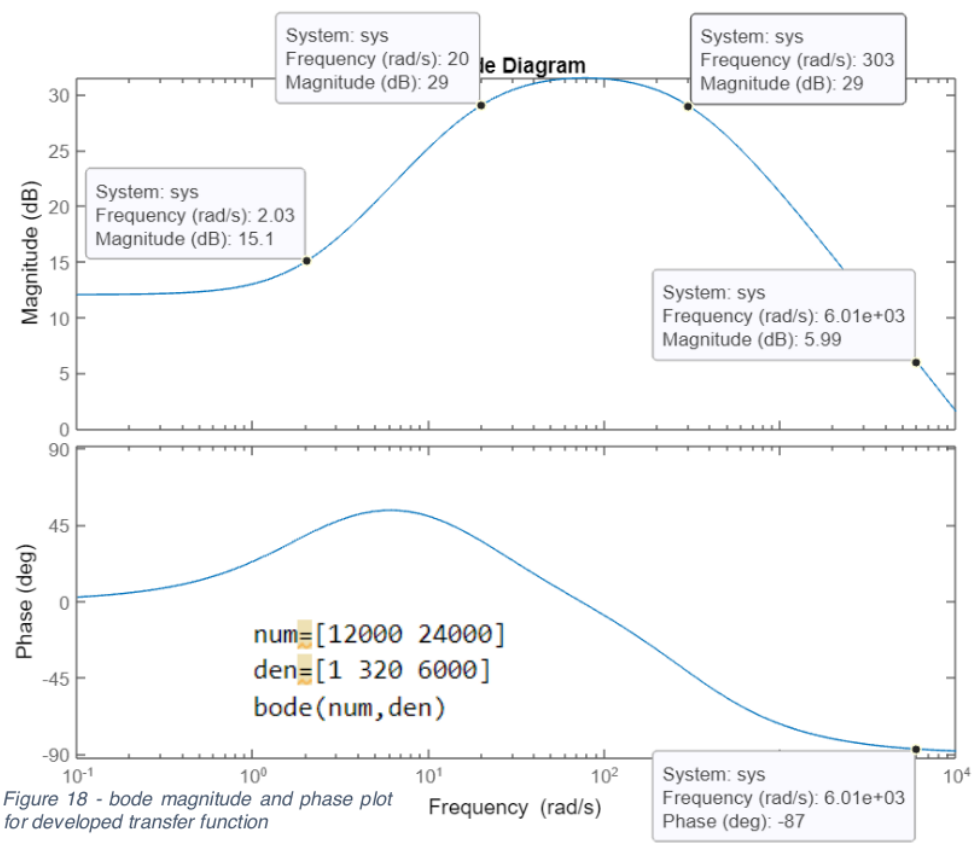


Figure 18 - bode magnitude and phase plot for developed transfer function

Figure 18 above shows the results when the values are put into MatLab. The results are always 3dbs off. As the magnitude climbs to its peak the phase angle steepens and decreases down to negative 90 degrees after the magnitude has peaked. Possibly, because the system would be increasing at this point and once the steady state is reached the system eases off.

5.1 Asymptote plots

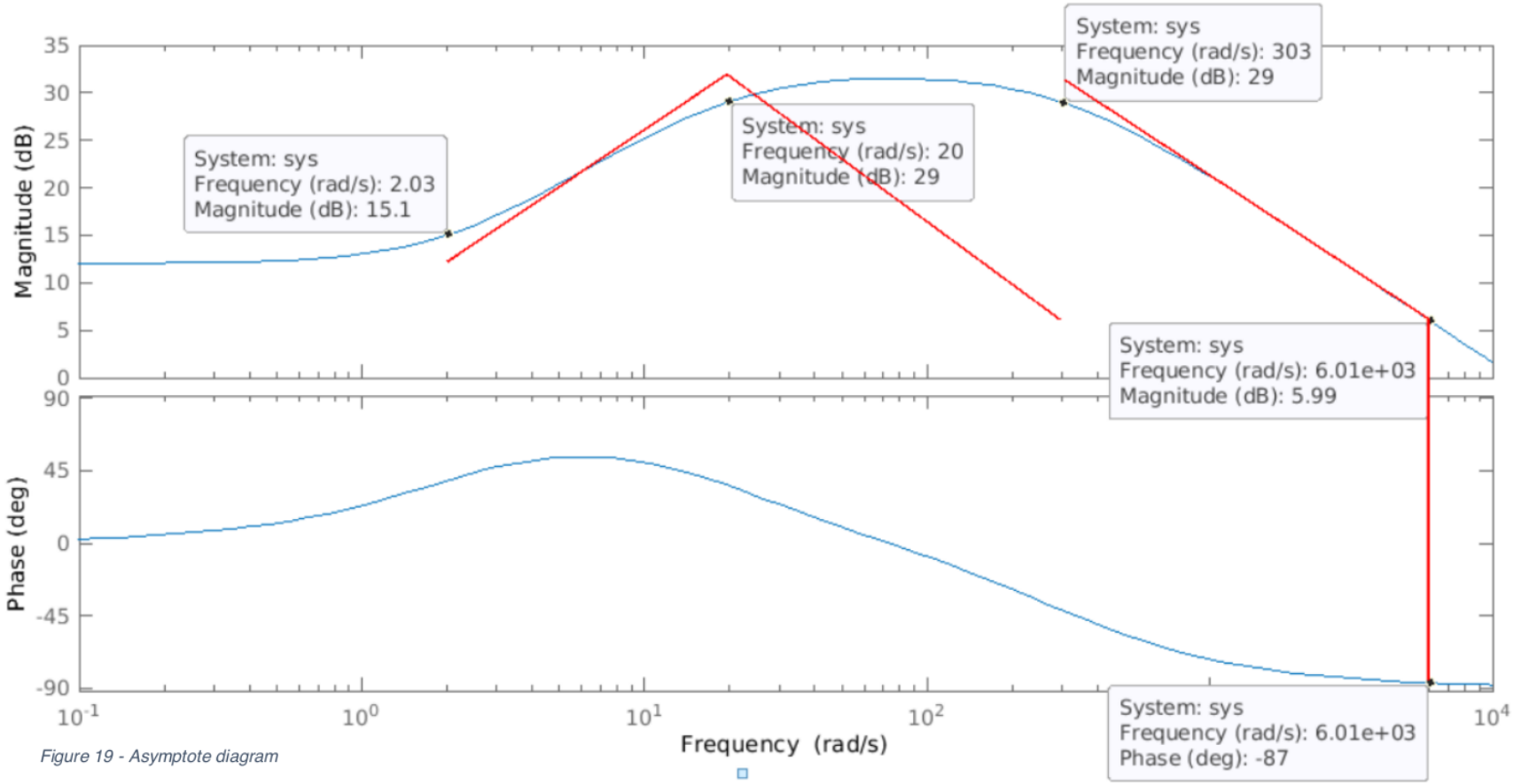


Figure 19 - Asymptote diagram

Regarding figure 19, the asymptote diagram shows that once the magnitude goes below the "K" value of 6000, the phase angle is approximately at its lowest point. (Or negative peak). The asymptote points above the p, q and below the r points show the error range of the real cut-off points of the response since the simulator has a 3db error range. Where the line coming from point "P" intersects the line coming from point "K" is the peak magnitude.

5.2 Relative stability

The phase does not reach -180 degrees. Meaning the gain margin method cannot be used since -180 degrees is required for this method. Therefore, the phase margin method is used.

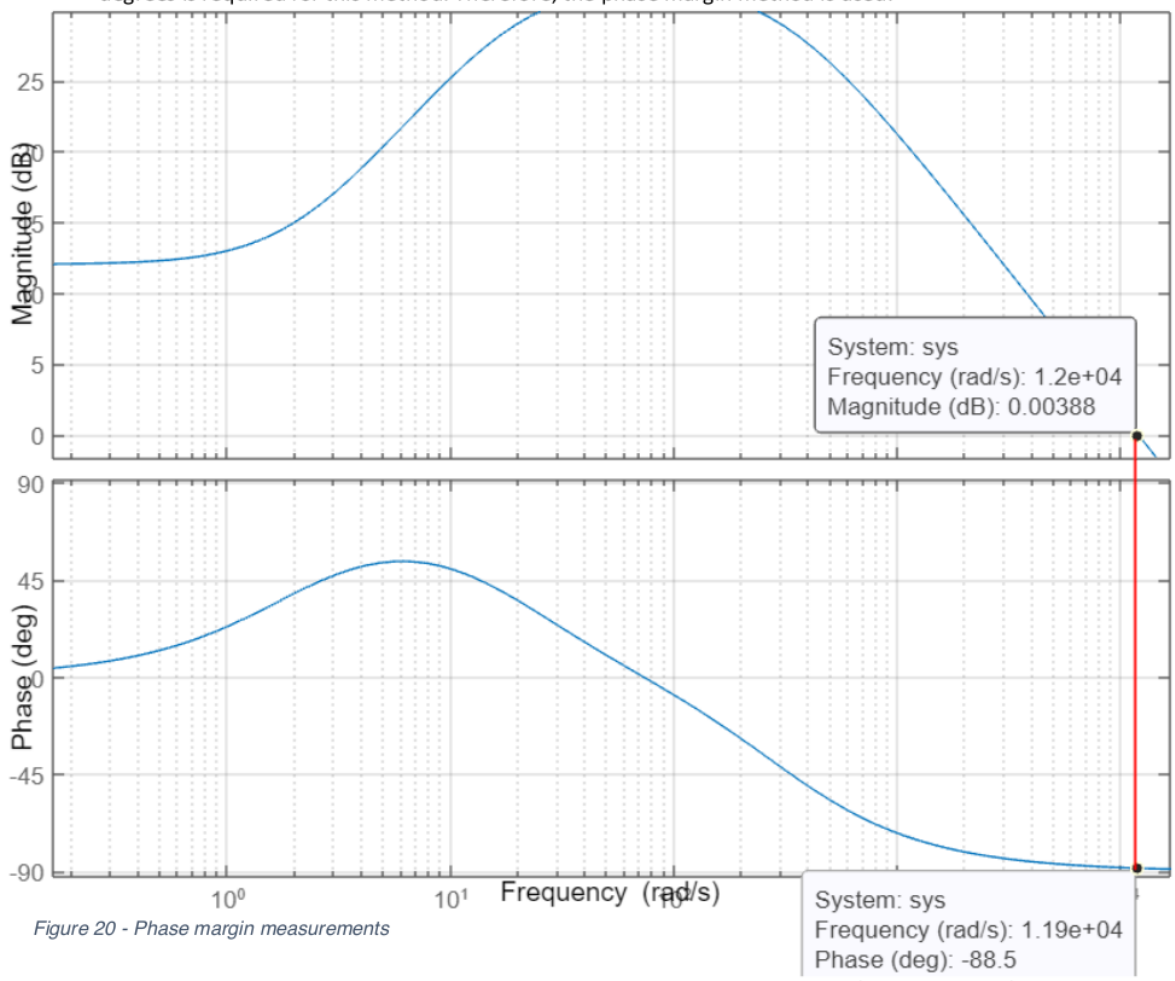


Figure 20 - Phase margin measurements

As shown in figure 20, when the gain is 0Db, the frequency is about 12000rad/s. At 12000rad/s the phase angle is -88.5 degrees. Therefore:

$$Phase\ margin = -180° - (-88.5°) = -91.5°$$

Good answer

Because the phase margin is greater than 45 degrees the system is proved to be stable.

6. Digital control systems

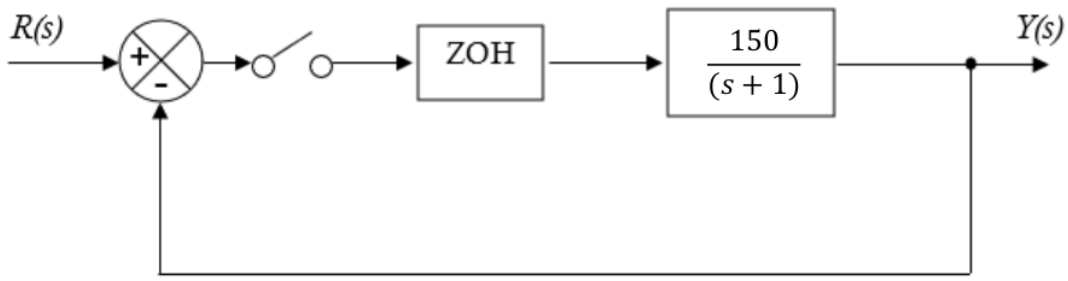


Figure 21 - Digital control system block diagram

ZOH Laplace transform where ZOH is zero order:

$$ZOH = \frac{1 - e^{-sT}}{s}$$

For an open loop transfer function:

$$G(s) = \frac{1 - e^{-sT}}{s} \times \frac{150}{(s+1)}$$

To produce Z transforms given that $z = e^{sT}$

$$G(z) = z\{G(s)\}$$

$$G(z) = (1 - z^{-1})Kz\left\{\frac{150}{s^2(s+150)}\right\}$$

Then with partial fractions method: *There is something wrong here are the open loop transfer function is first order*

$$\frac{70}{s^2(s+150)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+150)}$$

Combining A and B fractions

$$\frac{As(s+150) + B(s+150) + Cs^2}{s^2(s+150)}$$

When $s = 0$

$$0 + B(0 + 150) + 0 = 150$$

$$B = 1$$

When $s = -150$

$$0 + 0 + C(-150)^2 = 150$$

$$C = \frac{1}{150}$$

When s=1

$$A1(1+150)+1(1+150)+\frac{1}{150}s^2=150$$

$$151A+151+\frac{1}{150}=150$$

$$151A=150-151-\frac{1}{150}$$

$$A=\frac{\left(-1-\left(\frac{1}{150}\right)\right)}{151}=\frac{150+1}{150\times 151}=-\frac{1}{150}$$

Therefore:

$$\frac{150}{s^2(s+150)}=\frac{-\frac{1}{150}}{s}+\frac{1}{s^2}+\frac{\frac{1}{150}}{s+150}$$

$$z\left(\frac{150}{s^2(s+150)}\right)=z\left(\frac{-\frac{1}{150}}{s}+\frac{1}{s^2}+\frac{\frac{1}{150}}{s+150}\right)$$

$$z\left(\frac{150}{s^2(s+150)}\right)=\frac{Tz}{(z-1)^2}-\frac{z(1-e^{-70T})}{150(z-1)(z-e^{-70T})}$$

$$=(1-z^{-1})\times k\left[\frac{Tz}{(z-1)^2}-\frac{z(1-e^{-70T})}{150(z-1)(z-e^{-70T})}\right]$$

$$=\frac{\cancel{z-1}}{\cancel{z}}\times\frac{k\cancel{z}}{\cancel{z-1}}\left[\frac{T}{(z-1)}-\frac{1-e^{-70T}}{150(z-e^{-70T})}\right]$$

$$G_0(z)=k\left[\frac{T}{(z-1)}-\frac{1-e^{-70T}}{150(z-e^{-70T})}\right]$$

When T=0.1

$$G_0(z) = k\left[\frac{0.1}{(z-1)} - \frac{1-e^{-150\times 0.1}}{150(z-e^{-150\times 0.1})}\right]$$

$$= k\left[\frac{0.1}{(z-1)} - \frac{1-0.0000003059}{70(z-0.0000003059)}\right]$$

$$= k\left[\frac{0.1}{(z-1)} - \frac{0.9999996941}{70(z-0.0000003059)}\right]$$

$$= k\left[\frac{0.1}{(z-1)} - \frac{0.01429}{z-0.0000003059}\right]$$

$$= k\left[\frac{0.1(z-0.0000003059) - 0.01429(z-1)}{(z-1)(z-0.0000003059)}\right]$$

$$= k\left[\frac{0.1z - 0.00000003059 - 0.01429z + 0.01429)}{z^2 - 0.0000003059z - z + 0.0000003059}\right]$$

$$= k\left[\frac{0.08571z + 0.01428997}{z^2 - 1.0000003059z + 0.0000003059}\right]$$

With unity feedback:

$$G(z) = \frac{G_0(z)}{1+G_0(z)}$$

By making $G_0(z) = \frac{num}{den}$

$$G(z) = \frac{\frac{num}{den}}{1+\left(\frac{num}{den}\right)} = \frac{num}{den+num}$$

$$G(z) = \frac{k(0.08571z + 0.01428997)}{z^2 - 1.0000003059z + 0.0000003059 + k(0.08571z + 0.01428997)}$$

$$G(z) = \frac{k(0.086z + 0.0139)}{z^2 + z(0.08571k - 1.0000003059) + (0.0000003059 + 0.01428997k)}$$

Should be first order

For a closed-loop transfer function k=100:

$$G(z) = \frac{150(1 - e^{-T})}{z - e^{-T} + 150 - 150e^{-T}}$$

$$= \frac{150(1 - e^{-T})}{z + 150 - 151e^{-T}}$$

To find the poles:

$$z + 150 - 151e^{-T} = 0$$

So:

$$z = 151e^{-T} - 150$$

In order for the system to be stable, the pole needs to be within:

$$-1 < z < 1$$

For $z = -1$

$$-1 = 151e^{-T} - 150$$

$$149 = 151e^{-T}$$

$$\frac{149}{151} = e^{-T}$$

$$e^{-T} = 0.987$$

Using $\log_e$:

$$-T = -0.01333353087$$

$$T = 0.01333353087$$

For Z=1

$$1 = 151e^{-T} - 150$$

$$151 = 151e^{-T}$$

$$\frac{151}{151} = e^{-T}$$

$$e^{-T} = 1$$

$$T = 0$$

So the sample rate for stability is 0<T<0.01333353087

Natural log:

$$Minimum\ sampling\ period = \frac{1}{0.01333353087} = 75Hz$$

incorrect as you have been treating it as a second order system

where is your Simulink model?

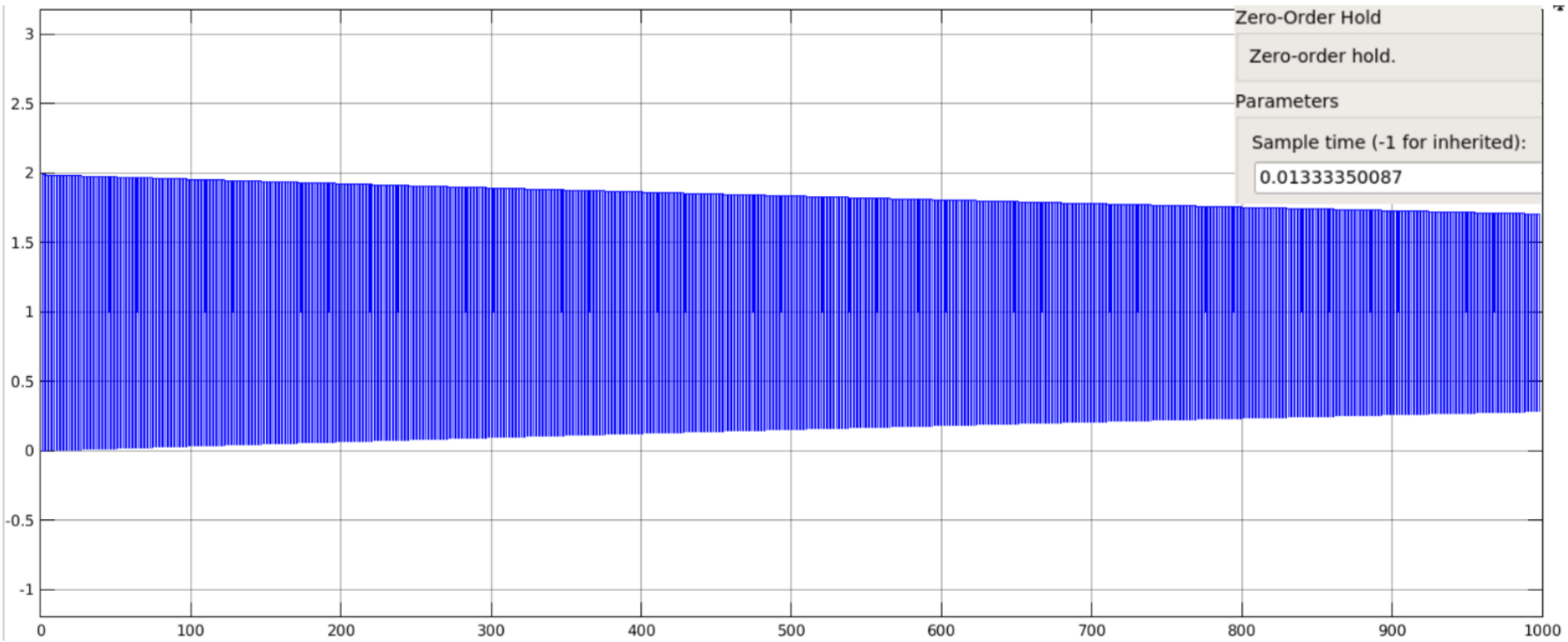


Figure 22 - Task 6 MatLab stable results

Figure 22 shows the MatLab results when the zero-order hold time is reduced by 0.00000003 seconds. Creating a stable waveform.

There is something wrong here

Even if you were correct with your 75Hz, the sampling time would be around 0.0133 seconds or 13.33 ms

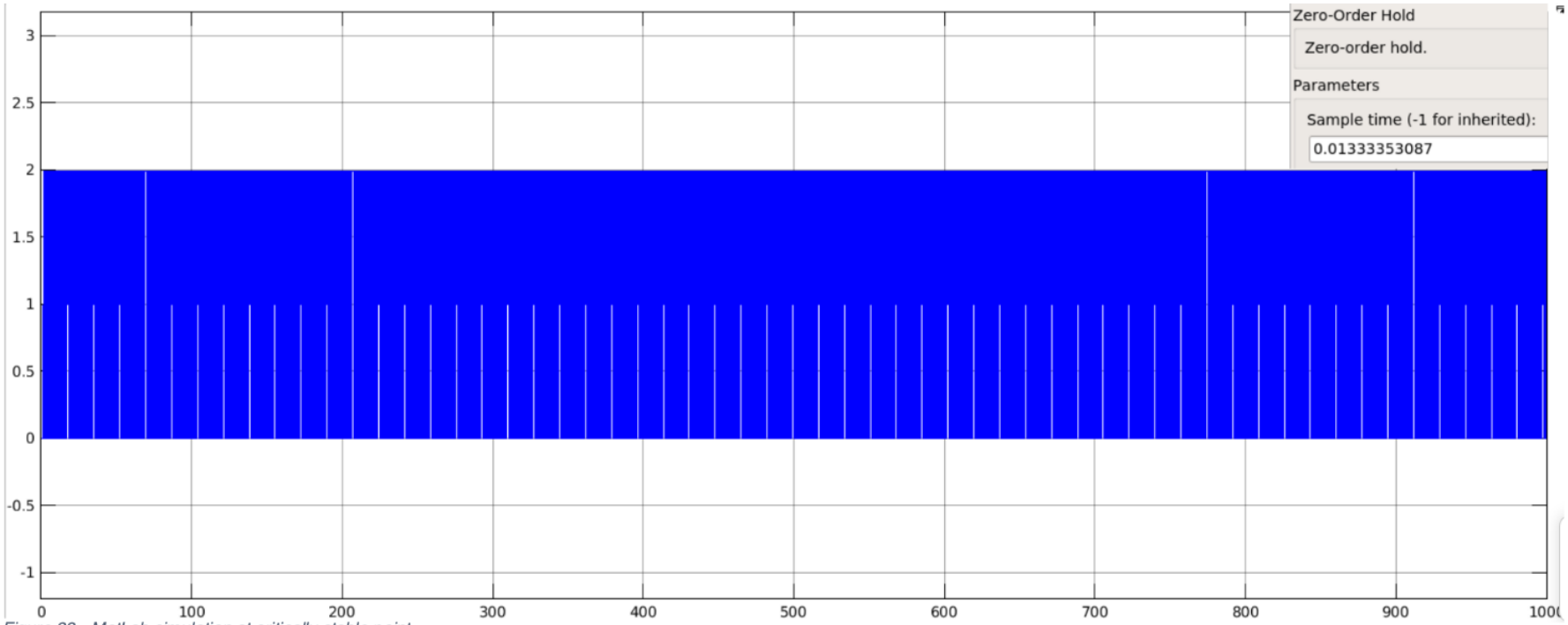


Figure 23 - MatLab simulation at critically stable point

Figure 23 shows the MatLab result at its critically stable point. To achieve this the zero-order hold parameter calculated to be 0.01333353087 seconds was inputted into MatLab without rounding down. By increasing the time, the waveform will start to go unstable. Shown in figure 24 set at 0.01333357087. Proving how small a change is needed to throw a waveform into an unstable state. In this case, to get the result shown it was 0.00000004 seconds.

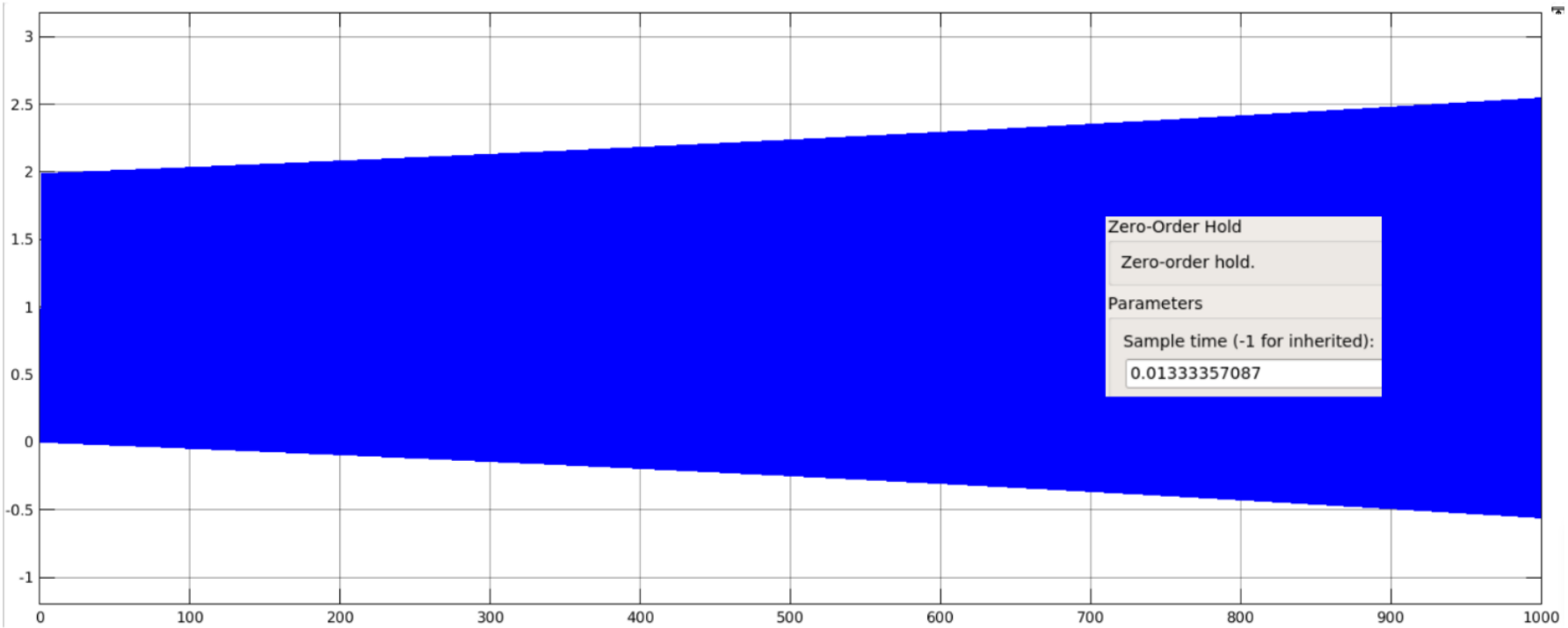


Figure 24 - MatLab result unstable

To calculate the stability in the z plane, the following equations are used:
Using $z = e^{st}$:

$$s = \sigma + j\omega$$
$$z = e^{(\sigma + j\omega)T}$$

Euler's formulae states:

$$z = e^{\sigma T}\big(\cos(\omega T) + jsin(\omega T)\big)$$
$$z = e^{\sigma T}\angle\omega T$$

When $\sigma = 0$, z is a circle with a radius of 1 and once $\sigma < 0$ the radius is less than 1. Then once $\sigma > 0$ the radius is more than 1.

7. Conclusions and recommendations

Throughout the tasks completed, the simulation and the calculated results always closely match. Once the transfer function has been created it can be tested without the need for a simulator to test if the waveform will be stable, unstable, underdamped or overdamped by plotting the pole and zero positions on an argand diagram and assessing the positions. Using the rlocus function in MatLab, the argand diagram can be checked to ensure the calculations are correct and can be used to determine other factors including the point of critical damping ("K") and the critical stability points.

The key differences in a step response is it being either overdamped or underdamped. Which drastically alters the resultant waveform. While they can both take similar times to reach a steady state they get there differently. Overdamping produces a steady rise and, in some cases, will produce an initial drop. Whereas underdamping will always produce a certain percentage overshoot and oscillation for a certain time before reaching steady state. The addition of negative feedback in this case will reduce the overshoot and settling time. Although, if the zeroes are located on the $j\omega$ axis the waveform will continue to oscillate at a steady rate.

However, any simulation done can always produce minor errors and is never correct. For example, a phase magnitude waveform will always be in error in a range of 0-3Dbs. But the differences in calculation and simulation results are usually negligible unless human error plays a factor.

8. Appendix

1809806

GRADEMARK REPORT

FINAL GRADE

64/100

GENERAL COMMENTS

Instructor

PAGE 1

PAGE 2

PAGE 3

PAGE 4

QM

Use the third Person

You are referring to first or second persons (I, we, you, us etc) whereas you should use the third person thus depersonalising it.

QM

Use the third Person

You are referring to first or second persons (I, we, you, us etc) whereas you should use the third person thus depersonalising it.

PAGE 5

Text Comment. Most of these calculations should be in the appendix as stated in the brief with a summary table of results placed in the main report.

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PAGE 16

Text Comment. When does critical damping occur?

PAGE 17

Text Comment. a good way of proving stability

PAGE 18

PAGE 19

PAGE 20

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PAGE 22

PAGE 23

PAGE 24

PAGE 25

Text Comment. Good evidence of testing

PAGE 26

PAGE 27

PAGE 28

PAGE 29

Text Comment. Good answer

PAGE 30

Text Comment. There is something wrong here are the open loop transfer function is first order

PAGE 31

PAGE 32

Text Comment. Should be first order

PAGE 33

Text Comment. incorrect as you have been treating it as a second order system

PAGE 34

Text Comment. where is your Simulink model?

Text Comment. There is something wrong here

Text Comment. Even if you were correct with your 75Hz, the sampling time would be around 0.0133 seconds or 13.33 ms

PAGE 35

PAGE 36

PAGE 37

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GRADING FORM: NEW CSG-RUBRIC

RICHARD HARRISON	64

1. ENGAGEMENT WITH LITERATURE

NA

2. KNOWLEDGE AND UNDERSTANDING

Knowledge is reasonably extensive. Exhibits very competent critical understanding of the well-established concepts and principles of the subject and the main methods of enquiry. Breadth and depth of knowledge. (12/20)

12

3. COGNITIVE AND INTELLECTUAL

Sound critical evaluation and analysis of concepts. Is selective in the range of evidence used and synthesises rather than describes. Ability to devise arguments that show awareness of different stances, and use evidence convincingly, to support appropriate and valid judgements. (20/30)

20

4. PRACTICAL APPLICATION

A very good application of a range of established methods, materials, tools and/or techniques. Very good consideration of the context of the application, with perceptive insights. Consistent, accurate and logical application of theory to practice, making appropriate links between the two. Can identify problems and propose appropriate solutions outside the context in which they were first studied. Evidence of some creativity. (20/30)

20

5. TRANSFERABLE SKILLS

Can communicate well, confidently and consistently in a suitable medium for the audience. Work is coherent, fluent, well-structured and organised. Can work very well autonomously and/or as part of a team, with very good contribution to group activities. Demonstrates very good skills for employment requiring the exercise of personal responsibility and decision-making, with just occasional minor weakness. (12/20)

12