

Assignment 2

by Gareth Wilkins

FILE	13876_GARETH_WILKINS_ASSIGNMENT_2_179202_1176764367.PDF (985.57K)		
TIME SUBMITTED	29-APR-2021 09:47PM (UTC+0100)	WORD COUNT	3889
SUBMISSION ID	151070495	CHARACTER COUNT	19870

Course: HND Electrical and Electronic Engineering
Module: SGEE5009 Control Engineering
Assignment: 2. Control Systems Analysis, Modelling and Simulation.
Name: Gareth Wilkins
University number: 1809804
Date submitted: 29 Apr 2021
Word count: 1860

Contents

1. Executive Summary	4
2. Transient Response of a Second Order System	5
3. Control System Modelling	8
4. Block Diagram Reduction Method	12
5. Frequency Response Methods	14
6. Digital Control Systems	16
7. Conclusion and recommendations	18
8. Appendix.....	19
8.1 Transient response second order system.....	19
8.2 Control System modelling	21
8.3 Block diagram reduction	25
8.4 Frequency response Calculations	28
8.5 Digital Control Systems Calculations	29
9. Works Cited	34

List of figures

Figure 1: Second order control system.....	5
Figure 2: Second order system transient response simulation on MATLAB.	6
Figure 3: Control system Argand diagram.....	9
Figure 4: Matlab Rlocus diagram.	10
Figure 5: Rlocus showing Critical stability	10
Figure 6: Block reduction argand diagram.....	13
Figure 7: Comparison of original and equivalent transfer functions	13
Figure 8: Bode diagram of second order system.....	14
Figure 9: Asymptotes applied to bode diagram.	15
Figure 10: Digital control system.	16
Figure 11: Stable response for digital system.....	17

List of tables

Table 1: Second order system natural frequency and gain values.....	5
Table 2: Calculations for the response to a step input for a second order control system.....	6
Table 3: Table of comparison of theoretical and simulated results.	7
Table 4: Control System Modelling selected values.	8
Table 5: Control system modelling Poles and Zeros calculated and Simulated comparison. .	9
Table 6: Control system stability prediction	11
Table 7: Block reduction control system pole and zero location	12
Table 8: Selected values for frequency response.....	14
Table 9: Maximum sampling period and minimum sampling frequency for digital control system	16

This is a good summary though it is a bit too long

1. Executive Summary

This report evaluates theoretical predictions with simulated results obtained from modelling software. A simple second order system behaviour was analysed when subjected to a step input. The single transfer function was compared to the standard second order transfer function and the systems characteristics were established. The calculated and simulated results were compared, and a comparable result was observed with any discrepancies due to rounding. An open loop control system was characterised by a differential equation which was translated to the frequency domain and the open loop transfer function was defined. The poles and zeros of the transfer function were calculated and plotted onto the argand diagram. The poles were placed along the real axis of the left-hand plane with the zeros placed in the right-hand plane as complex values. It was predicted that the control system would be stable but would oscillate. *If the poles are on the real axis, the system would be over damped.*

Negative feedback was introduced, and the argand diagram was reproduced illustrating the effect of different values of gain. *You are confusing critical damping with critical stability* A value of gain for critical damping was obtained. Instability was achieved by implementing a gain of greater than the critical value and stability was achieved by a value less than the critical value for stability. The root locus method was found to be advantageous to predict the performance of the control system. A complex control system incorporating six individual transfer functions was condensed into a single transfer function by using block diagram reduction principles. Values were provisioned and the poles and zeros were obtained. The poles were found to be in the left-hand plane deemed the system stable. A step input was connected to the original six transfer block control system with the equivalent single transfer function run in parallel. The scope produced an identical resultant output for both systems.

The frequency response of a second order control system with one zero and two poles was analysed. It was found that the transfer function variables constituted the cut off frequencies and might not be in the order of magnitude. The overall frequency response was established by the three complex numbers attributes being the magnitude and phases. To approximate the response with a max error of 3dB at cut off frequency asymptotes are applied to the bode diagram. The phase response determines the stability of the system and this was achieved by the phase margin method. A digital control system minimal sampling frequency was calculated to ensure system stability. A Simulink model was produced to confirm the calculated minimum sampling frequency value and values above and below were tested which corroborated the calculations. A conclusion was composed detailing the evident similarity obtained from the theoretical calculated results and simulated results.

2. Transient Response of a Second Order System

The values of 'k' and 'a' have been selected and inputted into the transfer function and is shown in figure 1.

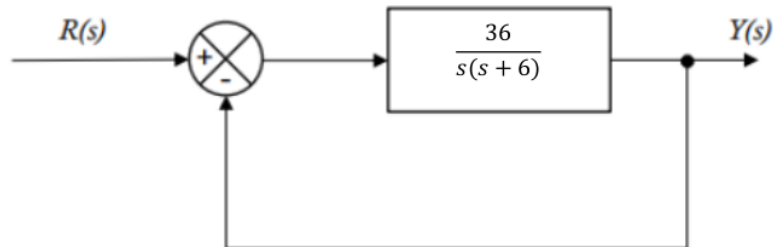


Figure 1: Second order control system

The second order system shown in figure 1 can be reduced to a single transfer shown below.

$$T(s) = \frac{G(s)}{1 + G(s) \times H(s)} = \frac{\frac{36}{s(s+6)}}{1 + \frac{36}{s(s+6)} \times 1} = \frac{36}{s^2 + 6s + 36}$$

Comparing this transfer function to the standard transfer function for a second order system.

$$\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{36}{s^2 + 6s + 36}$$

The system's natural frequency and gain values are tabulated in table 1.

Table 1: Second order system natural frequency and gain values.

Second order control system values		
ω_n^2	36	
ω_n	6	
$K\omega_n^2$	36	
K	1	

The system responses calculations are shown in appendix 8.1 and the values are shown in table 2.

Table 2: Calculations for the response to a step input for a second order control system.

	Calculated
Natural frequency	6
Damping factor	0.5
Damped natural frequency	5.196
Rise time from 0 to 100% of final value	0.403sec
Peak time	0.605sec
Percentage overshoot	16.303
Settling time within +2%	1.333
Number of oscillations during settling time	1.103

Matlab code was written and the transient response with annotations is shown in figure 2.

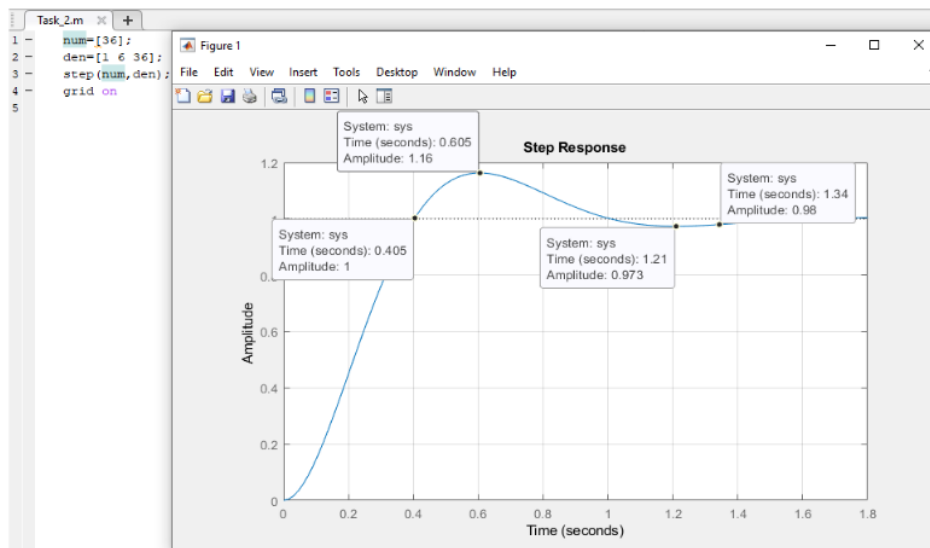


Figure 2: Second order system transient response simulation on MATLAB.

A comparison table for calculated and simulated results is shown in table 3.

Table 3: Table of comparison of theoretical and simulated results.

	Calculated	Simulated
Damped natural frequency	5.196	5.192
Rise time from 0 to 100% of final value	0.403sec	0.405sec
Peak time	0.605sec	0.605
Percentage overshoot	16.303	16
Settling time within +2%	1.333	1.34

The theoretical and simulated results shown in table 3 are comparable and any discrepancies are attributed to rounding to the nearest decimal place. With a damping factor of 0.5 the response was predicted to oscillate which was proven with the simulation. The number of oscillations was calculated to be 1.103 which was proven graphically on the simulation before reaching its settling time of 1.34 seconds which was comparable to the calculated prediction.

3. Control System Modelling

An open loop can be characterised by the following differential equation.

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx$$

The open loop transfer function $G(s)$ of the system is:

$$G(s) = \frac{s^2 - ps + q}{s^2 + as + b}$$

The method can be found in appendix 8.2.

The selected values for 'a' 'b' 'p' and 'q' are presented in table 4.

Table 4: Control System Modelling selected values.

Control Model System Values		
<i>These values set the location of the poles and zeros but are not in themselves poles and zeros</i>		
Poles	a	5
	b	2
Zeros	p	4
	q	6

The calculation shown in appendix 8.2 enabled the plotting of the poles and zeros on the argand diagram shown in figure 3.

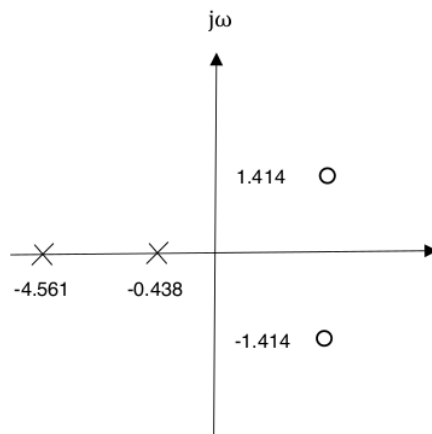


Figure 3: Control system Argand diagram

The characteristic equation is the term used for the denominator polynomial when set to zero as the roots of this equation establish the characteristic of the time response. These roots are named the poles of the system. The roots of the numerator polynomials are named the zeros of the system. The poles and zeroes are significant as at the poles the function of $Y(s)$ becomes infinite while the zero function becomes zero. (Richard C. Dorf, 2017).

The argand diagram indicates that the control system will be stable as the poles are positioned in the left-hand plane.

The open loop transfer function was coded to a step input on MATLAB which provided the positioning of the poles and zeroes. The MATLAB simulation can be viewed in appendix 8.2. The poles and zeroes calculated, and simulated results are tabulated in table 5. The calculated and simulated results are identical concluding the calculations are correct.

Table 5: Control system modelling Poles and Zeros calculated and Simulated comparison.

Poles and zeroes			
Poles		Zeroes	
Calculated	Simulated	Calculated	Simulated
-4.561	-4.5616	$2 + 1.414$	$2 + 1.4142$
-0.438	-0.4384	$2 - 1.414$	$2 - 1.4142$

The locations of the open loop poles are -4.561 and -0.438 . When the gain value is 0.23 the poles move together where they meet at -1.66 . This is termed critical damping where there is no overshoot or oscillation. When the gain exceeds 0.23 the poles have imaginary parts and move up while the other moves down towards. The calculations are shown in appendix 8.2 and the Matlab model is shown in figure 4.

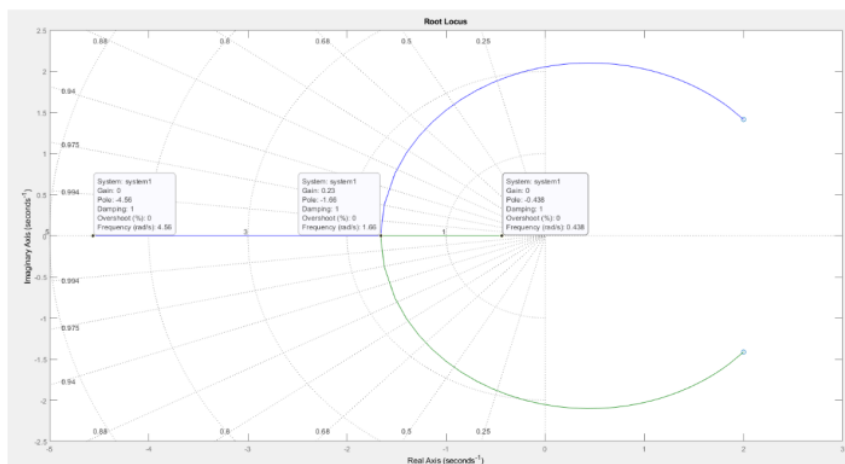


Figure 4: Matlab Rlocus diagram.

To obtain critical stability of the closed loop system the value of gain (k) was calculated which can be seen in appendix 8.2. To achieve stability the gain must be less than 1.25 ; to have an unstable system the gain must be greater than 1.25 . The calculations are shown in appendix 8.2 and the Matlab locus results supporting the calculations is shown in figure 5.

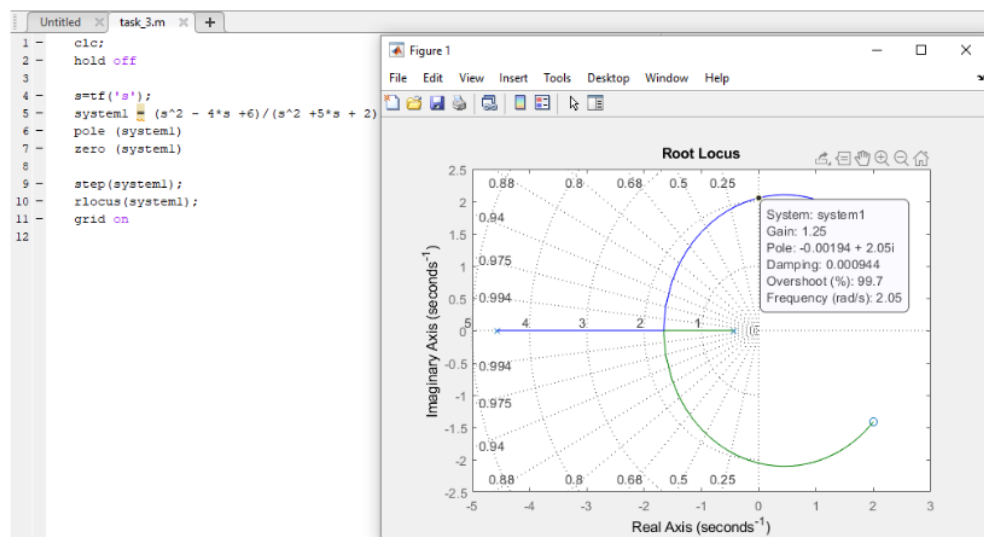


Figure 5: Rlocus showing Critical stability

A Simulink model shown in appendix 8.2 was constructed to corroborate the calculation for critical stability. Different values of gain were applied shown in table 6.

Table 6: Control system stability prediction

Control System response to different values of gain (k)	
Value of Gain	Predicted system response
0.5	Stable
1.25	Critical Stability
2.5	Unstable

The root locus is very beneficial to graphically depict the characteristics of a control system. By increasing the gain the poles moved together before reaching a point of critical damping. Increasing the gain further resulted in the poles having imaginary parts which would result in oscillations in the system. Increasing the gain beyond the calculated stable threshold resulted in the poles locating in the right-hand plane which caused the system to become unstable.

4. Block Diagram Reduction Method

Following block reduction methods shown in appendix 8.3 the expression below was derived.

$$T = \frac{G_1 G_3 G_4 + G_2 - G_2 G_3 G_4 H_1}{1 - G_3 G_4 H_1 + H_2 G_1 G_3 G_4 + H_2 G_2 - H_2 G_2 G_3 G_4 H_1}$$

The values chosen for the control system are shown below.

$$\begin{aligned} G_1 &= 8 \\ G_2 &= 5 \\ G_3 &= \frac{2}{s+1} \end{aligned}$$

$$\begin{aligned} G_4 &= \frac{1}{s} \\ H_1 &= \frac{1}{(s+1)} \\ H_2 &= 20 \end{aligned}$$

The derived transfer function is shown below the method can be found in appendix 8.3.

$$\frac{5s^3 + 10s^2 + 21s + 6}{101s^3 + 202s^2 + 421s + 118}$$

The poles and zeros for this system are tabulated below and the method can be found in appendix 8.3.

Table 7: Block reduction control system pole and zero location

Poles	Zeroes
-0.8356 + 1.7182i	0.8390 + 1.7100i
-0.8356 - 1.7182i	-0.8390 - 1.7100i
-0.3287 + 0.0000i	-0.3220 + 0.0000i

The argand diagram in figure 7 shows that the poles are situated in the left-hand plane which concludes that the system is stable. The poles are located on the real axis and the imaginary axes which concludes that oscillation will be present.

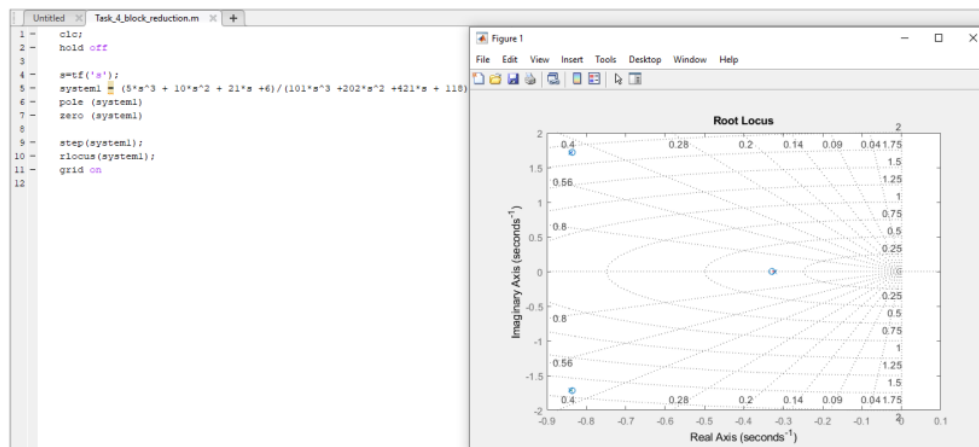


Figure 6: Block reduction argand diagram

The overall transfer function was tested in parallel with the six block control system. The Simulink model can be found in appendix 8.4. The two systems mirrored each other's response concluding that the equivalent transfer function accurately represented the original system.

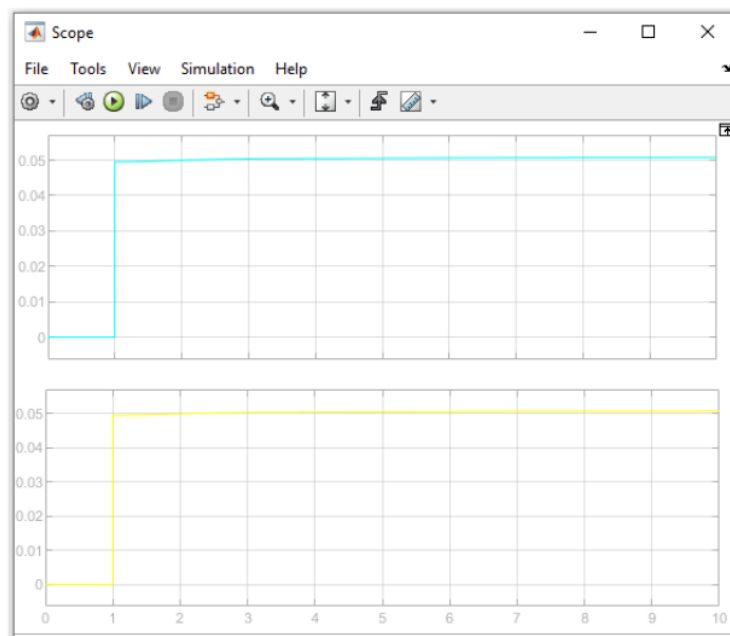


Figure 7: Comparison of original and equivalent transfer functions

5. Frequency Response Methods

A control system has the following transfer function.

$$G(s) = \frac{K(s + p)}{(s + q)(s + r)}$$

The selected values for p, q and r are tabulated below.

Table 8: Selected values for frequency response

	Value
K_1	1800
p	2
q	110
r	1700

The magnitude and phase of the transfer function over a range of frequency was simulated in Matlab illustrated in figure 8.

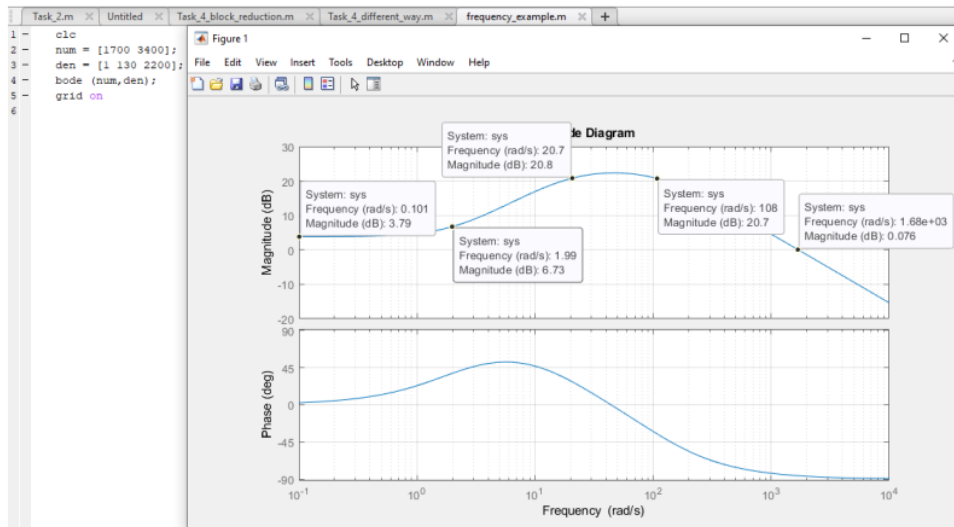


Figure 8: Bode diagram of second order system

The asymptotes have been drawn on the bode diagram shown in figure 9. The gain margin method cannot be employed as the phase does not reach -180 . Therefore, using the phase margin method, the control system is deemed stable as the phase margin is approximately 95 degrees which is greater than the 45 necessary to establish a stable system.

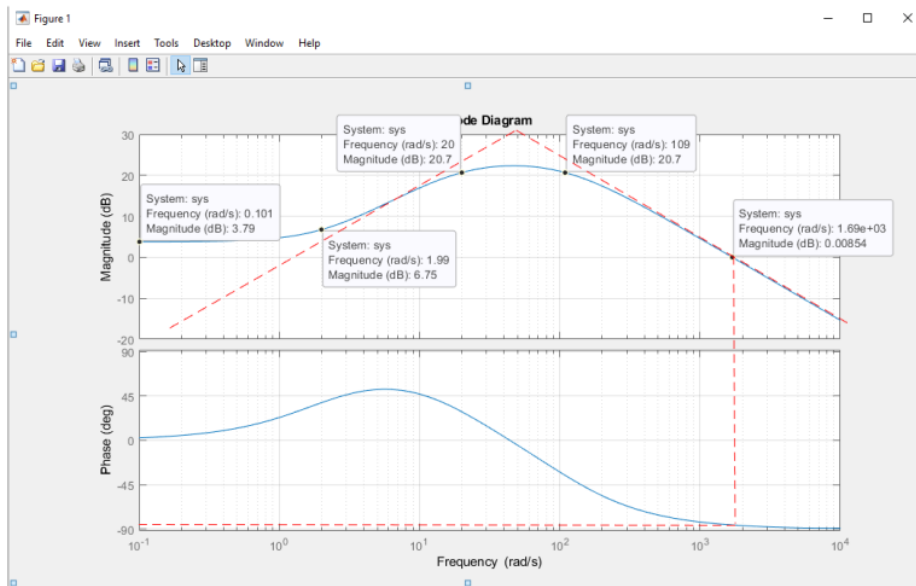


Figure 9: Asymptotes applied to bode diagram.

a point should be plotted on the phase diagram to find the phase margin. it looks about 85 which explains your answer of a phase margin of 95.

6. Digital Control Systems

The digital control system contrasts with the previously analysed systems as digital controllers operate from samples of information compared with continuous (Bolton, 2015).

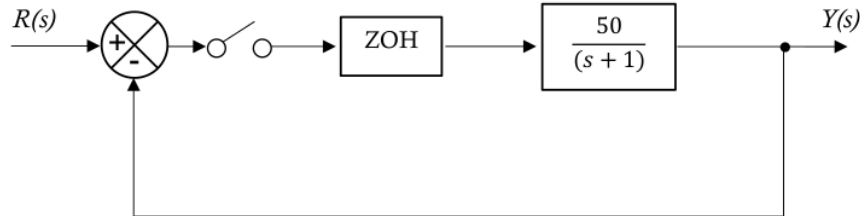


Figure 10: Digital control system.

The closed loop transfer function is shown below, and method can be found in appendix 8.6.

$$T(z) \frac{50(1 - e^{-T})}{z + 50 - 51e^{-T}}$$

The maximum sampling period and minimum sampling frequency was calculated see appendix 8.6 with the values tabulated in table 9.

Table 9: Maximum sampling period and minimum sampling frequency for digital control system

Maximum sampling period	0.04000533464 seconds
Minimum sampling frequency	24.99666629Hz

A Simulink model of the digital control system was constructed and can be viewed in Appendix 8.5. Entering the calculated value for the maximum sampling period in the ZOH produced a stable response shown in appendix 8.6 and a zoomed in image of the signal is shown in figure 11. This signal is stable as the saw-toothed signal is oscillating at a constant rate. Varying the value from the maximum sampling rate produced an uneven oscillation which eventually flatlined.

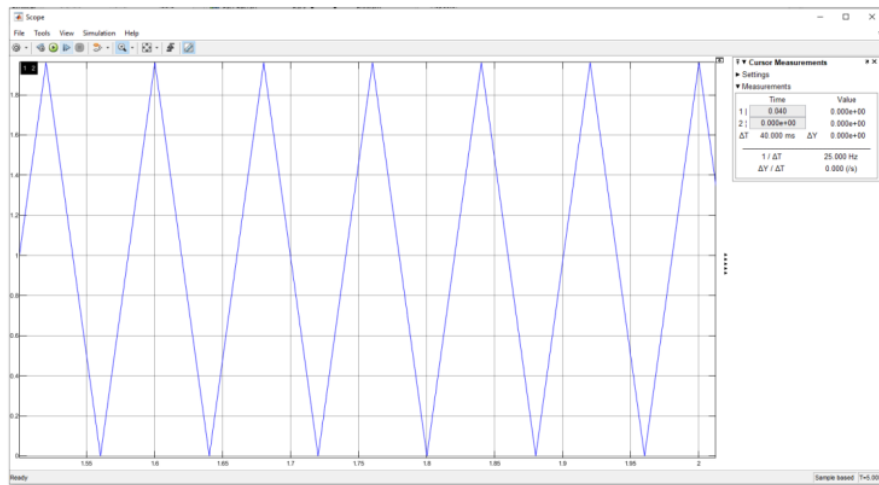


Figure 11: Stable response for digital system

7. Conclusion and recommendations

Analysis on a simple second order system deduced that the theoretical calculation produced the same results as the simulation software. The standard second order transfer function allows the prediction of the transient response. It was predicted that the control system would be underdamped as the damping factor was 0.5 which would incur oscillations and an overshoot of approximately 16%. Evaluation of an open loop system revealed the significance of the argand diagram with the positioning of the poles determining the system stability. A transfer function that yields negative poles constitutes an unstable system. The introduction of negative feedback demonstrated its ability to achieve critical damping where the poles move together. Critical damping provides optimal system behaviour as there are no oscillations or overshoot. Surpassing the value of critical feedback will result in the poles becoming imaginary values producing oscillations in the system. The root locus enables the prediction of the system behaviour. Analysing the root locus establishes the required value of gain for the application. Greater value of gain constitutes a quicker response time with the drawback of oscillations. This must be evaluated on the application of the system to produce optimal system performance. A gain value greater than critical stability will result in the poles entering the right-hand plane of the argand diagram resulting in an unstable system. Block reduction permitted the compression of a multi block system to a single transfer function. The original six block system was transformed into a third order transfer function. The single transfer function accurately replicated the multi block system. Compacting the system into a single transfer function simplifies the modelling of systems. Analysis of the frequency response of a second order system demonstrated that the transfer function variables established the cut off frequencies but may not be in the order of magnitude. Asymptotes were applied to the bode diagram which approximates the response with a 3dB tolerance. The stability of the system was ascertained by the phase margin method. Digital control systems comprising microprocessors have minimal sampling frequency to ensure stable transfer. Calculating the required frequency provided a smooth rate of saw-toothed oscillations. This report would benefit from further investigation into practical application of control systems. Using the simulating software does provide a graphical depiction of the transfer functions but does not provide the practical exposure to industrial applications.

8. Appendix

8.1 Transient response second order system

Natural frequency calculation

$$\therefore \omega_n^2 = 36$$

$$\omega_n = 6 \text{ rad/s}$$

Damping factor calculation

$$2\zeta\omega_n = 6$$

$$\zeta = \frac{6}{2\omega_n} = \frac{6}{2 \times 6} = 0.5$$

Damped natural frequency

$$\omega = \omega_n \sqrt{1 - \zeta^2}$$

$$= 6 \times \sqrt{1 - 0.5^2}$$

$$= 5.196 \text{ rad/s}$$

Rise time from 0 to 100% of final value:

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\omega_s \sqrt{1 - \zeta^2}}$$

$$t_r = \frac{\pi - \tan^{-1}\left(\frac{\sqrt{1 - 0.5^2}}{0.5}\right)}{5.196}$$

$$= \frac{\pi - 1.0471}{5.196}$$

$$= 0.403 \text{ sec}$$

Peak time

$$t_p = \frac{\pi}{\omega_n \sqrt{1 - \zeta^2}}$$

$$t_p = \frac{\pi}{5.196}$$

$$= 0.605 \text{sec}$$

Percentage maximum overshoot

$$100e^{-\left(\frac{\zeta \pi}{\sqrt{1 - \zeta^2}}\right)} = 100e^{-\left(\frac{0.5 \times \pi}{\sqrt{1 - 0.5^2}}\right)} = 16.303\%$$

Settling time within 2%

$$T_s = 2\% = \frac{4}{\zeta \omega_n} = \frac{4}{0.5 \times 6} = 1.333$$

Number of oscillations

$$\pm 2\% = n = \frac{2}{\pi} \sqrt{\frac{1}{\zeta^2} - 1} = \pm 2\% = n = \frac{2}{\pi} \sqrt{\frac{1}{0.5^2} - 1} = 1.103$$

8.2 Control System modelling

$$\frac{d^2y}{dt^2} + a \frac{dy}{dt} + by = \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx$$

$$\mathcal{L} \left\{ \frac{d^2y}{dt^2} + a \frac{dy}{dt} + by \right\}$$

$$= s^2Y(s) + asY(s) + bY(s) \text{ where } y(0)=0$$

$$\mathcal{L} \left\{ \frac{d^2x}{dt^2} - p \frac{dx}{dt} + qx \right\}$$

$$= s^2X - psX(s) + qX(s) \text{ where } X(0)=0$$

The equation becomes

$$s^2Y(s) + asY(s) + bY(s) = s^2X(s) - psX(s) + qX(s)$$

Factorising

$$Y(s)[s^2 + as + b] = X(s)[s^2 + ps + q]$$

$$\frac{Y(s)}{X(s)} = \frac{s^2 - ps + q}{s^2 + as + b}$$

$$G(s) = \frac{s^2 - ps + q}{s^2 + as + b}$$

Zeroes can be found from the numerator of G(s)

$$s^2 - 4s + 6$$

$$s = \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)}$$

$$s = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$s = \frac{4 \pm \sqrt{-16 - 24}}{2}$$

$$s = \frac{4 \pm j\sqrt{-8}}{2}$$

$$s = 2 \pm j1.414$$

Poles can be found by the denominator:

$$s^2 + 5s + 2$$

$$s = \frac{-b \pm \sqrt{(b)^2 - 4(ac)}}{2(a)}$$

$$s = \frac{-5 \pm \sqrt{5^2 - 4 \times 1 \times 2}}{2 \times 1}$$

$$= \frac{-5 \pm \sqrt{25 - 8}}{2}$$

$$= \frac{-5 \pm \sqrt{17}}{2}$$

$$=-0.438, -4.561$$

```

system1 =

      s^2 - 4 s + 6
      -----
      s^2 + 5 s + 2

Continuous-time transfer function.

ans =

    -4.5616
    -0.4384

ans =

    2.0000 + 1.4142i
    2.0000 - 1.4142i

```

The denominator is compared with the standard equation:

$$s^2 + 5s + 2 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

Natural frequency calculation:

$$\omega_n^2 = 2$$

$$\omega_n = 1.414$$

Damping factor calculation

$$2\zeta\omega_n = 5$$

$$\zeta = \frac{5}{2 \times 1.414} = 1.768$$

Negative feedback

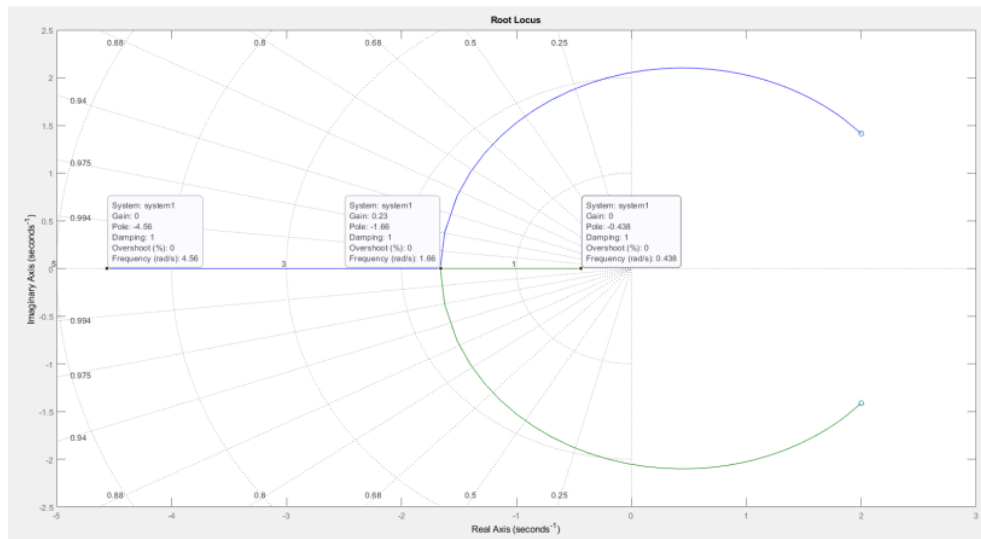
$$T(s) = \frac{G(s)}{1 + H(s)G(s)}$$

$$\frac{\frac{s^2 - 3s + 5}{s^2 + 4s + 2}}{1 + k \left(\frac{s^2 - 3s + 5}{s^2 + 4s + 2} \right)}$$

$$\frac{-\left(\frac{5-4k}{1+k}\right)}{2 \times 1} = 0 = \frac{-5+4k}{1+k} = 0 = -5+4k = 0, \quad 4k = 5, k = 1.25$$

$$T(s) = s^2 + \frac{1.25 - 1(0.23)}{(1 + (0.23))}s + \frac{(1 + 1.25(0.23))}{(1 + (0.23))}$$

$$T(s) = s^2 + 2.252s + 1.373$$



8.3 Block diagram reduction

$$G_5 = G_3 G_4$$

$$G_6 = \frac{G_5}{1 - G_5 H_1}$$

$$G_7 = G_1 \times G_6$$

$$G_8 = G_7 + G_2$$

$$T(s) = \frac{G_8}{1 + H_2 G_8}$$

$$T(s) = \frac{(G_7 + G_2)}{1 + H_2 (G_7 + G_2)}$$

$$T(s) = \frac{(G_1 G_6 + G_2)}{1 + H_2 (G_1 G_6 + G_2)}$$

=

$$T(s) = \frac{G_1 \frac{G_5}{(1 - G_5 H_1)} + G_2}{1 + H_2 \left[\frac{G_1 G_5}{1 - G_5 H_1} + G_2 \right]}$$

$$\frac{G_1 G_5 + G_2 (1 - G_5 H_1)}{1 - G_5 H_1 + H_2 G_1 G_5 + H_2 G_2 (1 - G_5 H_1)}$$

$$\frac{G_1 G_3 G_4 + G_2 (1 - G_3 G_4 H_1)}{1 - G_3 G_4 H_1 + H_2 G_1 G_3 G_4 + H_2 G_2 (1 - G_3 G_4 H_1)}$$

$$\frac{G_1 G_3 G_4 + G_2 - G_2 G_3 G_4 H_1}{1 - G_3 G_4 H_1 + H_2 G_1 G_3 G_4 + H_2 G_2 - H_2 G_2 G_3 G_4 H_1}$$

$$T = \frac{8 \times \frac{2}{s+1} \times \frac{1}{s} + 5 - (5 \times \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{(s+1)})}{1 - \left(\frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}\right) + \left(20 \times 8 \times \frac{2}{s+1} \times \frac{1}{s}\right) + (20 \times 5) - \left(20 \times 5 \times \frac{2}{s+1} \times \frac{1}{s} \times \frac{1}{s+1}\right)}$$

$$\frac{\frac{16}{s(s+1)} + 5 - \frac{10}{s(s+1)^2}}{1 - \frac{2}{s(s+1)^2} + \frac{320}{s(s+1)} + 100 - \frac{200}{s(s+1)^2}}$$

$$= \frac{16(s+1) + 5s(s+1)^2 - 10}{s(s+1)^2 - 2 + 320(s+1) + 100s(s+1)^2 - 200}$$

$$\frac{16s + 16 + 5s^3 + 5s^2 + 5s^2 + 5s - 10}{s^3 + s^2 + s^2 + s - 2 + 320s + 320 + 100s^3 + 100s^2 + 100s^2 + 100s - 200}$$

$$\frac{5s^3 + 10s^2 + 21s + 6}{101s^3 + 202s^2 + 421s + 118}$$

```
system1 =
```

$$\frac{5s^3 + 10s^2 + 21s + 6}{101s^3 + 202s^2 + 421s + 118}$$

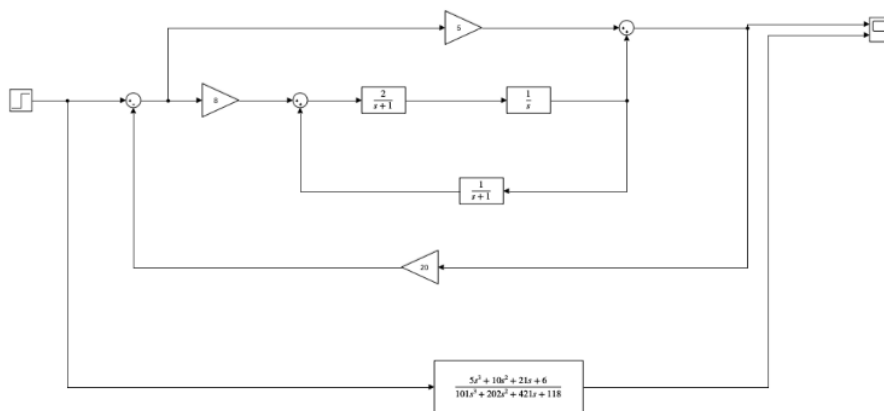
Continuous-time transfer function.

```
ans =
```

```
-0.8390 + 1.7100i  
-0.8390 - 1.7100i  
-0.3220 + 0.0000i
```

```
ans =
```

```
-0.8356 + 1.7182i  
-0.8356 - 1.7182i  
-0.3287 + 0.0000i
```



8.4 Frequency response Calculations

$$T(s) = \frac{K_1(s + \omega_{c1})}{(s + \omega_{c2})(s + \omega_{c3})}$$

$$T(j\omega) = \frac{K_1(j\omega + \omega_{c1})}{(j\omega + \omega_{c2})(j\omega + \omega_{c3})}$$

$$T(j\omega) = \frac{K_1 \left(\frac{j\omega}{\omega_{c1}} + 1 \right) \omega_{c1}}{\left(\frac{j\omega}{\omega_{c2}} + 1 \right) \omega_{c2} \left(\frac{j\omega}{\omega_{c3}} + 1 \right) \omega_{c3}}$$

$$T(j\omega) = \frac{K_1 \omega_{c1}}{\omega_{c2} \omega_{c3}} \times \frac{\left(1 + \frac{j\omega}{\omega_{c1}} \right)}{\left(1 + \frac{j\omega}{\omega_{c2}} \right) \left(1 + \frac{j\omega}{\omega_{c3}} \right)}$$

Overall gain $k = \frac{k_1 \omega_{c1}}{\omega_{c2} \omega_{c3}}$

$$K = \frac{K_1 P}{q \times r}$$

$$K = \frac{1700 \times 3}{20 \times 110} = 1.54545$$

8.5 Digital Control Systems Calculations

Laplace transform for ZOH

$$ZOH = \frac{1 - e^{-sT}}{s}$$

An open loop transfer function is given as:

$$G(s) = \frac{1 - e^{-sT}}{s} \times \frac{50}{(s + 1)}$$

Z transforms as $z = e^{sT}$

$$G(z) = z\{G(s)\}$$

$$G(z) = (1 - z^{-1})50z\left\{\frac{1}{s(s + 1)}\right\}$$

Patial fraction method:

$$\frac{1}{s(s + 1)} = \frac{A}{s} + \frac{B}{s + 1}$$

A and B partial fractions combined:

$$\frac{1}{s(s + 1)} = \frac{A(s + 1) + Bs}{s(s + 1)}$$

Multiplying both sides by:

$$1 = A(s + 1) + Bs$$

When $s = 0$

$$A = 1$$

When $s = -1$

$$B = -1$$

$\therefore$

$$\begin{aligned}\frac{1}{s(s+1)} &= \frac{1}{s} - \frac{1}{s+1} \\ z\left\{\frac{1}{s(s+1)}\right\} &= z\left\{\frac{1}{s} - \frac{1}{s+1}\right\} \\ &= 1z\left\{\frac{1}{s} - \frac{1}{s+1}\right\}\end{aligned}$$

By using the z transfer table:

$$\begin{aligned}&= \left\{\frac{z}{z-1} - \frac{z}{z-e^{-T}}\right\} \\ G(z) &= (1-z^{-1})50 \times 1z\left\{\frac{z-e^{-T}-z+1}{(z-1)(z-e^{-T})}\right\} \\ G(z) &= \left(\frac{z-1}{z}\right)50 \times 1z\left\{\frac{1-e^{-T}}{(z-1)(z-e^{-T})}\right\} \\ G(z) &= \frac{50(1-e^{-T})}{z-e^{-T}}\end{aligned}$$

Closed loop transfer function:

$$\begin{aligned}T(z) &= \frac{G(z)}{1+G(z)} \\ T(z) &= \frac{\left(\frac{50(1-e^{-T})}{z-e^{-T}}\right)}{1+\frac{50(1-e^{-T})}{z-e^{-T}}}\end{aligned}$$

Multiply by $z - e^{-T}$

$$\begin{aligned}\frac{50(1-e^{-T})}{z-e^{-T}+50(1-e^{-T})} \\ \frac{50(1-e^{-T})}{z+50-51e^{-T}}\end{aligned}$$

To locate pole

$$z + 50 - 51e^{-T} = 0 \quad \text{therefore,} \quad z = 51e^{-T} - 50$$

$$-1 < z < 1$$

Z=1:

$$1 = 51e^{-T} - 50$$

$$51 = 51e^{-T}$$

$$e^{-T} = \frac{51}{51} = 1$$

Logs to the base e:

$$-T = \ln(1)$$

$$T = \frac{0}{-1}$$

$$T = 0 \text{ seconds}$$

Z=-1:

$$-1 = 51e^{-T} - 50$$

$$49 = 51e^{-T}$$

$$e^{-T} = \frac{49}{51} = 0.9607843137$$

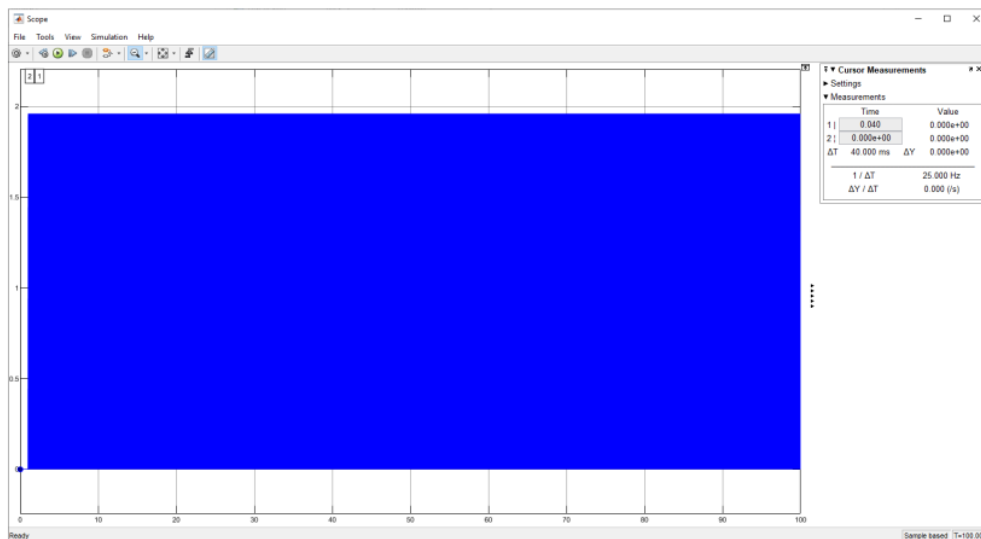
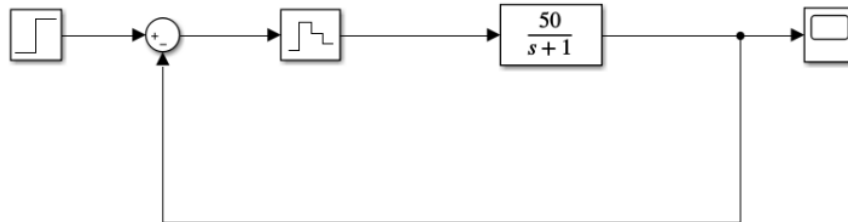
Natural log:

$$-T = -0.04$$

$$T = 0.04000533464 \text{ seconds}$$

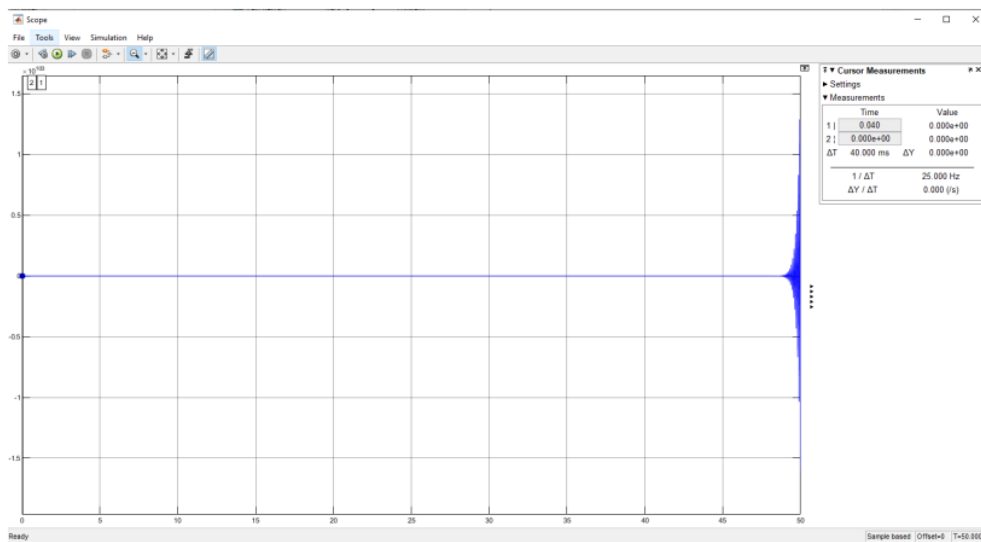
$$\text{Maximum sampling period} = 0.04000533464 \text{ seconds}$$

$$\text{Minimum sampling period} = \frac{1}{0.04000533464} = 24.99666629\text{Hz}$$



Stable

Unstable 0.45



Unstable 0.459



9. Works Cited

Bolton, W., 2015. *Instrumentation and Control Systems*. Second ed. Oxford: Joe Hayton.
Richard C. Dorf, R. H. B., 2017. *Modern Control Engineering*. Thirteenth Edition ed. Essex: Pearson Education Limited.

Assignment 2

GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

73/100

PAGE 1

PAGE 2

PAGE 3

PAGE 4

Text Comment. This is a good summary though it is a bit too long

Text Comment. If the poles are on the real axis, the system would be over damped.

Text Comment. You are confusing critical damping with critical stability

PAGE 5

Text Comment. plus signs are missing

PAGE 6

PAGE 7

PAGE 8

Text Comment. These values set the location of the poles and zeros but are not in themselves poles and zeros

PAGE 9

PAGE 10

PAGE 11

PAGE 12

PAGE 13

PAGE 14

PAGE 15

Text Comment. a point should be plotted on the phase diagram to find the phase margin. it looks about 85 which explains your answer of a phase margin of 95.

PAGE 16

PAGE 17

PAGE 18

Text Comment. Incorrect, negative poles give a stable system

PAGE 19

PAGE 20

PAGE 21

PAGE 22

PAGE 23

PAGE 24

PAGE 25

PAGE 26

PAGE 27

PAGE 28

PAGE 29

PAGE 30

PAGE 31

PAGE 32

PAGE 33

PAGE 34

GARETH WILKINS**73****1. ENGAGEMENT WITH LITERATURE**

N/A

2. KNOWLEDGE AND UNDERSTANDING

Knowledge is reasonably extensive. Exhibits very competent critical understanding of the well-established concepts and principles of the subject and the main methods of enquiry. Breadth and depth of knowledge. There are places where you make very good deductions but these are contradicted in other places. This seems to occur in the summary and conclusion section while the main analysis is sound. (12/20)

12**3. COGNITIVE AND INTELLECTUAL**

Excellent critical evaluation and analysis of concepts and principles leading to logical, evidence-based, reasoned arguments and judgements. Explicit recognition of other stances and a strong awareness of the limits of their knowledge. A capacity for independent thought

23

and ability to 'see beyond the question', suggesting some grasp of the broader field and wider concepts. (23/30)

4. PRACTICAL APPLICATION



An advanced application of a range of established methods, materials, tools and/or techniques. The context of the application is well considered, and insightful. Consistent, accurate and logical application of theory to practice, making well-developed links between the two. Can identify problems and propose excellent, creative solutions outside the context in which they were first studied. (23/30)

23

5. TRANSFERABLE SKILLS



Can communicate professionally confidently and consistently in a suitable medium for the audience. Work is coherent, very fluent and is presented proficiently. Can work autonomously with initiative. Where relevant can work professionally within a team, showing leadership skills as appropriate, and meeting obligations. Demonstrates excellent skills for employment requiring the exercise of personal responsibility and decision-making and an appetite for further development. (15/20)

15