

Micromechanical Analysis of a Lamina

When analyzing composites there are two main approaches which can be taken – **Micromechanical** and **Macromechanical**. Macromechanical approaches calculate the behavior of the composite material by considering the global behavior of the material and Micromechanical approaches calculate the behavior of the composite material based on the properties of the component parts. These notes demonstrate how the Micromechanical approach can be used to calculate the mechanical properties of a unidirectional composite. Typical values of fibers and matrices are shown in tables 1 and 2;

Typical Properties of Fibers (SI System of Units)

Property	Units	Graphite	Glass	Aramid
Axial modulus	GPa	230	85	124
Transverse modulus	GPa	22	85	8
Axial Poisson's ratio	—	0.30	0.20	0.36
Transverse Poisson's ratio	—	0.35	0.20	0.37
Axial shear modulus	GPa	22	35.42	3
Axial coefficient of thermal expansion	$\mu\text{m}/\text{m}/^{\circ}\text{C}$	-1.3	5	-5.0
Transverse coefficient of thermal expansion	$\mu\text{m}/\text{m}/^{\circ}\text{C}$	7.0	5	4.1
Axial tensile strength	MPa	2067	1550	1379
Axial compressive strength	MPa	1999	1550	276
Transverse tensile strength	MPa	77	1550	7
Transverse compressive strength	MPa	42	1550	7
Shear strength	MPa	36	35	21
Specific gravity	—	1.8	2.5	1.4

Table 1: Typical properties of fibers

Typical Properties of Matrices (SI System of Units)

Property	Units	Epoxy	Aluminum	Polyamide
Axial modulus	GPa	3.4	71	3.5
Transverse modulus	GPa	3.4	71	3.5
Axial Poisson's ratio	—	0.30	0.30	0.35
Transverse Poisson's ratio	—	0.30	0.30	0.35
Axial shear modulus	GPa	1.308	27	1.3
Coefficient of thermal expansion	$\mu\text{m}/\text{m}/^{\circ}\text{C}$	63	23	90
Coefficient of moisture expansion	$\text{m}/\text{m}/\text{kg}/\text{kg}$	0.33	0.00	0.33
Axial tensile strength	MPa	72	276	54
Axial compressive strength	MPa	102	276	108
Transverse tensile strength	MPa	72	276	54
Transverse compressive strength	MPa	102	276	108
Shear strength	MPa	34	138	54
Specific gravity	—	1.2	2.7	1.2

Table 2: Typical properties of matrices

Volume fractions, Mass fractions and density

The volume fraction of the fibers is defined as;

$V_f = \frac{V_f}{V_c}$ where V is the volume fraction, v is the volume and the subscripts f and c are for the fibers and composite respectively.

Similarly the volume fraction for the matrix is, $V_m = \frac{V_m}{V_c}$, where the subscript m is for the matrix.

Note that $V_f + V_m = 1$ and $v_f + v_m = v_c$.

In the same way, the mass fraction for the fibers is defined as $W_f = \frac{w_f}{w_c}$, and for the

matrix is defined as $W_m = \frac{w_m}{w_c}$ such that $W_f + W_m = 1$ and $w_f + w_m = w_c$.

Density of a composite is defined as $\rho_c = \frac{w_c}{V_c}$, the density of the composite can therefore be shown as;

$$\rho_c = V_m \rho_m + V_f \rho_f$$

Voids

During manufacture voids are introduced into the matrix meaning that the actual density, ρ_{ac} is lower than the theoretical density, ρ_{th} as can be seen below;



Not only do these voids influence the density they also are also detrimental to the mechanical properties. These include lower;

- Shear strength and stiffness
- Compressive strengths
- Transverse tensile strengths
- Fatigue resistance
- Moisture resistance

A decrease of 2 to 10% in the preceding matrix-dominated properties generally takes place with every 1% increase in the void content.

The volume fraction of the voids, V_v is defined as; $V_v = \frac{V_v}{V_c}$

Meaning that the total volume is now given by; $V_c = V_f + V_m + V_v$

By definition the actual density is given by;

$$\rho_{ac} = \frac{W_c}{V_c}$$

and the theoretical density is given by;

$$\rho_{th} = \frac{W_c}{V_f + V_m}$$

Thus the volume of the voids is given by;

$$V_v = \frac{W_c}{\rho_{ac}} \left(\frac{\rho_{th} - \rho_{ac}}{\rho_{th}} \right)$$

And the volume fraction of the voids is;

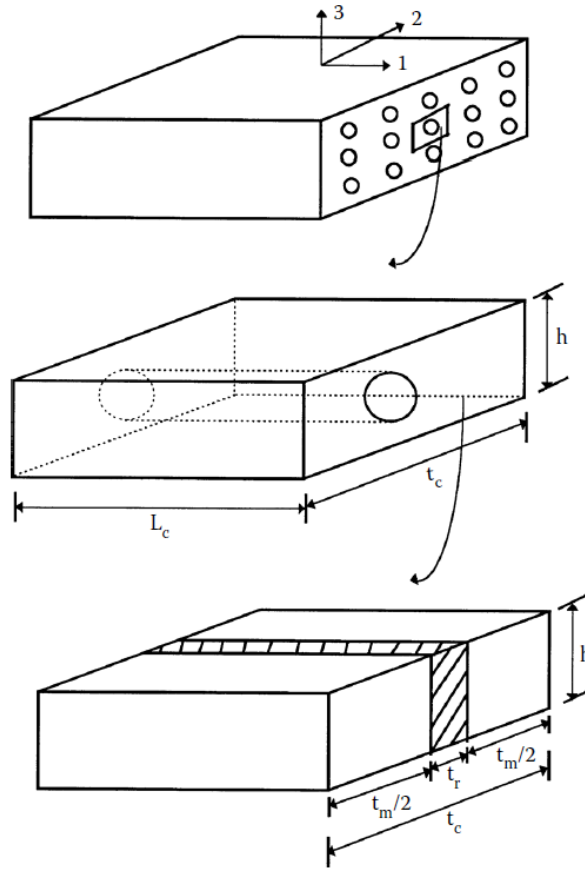
$$V_v = \frac{\rho_{th} - \rho_{ac}}{\rho_{th}}$$

Evaluation of the four elastic moduli

There are four elastic moduli of a unidirectional lamina;

- Longitudinal Young's modulus, E_1
- Lateral Young's modulus, E_2
- Major Poisson's ratio, ν_{12}
- In-plane shear modulus, G_{12}

These can be calculated using a strength of materials approach shown below. Consider a unidirectional lamina and take a Representative Volume Element (RVE) that consists of a fiber surrounded by the matrix. This representative volume element can further be represented as rectangular blocks. The fibre, matrix and the composite are assumed to be of the same width, h , but of thicknesses t_f , t_m and t_c respectively.



Thus the areas are given by;

$$A_f = t_f h$$

$$A_m = t_m h$$

$$A_c = t_c h$$

And the volume fractions are given by;

$$V_f = \frac{t_f}{t_c}$$

$$V_m = \frac{t_m}{t_c}$$

And by definition $V_m = 1 - V_f$

Using these definitions, the elastic moduli can be calculated making the following assumptions;

- The bond between fibers and matrix is perfect
- The elastic moduli, diameters and space between the fibers are uniform
- The fibers are continuous and parallel
- The fibers and matrix follow Hooke's law (they are linearly elastic)
- The fibers possess uniform strength
- The composite is free of voids

Calculating E_1

Consider our RVE being loaded in the longitudinal direction, such that;

$$F_c = F_f + F_m$$

Therefore;

$$\begin{aligned}\sigma_c A_c &= \sigma_f A_f + \sigma_m A_m \\ E_1 \varepsilon_c A_c &= E_f \varepsilon_f A_f + E_m \varepsilon_m A_m\end{aligned}$$

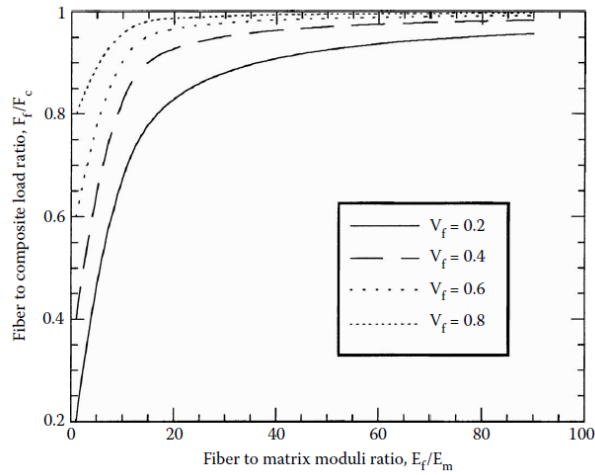
Hence, assuming that $\varepsilon_c = \varepsilon_f = \varepsilon_m$;

$$E_1 = E_f V_f + E_m V_m$$

This relationship is called the 'rule of mixtures'. When using unidirectional composites it is often useful to state the ratio of the load taken by the fibers to the load taken by the composite;

$$\frac{F_f}{F_c} = \frac{E_f}{E_1} V_f$$

The below figure which plots this ratio against the ratio of the fiber to matrix moduli ratio, shows that as the fiber to moduli ratio increases, the load taken by the fiber increases tremendously;



Calculating E_2

If you now consider that our RVE is loaded in the lateral direction, then the lateral extension is given by;

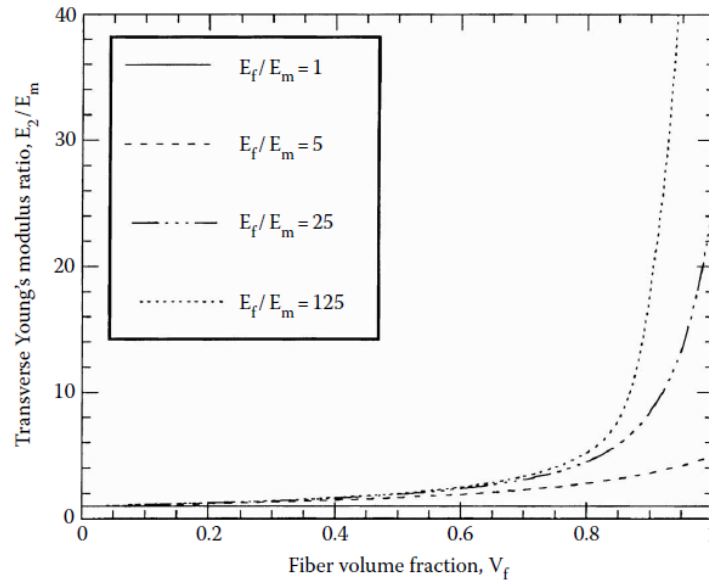
$$\begin{aligned}\Delta_c &= \Delta_f + \Delta_m \\ t_c \varepsilon_c &= t_f \varepsilon_f + t_m \varepsilon_m \\ t_c \frac{\sigma_c}{E_2} &= t_f \frac{\sigma_f}{E_f} + t_m \frac{\sigma_m}{E_m}\end{aligned}$$

Thus;

$$\frac{1}{E_2} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

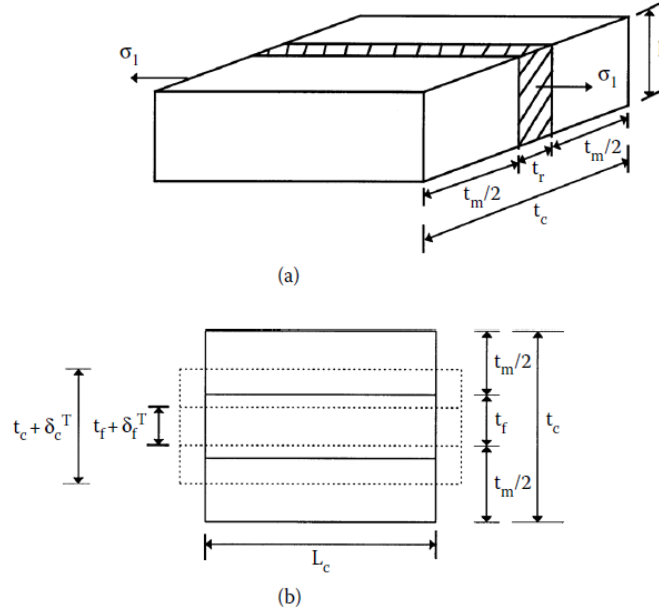
The below figure plots the transverse Young's modulus as a function of fiber volume fraction for constant fiber-to-matrix elastic moduli ratios. Note that for metal and ceramic matrix composites $\frac{E_f}{E_m}$ is low (eg for SiC/aluminium metal matrix composites this value is

4), however for polymeric composites this value is high (eg for glass/epoxy polymer matrix composites this value is 25). The graph shows that for polymeric composites the contribution of the fiber modulus only increases substantially for a fiber volume fraction greater than 80%.



Calculating ν_{12}

Consider our RVE, this time being loaded in the longitudinal direction. The plan view is shown below with the associated lateral dimensional changes, hence we can calculate the major poisons ratio.



The deformations in the lateral direction of the composite is the sum of the lateral deformations in the fiber and the matrix;

$$\delta_c^T = \delta_f^T + \delta_m^T$$

$$\varepsilon_c^T t_c = \varepsilon_f^T t_f + \varepsilon_m^T t_m$$

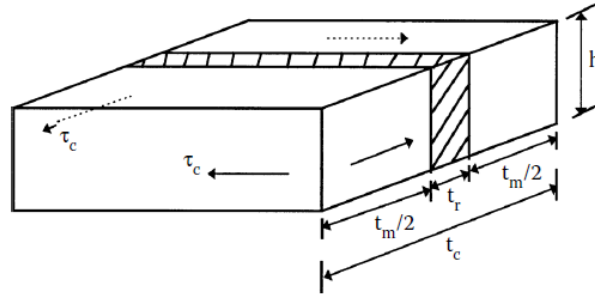
$$-\nu_{12} \varepsilon_c^L t_c = -\nu_f \varepsilon_f^T t_f - \nu_m \varepsilon_m^T t_m$$

Therefore assuming that $\varepsilon_c^L = \varepsilon_f^L = \varepsilon_m^L$;

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

Calculating G_{12}

Finally, consider the RVE being subjected to shear stress as shown below;



The shear deformation of the composite will be the sum of the shear deformation of the fibers and the matrix;

$$\delta_c = \delta_f + \delta_m$$

$$\gamma_c t_c = \gamma_f t_f + \gamma_m t_m$$

$$\frac{\tau_c}{G_{12}} t_c = \frac{\tau_f}{G_f} t_f + \frac{\tau_m}{G_m} t_m$$

Thus, assuming that $\tau_c = \tau_f = \tau_m$

$$\frac{1}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m}$$

Examples

1. A glass/epoxy lamina consists of a 70% fiber volume fraction. Using properties of glass and epoxy from tables 1 and 2 determine the;
 - a. Density of the Lamina
 - b. Mass fractions of the glass and epoxy
 - c. Volume of composite lamina if the mass of the lamina is 4kg
 - d. Volume and mass of glass and epoxy in part c.

[2110kg/m³, 0.8294, 0.1706, 1.896e-3 m³, 1.327e-3 m³, 0.5688e-3 m³, 3.318kg, 0.6826kg]

2. A graphite/epoxy cudoid specimen with voids has dimensions of a x b x c and its mass is M_c. After it is put into a mixture of sulfuric acid and hydrogen peroxide, the remaining graphite fibers has a mass of M_f. From independent tests, the densities of a graphite and epoxy are ρ_f and ρ_m , respectively. Find the volume fraction of the voids in terms of a, b, c, M_f, M_c, ρ_f and ρ_m

$$[V_v = 1 - \frac{1}{abc} \left[\frac{m_f}{\rho_f} + \frac{m_c - m_f}{\rho_m} \right]]$$

3. Find the longitudinal elastic modulus of a unidirectional glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from table 1 and 2. Also, find the ratio of the load taken by the fibers to that of the composite.
[60.52GPa, 0.9831]
4. Find the lateral Young's modulus of a glass/epoxy lamina with a fiber volume fraction of 70%. Use the properties of glass and epoxy from table 1 and 2.
[10.37GPa]
5. Find the major and minor Poisson's ratio of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from table 1 and 2.
[0.230, 0.03941]
6. Find the in-plane shear modulus of a glass/epoxy lamina with a 70% fiber volume fraction. Use the properties of glass and epoxy from table 1 and 2.
[4.014GPa]

Notes adapted from/further reading:

Mechanics of composite materials, 2nd edition, Autar K Kaw