

2ai) Strain energy

$$U = \frac{1}{2} Q \delta$$

$$= \frac{1}{2} Q \gamma \delta s$$

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\gamma}$$

$$Q = \tau A \quad \tau = \frac{Q}{A}$$

$$\therefore G = \frac{Q}{\gamma A} \quad \text{and} \quad \gamma = \frac{Q}{GA}$$

2 a ii)
then

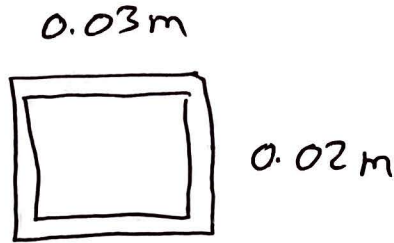
$$U = \frac{1}{2} Q \times \frac{Q}{AG} \times \delta s = \frac{1}{2} \times \frac{Q^2}{AG} \times \delta s$$

$$u = \frac{Q^2}{2AG} \times \delta s$$

From this total strain energy can be shown as

$$U = \frac{Q^2 L}{2AG}$$

2bi)

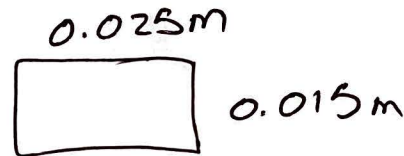


Thickness
 $25 \times 10^{-3} \text{ m}$

Length
 2.4 m

$$U = \frac{P^2 L}{2AE}$$

Internal dimensions



$$\begin{aligned} A &= (0.03 \times 0.02) - (0.025 \times 0.015) \\ &= 6 \times 10^{-4} - 3.75 \times 10^{-4} \\ &= 2.25 \times 10^{-4} \text{ m}^2 \end{aligned}$$

$$U = \frac{(480 \times 10^6)^2 \times 2.4}{2 \times 2.25 \times 10^{-4} \times 190 \times 10^9}$$

$$= \frac{5.5 \times 10^{17} \text{ ⑧}}{85.5 \times 10^6 \text{ ⑨}}$$

$$= 6.47 \times 10^9 \text{ J}$$

2 b ii) Potential energy = Mgh

$$PE = mgh$$

$$U = \frac{P^2 L}{2AE}$$

energy
in tension

$$\frac{\delta L}{L} = \frac{\sigma}{E} \quad \therefore \quad \delta L = \frac{\sigma L}{E}$$

Assuming all potential energy becomes strain energy

$$mgh = \frac{P^2 L}{2AE}$$

$$mgh = \frac{P^2 L}{2AE}$$

$$\therefore mgh = \frac{\sigma^2 AL}{2E}$$

$$h = \frac{\sigma^2 AL}{2Emg} \quad \leftarrow \text{Volume}$$

weight

$$h = \frac{\sigma^2 AL}{2EW}$$

$$h = \frac{\sigma^2 AL}{2EW}$$

2biii)
$$h = \frac{\sigma^2 V}{2EW}$$

$$\sigma^2 = \frac{2EW h}{V}$$

$$\sigma = \sqrt{\frac{2EW h}{V}}$$

$$\sigma = \frac{2 \times 190 \times 10^9 \times 20 \times 9.81 \times 0.45}{2.25 \times 10^{-4} \times 2.4}$$

$$\sigma = \frac{3.36 \times 10^{13} \text{ (A)}}{5.4 \times 10^{-4}} = 6.213 \times 10^{16} \text{ Pa}$$

2biv) As all parameters are the same except ~~the~~ dimensions of bar / pipe from box to circle the area must ~~be~~ must ~~be~~ be the same

~~3.25×10^{-4}~~ = ~~find~~ find inner diameter

$$2.25 \times 10^{-4} = \pi d_1^2 - \pi d_2^2$$

$$\pi d_2^2 = \pi d_1^2 - 2.25 \times 10^{-4}$$

$$2b) iv) \quad \sigma_2^2 = \frac{\pi \sigma_1^2 - 2.25 \times 10^{-4}}{\pi}$$

$$\sigma_2 = \sqrt{\frac{\pi \sigma_1^2 - 2.25 \times 10^{-4}}{\pi}}$$

$$= \sqrt{\frac{\pi 0.02^2 - 2.25 \times 10^{-4}}{\pi}}$$

$$\sigma_2 = 0.018 \text{ m}$$

$$\text{wall thickness} = \frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{0.02 - 0.018}{2}$$

$$= 0.001 \text{ m}$$

$$4 a) \quad I_{xx} = 2(I_{xx} \text{ C-sections}) + 2(I_{xx} \text{ Plates})$$

$$\begin{aligned} I_{xx} &= (2 \times 26.2 \times 10^6) + 2 \times (I_g + Ah^2) \\ &= 52.4 \times 10^6 + 2 \times \left(\frac{300 \times 12^3}{12} + (300 \times 12 \times 94^2) \right) \\ &= 52.4 \times 10^6 + ~~76.32 \times 10^6~~ 63.7 \times 10^6 \\ &= 116.1 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$I_{yy} = 2(I_{yy} \text{ C-sections}) + 2(I_{yy} \text{ Plates})$$

$$\begin{aligned} I_{yy} &= 2 \times (I_g + Ah) + 2 \left(\frac{bd^3}{12} \right) \\ &= 2 \times (1.57 \times 10^6 + 4400 \times 79) + 2 \times \left(\frac{12 \times 300^3}{12} \right) \\ &= 3.84 \times 10^6 + 54 \times 10^6 \\ &= 57.84 \times 10^6 \text{ mm}^4 \end{aligned}$$

$$I_{yy} < I_{xx}$$

the strut will tend to buckle about the ~~y~~ axis

$$4b) \quad k = \sqrt{\frac{I}{A}}$$

$$I = 57.84 \times 10^6 \text{ mm}^4$$

$$A = 2 \times 4400 + 2(300 \times 12)$$

$$A = 16,000 \text{ mm}^2$$

$$k = \sqrt{\frac{57.84 \times 10^6}{16000}}$$

$$k = 60.12 \text{ mm}$$

$$4c) \quad \text{Slenderness ratio} = \frac{L}{k}$$

$$S_r = \frac{8}{60.12 \times 10^{-3}}$$

$$S_r = \underline{\underline{0.133}} \quad 133.07$$

$$4d) \quad P_e = \frac{\pi^2 EI}{L^2} \quad \begin{aligned} &57.84 \times 10^6 \text{ mm}^2 \\ &= 57.84 \times 10^{-6} \text{ m}^2 \end{aligned}$$

$$P_e = \frac{\pi^2 \times 200 \times 10^9 \times 57.84 \times 10^{-6}}{8^2}$$

$$P_e = 1.78 \times 10^6 \text{ N}$$

$$4e) \quad P_R = \frac{\sigma_y A}{1 + a(L/L)^2}$$

$$P_R = \frac{320 \times 10^6 \times \cancel{16 \times 10^{-3}} 0.016}{1 + \frac{1}{7500} \times 133.07^2}$$

$$P_R = 1.52 \times 10^6 \text{ N}$$

$$4f) \quad P_e = \frac{\pi^2 E I}{L^2}$$

$$I = 57.84 \times 10^6 + 2 \times \left(\frac{bd^3}{12} + Ah^2 \right)$$

$$= 57.84 \times 10^6 + 2 \left[\frac{224 \times 6^3}{12} + (224 \times 6 \times 153^2) \right]$$

$$= 57.84 \times 10^6 + 62.9 \times 10^6$$

$$= 120.78 \times 10^6 \text{ mm}^4$$

$$I = 120.78 \times 10^{-6} \text{ m}^4$$

$$P_e = \frac{\pi^2 \times 200 \times 10^9 \times 120.78 \times 10^{-6}}{8^2}$$

$$P_e = 3.725 \times 10^6 \text{ N}$$

$$3a) \quad \frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$$

$$\cancel{\frac{T}{J} = \frac{\tau R}{J}} \quad T = \frac{J}{R}$$

$$\sigma_1 - \sigma_3 = \sigma_y$$

$$325 \times 10^6 - - 325 \times 10^6 = 650 \times 10^6$$

$$\sigma_y = 650 \times 10^6 \text{ Pa}$$

$$\text{Safety factor} = 6 = \frac{\sigma_y}{\text{Allowable working Stress}} \quad (\text{AWS})$$

$$\text{AWS} = \frac{\sigma_y}{6} = \frac{650 \times 10^6}{6}$$

$$\text{AWS} = 108.3 \times 10^6 \text{ Pa}$$

$$J = \frac{\pi d^4}{32}$$

$$J = \frac{\pi \times (37.5 \times 10^{-3})^4}{32}$$

$$= 1.94 \times 10^{-7} \text{ m}^4$$

$$\tau = \frac{V}{R}$$

$$\tau = \frac{108.3 \times 10^6 \times 1.94 \times 10^{-7}}{3.5 \times 10^{-3}}$$

$$\tau = \cancel{560.3 \text{ Nm}} \cancel{598.6 \text{ Nm}} 600.3 \text{ Nm}$$

$$3b) \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right) = 2\sigma_y^2$$

$$\sigma_1 = 325 \times 10^6$$

$$\sigma_2 = 0$$

$$\sigma_3 = -325 \times 10^6$$

$$2\sigma_y^2 = (325 \times 10^6)^2 + (325 \times 10^6)^2 + (-\cancel{325}^{650} \times 10^6)^2$$

$$2\sigma_y^2 = 6.3375 \times 10^{17}$$

$$\sigma_y = \sqrt{\frac{6.3375 \times 10^{17}}{2}}$$

$$\sigma_y = 562.9 \times 10^6 \text{ Pa}$$

$$AWS = \frac{562.9 \times 10^6}{6}$$

$$AWS = 93.8 \times 10^6 \text{ Pa}$$

$$T = \frac{\cancel{108.9} \cdot 93.8 \times 10^6 \times 1.94 \times 10^{-7}}{3 \cdot 5 \times 10^{-3}}$$

$$T = \cancel{485.3} \text{ Nm} \quad 519.9 \text{ Nm}$$

$$3c) \left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right) = 2\sigma_y^2$$

$$\sigma_1 = 880 \times 10^6$$

$$\sigma_2 = 0$$

$$\sigma_3 = -880 \times 10^6$$

$$\left((880 \times 10^6)^2 + (880 \times 10^6)^2 + (-1760 \times 10^6)^2 \right) = 2\sigma_y^2$$

$$2\sigma_y^2 = 4.6464 \times 10^{18}$$

$$\sigma_y = \sqrt{\frac{4.6464 \times 10^{18}}{2}} = 1.52 \times 10^9 \text{ Pa}$$

$$3c) \quad \tau = \frac{552.7 \times 10^6 \times 6.136 \times 10^{-7}}{23.5 \times 10^{-3}}$$

$$\tau = 14.4 \times 10^3 \text{ N}$$

$$J = \frac{\pi \times (50 \times 10^{-3})^4}{32}$$

$$= 6.136 \times 10^{-7}$$

$$AWS = \frac{1.52 \times 10^9}{2.75}$$

$$= 552.7 \times 10^6$$

$$3d) \quad \frac{14.4 \times 10^3 - 519.9}{519.9} \times 100 = 2722$$

on increase of 2722% is seen