

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 4

LEFEL 4

EXAMINATION PERIOD
MAY 2025

CYFNOD ARHOLIADAU
MAI 2025

SOLUTIONS
MODULE CODE: GOEN4001

MATHEMATICS TEST 2

TIME: 2 HOURS

ANSWER ALL QUESTIONS

Show all workings clearly

Equipment or materials required – calculators & formula book

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Question One (34 Marks)

a) If $A = \begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix}$

i. The Determinant of A

$$|A| = \begin{vmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{vmatrix} = 2(0 - 6) - 0(0 - 0) + 3(2 - 0) = -12 - 0 + 6 = -6$$

[4]

ii. The Determinant of C

Not possible

[1]

iii. The Determinant of D

$$D = \begin{vmatrix} 2 & 4 \\ -3 & 8 \end{vmatrix} = 16 + 12 = 28$$

[2]

iv. AB

$$\begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 9 & 22 \\ 22 & 49 & 20 \\ 4 & 10 & 0 \end{bmatrix}$$

[4]

v. BC

$$\begin{bmatrix} 3 & 0 & 2 \\ 2 & 5 & 0 \\ 1 & 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 14 \\ 12 & -17 \\ 25 & -5 \end{bmatrix}$$

[3]

vi. DC

$$\begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 1 \end{bmatrix} = \text{Not Possible}$$

[1]

vii. $A + D$

$$\begin{bmatrix} 2 & 0 & 3 \\ 1 & 8 & 3 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 & 4 \\ -3 & 8 \end{bmatrix} = \text{Not possible}$$

[1]

b) Find the Inverse of the following matrix:

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 4 & 7 \\ 0 & 5 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$|A| = 2(8 - 35) - 4(2 - 0) + 1(5 - 0)$$

$$|A| = -54 - 8 + 5$$

$$|A| = -57 \neq 0 \quad [2]$$

Calculations for Minors,

$$1. (4 \times 2) - (7 \times 5) = -27$$

$$2. (1 \times 2) - (7 \times 0) = 2$$

$$3. (1 \times 5) - (4 \times 0) = 5$$

$$4. (4 \times 2) - (1 \times 5) = 3$$

$$5. (2 \times 2) - (1 \times 0) = 4$$

$$6. (2 \times 5) - (4 \times 0) = 10$$

$$7. (4 \times 7) - (1 \times 4) = 24$$

$$8. (2 \times 7) - (1 \times 1) = 13$$

$$9. (2 \times 4) - (4 \times 1) = 4$$

$$\begin{bmatrix} -27 & 2 & 5 \\ 3 & 4 & 10 \\ 24 & 13 & 4 \end{bmatrix} \rightarrow \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \rightarrow \begin{bmatrix} -27 & -2 & 5 \\ -3 & 4 & -10 \\ 24 & -13 & 4 \end{bmatrix}$$

[6 marks (-1) for each wrong]

$$\text{adj } A = \begin{bmatrix} -27 & -3 & 24 \\ -2 & 4 & -13 \\ 5 & -10 & 4 \end{bmatrix} \quad [1]$$

$$A^{-1} = \frac{1}{-57} \begin{bmatrix} -27 & -3 & 24 \\ -2 & 4 & -13 \\ 5 & -10 & 4 \end{bmatrix} \quad [1]$$

c) Use Cramer's Rule to solve the following system of simultaneous equations:

$$3x - 2y + z = 2$$

$$x - 3y + 2z = 1$$

$$2x + y - 3z = -5$$

[8]

$$\begin{bmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ 2 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -5 \end{bmatrix} \quad [1]$$

$$\begin{vmatrix} 3 & -2 & 1 \\ 1 & -3 & 2 \\ 2 & 1 & -3 \end{vmatrix} = 3(9 - 2) + 2(-3 - 4) + 1(1 + 6)$$
$$\Delta = 14 \quad [1]$$

$$\begin{vmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ -5 & 1 & -3 \end{vmatrix} = 2(9 - 2) + 2(-3 + 10) + 1(1 - 15)$$
$$\Delta_x = 14 \quad [1]$$

$$\begin{vmatrix} 3 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & -5 & -3 \end{vmatrix} = 3(-3 + 10) - 2(-3 - 4) + 1(-5 - 2)$$
$$\Delta_y = 28 \quad [1]$$

$$\begin{vmatrix} 3 & -2 & 2 \\ 1 & -3 & 1 \\ 2 & 1 & -5 \end{vmatrix} = 3(15 - 1) + 2(-5 - 2) + 2(1 + 6)$$
$$\Delta_z = 42 \quad [1]$$

$$x = \frac{\Delta_x}{\Delta} = \frac{14}{14} = 1 \quad [1]$$

$$y = \frac{\Delta_y}{\Delta} = \frac{28}{14} = 2 \quad [1]$$

$$z = \frac{\Delta_z}{\Delta} = \frac{42}{14} = 3 \quad [1]$$

Question Two (34 Marks)

a) Given the following complex numbers:

$$Z_1 = 3 + 4j, \quad Z_2 = 5 + 3j, \quad Z_3 = 2 + 6j$$

Calculate the following, expressing your answers in rectangular form:

i. $Z_1 + Z_2$ [1]

$$\mathbf{Z_1 + Z_2 = 3 + 4j + 5 + 3j = 8 + 7j} \quad [1]$$

ii. $2Z_2 + 3Z_3$ [2]

$$\begin{aligned} 2Z_2 + 3Z_3 &= 2(5 + 3j) + 3(2 + 6j) \\ &= \mathbf{10 + 6j + 6 + 18j = 16 + 24j} \end{aligned} \quad [2]$$

iii. $Z_1^2 + Z_3$ [3]

$$\begin{aligned} Z_1^2 + Z_3 &= (3 + 4j)(3 + 4j) + (2 + 6j) \\ \mathbf{Z_1^2 + Z_3} &= \mathbf{(9 + 12j + 12j - 16) + 2 + 6j} \\ \mathbf{Z_1^2 + Z_3} &= \mathbf{-5 + 30j} \end{aligned} \quad [3]$$

iv. $Z_1Z_2Z_3$ [4]

$$\begin{aligned} Z_1Z_2Z_3 &= (3 + 4j)(5 + 3j)(2 + 6j) = (15 + 9j + 20j - 12)(2 + 6j) \\ Z_1Z_2Z_3 &= (3 + 29j)(2 + 6j) \\ \mathbf{Z_1Z_2Z_3} &= \mathbf{6 + 18j + 58j - 174 = -168 + 76j} \end{aligned} \quad [4]$$

v. $|Z_1 Z_2 Z_3|$ [3]

$$Z_1 Z_2 Z_3 = -168 + 76j$$

$$|Z_1 Z_2 Z_3| = \sqrt{(-168)^2 + (76)^2}$$

$$|Z_1 Z_2 Z_3| = \mathbf{184.39}$$

vi. $\frac{Z_1}{Z_2}$ [5]

$$\frac{Z_1}{Z_2} = \frac{3 + 4j}{5 + 3j}$$

$$= \frac{3 + 4j}{5 + 3j} \times \frac{5 - 3j}{5 - 3j} = \frac{15 - 9j + 20j + 12}{25 + 9} \quad [3]$$

$$\frac{Z_1}{Z_2} = \frac{27 + 11j}{34} \quad [1]$$

$$\frac{Z_1}{Z_2} = \frac{27}{34} + \frac{11}{34} j \quad [1]$$

b) Solve the following quadratic equation, expressing your answers in Polar Form:

$$5x^2 - 8x + 5 = 0$$

[9]

$$a = 5 \quad b = -8 \quad c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(5)(5)}}{2(5)} \quad [1]$$

$$x = \frac{8 \pm \sqrt{-36}}{10} = \frac{8 \pm 6j}{10} \quad [2]$$

$$x = \frac{8 + 6j}{10} = \frac{4}{5} + \frac{3}{5}j$$

Or

$$x = \frac{8 - 6j}{10} = \frac{4}{5} - \frac{3}{5}j \quad [1]$$

in polar form

$$x = 0.8 + 0.6j$$

$$|0.8 + 0.6j| = \sqrt{0.8^2 + 0.6^2} = 1 \quad [1]$$

$$\phi = \tan^{-1}\left(\frac{0.6}{0.8}\right)$$

$$\phi = 36.87^\circ \quad [1]$$

$$x = 0.8 + 0.6j \text{ rectangular form}$$

$$x = 1(\cos 36.87^\circ + j \sin 36.87^\circ) \text{ polar form} \quad [1]$$

$$x = 0.8 - 0.6j$$

$$|0.8 - 0.6j| = \sqrt{0.8^2 + 0.6^2} = 1$$

$$\phi = \tan^{-1}\left(\frac{-0.6}{0.8}\right)$$

$$\phi = 36.87^\circ \text{ not a solution}$$

$$\phi = 360^\circ - 36.87^\circ$$

$$\phi = 323.2^\circ \quad [1]$$

$$x = 0.8 - 0.6j \text{ rectangular form}$$

$$x = 1(\cos 323.2^\circ + j \sin 323.2^\circ) \text{ in polar form} \quad [1]$$

- c) Use de Moivre's theorem to calculate $(3 + 4j)^4$, giving your answers in rectangular form.

[7]

$(3 + 4j)$ in polar form :

$$|(3 + 4j)| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \quad [1]$$

$$\phi = \tan^{-1}\left(\frac{4}{3}\right)$$

$$\phi = 53.13^\circ \quad [1]$$

$$\text{so } (3 + 4j) \text{ in polar form is } 5(\cos 53.13^\circ + j \sin 53.13^\circ)$$

$$\text{Calculate } [5(\cos 53.13^\circ + j \sin 53.13^\circ)]^4$$

Applying de Moivre's theorem gives $5^4(\cos 4 \times 53.13^\circ + j \sin 4 \times 53.13^\circ)$ [3]

$$625(\cos 212.52^\circ + j \sin 212.52^\circ) \quad [1]$$

$$(3 + 4j)^4 = -527 - j336 \text{ in rectangular form} \quad [1]$$

Question Three (32 Marks)

a) Integrate the following:

i. $\int (25x^4 + 12x^2 + 6x - 10) dx$ [4]

$$= \frac{25x^5}{5} + \frac{12x^3}{3} + \frac{6x^2}{2} - 10x + C = 5x^5 + 4x^3 + 3x^2 - 10x + C$$

ii. $\int (\sin 7x + 6\cos 3x + 5e^{5x}) dx$ [3]

$$= -\frac{1}{7}\cos 7x + \frac{6}{3}\sin 3x + \frac{5}{5}e^{5x} + C = -\frac{1}{7}\cos 7x + 2\sin 3x + e^{5x} + C$$

iii. $\int \frac{10\theta^4 + 2}{2\theta^5 + 2\theta + \pi} d\theta$ [2]

$$= \int \frac{10\theta^4 + 2}{2\theta^5 + 2\theta + \pi} d\theta$$

$$= \ln|2\theta^5 + 2\theta + \pi| + C$$

iv. $\int \left(\frac{16}{f} + \ln(e^2) + 3 \right) df$ [2]

$$= 16 \ln|f| + 2f + 3f + C = 16 \ln|f| + 5f + C$$

v. $\int_1^5 (2x^3 + 5x^2 + 3x - 9) dx$ [4]

$$= \left[2\frac{x^4}{4} + 5\frac{x^3}{3} + 3\frac{x^2}{2} - 9x \right]_1^5 = \left[\frac{2x^4}{4} + \frac{5x^3}{3} + \frac{3x^2}{2} - 9x \right]_1^5$$

$$= \left[\frac{2(5)^4}{4} + \frac{5(5)^3}{3} + \frac{3(5)^2}{2} - 9(5) \right] - \left[\frac{2(1)^4}{4} + \frac{5(1)^3}{3} + \frac{3(1)^2}{2} - 9(1) \right] =$$

$$= \left[\frac{2(625)}{4} + \frac{5(125)}{3} + \frac{3(25)}{2} - 45 \right] - \left[\frac{2}{4} + \frac{5}{3} + \frac{3}{2} - 9 \right]$$

$$= [513.333] - [-5.333] = 518.666$$

vi. $\int_0^4 (2e^{2x} + \sin \frac{x}{2}) dx$ [3]

$$= \left[2 \frac{e^{2x}}{2} + \frac{\cos \frac{x}{2}}{\frac{1}{2}} \right]_0^4$$

$$= \left[2 \frac{e^8}{2} + \frac{\cos 2}{\frac{1}{2}} \right] - \left[2 \frac{e^0}{2} + \frac{\cos 0}{\frac{1}{2}} \right] = [e^8 + 2\cos 2] - [1 + 2] = 2979.957$$

[18]

b) Using the substitution method, let $u = 4x^3 + 9$ find :

$$\int x^2 (4x^3 + 9)^5 dx$$

[6]

$$u = 4x^3 + 9$$

$$\frac{du}{dx} = 12x^2 \quad du = 12x^2 dx \quad dx = \frac{du}{12x^2}$$

[2]

$$\int x^2 (4x^3 + 9)^5 dx = \int x^2 \cdot u^5 \frac{du}{12x^2}$$

$$= \frac{1}{12} \int u^5 du$$

[2]

$$= \frac{1}{12} \int u^5 du = \frac{1}{12} \frac{u^6}{6} + C$$

$$= \frac{1}{72} (4x^3 + 9)^6 + C$$

[2]

c) Use Integration ‘by Parts’ to find

$$\int 5x e^{5x} dx \quad [8]$$

$$u = 5x \rightarrow \frac{du}{dx} = 5 \rightarrow du = 5 dx \quad [2]$$

$$dv = e^{5x} dx \rightarrow v = \int e^{5x} dx = \frac{1}{5} e^{5x} \quad [2]$$

$$\int 5x e^{5x} dx = 5x \left(\frac{1}{5} e^{5x} \right) - \int \left(\frac{1}{5} e^{5x} \right) 5 dx = x e^{5x} - \int e^{5x} dx \quad [2]$$

$$\int 5x e^{5x} dx = x e^{5x} - \frac{1}{5} e^{5x} + C = x e^{5x} - \frac{e^{5x}}{5} + C \quad [2]$$

END