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Designing a two-objective model for location – routing problems by considering a timeframe for each customer to design a distribution network

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Abstract

Nowadays, supply chain management is one of the most critical issues of the day, and the integrated integration of logistics systems is one of the essential problems in supply chain management in recent decades. In an integrated approach, a set of strategic, tactical and operational level decisions are taken at the same time, and this approach prevents local achievement. One of the issues in the field of integrated logistics, location - routing, which includes a choice of facility location and vehicle routing with the aim of minimizing costs is. In this research, a new two-objective model for the problem (location-routing) is presented, in which a time window is considered for each customer. The time window is the service interval, and the service time reduction helps to increase the service level. In addition, in this model, simultaneous collection and delivery are only between central and intermediate depots and only delivery for customers. One of the features of this model is considering two different aims: one that reduces the cost of the entire system, which is the common goal of most research and minimizes the time of service to each customer. Also, according to the type of problem is NP-HARD. The decision to provide a genetic algorithm for solving the model and to evaluate and compare the responses by two multi-objective genetic algorithms (NSGA-II) and the firefly algorithm (MOFA).

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DECLARATION

This work has not been previously been accepted in substance for any degree and is not being concurrently submitted in candidature for any degree.

Signed (Candidate)

Date

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First chapter

Introduction

Nowadays, with the emergence of the importance of supply chain management to industrialists, the role of coordination and integration of the various components of the supply chain has become more critical in creating a competitive advantage. As it identifies and eliminates the high distribution costs and opportunity costs, such as unused capacity, throughout the supply network. Physical distribution costs account for 7.5% of the total US sales to give (La Londe and Masters, 1994). One of the significant issues involved in the design and implementation of logistics systems. Determine the location of distribution centre. Considering this, in optimizing location decisions, the balance between equipment costs, inventory costs, shipping costs, and customer satisfaction has to be carefully considered. Because determining the location of distribution centers and appropriate routes with minimum time is not only useful for the profitability of the organization but also the organization's ability to improve the quality and level of service and customer satisfaction. Locations and logistics costs, although they are highly dependent on the industry, are less relevant in the subject literature and worked on them. In fact, the location of the flow of materials and the mode of transportation are highly interconnected and determining the optimal values of these variables in interaction, which can lead to finding an optimal logistics system with minimum possible costs. One of the most important strategic decisions involved in the design of distribution networks Related to the distribution location. Places that are considered as candidates for construction of depots.

1 Definition of the problem and the subject of the research

Over the past three decades, the integrated logistics system optimization approach has become one of the most critical optimization issues in supply chain management. This approach focuses on simultaneously optimizing strategic, tactical and operational decisions such as facility location, customer allocation to facilities, fleet and transportation routes, and inventory planning and control. This approach is based on the principle that ignoring the dependency between the above decisions in a logistic system may lead to local optimization and cost increases. Integrated issues in the field of logistics have a variety of types that can be found location-routing, location-inventory, and routing-inventory, which has been exploring these issues with increasing growth. In the last three decades, the increase in computational power has provided a framework for studying integrated problems with high computational complexity. In this study, some complex issues will be introduced in the design of the distribution network. These issues include locating facilities, vehicle routing, and inventory decisions. In this research, a scenario will be selected and thoroughly analyzed. This scenario is based on the model of Ambrosiano and Scutella, 2005 and structurally considers inventory flows between distribution canterers and transmission points. With the difference that in the given problem two different objects are considered. One of the aims is the total cost of the system, which is a common aim in the subject literature and aims to minimize the total time of service to customers, or in other words, maximize the service. This model is an issue developed. In this research, a new model for location-routing is presented in a distribution network in which goods or services are provided to customers in defined time windows.

1.1 Research hypotheses

- The demand for each customer is in the form of delivery.
- The relationship between central depots and middle depots is of the type of collection and delivery.

- The fleet is not homogeneous (it is not used from a type of vehicle, for example, a truck can be used as a van).
- The starting path of any vehicle will be from the center of distribution (Central Depot) and will end at the same center.

1.2 Research objectives

- ✓ The purpose of this research is to provide an integrated location-routing model taking into account the time frame for each customer. Transportation costs by taking the time window is a large part of the value of a distribution network, and this cost is dependent on the location of the facility. Therefore, locating and routing sub-issues in sequential order will lead to local optimization. The purpose of the proposed model is to minimize costs in the distribution network. So that the service intervals are reduced to each customer, or in other words reach the maximum service level.
- ✓ The results of the survey help managers make decisions.
- ✓ Another object of this research is the design of meta-innovative methods such as the genetic algorithm for solving the proposed model given the NP-HARD problem.
- ✓ Which depots to choose and which path of each vehicle is selected to meet the objectives of the model.
- ✓ Finding research gaps



Generally, ideal goals include two goals of reducing costs and reducing customer service time and waiting time, and the purpose of this thesis is to present a mathematical model and solving algorithm.

1.3 Possible applications of research

The proposed model is used in the optimal design of distributive networks in which it is contemplating deciding on the establishment of facilities, the allocation of customers to facilities and vehicle routing simultaneously. And in parallel with this thinking, there is another way of thinking about customer satisfaction by reducing the time of service, or in other words, increasing the level of service by the time frame for each customer (time window). Also, this model can be used to design a network in which there is a demand for collection as well as delivery demand, such as an integrated logistics network.

1.4 Users of thesis results

- Manufacturing and services companies that are interested in designing or modifying their distribution network to reduce costs by increasing customer satisfaction by reducing customer service time or the same level of service to customers.
- Researchers interested in the field of integrated issues in the design of integrated logistics networks, in particular, the problem of locating-Routing

1.5 New aspects of innovation

- Considering the time window in the locating - Routing problem: An overview of the models of location Routing shows that only 17% of models, the time window is considered
- Considering two different aims, one reducing the total cost of the system, and the other reducing the service time in an integrated distribution model with a time window.
- Presentation of the genetic algorithm for solving the model and review and comparison of the results by two multi-objective genetic algorithms (NSGA-II) and firefly algorithm (MOFA)

1.6 Research steps

1. Definitions of theoretical fundamentals and review of the literature on the subject of locating-routing
2. Designing a mathematical model concerning the opportunities and research gap in the problem of locating-routing
3. Use of Multi-objective Genetic Algorithm and Firefly Algorithm
4. Solving Examples of a Model with Genetic Algorithm and Comparing Answers by Firefly Algorithm
5. Reviewing and analysing the results and conclusions

1.7 Method and tool for collecting information

- Studies of articles published in prestigious journals
- library study
- Study-related dissertations

1.8 The strategy of selecting articles for the review of the literature

The scope of the review is articles published in this area and part of this selection according to the strategy used by Min et al., 1998. The following logical approach has been used to investigate issues and issues related to location and routing issues about the limitations of access to various journals.

- ❖ All published articles are divided into two sections.
 1. Articles published before 1996
 2. Articles published after 1996

In the case of the first category articles, Min et al., 1998 mentioned a comprehensive study. And articles published after that year are provided in full text and through various Internet databases and searches for these articles have been used based on keywords.

1.9 Methods of solving the model

The problem location-routing has two sub-issues .one of them is allocation another one is routing. These two issues are NP-HARD. So a problem of location -routing is NP-HARD as well. In addition to considering the window of time, the heterogeneous fleet and the collection and delivery of the same time between the central and intermediate depots and only delivery to the customer in the problem of locating - routing has increased the complexity of the problem. Therefore, only small samples can be solved using precise methods. Therefore, the design of a metaheuristics-algorithm is necessary to solve the proposed model.

1.10 Research structure

In this research, the second chapter describes the definitions, theoretical fundamentals, and the literature on the issue of locating-Routing.

In Chapter 3, the proposed mathematical model and the proposed method will be expressed.

In Chapter 4, their results and their interpretation will be discussed, and the proposed algorithm NSGA-II will present the results from solving examples of the model, the complete fitting of the model.

Second chapter

Theoretical fundamentals and literature review

Introduction

In this chapter, we first get acquainted with the supply chain and its concepts. Because designing an optimal distribution system helps to realize the goals of the chain and a tool for achieving macro and overall goals of the supply chain. After explaining the concepts of the supply chain, we describe the locating and routing issues of the vehicle. Because these are sub-issue of the locating -routing problem. After that, we explain the main problem of locating-routing. We review the methods of solving it. Finally, in the conclusion section, we will address the research gap in the problem of locating-routing.

2.1 Logistics and supply chain management

The concept of supply chain was first introduced in the late 1980s by Jones and Riley, and then expanded in the 1990s. Since about 30% of the finished product, cost includes logistics costs. This concept can be considered one of the most important management and operational tendencies in the last two decades. Chopra and Meindl defined the supply chain as follows "A supply chain includes all steps that are directly or indirectly involved in fulfilling a customer's request. The supply chain not only includes suppliers and manufacturers, but also includes a transport system, facilities, retailers, and customers. "

In a typical supply chain, raw materials are shipped from suppliers to factories, then from there, it reaches retailers and ultimately sends to the final customer or consumer. In some of these steps, the goods are stored and some other steps shipped. A supply chain is a collection of storage and transportation. The components of a typical supply chain include suppliers of raw storage, production centers, distributors, retailers, End-customer.

2.1.1 Supply Chain Management

Supply chain management is a process of planning, implementing a supply chain control operation efficiently. Supply chain management is a set of solutions for integrating chain components (suppliers, manufacturers, distributors, retailers, end customers) to reduce system costs and increase customer service levels (Toyoglu, Karasan and Kara, 2011).As the supply chain management focuses on integrating the suppliers' activities, manufacturers, distributors, end customers, so it involves many activities of companies at various levels.

2.1.2 Phases of decision making in the supply chain

Successful management of the supply chain requires decisions related to the flow of product information and financial resources. All of these decisions are divided into three categories or phases, depending on the abundance of each decision and time horizon a decision affects (Toyoglu, Karasan and Kara, 2011).

2.1.2.1 Strategic Phase

During this phase, Decide on how to structure the chain. What processes and with what combination each step will take place. These decisions include:

- The location and capacity of production facilities and warehouses
- Products that should be produced or stored at different locations
- Shipping methods required at different stations.
- Type of information system required

The company must ensure that the components of the chain support its strategic objectives during this phase.

2.1.2.2 Tactical Phase (Planning)

In this phase, companies define a set of operational policies that cover their short-term operations.

It is assumed that strategic decisions do not change. Companies will begin the tactical phase with the forecast of different market demand in the coming year.

- Which markets and what facilities will be provided?
- How large is the accumulation size
- Construction contractor
- Ordering and inventory requirements policies
- approved policies required for backup facilities when faced with a shortage of inventory
- How to schedule and promote marketing

Planning phase, the parameter specifies that a chain for a given period works. In this phase, companies need to consider uncertainty in demand, exchange rates, and competition in the relevant period in their decisions.

2.1.2.3 Operational phase

The time horizons of this phase are equivalent daily or weekly, and companies decide on specific orders of the customer. At the operational level, the composition of the chain is constant, and planning systems are predefined. The main purpose of the chain in this phase is to achieve the best of possible operational policies. During this phase, companies set specific orders for inventories or production and set the deadline for order fulfilment. Since short-term operational decisions are faced with uncertainty less. The goal of this phase is to reduce uncertainty and optimize performance concerning the limitations imposed by planning policies.

2.1.3 Supply Chain Performance Conductors

In this section, four major drivers are introduced: inventories, transportation, facilities and information, which determine the performance of a supply chain. Chain managers should make a comparison between cost and chain responsiveness concerning each driver. The combined effect of conductors determines the cost and responsibility of the entire chain (Toyoglu, Karasan and Kara, 2011).

2.1.4 Inventory

The inventory includes all raw materials, goods in the process flow and finished goods in the chain.

Increasing inventory will increase customer responsiveness, but, on the other hand, increase the cost of the supply chain. Because there is a slight adaptation between supply and demand in different parts of the supply chain. The most important role of inventory in the chain is to increase the demand that can be met due to the availability of the products when demanded by the customer. Another critical role is to reduce costs by exploiting economies of scale during production and distribution.

2.1.5 Transportation

Transportation leads to the transfer of inventory from point to point in the chain. In other words, it puts products between different parts of the chain. In addition to the speed of transport that responds

to the cost and cost of the chain, the type of transportation that a company uses also affects the inventory and location of the chain facility. Transport decisions that companies should consider when designing and operating the chain are the method of transport, domestic, or contracting transport, the choice of route and network.

- 1) Shipping method: There are six commonly used transportation methods: air, road, rail, sea, pipeline, electronic.
- 2) Domestic or transport contract: The purpose of an internal or contractual shipping arrangement is that the company owns and utilizes transportation equipment, or uses a third-party transportation company.
- 3) Route selection: The route selection means that if there are different routes for transportation, which one is optimal.

2.1.6 Facility

Facilities are places in the chain in which they are stored, assembled or produced. Two of the most famous facilities are production and storage facilities. Decisions on location and capacity location will have a significant impact on chain performance. If there are more facilities in the chain, so there are more responsive, but the costs increase. Decisions made in the area of the chain facility include facility location, facility capacity, facility production method, and storage facility in the facility.

1. Facility Location: Typically, vacancies are selected as the candidate's location for the facility, and then, depending on the features, the appropriate location is selected. These features include macroeconomic factors, strategic factors, worker quality, cost of facilities, access to necessary infrastructure, proximity to clients and other network members, tax effects.
2. Facility Capacity: If an additional capacity facility is available, it can use this extra capacity regarding demand fluctuations when needed. But without using the capacity, the cost per unit of production is facilitated. Therefore, facilities that do not have extra capacity will result in fewer costs and less responsiveness.
3. The storage method in the facility There are several different methods for warehousing in storage facilities, some of which are storage of unit of maintenance - inventory, storage accumulation, and simultaneous loading and unloading.

2.1.7 Information

Information includes data and processing data in the field of inventory, transportation and facilities, and customers throughout the chain Information is the most important chain Conductor because it directly affects other drivers. Improving the level of information in the chain increases responsiveness and reduces costs. Information as an interconnection between different chain members creates coordination between chain members and reduces the cost of the chain. The decisions made in the area of information include: determining the tension or compression of the chain, the level of coordination and information sharing, the prediction and planning of integration, and the use of new technologies.

2.1.8 Network design in the supply chain

Chain facility decisions, including locating production, storage, mode of transportation between the facilities, and the determination of the capacity of each facility are known as network design decisions, are categorized as follows. (Toyoglu, Karasan and Kara, 2011).

- Determine the role of each facility

- Locating facilities
- Allocation of capacity to each facility
- Allocation of resources and markets to the facility

The supply chain is divided into three main categories: supply, production, and distribution.

Among these three, the distribution network is very important. Because only cost factors are considered in the design of the supply and production sectors. But to plan a distribution network in addition to cost factors, factors related to customer service should also be considered. Usually, plans and strategies that have lower cost will create lower customer service levels. Therefore, the distribution network must be designed in such a way as to satisfy the customers' requirements at the lowest possible cost. Distribution involves planning and controlling the physical movement of goods from the manufacturer to the final consumer. Distribution is the availability of products in markets. In other words, distribution means bringing the right product to the right place at the right time. Chain distribution channels from different companies, including the main manufacturer, final consumer and intermediaries (wholesalers and retailers).

Designing distribution network is to make decisions about the facilities and facilities of the distribution network, including the location of production, the storage, the mode of transportation between the facilities, and the determination of the capacity of each facility. The two main types of factors affect the design of the distribution network are factors related to cost and factors associated with customer satisfaction. As customer service levels increase, logistics costs also increase. Generally, if the level of service to the customer is going to higher, the demand will increase. There are six common strategies for designing a distribution network: (Toyoglu, Karasan and Kara, 2011)

1. Store inventory by the manufacturer and send directly to customers
2. Store Inventory by the manufacturer and direct shipping with product integration during shipment
3. Store inventory by the manufacturer and send directly to customers by the transport fleet
4. store inventory by the distributor and delivery to the customer by post system
5. store Inventory by distributor or manufacturer and delivery to the customer via delivery centers
6. store inventory by retail and delivery to the customer

2.1.9 Integration levels

As discussed earlier, a supply chain can be efficient and responsive to customers, if it can create appropriate communications between different levels of itself, the decisions made in the chain are integrated, and each one is not programmed individually, the levels of integration are different and can be done in a variety of ways, depending on the circumstances. All models operations research can be associated with the construction of new facilities for the following categories:

- Facility locating problem

This types of models seek to minimize transportation costs and set up facilities.

- Routing Problem

These types of models seek to minimize transportation costs and costs of distribution cars.

- Locating Routing Problem

This model is a combination of facility locating and distribution vehicles routing

- locating-inventory problem

These models seek to minimize transportation costs, facilities, and store inventory costs.

- Inventory Routing Problem(IRP)

The model also seeks to minimize transportation costs and inventory costs.

2.1.10 Concept of locating facilities in the supply chain

Perhaps locating facilities is the most critical and difficult decision that is needed in designing an efficient supply chain. Transport, inventory and information sharing decisions can be changed in response to changes in supply chain conditions to remain an optimal chain. While decisions made in locating the facility cannot be easily changed due to changes in the conditions. These decisions are strategic and will have a long-term impact on the supply chain. If we assume that inventory is something that passes through the chain and considers shipping as a way of moving them, location as where to take them.

- ❖ Also Considering the variable of the capacity level of the facility the location problem can be divided into two types with the limited and unlimited capacity.

If the capacity of the facility remains unused the cost of each produced unit in that facility increases. In some locating models, the capacity of each facilitator is considered as a decision variable.

2.1.11 Concept of vehicle routing in the supply chain

Finding a vehicle's route efficiently is an important logistical issue that has been investigated in the last decade. The success of each supply chain depends on the proper utilization of transport. Transportation is a critical connection between the chain members. Transportation costs are an important part of the expenses incurred by a chain. A manufactory that can reduce the delivery routes or increase the number of its vehicles can provide better service to its customers and operate more efficiently, and it may gain more market share. For products sold through the Internet, shipping costs are an important part of the price of the product. Supply chains use responsive transport to focus and carry out their operations with fewer facilities. An example of the problem of vehicle routing involves the simultaneous determination of the route for several vehicles from the central terminal to a number of customers and returning to the terminal without violating the capacity of the vehicles. It is economically important in business. Because time and costs depend on the fleet of vehicles to deliver goods to a set of geographically dispersed customers. Also, such issues are important in public services, such as routing vehicles on buses, posts and other public vehicles. In each delivery carried out in the supply chain, there are two key elements: the sender and the carrier. The carrier must make decisions about investing in transportation infrastructure and then operating decisions to maximize the return on these assets. The cost of transportation is related to fuel costs and human resources. When a loader has the highest cost efficiency, its full capacity is used. On the other hand, the sender

also uses shipping to minimize the total cost and have a good level of responsiveness to customers at the same time. (Toyoglu, Karasan and Kara, 2011)

Fuel costs are directly related to the cost of the travelled distance, which is why reducing the cost of the travelled distance by finding suitable routes for better service and customer satisfaction, is seen in most examples, company goals and issues. There are six common ways to transport: air, road, rail, sea and electronic. The operational specifications of a transportation method include the extent of the steps, the speed, the availability and the load-transferability of the load by that method. These factors, along with the price, are the most critical factors in choosing the method and shipping carriers by the senders. In order to determine the transport decisions in the chain, the effects on the cost of inventory, cost of the facility, the cost of processing, the cost of synchronization of operations and the level of response to customers should be considered. Managers should consider the following cost comparisons when making transport decisions. (Toyoglu, Karasan and Kara, 2011)

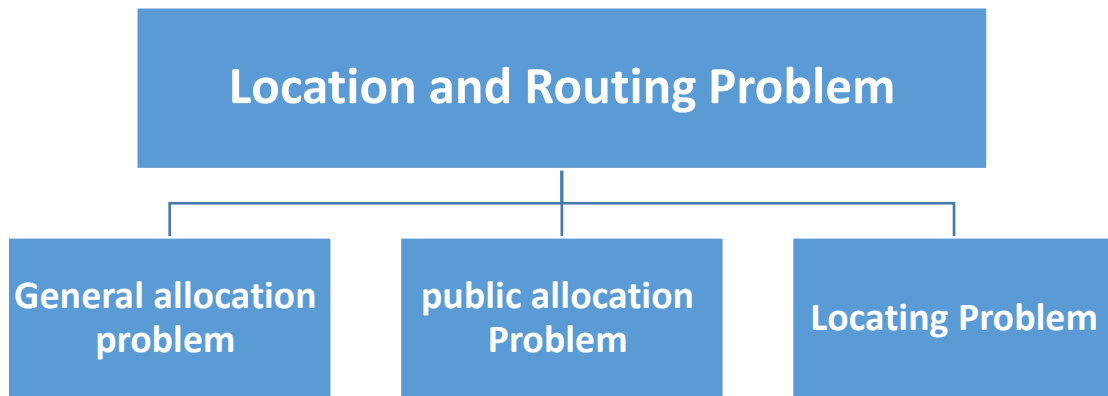
1. Compare the cost of inventory with shipping cost.
2. Compare transportation costs with customer satisfaction

2.2 Definition and recognition of the components of the problem of location-routing (locating-routing problem)

Considering the importance of identifying the problem, it is better to provide a general definition of that: it is defined as the establishment of one or more new facilities in the candidate's places and the choice of delivery routes from the facility to customers (consumer or recipient). Given the definition of the problem, it is clear that decision making and routing are two complex decisions taken at one time. According to the definition it is quite clear that the decision of locating and routing are two complex decisions taken at one time.

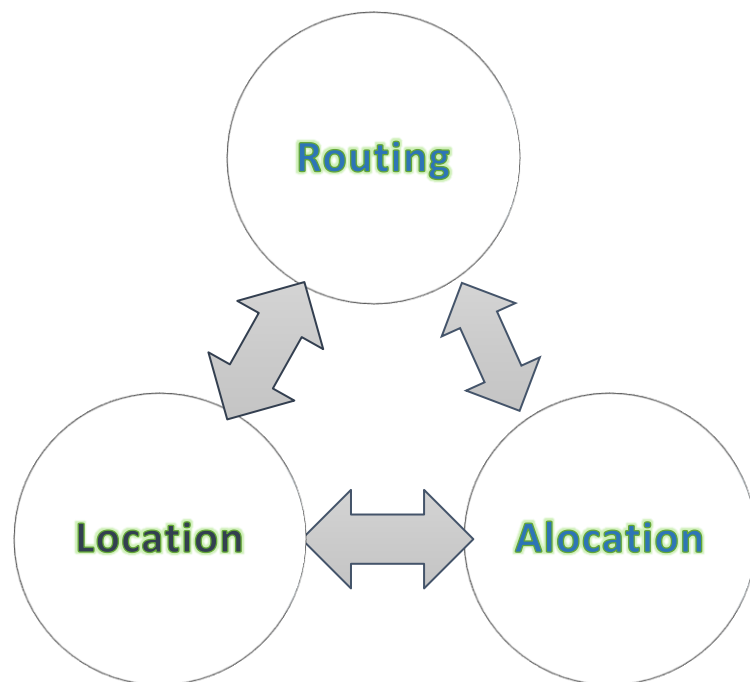
Another definition of (Nagi and Salhi, 2007) presented in their article is a hierarchical viewpoint, based on which the goal is to solve the problem of locating (the main problem) but to simultaneously solve the problem of vehicle routing (the sub problem). This indirectly implies the integrity of the issue. Therefore, this decision is a combined decision and this issue is a combined issue (Min et al. 1998). The following figure shows the components of the hybrid problem as the subsets (Sambola, 2003).

Figure (2.1)



Below figure helps to understand this combination issue and why for achieving the optimal answer, we cannot solve one of the sub-issues alone. This figure shows the internal relation between this sub-issues (Min et al., 1998)

Figure (2.2)



2.2.1 Location and routing issues from the service viewpoint

The ultimate output for location issues is the deployment of a number of new devices (facilities) that are classified into two categories in terms of serving customers.

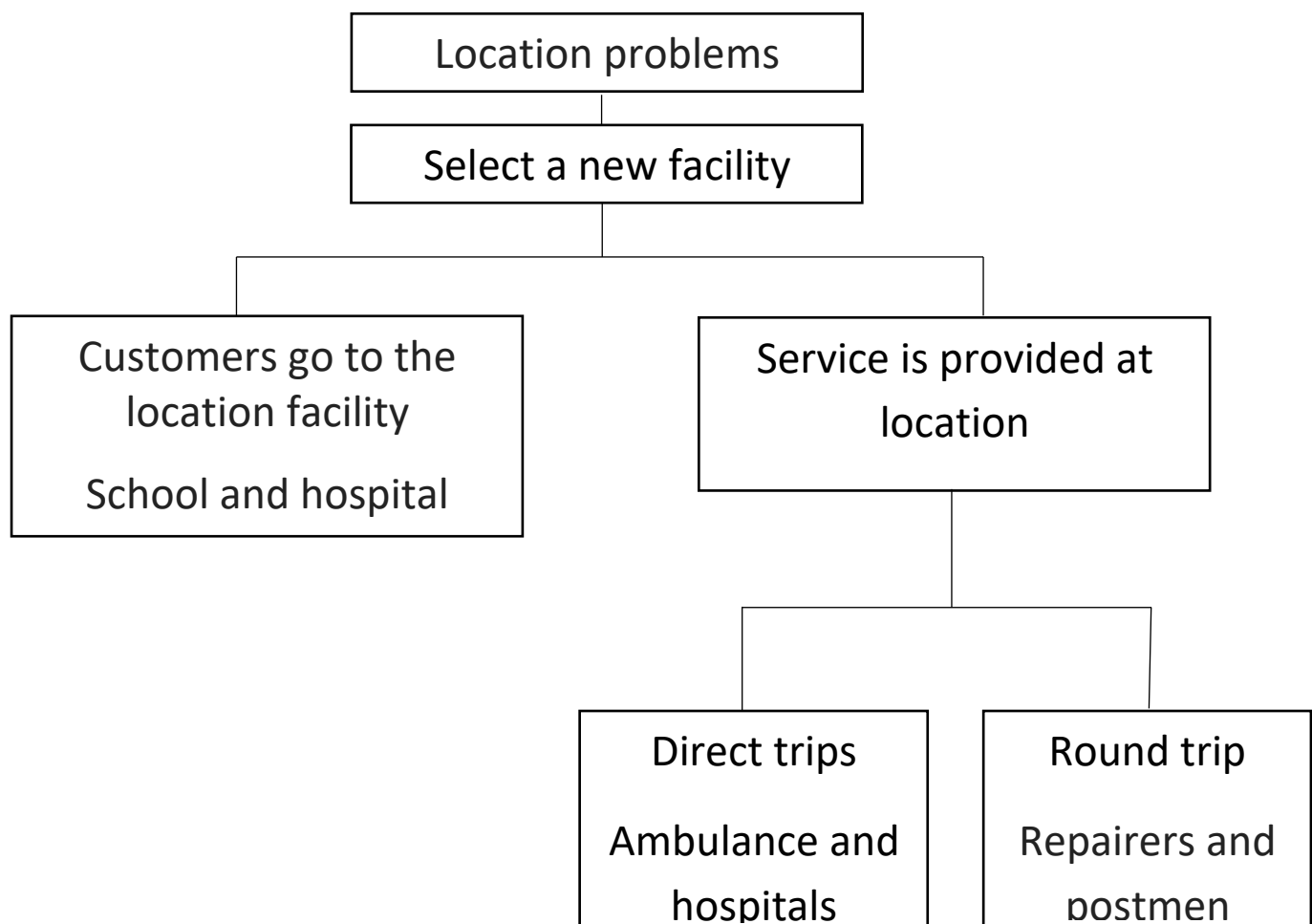
- Servicing is carried out at the customer's place (meaning that customers do not go to the facility for service).
- Customers go to the facility for receiving service. For such issues, examples such as the school, the hospital will clarify these issues.

If the first category is to be more precise, then it can be seen that customer service at the customer location is generally divided into two categories. In the first section, after serving, the service provider must necessarily be relocated to the deployment location. Such as firefighting cars and relief teams that are returned to their station and then ready for operation. These trips are the same direct trips, but in the latter case, service can be carried out at customer premises one after the other. For better understanding, repairers and postmen can be named (Symbla , 2003). These trips are the same as round trips. Figure (2-3)] (Zanjirani Farahani, R, 2009) Displays the service, from the facility to the customer. Regarding the division, it seems evident that considering the costs of the direct and round trip, the nature of the service can be expressed as follows (Min et al., 1998, Lane and Kowak 2006).

The problem of locating and routing is related to classical location problems and is related to routing issues. In fact, both groups can be considered as a particular case of routing-location models.

- Assuming that all customers are directly connected to the distribution center (straight trip), the problem of locating - routing becomes the standard location problem (locating-allocation).
- Assuming that the location and distribution points are specified and fixed, this problem is converted into routing models.

Figure (2.3)



A number of researchers disagreed with an integrated look at two issues of location and routing in a model. And they believe that these two issues are two different levels of decision making. These

people believe that locating is a strategic issue and routing is a tactical problem, and so the time horizons of the implementation of these two decisions are different. In other words, routing is a decision that can be reconsidered periodically and in non-prolonged periods. But location decisions are usually taken for a longer time horizon. Nevertheless, the findings show that considering these two issues like an integrated issue and in a comprehensive decision framework can reduce the costs of the distribution network over a long time horizon. There are two issues of locating and routing in most scenarios of issues, and combining them in one problem is very complicated and difficult. Therefore, many of the existing research in this field used inaccurate methods to solve. Because in most cases, even in the medium size, they cannot be solved with accurate methods in the reasonable time (logic time). **LA Porte** presented a summary of the research that published before 1988. **Min et al. 1998** also categorized and presented solutions of the raised issues in this area. An overview of the concepts, models, and methods of solving proposed research in the field of location-routing have presented by Nagi and Salhi, 2007.

2.2.2 Application fields location - routing problem

The field of application of location-finding problems is often in the distribution of consumer goods or postal packages. Nevertheless, the application of these issues are in military, health, and communications areas. also they are seen in the research presented, distribution of food, distribution of goods Consumers, locating blood banks, distributing newspapers, locating mailboxes, disposing of medical wastes, delivering postal packets, collecting waste, and transporting hazardous materials . Some important and practical work of this category of issues are summarized in the table below. The following table shows the larger dimensions of the problem regarding the number of facilities and customers. It has been shown that 20% of the location and routing articles were for real samples, indicating that this is a matter of fact in the real world, not just the use of researchers in academic sciences.

The applications in the table below have been selected from the Nagy and Salhi reference. These four other applications have been added.

Table (2.1)

country	Application areas	authors	year
England	Distribution of Soft Drinks and Food	Watson-Gandy & Dohrn	1973
Australia	Distribution of customer goods	Strohmeier	1978
America	Blood bank locating	pires cala,et al	1978
Denmark	Distribution of newspaper	Jacobson and Madsen	1984
Malaysia	Locating the rubber machine	Nambiar et al	1981
America	Distribution of goods	perl and danskin	1985
Belgium	Locating post boxes	Lobe and Laporte	1995
Switzerland	Distribution of beans	Semet and Taillard	1998

Belgium	Selection of waste	Kulkar	2000
America	Locating military equipment	Jang et al	2000
Switzerland	Delivery of mail packages	Bronze et al	2001
America	Disposal of medical waste	Chan et al	2001
Hong Kong	Delivery of bill	Lin et al	2002
Korea	Optical network design	Lee et al	2003
Austria	Delivery of mail packages	Wasner and Zapfel	2004
Spain	Locating destroy of livestock waste	Cabllero et al	2007
Italy	Locating vehicle	Amborosino et al.	2005
Spain	Locating rail networks	Marian and garcia - Rodenza	2009
Brazil	vehicle routing with Heterogeneous fleet	Balfour and Yoshizaki	2009
Iran	Locating and managing inventory of crisis management centers	Zanjirani Farahani et al	2008
Iran	Reduce energy consumption	Alinaghian et al	2016
Iran	Relief process	Talebian sharif et al	2013
Iran	The drink and food industry	Setak et al	2014

2.2.3 The history and division of locating and vehicle routing issues

The history and division of locating and vehicle routing issues include the following two parts:

- History and division of location issues
- History and Division of Vehicle Routing Issues

2.2.3.1 History and division of location issues

Using mathematical science to optimize discrete and continuous locating has devoted a lot of research to more than four decades. The word locating or location is attributed to the modelling and the solution to a bunch of issues that are the establishment of facilities at specific locations.

Roll and Islet in 2005 stated that the disassembly was different from the layout. Because in locating, the level of establishment of facilities relative to the places they are intended is small for them, while in the layout, the opposite is true. On the other hand, maybe there is no connection between the

facilities, while the existence of connection in the layout is a rule. They also included four components for this category of issues.

- Customers, who are supposed to be deployed in a series of locations and routes.
- The facility, which should be located and deployed.
- The distance between customers and the candidate location of the facility deployment.
- A scale that measures the distance or time between customers and facilities.

In 2005, Roll and Islet argued that the first location issue in the subject literature was attributed to a mathematician named Torisley in the period 1608 and 1674. His issue included a customer and one facility. From the early twentieth century until the 1960s, geoscientists and economics had a tremendous impact on these issues. And one of the obvious examples of this was Weber's work in 1907. The problem of locating has wide literature and different types of problems are defined. This problem is categorized in different ways. In 1974, Francis and White categorized the problem of location for the first time. Subsequently, various categories are also presented with the growth of the problem of location. Each of these categories has used a set of criteria and index. The following is a list provided by Deskin in 1995.

In order to categorize these issues, the division of Ham Machar and Nicol in 1998 were used. The structure of this classification includes five positions, each of which could include the following.

Pos1/pos2/pos3/pos4/pos5

Pos1: Information about the number and type of facility

Pos2: The type of location model according to the decision of the desired space, which can be discrete, continuous, and network.

pos3: Descriptions of some model specifications such as capacity constraints for facilities

Pos4: The relation between new and existing facilities, this relationship can be a function of distance or cost

Pos5: An explanation of the objective function

If there is no specific assumption at each position, the symbol is used. For each location, a symbol is defined to represent a useful attribute for each characteristic.(Ham Machar et al., 1998)

2.2.3.2 History and classification of vehicle routing problems

One of the most critical issues in this area is the problem of routing transport vehicles, which has been used dramatically in the transport industry in the 1960s (Tarantilis et al., 2005). The reason for these issues of vehicle routing comes from the fact that the transportation process at all levels of production and distribution includes about 10 to 20 percent of the cost of goods. Distribution of goods, in fact, serves a group of customers over a specified period by a number of vehicles and drivers in one or more distribution centers on a road network. Most of the distribution costs are for vehicle drivers and a small part is related to routing (80% and 20% distribution costs respectively) (Lynn and Kowack, 2006). In a particular case, the answer to the vehicle routing issue can be as follows: Determine the set of paths that are used by each vehicle. (These vehicles directly transfer from the distribution center and eventually return to the same distribution center.). In such a way that the customer's needs are met,

and all operational constraints are also met, resulting in a minimization of overall costs. Toç and Vigo introduced five elements for the following issues in 2002:

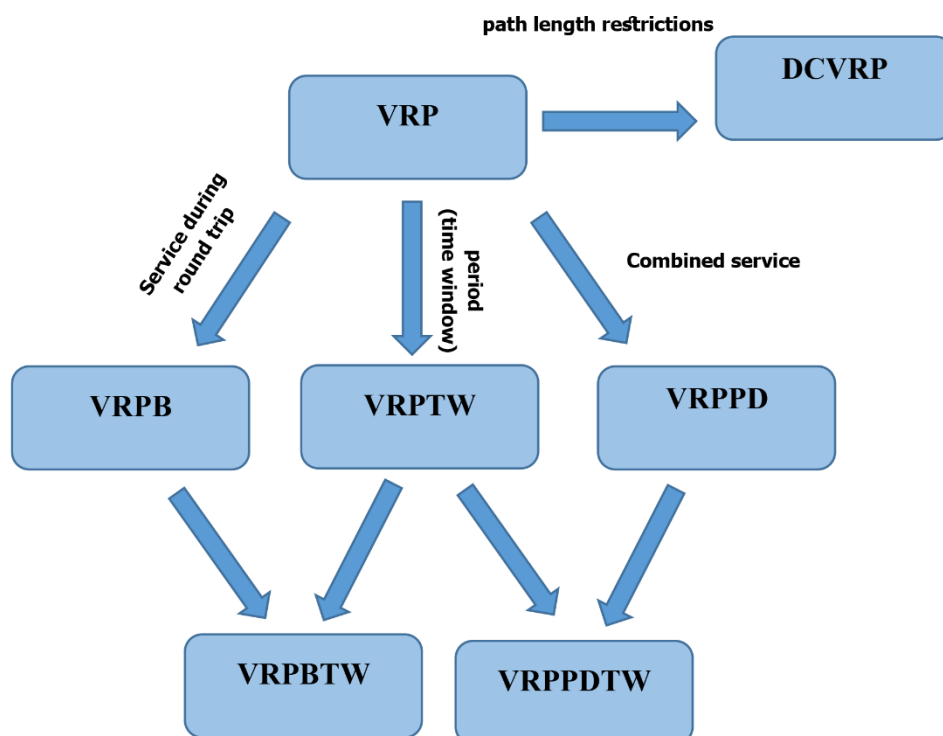
- road network
- Customers
- Distribution centres
- Vehicles
- Drivers

A number of basic issues arise in this regard. Before discussing the basic issues of vehicle routing, it is better to define the following concepts:

- **PICUP AND DELIVERY:** Issues related to this category occur when customers of a route receive two types of services (delivery of a series of goods and the collection of a series of other goods)
- **BACK HIALING** Issues related to this category occur when, on a route, we have to deliver some goods to some customers, which is referred to as LINEHAUL, and from some of them we collect goods that are referred to as BACKHAUL.

According to this definition can be given a classification for these issues:

- Vehicle Routing problem by Capacity (**CVRP**)
- vehicle routing problem with regard to domain capacity along the route (**DCVRP**)
- Vehicle Routing problem by Time window (**VRPTW**)
- Vehicle Routing problem by Delivering and Receiving Goods along the paths (**VRPB**)
- Vehicle Routing Issues by Delivering and Receiving Goods along Distribution Paths(**VRPBD**)



The internal relation between vehicle routing problems (R.Zanjirani Farahani. et al.,2009) **Figure (2.4)**

Routing and scheduling are the types of transportation operational decisions that usually their time horizon is daily.

There are several types of routing issues. The most important of them are:

1. Issues in which a point is the start of a trip and a point as the destination of the trip. The most common technique for solving this problem is the shortest method
2. Issues that have several starting points and several destination points(Transport issues)
3. Problems which destination points of them the same as the starting point, and the vehicle must return to the starting point after the completion of travel. This issue is known to the seller of the roundabout, and the goal is finding the shortest route and can be used to go to all points and then return to the starting point.

In this research, the third type is sought. The problem of routing the vehicle can be expressed as a **graph theory problem**. In this case, the following assumptions are established:

Assume that $(V, A) = G$ is a complete graph. A set of edges (also called an arc or line) and $V = \{0, 1, \dots, n\}$ is the set of nodes. The nodes $j = 1, \dots, n$ correspond to the customers, and node 0 corresponds to the distribution center.

- The weight of each edge (line) corresponds to the distance between the two nodes.
- Each customer has a specific demand.
- Each route starts from a distribution center and ends in the distribution center.
- Only one route is assigned to each customer.
- The purpose of the problem is to find the routes of vehicles so that the distance traveled is minimal and the demand of all customers is met.

The routing problem includes different types. The last category presented for the routing problem is by Burake and others in 2009 (B.Eksioglu. et al., 2009), which is presented in the table below.

Types of criteria in the routing problem (B.Eksioglu et.al., 2009) [Table (2-2)]

CRITERIA	TYPE		
1. Number of vehicles	A: Exactly a vehicle	B: Maximum N vehicles	
2.Type of time window	A: Customers	B: Routes (path)	C:Depots D: Drivers
3. Time window structure	A: Soft	B: Hard	C:mix
4. Customer locations	A: Nods	B: Edges (arc)	
5. Capacity of Vehicle	A: Limited	B:Unlimited	
6.Type of vehicles	A: Homogenous	B: Heterogeneous	
7. Type of demand	A: Collect	B: Delivery	C: Collect and Delivery

8. Load Separation	A: Load separation B: Load without separation
9. Cost	A: Depending to trip time B: depending to distance C: Depending to type of vehicle D: Depending to late
10. Number of facilities	A: Single facility B: Multi facility
11. Model type	A: Certain B: Static
12. Planning horizons	A: Single period B: Multi period
13. Solving method	A: Exact B: Innovative (Meta-heuristic)

1. Routing with considering the time window

In most real-world issues, the distribution of goods and the provision of services must be specified at the specified time, which is determined by the pattern of the organization or company. One of the criteria that determine the level of customer satisfaction or service provider level in a distribution network. The window is an interval of time $[e_i, l_i]$. e_i is the earliest acceptable time, and l_i is the latest acceptable time to receive the service. The issue of routing with regard to time windows is one of a kind of routing problem, and its applications can be the distribution of money between branches of banks and ATMs, distribution of postal items, industrial waste collection, school routing, fuel distribution, the supply of materials Initial for factories, JIT. Windows can be considered as a soft window, a hard window, or a soft and hard window (blended). In the hard windows, the service must be specified only within the range. But in the soft window, a larger period of time including a hard interval is also considered; in this case, if the service time is out of hard range, but still within the soft range, the service is acceptable, but a fine is imposed. In soft and hard windows one side of the timeline is soft, and the other side is hard.

2. Routing with consideration of collection and delivery

In routing issues, customer demand can be the collection, delivery, or collection and delivery. Collection and delivery issues can be divided into two general categories (B.Eksioglu et.al 2009). In the first type, the goods must be delivered to customers are loaded in one or several warehouses, and goods that are collected from customers are also unloaded into one or more warehouses. This type of problem is called the Routing Consideration of Returning Goods (VRPBS). The routing problem with regard to returning goods consists of four categories.

In the first and second group, customers are requesting collection or delivery, but in the last two categories each customer can request to collect and deliver together. The routing problem concerning the return of the goods are:

1) Vehicle Routing Problem with clustered Backhauls (VRPCB): In this case, firstly, customers who have demand for delivery and then customers who have demand for collection are met on every route.

2) Vehicle Routing Problem with Mixed Line hauls and Backhaul (**VRPMB**): In this type of problem, customers have a demand for collection and delivery can be served with any arrangement. And do not need to be delivered before collection.

3) Vehicle Routing Problem Divisible Delivery and Pickup (**VRPDDP**)

In this type of issue, each customer has any type of collection and delivery request. And delivery and collection for a customer may occur simultaneously and in a non-synchronous manner.

4) Vehicle Routing Problem Simultaneous Delivery and Pickup (**VRPSDP**)

In this case, the process of collecting and delivering to a customer is carried out simultaneously. It means that each customer only is visited once. Maine first raised this issue in 1989.

In the second type of collection and delivery issue, goods are moved between points of collection and delivery, that is, goods are collected from a customer and delivered to another customer. Transport between the collection point and the corresponding delivery takes place. This type of routing problem is called collection and delivery (**VRPPD**). Vehicle Routing Problem Pickups and Deliveries (**VRPPD**) are divided into two categories:

1) Pickup and Delivery Vehicle Routing Problem (**PDVRP**)

In this case, collection and delivery locations are not defined accordingly, and the goods are considered the same. Each item that is collected can be used to meet the demands of any customers who have demand delivery, is used.

2) Pickup and Delivery Problem (**PDP**): In this case, for any place where goods are collected, a destination is considered to be the delivery of the goods. That is, there is a collection of ordered pairs of destinations and origins.

3. Routing by considering non-homogeneous fleets

In the routing problem, vehicles may be homogeneous or non-homogeneous. Homogeneous vehicles are the same in terms of all features, such as capacity, speed and cost, and so on, but in a non-homogeneous fleet, devices have different characteristics.

In the case of routing with heterogeneous fleets (non-homogeneous), fleets of vehicles are composed of different types of vehicle types $M = \{1, \dots, m\}$. Any kind of vehicle $k \in M$, m_k vehicles are available with a capacity each of them is Q_k . Each path is defined as a regular pair (R, K) . R is a set of customers and a warehouse, and k is the type of device used. Of course, by applying any of the following hypotheses, a certain type of problem is created.8)

1. The number of the vehicle of any type is unlimited. $m_k = +\infty, \forall k \in M$

2. The fixed cost of the vehicle is not considered. $F_k = 0, \forall k \in M$

3. Routing costs are not dependent on the type of vehicle.

$$C_{ij}^{K_1} = C_{ij}^{K_2} = c_{ij}, \forall k_1, k_2 \in M, K_1 \neq K_2, \forall (i, j) \in A$$

Therefore, the routing issue is divided into several categories according to the fleet used, as shown in the below table. As you can see in the table, problems with a limited fleet size are called routing issues with heterogeneous fleets, and the issues of which the size of the fleet is unlimited are called Feet size and mix vehicle routing problem(FSMVRP).

4. Feet size and mix vehicle routing problem (FSMVRP)

In this case, the number of vehicles of any kind is unlimited, and in addition to routing, it should decide on the composition and size of the fleet. The purpose of this issue is to use the Fleet to meet customer demand. Three different cost structures can be considered for this issue:

1. Variable cost vehicle is considered to be the same. While the fixed costs of the vehicle are considered different
2. The fixed cost for vehicles is not considered, and variable costs for vehicles are considered differently.
3. Both fixed costs are considered, as well as variable costs of vehicles are different.

Types of routing issues with heterogeneous fleets (non-homogeneous fleets). (B.Eksioglu. et al., 2009)

Table 2.3

Routing costs depend on type	Considering the fixed cost of vehicles	Limited fleet size	Issue type
✓	✓	✓	Routing by considering heterogeneous fleets, fixed costs and routing costs depending on the type of vehicle (HVRPFD)
✓		✓	Routing with regard to heterogeneous fleet and routing charges depending on the type of vehicle (HVRPD)

✓	✓		Fleet Size and Mix with Fixed and Dependent costs (FSMFD)
✓			Fleet Size and Mix with Dependent costs (FSMD)
✓	✓		Fleet Size and Mix with Fixed costs (FSMF)

2.3 Location - Routing Problem

The problem of locating - routing involves identifying suitable locations for facilities, allocating customers to facilities, and routing vehicles in order to meet customer demand. From the applied point of view, the problem of locating-routing, a component of distributed management and from a mathematical point of view, is a combination of optimization problems and NP-Hard. Facility location and vehicle navigation are two fields of research related to each other. For the first time, this view was raised in 1964, and then Rand in 1976 referred to the need to consider these routing and locating issues as an integrated problem. He believed that considering these issues individually could lead to local optimality. However, this interaction is usually ignored, and location and routing issues are studied individually. The reason for this subject is one of the following. (1)

- In many real-world issues, the problem of locating is not a routing aspect.
- Some scholars believe that these two issues are not compatible and cannot be considered integrally. The reason for this inconsistency is that the problem of locating a strategic type and the routing problem is tactical. The routes can be designed alternately (even every day), while the facility location cannot be changed so quickly. In other words, the planning horizon is long-term for locating problems and for routing issues is short-term.
- Conceptually, locating - routing Issues is much more difficult than any locating and routing problem. In locating- routing issue, the location of the facility should be determined according to the paths so that the total cost is minimal, while in the case of classical locating issues, facilities should be located according to the distance from the points of demand. Therefore, the problem is simpler and this can be one of the reasons for the slower progress of locating-routing issues. The problem of locating - routing has a variety of applications, especially in the

supply chain. In many studies on location-routing, examples and real data have been used. These articles are diverse, and most of them have been focusing on the distribution of goods and foodstuffs. Other applications of this issue are the design of healthcare services networks, the locating and routing of military installations and equipment, the distribution of blood products, the distribution of postal services, the collection of waste, etc. With regard to literature, applied articles, about one-fifth of articles make.

The problem of locating - routing in its general form on a complete, weighted, and no directional graph $\{G = (V, E)\}$ is defined. The hypotheses of the problem are:

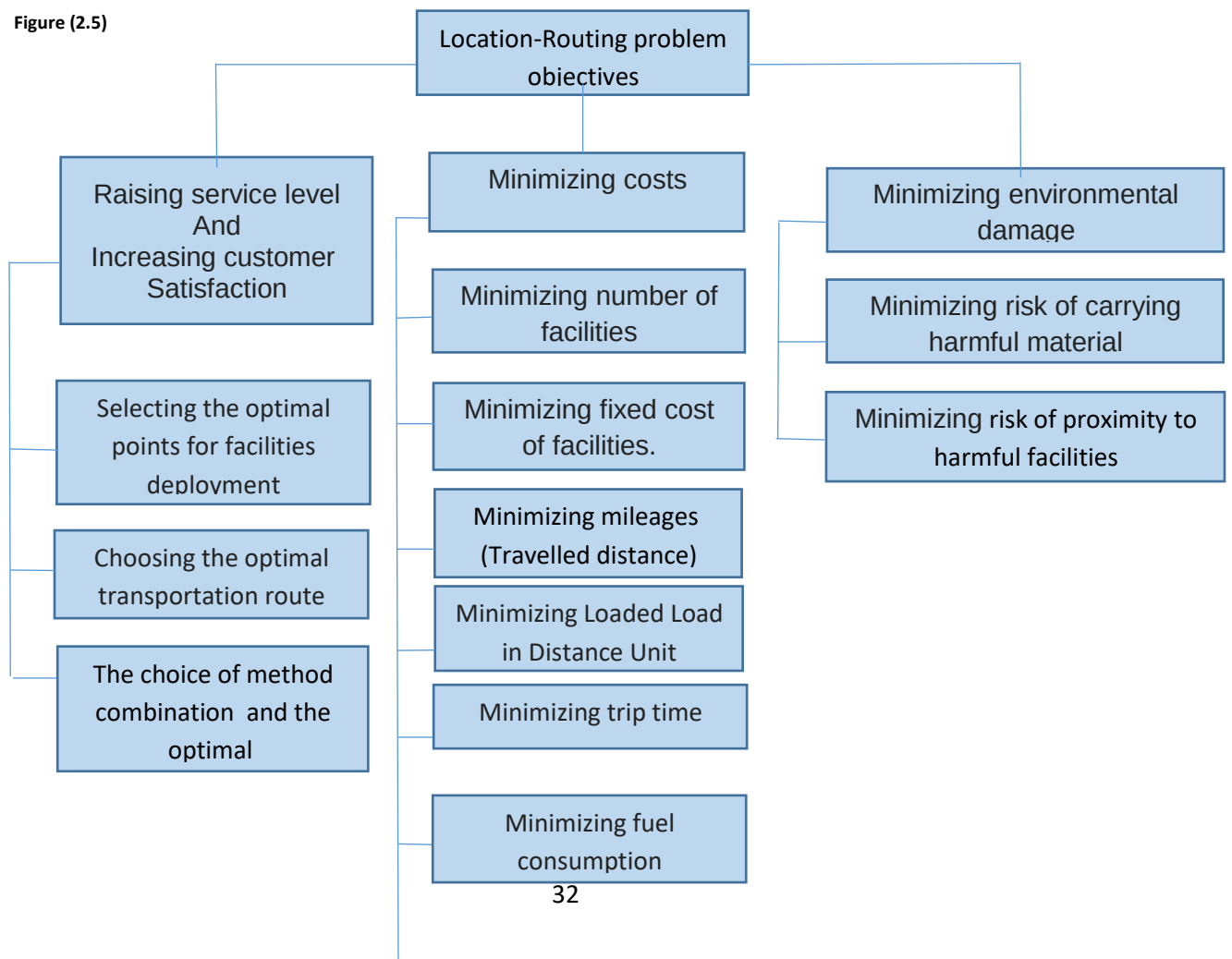
- **V** is a collection of graph nodes and consists of two sub-sets **I** and **J**. The subset **(I)** contains **m** nodes and each node corresponding to a candidate location for the facility and the subset **J** contains **n** corresponding node of each client.
 - **E** is the edge (Arc) of the graph. The graphs of the field correspond to the direct paths between the locations. The weight of the edges (Arc) in the graph is equal to the distance between the two vertices of the edges (Arc). The size of a limited fleet of assumptions is that the triangular inequality is supposed to be, i.e. the direct distance between two locations is less than the distance of each other route that correlates the two locations.
 - Establishing any facility has cost for the system.
 - A set of identical vehicles is intended for routing; each vehicle introduces two types of costs to the system: fixed cost and variable cost. The variable cost depends on the distance travelled by the vehicle.
 - Each path begins with a facility and ends with the same facility.
 - Each customer has a specific demand that must be met by only one vehicle.
 - The aim is to find the optimal number and location for facilities and find the optimal paths corresponding to each facility, so that the set of costs is minimal and the demand of all customers is met.
- Usually, the aim of a locating-routing problem is minimizing the linear combination of costs. These costs include the cost of setting up warehouses, the operational costs of warehouses and fixed and variable costs of vehicles.

2.3.1 Types of objectives in the Location - Routing problem can be divided into three categories:

- Minimizing costs
- Minimizing environmental damage
- Raising the service level by selecting the optimal points for deploying optimal facilities and routes in transportation services.

Types of targets in the problem of location-routing are shown in Figure (2.5)

Figure (2.5)



Minimizing
transportation costs

2.3.2 Type of Location-Routing problem

There are many types of location - routing issues. Mine et al. (1998) presented the first category for routing issues, **Nagi and Salhi, 2007** completed this categorization in 2007. According to the objective function, the decision variables, parameters and problem limitation are criteria of locating - routing problem. In fact, location-routing criteria are a combination of locating problem criteria and routing problem criteria. Table 2.4 shows the location-routing criteria. One of the criteria that categorizes location- routing issues is the problem structure. Locating-routing issues according to their structure are divided into standard and non-standard categories. Problems with the standard structure are also divided into two categories. These are issues in terms of certainty and issues in the conditions of uncertainty. (Nagi and Salhi, 2007). According to the literature, among the issues of locating - routing with a standard structure and in conditions of uncertainty, Most studies and papers address the problem of general location - routing, which has already been considered.

Types of criteria in the Location-Routing Problem (table 2.4)

Criteria	Type	
1. Capacity of the facilities	A. Unlimited	B. Limited
2. facility candidate place	A. Finite (discrete)	B. Infinite (Continuous)
3. Number of facilities level	A. Single level	B. Multi-level
4. costumers demand	A. Collect/Delivery	B. Collect/Delivery
5. Customer locations	A. Nods	B. Edges (arc)
6. Time window	A. With time window	B. Without time window
7. Type of vehicles	A. Homogenous	B. Heterogeneous
8. Capacity of vehicle	A. Unlimited	B. limited
9. Cost of vehicle	A. Variable	B. Fixed and Variable
10. Distance / time trip of vehicle	A. Unlimited	B. limited
11. Products	A. Single type	B. Multi type
12. Issue Problem structure	A. Standard	B. Non-standard
13. Type of model	A.1. Static A.2. Certain	B.1. Dynamic B.2. Probably

14. Planning horizons	A. Single period	B. Multi period
15. Objective function	A. Single objective	B. Multi objective
16. Solving method	A. Exact	B. Innovative
17. Type of data	A. Hypothetical	B. Real

2.3.2.1 Location - Routing Issues with standard structure and in terms of certainty

- 1) Location Problem - Routing with Closed Ring: In this case, each vehicle starts from a depot, collects goods from some customers and delivers them to other customers, and then returns to the depot.
- 2) Two-dimensional Location-Routing Problem: Unlike positioning another locating- routing problems that are discrete in space, this issue involves in a continuous space.
- 3) Hamiltonian P-median location-routing problem: This is a combination of the locating problem and Hamiltonian path p-median and travelling salesman problem.
- 4) Problem locating - arc-directional routing: In this regard, unlike all the issues of locating - routing in which customers are located on network nodes, Customers are on the edge (arc) of the network.

2.3.2.2 Location - Routing Issues with standard structure and in terms of uncertainty

- 1) Locating - Travelling salesman problem: In this case, the aim is to find the location of a traveling sales man (Considered a vehicle and a facility), and customer demand is random.
- 2) Random location-routing problem: In this case, one or more parameters are considered randomly.
- 3) Dynamic Locating - Routing Problem: In this type of problem, Locating - Routing with the assumption of system dynamism and It is studied in a planning horizon that includes several periods of time.

2.3.2.3 Location-Routing issues with non-standard structure

- 1) Shipping - Locating issue: In this case, the location of facilities and the transportation of goods between points of supply and demand points are studied seamlessly.
- 2) Routing - Allocation problem: This issue includes the design of the route between the facilities. Customers are assigned to facilities that are designed on the route. This issue is not any routing on the level of customers.
- 3) Location - Multi-level routing Problem: In these issues, more than one level is considered for facilities. For example, if three layers are considered for the distribution network. The first layer is related to the manufacturers (initial facilities), the second layer includes distributors (Secondary facilities), and the third layer includes customers. Usually, the location of the Secondary facilities and customers is defined and Secondary facilities are located. Two modes can be considered for distributing goods by the vehicle from the t layer to $t + 1$. The two modes are: Route designing and considering rout of round trip. In some cases, customers come to the facility to receive services and goods. And or any path contains a customer. In any case, the distribution method is shown as $\lambda/M_1/M_2/.../M_{\lambda-1}$, where λ represents the number of layers.

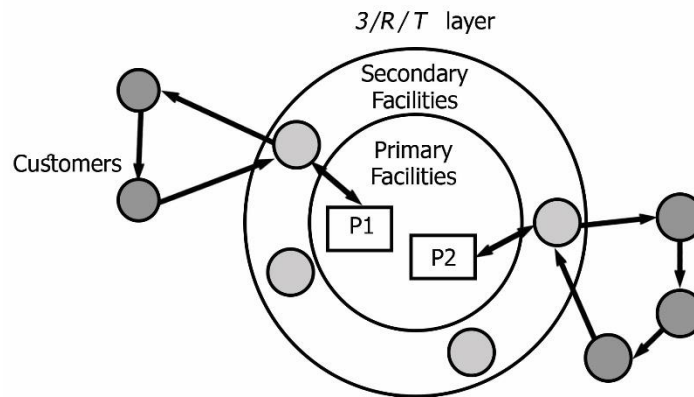


Figure (2.6)

2.4 Location - Routing Problem Solving Methods

In the solution of the location-routing problem, exact (Rigorous) and innovative methods have been used. Accurate solving methods used for location-routing problems include Branch and Bound Methods, dynamic programming, linear programming. In summary, precise methods, due to the complexity of location-routing problems, are only useful for small-sized samples.

The problem of locating-routing is an NP-HARD problem because it includes two NP-HARD problems, location-routing. Innovative solution methods for location-routing issues are divided into three categories: Lagrangian relaxation, Meta-Innovative Methods, and Fuzzy Planning.

On the other hand, considering the structure of the innovative method, four groups can be considered (Nagi and Salhi, 2007). They are:

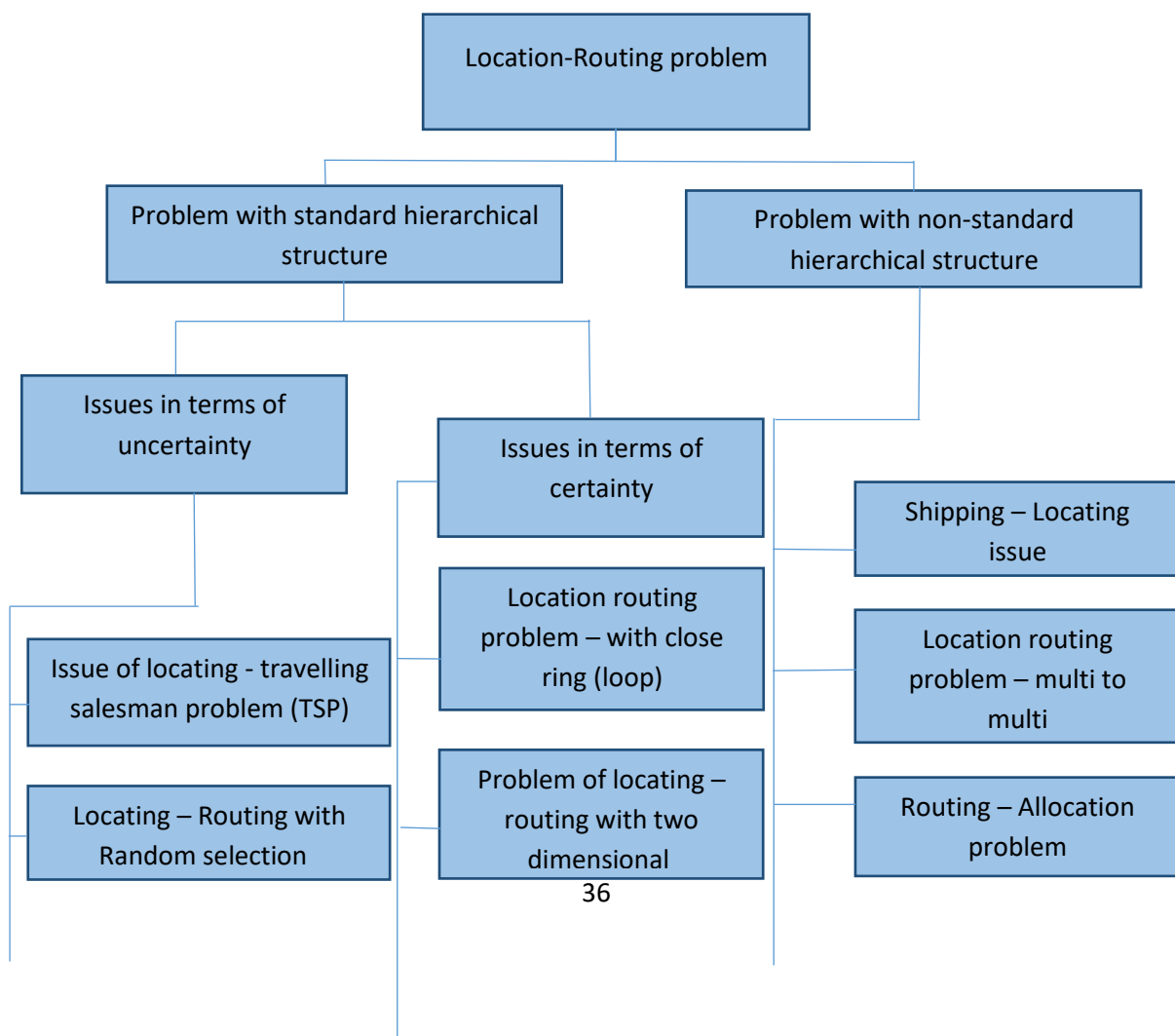
- 1) Sequential methods: This method first solves the problem of locating and then routing, or first, routing and then location. In the issue of locating, the goal is minimizing the total distance between warehouses and customers. In the sequential method, there is no routing phase to feed-back phase. The sequential solving method for location-routing problems may lead to a local optimal response. In practice, this result is proved in a static and dynamic state. Of course, it should be noted that this method is a criterion for other innovative methods and is not usually used.
- 2) Cluster-based methods: Cluster-based methods start by splitting customers into clusters. Two options are available in the following:
 1. Locating a warehouse in each cluster and then solving the vehicle's routing problem for each cluster.
 2. Solve Travelling salesman problem (TSP) for each cluster and then locate depots.
- 3) Repeat method: The innovative method of repetition in the problem of locating-routing consists of two sub-issues. In this way, two sub-issues are solved sequentially. Each phase is solved based on another phase information. The problem with this method is how to use one phase information in another phase. Repetitive innovative methods are better methods than

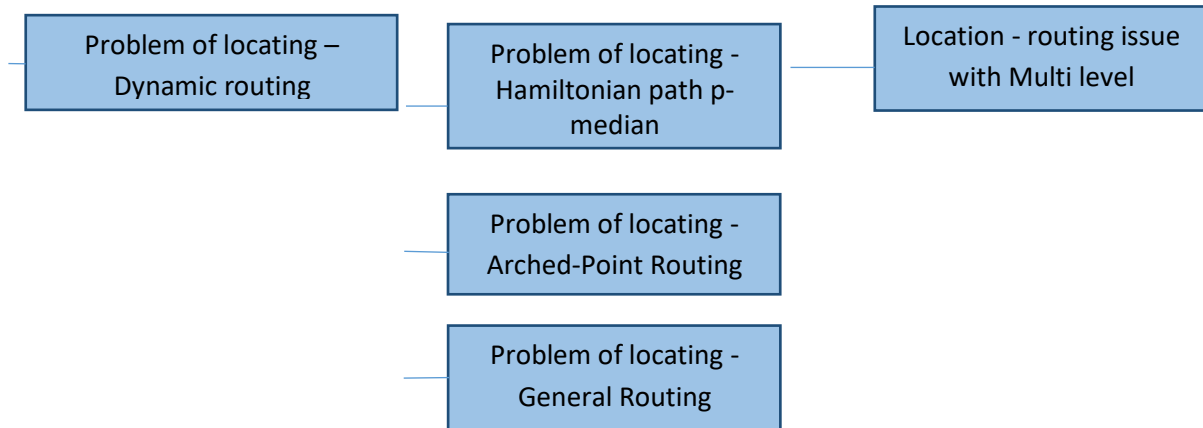
sequential methods, but they also have disadvantages. The problem with this method is that if the routing information in the location phase is not beneficial, this method goes away. In this way, both sub-issue (location and routing issues) have the same position, but not in the hierarchical way.

- 4) Hierarchical method: In this method, the problem of location, the main problem and the problem of routing is the follower (function) problem. The main algorithm is the location problem solving, which refers to the sub-process that relates to the routing problem.

➤ It is clear that the hierarchical approach provides a better model of real situations as well as better answers.

Figure (2.7)





2.5 A review of research in the field of location-routing

As seen in the previous sections, in each of the researches in the field of locating-routing, certain assumptions are considered, and different ways of solving this problem have been proposed. Toyoglu and colleagues reviewed and categorized locating -routing issues until 2009(Toyoglu.et al.,20011). In this research, the problem of locating-routing after 2008 has been studied, which is presented in the table below. By reviewing the provided tables, it is easy to see the research gaps in the locating -routing problem that are referred to in the Conclusion section.

Authors / Published year	Inventory	Multi product	Continuous response space	Multi objective	Real data	Time window	Heterogeneous fleet	Limitations vehicle	limitation of distribution	Multilevel distribution network	Multi period	Probable	Collect and deliver
Bumess and White (1976)			✓									✓	
Jacobsen and madsen (1980)								✓		✓	✓		
Nambier et al (1981)				✓		✓			✓	✓			
Oil (1983)						✓		✓	✓	✓			
Perl (1984)				✓				✓	✓	✓			
Perl and Daskin (1985)				✓				✓	✓	✓			
Berman (1988)												✓	
Bookbinder and Reece (1988)		✓						✓	✓	✓	✓		
Laporte . (1989)							✓	✓					
Laporte et al. (1989)								✓		✓		✓	
Levy (1989)				✓		✓				✓			
Nambiar et al (1989)				✓		✓	✓		✓	✓			
Zografos (1989)					✓				✓				
Srivastava and Beneton (1990)						✓				✓	✓		
List (1991)		✓		✓	✓								
ReVelle et al. (1991)				✓	✓								
Simchi – Levi (1991)										✓		✓	

Table 2.5

Table 2.6

Authors / Published year	The name of the Innovative and Metaheuristic method is used
Jacobsen and Madsen(1980)	Tor construction method with implicit transfer point location
Oli (1983)	Saving –drop procedure
Bookbinder and Reece (1988)	Benders decomposition

Authors / Published year	Inventory	Multi product	Continuous response space	Real data	Multi objective	Time window	Multi period	Multilevel distribution network	limitation of distribution centers	Limitations vehicle	Heterogeneous fleet	Collect and deliver	Probable
Simchi – Levi (1991)				✓		✓		✓		✓	✓		
Semet and Taillard (1993)												✓	
Averbakh et al (1994)					✓		✓		✓				
Jacobs and Warmedam (1994)												✓	
Jamil and Malon (1994)												✓	
Averbakh (1995)								✓		✓	✓		
Seme (1995)				✓					✓	✓			✓
Min (1996)					✓				✓				
Lonnis (1998)						✓	✓			✓			✓
Nagy and Salhi (1998)										✓	✓		
Tuzun and Burke (1999)				✓		✓				✓		✓	
Chan et al (2001)				✓		✓			✓	✓			
Lin et al (2002)									✓	✓	✓		
Wu et al (2002)	✓									✓		✓	
Liu and Lee (2003)				✓		✓		✓		✓	✓		✓

Table 2.7

Authors / Published year	The name of the Innovative and Metaheuristic method is used
Tuzun and Burke (1999)	Extended Clarke-Wright heuristic
Simchi –Levi (1991)	Tabu search
Averbakh (1995)	Lagrangian relaxation
Semet (1995)	Multiphase decomposition heuristic procedure
Nagy and Salhi (1998)	Two-phase tabu search approach
Chan et al (2001)	Simulated annealing
Lin et al (2002)	Simulated annealing
Wu et al(2002)	Route-first,location-allocation second approach

Table 2.8

/ Authors Published year	Inventory	Multi product	Continuous response space	Real data	Multi objective	Time window	Multi period	Multilevel distribution network	limitation of distribution centers	Limitations vehicle	Heterogeneous fleet	Collect and deliver	Probable
Wasner and Zapfel (2004)						✓		✓		✓			
Aksen and Altinkemetr(2005)	✓						✓	✓	✓	✓	✓		
Ambrosino and Secutella(2005)									✓	✓			
Melechovsky et al. (2005)									✓				
Alberda and Fernandez (2005)			✓	✓						✓			
Dethloff (2005)				✓		✓			✓	✓			
Lin and Kwok (2006)		✓		✓	✓			✓	✓				
Alumur and Kara (2007)				✓	✓	✓				✓			
Cabllero et al (2007)	✓									✓			
Qzurt and Aksen (2007)									✓			✓	
Alberda et al (2007)	✓							✓		✓		✓	

Shen and Qi (2007)		✓					✓	✓	✓	✓	✓		✓
Yi and Qzdamar (2007)									✓	✓			
Duhamel et al. (2008)				✓				✓	✓	✓	✓		
Amborosino et al. (2009)					✓							✓	

Table 2.9

Table 2.10

Authors / Published year	The name of the Innovative and Metaheuristic method is used
Wasner and Zapfel(2004)	Augment Lagrangian relaxation
Ambrosino and Scutella(2005)	Hybrid metaheuristic merging Variable Neighborhood Search and Tabu Search
Melechovsky et al.(2005)	Tabu search
Dethloff(2005)	Tabu search and simulated annealing
Alumur and Kara(2007)	Tabu search
Cabllero et al(2007)	Tabu search
Alberda et al(2007)	Lagrangian relaxation
Yi and Qzdamar(2007)	Memetic algorithm
Duhamel et al.(2008)	A heuristic based on multi –exchanger techniques
Ambrosino et al(2009)	Hybrid simulated annealing

Authors / Published year	Inventory	Multi product	Continuous response space	Real data	Multi objective	Time window	Multi period	Multilevel distribution network	limitation of distribution centers	Limitations vehicle	Heterogeneous fleet	Collect and deliver	Probable
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Hassan-pour et al.(2009)				✓							✓	
Peng and Chen(2009)								✓	✓			
Prodhon (2009)						✓		✓	✓			
Song et al (2009)											✓	
Schittekat and Sorensen(2009)			✓					✓	✓			
Lee et al (2010)							✓					
Bozkaya et al (2010)						✓		✓				
Yu et al (2010)								✓	✓			
Mete and Zabinsky(2010)	✓		✓								✓	
Tavakoli et al (2010)				✓				✓	✓			
Ghods and Amiri (2010)		✓										✓
Ahmadi and Azad (2010)	✓							✓	✓		✓	
Duhamel et al (2010)			✓					✓	✓			
Nguyen et al.(2010)								✓				
Aras et al.(2010)			✓					✓	✓			
Boccia et al.(2010)									✓			
Cetiner et al (2010)								✓	✓			

Table 2.11

Authors / Published year	The name of the Innovative and Metaheuristic method is used
Hassan-pour et al.(2009)	Hybrid simulated annealing
Peng and Chen(2009)	Two – phase particle swarm optimization
Prodron (2009)	Evolutionary local search
Schittkat and Sorensen(2009)	Tabu search
Bozkaya et al (2010)	Genetic algorithm
Yu et al (2010)	simulated annealing
Tavakoli et al (2010)	Multi – objective scatter search algorithm
Ghods and Amiri (2010)	Variable neighborhood search algorithm
Ahmadi and Azad (2010)	Hybridization of Tabu search and simulated annealing
Duhamel et al (2010)	Greedy randomized adaptive search procedure, calling an evolutionary local search
Nguyen et al.(2010)	Multi – start evolutionary local search
Arash et al.(2010)	Tabu search

Table 2.12

Authors / Published year	Inventory	Multi product	Continuous response space	Real data	Multi objective	Time window	Multi period	Multilevel distribution network	limitation of distribution centers	Limitations vehicle	Heterogeneous fleet	Collect and deliver	Probable
Li et al .(2010)						✓			✓				
Burks et al (2010)						✓							✓
Karagolan et al (2010)				✓					✓	✓			✓
Wang et al(2010)									✓	✓		✓	
Zhang(2011)					✓								
Guduz(2011)						✓			✓	✓			
Perugia et al.(2011)					✓	✓							
Nguyen et al(2011)									✓	✓			
Zarandi et al(2011)						✓			✓	✓			
Karaoglan et al (2011)									✓	✓			✓
Mar – Ornz et al (2011)			✓								✓		
Derbel et al (2011)									✓	✓			
Nadizadeh et al .(2011)									✓	✓			
Tang et al .(2011)	✓						✓						
Prodhon.(2011)							✓						
Nguyen et al.(2012)				✓					✓	✓			
Derbel et al .(2012)									✓				
Albareda et al.(2012)							✓						
Karaoglan et al(2012)									✓	✓			✓

Mhammadi shad and Fatahi(2013)						✓				✓			
Setak et al(2014)						✓							✓

Table 2.13

Authors / Published year	The name of the Innovative and Metaheuristic method is used
Li et al .(2010)	Genetic algorithm
Burks et al (2010)	Tabu search
Karagolan et al (2010)	genetic algorithm Hybrid
Wang et al(2010)	Genetic algorithm
Zhang(2011)	Multi – objective genetic algorithm
Guduz(2011)	Tabu search algorithm
Perugia et al.(2011)	Tabu search algorithm
Nguyen et al(2011)	Greedy randomized adaptive search procedure
Zarandi et al(2011))	Simulated annealing
Derbel et al (2011))	Variable neighborhood search
Nadizadeh et al .(2011)	Greedy clustering method
Tang et al .(2011)	Location –allocation first ,route and inventory second approach
Prodhon.(2011))	Hybrid evolutionary algorithm
Nguyen et al.(2012	Multi – start iterated local search with tabu list and path relinking
Derbel et al .(2012)	Genetic algorithm
Karaoglan et al(2012)	Simulated annealing
Mhammedi shad and Fatahi(2013)	Variable Neighborhood Search algorithm
Saadi and Mesgar(2013)	Genetic algorithm
Setak et al(2014)	Tabu search algorithm

Table 2.14

Third chapter

Introduction

In this chapter we first describe the problem solving algorithms (genetic algorithms (NSGA-II) and the firefly algorithm (MOFA). Then, to clarify the problem, some of the most critical assumptions of the proposed model are presented. And the appropriate parameters and the overall scenario of the problem are described. Finally, the mathematical model of the problem is given according to the definition.

3.1 NSGA-II algorithm

The NSGA-II algorithm is one of the most utilized and powerful algorithms available for solving multi-objective optimization problems and its efficiency in solving various problems has been proven. Deb and colleagues in 1995 introduced the NSGA optimization method for solving multi-objective optimization problems. The highlights of this optimization method are:

- The answer that no other solution, definitely better than it, has more points. The answers are ranked and sorted based on how many solutions are better than them.
- The merit (fitness) for the answers, according to their rank and failure to overcome other responses.
- The method of **fitness sharing** is used for close answers. So Standard deviation of the answers is optimally adjusted. The answers are distributed normally in the search space.

Regarding the relatively high sensitivity of the performance and quality of the NSGA algorithm to the fitness sharing parameters and other parameters, the second version of the NSGA algorithm, NSGA-II, was introduced by Deb and colleagues in 2000. Along with all the performance of the NSGA-II, It can be considered a model for the formation of many multi-objective optimization algorithms. This algorithm and unique way of dealing with multi-objective optimization problems, over and over again by different people to create more new multi-objective optimization algorithms, have been used. Undoubtedly, this algorithm is one of the most basic collection members of the evolutionary multi-objective optimization algorithm that can be called the second generation of these methods.

The main features of this algorithm are:

- Define the **Crowding Distance** as an Alternative feature For methods such as fitness sharing
- using binary tournament selection operator
- Save and archive the winning answers that were obtained in the previous stages of the algorithm (Elitism).

In the NSGA-II algorithm, among the responses of each generation, some of them are selected by binary tournament selection method. In the binary selection method, two random responses are selected from among the population. Then two answers are compared with each other; finally, each one of them is chosen to be better. The selection criteria in the NSGA-II algorithm are firstly the response rank and secondly, the crowding distance is the related to the answer. When the answer is less than the rank and has a greater **crowding distance** is more favourable. By repeating the binary selection operator on the population of each generation, a set of people of that generation is chosen to attend **Crossover** and **Mutation**. On the part of the set of individuals selected, **Crossover** action and **Mutation** Action are performed on the other part. And a population of children and mutants is created. Subsequently, this population is merged with the main population. The first members of the newly formed based on rank and sorted by ascending. Members of the population that has the same

rank arranged in terms of **crowding distance** and descending. Now, members of the population are sorted first and second regarding rank and crowding distance respectively. A number of members of the population are selected from the top of the ordered list and the remaining members of the population are thrown away. Selected members make up the next generation. And the cycle is repeated in this section to complete the termination conditions. The winning solutions obtained from solving the multi-objective optimization problem are commonly known as the Pareto Front. None of Pareto Front answer are superior to other. Depending on the circumstances, one of them can be considered as an optimal decision (Dob et al., 2000).

3.2 Firefly algorithm (MOFA)

The firefly algorithm has been modelled from natural and social behaviour is simulated fireflies. This algorithm was introduced by Yang in 2008 (Yang, 2008). Fireflies produce light that the light patterns is different from each other. They use this light to attract pair and hunt. The amount of this light is directly related to the attractiveness of the firefly. With considering the amount of light each worm as an objective function value, the behaviour of worms can be modelled as an optimization algorithm (Yang, 2008). This is a meta-heuristic algorithm and radiant and social behaviour inspired by fireflies. This algorithm uses a population-based recurrence procedure with countless factors (fireflies). These factors are allowed to evaluate space of cost function more effectively than distributed random searches. The intelligent optimization technique is based on the hypothesis that the solution to an optimization problem can be considered as a firefighting factor, which is inextricably linked to its quality. Consequently, each firefly their counterparts (regardless of gender) attracts, which will examine the search space more effectively (Yang, 2009).

For ease of simulating the life of fireflies, three basic assumptions are considered in the modelling process:

- Fireflies all are single-sex. Therefore, gender plays no role in attracting them to each other.
- The attraction between the two fireflies with their brightness is directly proportional and with the distance between them is inversely proportional.
- So, the firefly, which is brighter than other Fireflies, absorbs them to itself. And if none were brighter than other, their movement is done randomly. The radiance of fireflies is determined based on the value of the objective function of them (Golestan et al., 2013)

As mentioned, the attractiveness of fireflies (β) relative to the distance between the two fireflies (r) and the light absorption coefficient (γ) is calculated depends on the following equation:

$$\beta_{(r)} = \beta_0 e^{-\gamma r^2}$$

In this formula, β_0 is the attractiveness value of firefly brighter at $r = 0$

The position of the worm i after moving to the worm j that is brighter is calculated from the following equation:

$$x_i^n = x_i^{n-1} + \beta_0 e^{-\gamma r_{ij}^2} (x_j^{n-1} - x_i^{n-1}) + \alpha e_i^{n-1}$$

$$r_{ij} = \|x_i - x_j\|$$

x_i = Fainter firefly location

x_j = Brighter firefly location

N= Repeat number

α = Random number

e_i^{n-1} = A vector of random numbers that can have a uniform distribution or Gaussian distribution

Is a random term relationship that leads to a more comprehensive search space is determined by the algorithm. The absorption of fireflies lasts as long as they are absorbed into the brightest firefly.

The firefly creates the best value for our objective function. The following figure shows the process of this algorithm. In determining the optimal parameters of the MOFA algorithm, usually α and β are usually in the interval [0, 1] and the value of γ in the range [0, 1]. (Moghari et al.2014)

The algorithm is based on communication between all over particles. The algorithm is based on communication between all over particles.so; it is more efficient and effective in multi-objective optimization.

3.3 Explaining the problem

To make reader's understanding of the dimensions of the designed model more clearly, the following is a list of some of the most important model presuppositions:

- ✓ Single-stage hierarchy: Single-step means the existence of distribution paths, in which the collection of goods is not possible.
- ✓ Nature of demand: The nature of demand is decisive in that the demand of customers does not change during the planning horizon.
- ✓ The number of facilities: In the issue of locating facilities, there are several facilitations to be deployed.
- ✓ The fleet of transport: It is multi-vehicle and consists of more than one vehicle.
- ✓ Facility capacity: facility capacity: The facility will be considered in terms of size.
- ✓ Facility Layer: The second layer (which means it is located in the middle layers of the facility.)
- ✓ The time interval: static (the time horizon is static, and the values of the parameters do not change during this period).
- ✓ Time limit: No deadline set (the time limit for serving consumers is not considered.)
- ✓ Purpose function: multi-objective (in this case, there are two objective functions, one is to minimize the total cost of the system, and the other to maximize the service level.)

According to the classification, it is assumed that the location and demand of customers are defined and definite. Also, described as the unit of goods. Also, the candidate points and the capacity of each facility for deployment are also apparent. In addition, it is assumed that the number of vehicles in the distribution network is already known and are supposed to be different from one another for being close to reality (Vehicle capacity is different). The distinctive feature of this issue is its integrity. Because inventories are also considered in decision making, and the problem becomes a problem of locating and routing with consideration of inventories. The problem is structurally similar to the issue of Ambrosiano and Scutella, 2005. And considers the flow of inventories between distribution canterers and transmission points. With the difference that this issue has two different purposes, one is to minimize costs and another to maximize service levels. There are several ways to improve the service level, which, in the proposed model, minimizes the total customer waiting time. In addition, the model is the base model completer.

To better understand this issue, we use the layer model that La Porte wrote in 1988.

3.4 Describing the model with using a layer diagram

Many distribution systems can be expressed through a layered model. The general symbol is shown in the layer model as follows.

$$\lambda / M_1 / M_2 / \dots / M_{t-1}$$

λ is the number of T layers

$M_1 = R$ is the travel type , t is the layer index

$M_1 = T$ Type of travel between layers between t and $t + 1$

$M_1 = R$ This is a type of round trip. And as shown in the position of location and routing issue, it is a journey that is done in a reciprocal way between the consumer and the facilitator.

$M_1 = T$ Circular travel is a journey that starts with a facility and returns to the same facility after passing a few consumers.

According to the layered diagram, the distribution network has three layers and the number of paths T . First, network nodes are defined and then the types of paths will be expressed. This form of classification is very general, and it can be used to extract the various scenarios. This general form is based on the network provided by Ambrosiano and Scutella, 2005). Network distribution nodes are divided into: Central Depot (CD), Regional Depot (RD), Transition Points (TP), Customers (C), Major Customers (BCs). On the Central Depot network, goods are shipped to the Regional depot and transit points, or they may directly serve customers. But usually customers are served by regional depot and transit points. Central departments usually serve significant customers because they are in high demand. Transmission points are like regional depots, with the difference that they are not capable

of holding inventory. And in fact, transition Points take the inventory and send it to customers. Therefore, the capacity for central and regional depots is defined as the maximum inventory that can be kept, and for the transfer points, the maximum inventory that can be accepted at the entry point. The following figure shows a justifiable answer to the general problem concerning the layered diagram. According to the definitions, the paths are defined as follows.

- ✓ Circling routes from central depots to regional depots or transfer points that only include a central depot.
- ✓ A trip from central depots to significant customers that only includes a central depot.
- ✓ The combined trip, from central depots to significant customers, transfer points and regional depots that only include a central depot.
- ✓ A route from central depots, transfer points, regional depots to customers, which only include a central depot, a transit point or a regional depot.

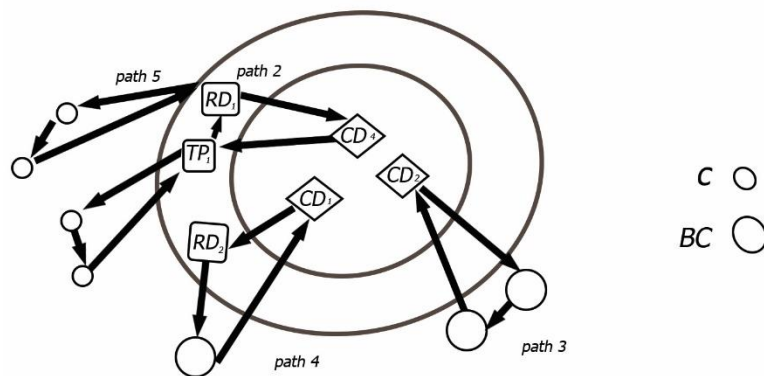


Figure 3.1

3.5 Describe the scenario examined

The problem under consideration is similar to the scenario defined (Amosino and Scott, 2005). This scenario can be described as follows:

There are a number of central depot in the first layer that provides all the requirements of the distribution network. Also, inventories are kept. Therefore, there is no difference between the regional distribution center and the transmission point. And the minimum inventory is not considered.

In this scenario, the distribution network has the following structure.

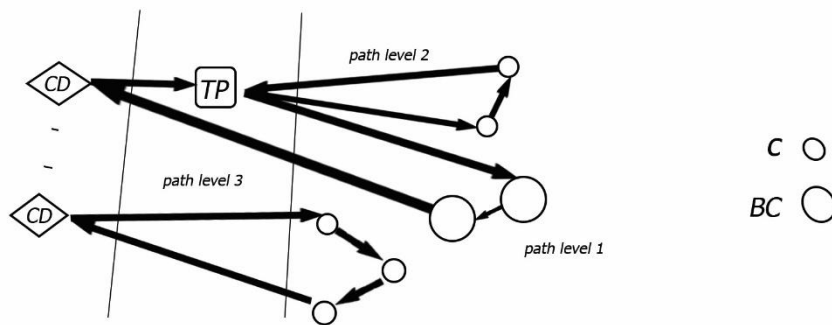
- ✓ **First and second layer:** It includes central depots for the candidate to deploy. The central depots provide all the requirements of the transition points of second-layer as well as the significant customers in the third layer; also, it can also serve customers directly.

- ✓ **Third Layer:** Includes Candidate Points to Deploy Transition Points.
- ✓ **Fourth layer:** composed of major customers.

In order to better describe the problem, look at below Figure. In this figure, a justifiable answer is given to the scenario under consideration. In general, two types of paths are defined, and they are referred to as the first and second level paths.

- First level path: Combined round trips started from a central depot and returning to the same depot after service to transition points and major customers.
- Second level patch: The service routes to customers are started as trips from the point of transit or central depot and after service to customers return to the same point of transfer or depot.

The scenario routes are also shown in the figure.



The justified answer from the desired scenario

Figure 3.2

Considering the scenario described, the design of an integrated distribution network can include the following decisions.

- Locating Decisions: Where and how many central depot and transfer points are located.
- Allocation decisions: How major customers and transition points are allotted to the depot central depot. How to allocate customers to the facility (central depot and transfer points).
- Routing decisions: These decisions include determining the first and second level routes.
 1. Determine the first level paths: Combination trips between more than two layers Start from a central depot and return to the depot after service to the points of transfer and major customers. So that major customers and customers will receive only one service vehicle and their capacity limits are preserved.
 2. Determine the second level paths: Round trips start from the central depot or transfer point and return to the same depot or point of departure after service to customers.

- Inventory decisions; the amount of goods shipped from depots and transfer points to capacity constraints as the central depot and the transition is satisfied.

Based on the specifics of the problem in this section, the issue includes the decision making of facilities, routing, and inventories. Which, like Ambrosiano and Scutella, 2005, refer to the issue of designing an integrated distribution network.

3.5.1 Scenario modelling

To model the problem, the vehicle flow model has been used that La porta classified them base on the index. (LaPorte , 1988).

Three-index models have high flexibility and can consider the type of route or class of vehicles. For this reason, we use them in problem modelling.

Symbols

Set of existing customers $C = \{1, 2, \dots, n\}$

Set of existing major customers $BC = \{1, 2, \dots, n\}$

Set of candidate points for the establishment of transfer points $TP = \{1, 2, \dots, m\}$

The set of candidate points for the establishment of central depots $CD = \{1, 2, \dots, h\}$

Set of vehicles $V = \{1, 2, \dots, v\}$

To model the distribution network, a directed graph $G = (N, A)$ is used where the nodes and arcs are as follows:

Set of nodes $N = BC \cup C \cup TP \cup CD$

Set of nodes that participate in the first level routing $N_1 = BC \cup TP \cup CD$

Set of nodes that participate in the first level routing $N_2 = C \cup TP \cup CD$

Arc set (Node connectors in the distribution network) $A = \{1, \dots, n\}$

Parameters

d_i = Customer demand $\forall i \in C \cup BC$

I_{gh} = The distance between nodes g and h per kilometre. $\forall g, h \in N$

t_{ij} = Time of trip from node i to j $\forall i, j \in N$

e_i = Maximum path length for the i level path $i = 1, 2$

k_k =Vehicle capacity k $\forall k \in V$

m_j =Facilitate capacity j $\forall j \in TP \cup CD$

v_j =The variable cost of each output unit is from facilitating j to major customers. $\forall j \in TP \cup CD$

b_k =The cost of transport per kilometre for vehicle of k $\forall k \in V$

c_k =The fixed cost of using the vehicle of k $\forall k \in V$

f_j =Facility establishment cost of j $\forall j \in TP \cup CD$

T_K =The time required to unload the k vehicle in place $\forall k \in V, \forall j \in C \cup BC \cup TP$

$PT_j =$ kth Customer operating time

t_{je} =The time between e and j customers

l_j =The latest possible time to start the service for jth customer

e_j =The earliest possible time to start the service for jth customer

Variables

If the node **h** in the **Kth** path after the node **g** is in the second level routing $\forall g, h \in N_1, \forall k \in V$

Otherwise $X_{ghk} \begin{cases} 1 \\ 0 \end{cases}$

This binary variable is used to represent each second level path.

If the node **h** in the **Kth** path after the node **g** is in the first level path. $\forall g, h \in N_1, \forall k \in V$

Otherwise $R_{ghk} \begin{cases} 1 \\ 0 \end{cases}$

This binary variable is used to represent each first level path.

If customer **i** receive service from facility j $\forall i \in C, \forall j \in TP \cup CD$

Otherwise $Y_{ij} \begin{cases} 1 \\ 0 \end{cases}$

This binary variable is used to show the relationship between customers and facilities

If facility j is established

$$\forall j \in TP \cup CD$$

Otherwise

$$Z_{ij} \begin{cases} 1 \\ 0 \end{cases}$$

This binary variable is used to display facility placement in locations.

If the vehicle k is used in round trip of the first level route

$$\forall k \in V$$

Otherwise

$$V_k^1 \begin{cases} 1 \\ 0 \end{cases}$$

If the vehicle k is used in round trip of the second level route

$$\forall k \in V$$

Otherwise

$$V_k^2 \begin{cases} 1 \\ 0 \end{cases}$$

These two variables show the state of use of vehicles on two levels.

If customer e is provided immediately after customer j by vehicle k

$$S_{jek} \in \{0,1\}, \forall j, e, h$$

Otherwise

$$S_{jek} \begin{cases} 1 \\ 0 \end{cases}$$

This variable represents the sequence of customers assigned to a device.

The number of goods is shipped from deck i to the transfer point j through route k

$$\forall j \in TP, \forall i \in CD, k \in V$$

$$F_{ijk} \begin{cases} 1 \\ 0 \end{cases}$$

This variable displays the transfer volume from Depo to customers in the paths.

W_{jk} = The start time of the service to the node j by the vehicle k $\forall j \in TP \cup BC \cup C, k \in V$

$W_{jk} = 0$ The start of service from the Central depot J by the vehicle k $\forall j \in CD, k \in V$

3.6 Problem model

Given the above, the problem model will be as follows.

$$\begin{aligned} \text{Min} \left[\sum_{j \in (TP \cup CD)} f_j \times z_j + \sum_{j \in (TP \cup CD)} \sum_{i \in (TP \cup CD)} v_j \times d_i \times Y_{ij} + \sum_{k \in K} c_k \times (v_k^1 + v_k^2) \right. \\ \left. + \sum_{k \in V} \sum_{g \in N_2} \sum_{h \in N_2} l_{gh} \times b_k \times X_{ghk} + \sum_{k \in V} \sum_{g \in N_t} \sum_{h \in N_t} l_{gh} \times b_k \times R_{ghk} \right] \end{aligned} \quad (1)$$

$$\text{Min} \sum_{k \in V} \sum_{j \in (BC \cup C)} w_{jk} \quad (2)$$

In this model, two objective functions are considered. The first objective function minimizes the total system cost. This function of purpose consists of 5 phrases.

- The first calculates the cost of establishing the facility.
- The second calculates the total cost of leaving the goods from the central depots and the transfer points.
- The third phrase calculates the total cost of using vehicles.
- The fourth and fifth phrases calculate the total cost of routing in the first and second level paths.

The second objective function minimizes waiting time of customers in the distribution network. (One of the ways that you can increase the level of service.). This objective function includes of a phrase.

S.t.

$$\sum_{k \in V} \sum_{g \in N_2} X_{ghk} = 1, \quad \forall h \in C \quad (3)$$

Restrictions (3) ensure that third-layer customers are only one path

$$\sum_{g \in N_2} X_{ghk} - \sum_{g \in N_2} X_{hgk} = 0, \quad \forall h \in N_2, \forall k \in V \quad (4)$$

Limitations (4) guarantee that if the vehicle enters into a node of member of N_2 , The same device also leaves that node.

$$\sum_{k \in V} \sum_{g \in S} \sum_{h \in S} X_{ghk} \geq 1, \quad \forall S \subset N_2, (TP \cup CD) \subseteq S \quad (5)$$

Limitations (5) assure that each customer is placed on a route link to the central depot or transfer point. They are known for connection restrictions.

$$\sum_{g \in N_2} \sum_{h \in (TP \cup CD)} X_{ghk} \leq 1, \quad \forall k \in V \quad (6)$$

Limitations (6) ensure that each vehicle is only dispatched from facilities located in the second level routing. Or, in other words, between customers in the third layer and the points of transfer in the second layer or the central depots in the first layer is at most one path.

$$\sum_{k \in V} \sum_{g \in N_1} R_{ghk} = 1, \quad \forall h \in BC \quad (7)$$

Limitations (7) ensure that the main third layer customers are only on one path.

$$\sum_{g \in N_1} R_{ghk} - \sum_{g \in N_1} R_{h g k} = 0, \quad \forall h \in N_1, \forall k \in V \quad (8)$$

Limitations (8) guarantee that if the vehicle enters into a node of member of N_1 , The same device also leaves that node.

$$\sum_{k \in V} \sum_{h \in N_1} R_{ghk} = Z_g, \quad \forall g \in TP \quad (9)$$

Limitations (9) show that if a path passes through the points of transition, that transfer point must be deployed.

$$\sum_{k \in V} \sum_{g \in S} \sum_{h \in S} R_{ghk} \geq 1 \quad \forall S \subset N_1, CD \subseteq S, BC \cap \bar{S} \neq \emptyset \quad (10)$$

Limitations (10) ensure that each major customer in the third layer is connected to a central depot in the first layer.

$$\sum_{k \in V} \sum_{g \in S} \sum_{h \in S} R_{ghk} \geq Z_j, \quad \forall j \in TP, \forall S \subset N_1, CD \subseteq S, \{j\} \cap \bar{S} \neq \emptyset \quad (11)$$

Limitations (11) ensure that the connection limit is established if a distribution center is established.

$$\sum_{g \in N_1} \sum_{h \in (CD)} R_{ghk} \leq 1 \quad \forall k \in V \quad (12)$$

Limitations (12) ensure that any vehicle can be sent only one time from facilities in the second layer. Or in other words there is a one route maximum between major clients and transfer points in the second and third layers and central depot in the first layer.

$$K_k \times \sum_{h \in N_1} R_{jhk} - F_{ljk} \geq 0, \quad \forall j \in TP, k \in V, \forall l \in CD \quad (13)$$

$$K_k \times \sum_{h \in N_j} R_{ljk} - F_{ljk} \geq 0, \quad \forall j \in TP, k \in V, \forall l \in CD \quad (14)$$

Limitations (13) and (14) ensure that the flow of goods from the central depot to the transfer point will be. If and only one path passes through them.

$$\sum_{i \in C} \sum_{j \in N_2} d_i \times X_{ijk} \leq V_k^2 \times K_k, \quad \forall k \in V \quad (15)$$

Limitations (15) ensure that vehicle capacity limits are satisfied on second-level routes.

$$\sum_{i \in BC} \sum_{h \in N_1} d_i \times R_{ihk} + \sum_{l \in CD} \sum_{j \in TP} F_{ljk} \leq V_k^1 \times K_k, \quad \forall k \in V \quad (16)$$

Limitations (16) ensure that vehicle capacity limits are considered on first-level routes.

$$V_k^1 + V_k^2 \leq 1, \quad \forall k \in V \quad (17)$$

Limitations (17) ensure that each vehicle can only be assigned a type of route.

$$\sum_{k \in V} \sum_{l \in CD} F_{ljk} - m_j \times Z_j \leq 0, \quad \forall j \in TP \quad (18)$$

Limitations (18) and (22) guarantee the capacity of the transfer points and central depots.

$$\sum_{h \in N_2} X_{ihk} + \sum_{h \in N_2} X_{hjk} - y_{ij} \leq 1, \quad \forall i \in C, \forall j \in (TP \cup CD), \forall k \in V \quad (19)$$

Limitations (19) ensure that each customer is assigned transfer point or central depot.

$$\sum_{h \in N_j} R_{ihk} + \sum_{h \in N_j} R_{hjk} - y_{ij} \leq 1, \quad \forall i \in BC, \forall j \in CD, \forall k \in V \quad (20)$$

Limitations (20) show that if the major customer and central depot are on a path, then the client is assigned to the depot. also , the limits (19) and (20) lose their effectiveness if used in the second objective function And thus, constraints (27) are added to the model, as it ensures the allocation of each customer to a depot or transfer point.

$$\sum_{i \in CD} \sum_{k \in V} F_{ijk} - \sum_{i \in C} d_i y_{ij} = 0, \quad \forall j \in TP \quad (21)$$

Limitations (21) ensure that the flows of the input goods to the points of transmission are equal to the flows of the outgoing goods from the transferring points.

$$\sum_{j \in TP} \sum_{k \in V} F_{ijk} + \sum_{i \in (C \cup BC)} d_i \times Y_{ij} \leq m_j \times Z_j \quad \forall l \in CD \quad (22)$$

Limitations (22) guarantee the capacity of the transfer points and central depots.

$$\sum_{h \in N_2} \sum_{g \in N_2} l_{gh} \times X_{ghk} \leq e_2, \quad \forall k \in V \quad (23)$$

$$\sum_{h \in N_1} \sum_{g \in N_1} l_{gh} \times R_{ghk} \leq e_1 \quad \forall k \in V \quad (24)$$

Limits (23) and (24) ensure that the lengths of the first and second level's path do not exceed the prescribed limit.

$$(W_{ij} + u_{ij} + t_{ij} - W_{jk}) \times R_{ijk} = 0, \quad \forall i \in N_1, j \in C, \forall k \in V \quad (25)$$

$$\left(\sum_{o \in V} W_{io} + u_{ik} + t_{ij} - W_{jk} \right) \times X_{ijk} = 0, \quad \forall i \in N_2, j \in C, \forall k \in V \quad (26)$$

Limitations (25) and (26) set arrival time of vehicles to customers, major customers and central depots. Because if two nodes are located immediately in a path, the waiting time for the second node is equal to the waiting time for the first node, also the service time and travel time of between them are equal. If a second objective function is used, a point without serving the client may appear on a first level path Therefore, limitations (28) make it possible to communicate between first and second level paths for transmission points.

$$\sum_{i \in (C \cup BC)} Y_{ij} = 1, \quad \forall j \in (TP \cup CD) \quad (27)$$

$$\sum_{i \in N_2} \sum_{k \in V} X_{ijk} - \sum_{i \in N_1} \sum_{k \in V} R_{ijk} \geq 0 \quad \forall i \in TP \quad (28)$$

$$e_j \leq W_{jk} \leq l_j \quad (29)$$

The limitation (29) shows that the start time of each customer's service will occur in the specified time window.

$$W_{jk} + PT_j + t_{je} - l_e \leq 2 * (1 - S_{jek}) * l_j \quad (30)$$

Limit (30) indicates the relationship between two consecutive customers served by a vehicle.

In this regard, the customer e can start after the total start time of j^{th} customer service and operation time j and the time interval of second client.

$$X_{ghk} \in \{0, 1\}, \quad \forall g, h \in N_2, \forall k \in V \quad (31)$$

$$S_{jek} \in \{0, 1\}, \quad \forall j, e, h \quad (32)$$

$$R_{ghk} \in \{0, 1\}, \quad \forall g, h \in N_1, \forall k \in V \quad (33)$$

$$Y_{ij} \in \{0, 1\}, \quad \forall g \in (C \cup BC), \forall j \in (TP \cup CD) \quad (34)$$

$$Z_j \in \{0, 1\}, \quad \forall g \in (TP \cup CD) \quad (35)$$

$$V_k^1, V_k^2 \in \{0, 1\}, \quad \forall k \in V \quad (36)$$

$$F_{ljk} \geq 0, \forall l \in CD, \forall j \in TP, \forall k \in V \quad (37)$$

$$W_{jk} \geq 0, \forall j \in N, \forall k \in V \quad (38)$$

Limitations (31) to (38) determine the types of variables. The path length constraints are considered in the proposed structure algorithm modelling. But due to the complexity of the problem, computational results are obtained without these constraints.

Regarding the second objective function, the restrictions (25) and (26) can be linearized in this way.

$$W_{ik} + u_{ik} + t_{ij} - W_{jk} \leq M \times (1 - R_{ijk}), \quad \forall i \in N_1, j \in (TP \cup BC), \forall k \in V \quad (39)$$

$$\sum_{o \in V} W_{io} + u_{ik} + t_{ij} - W_{jk} \leq M \times (1 - X_{ijk}), \quad \forall i \in N_2, j \in C, \forall k \in V \quad (40)$$

To use the objective function of the total time of service to customers, limitations (25) to (28) have been added to the general structure of the mathematical model.

Fourth chapter

As stated, the problem is NP-HARD, so in large-scale dimensions, the problem cannot be solved by solving exact methods such as branching and bounding, dynamic programming and so on. As a result of the development of the meta-heuristic method for solving this problem. In the previous chapter, we examined two NSGA-II and (MOFA) algorithms, and this chapter will explore two algorithms for problem-solving. In this chapter, the main innovation used will be that we will have two objectives function, one that seeks to minimize the cost of the payment, and the other to maximize the service level. We will deal with two NSGA-II and Firefly (MOFA) algorithms. By examining the algorithm, recognizing its advantages and disadvantages, a powerful approach will be presented that includes problem-solving, comparison of two algorithms and Taguchi test design in the parameter setting section.

4. How to display the solution of the proposed algorithm in the study problem

One of the most important decisions in the design of meta-heuristic methods how to display problem-solving and how to convert them into real solutions to the structure of the problem. The answer should be displayed in a way that will reduce costs and time by using algorithms. In this issue, each solution is a matrix consisting of several components, and n is the number of solving tasks in the problem.

The parameters of the problem variables in the matrix example

NC: Number of retail customers

nbc: Number of Customers

Nk: number of vehicles

nvar: The number of answer columns (equal to the number of customers + the number of vehicles - 1)

nrd: The number of regional depots

ntp: The number of transfer points

ncd: number of central depot

Based on the above parameters, the following example is presented to illustrate the problem

NC: 4

nbc: 2

Nk: 3

nvar: 8

nrd: 3 indicates whether the vehicle being transported goes to the regional depot and can accept equivalent values (-n to n).

ntp: 3, the transfer point can have values like a (-n to n).

ncd: 4, can set values such as 1 to n.

The matrix of the issue answer :

veh	6	3	7	2	5	4	1	8
RD	-3	0	-2	1	2	1	-2	-3
TP	-1	-2	0	-3	1	2	-3	-2
CD	1	3	1	2	1	3	2	1

Table 4.1

4.1 The matrix of the issue answer

The matrix-vector or the problem answer consists of 4 rows. The first row represents the arrangement of customers to the vehicle. And in fact, the allocation of the vehicle to customers. In the analysis of row 1 it can be written that the customer 6 and 3 will be shipped to vehicle 1 (No. 7), customer 2 and 5 to second vehicle (8) and other remaining customers with vehicle 3. in the other words, First, the first vehicle to the customer 6 then to the customer 1 and then to the depot, the second vehicle also went first to the customer 2, then to customer 5 and returns to depot for the vehicle 3, the same mode is established.

Note: According to the limitations mentioned in the model, the major customers must be carried by a vehicle (retail customers of 1 to 4 customers and major customers are 5 to 6. But as shown in the figure above. This action is not possible. So we will continue to solve this problem.

The second row represents our regional depot. As shown in the example table, the first vehicle from Depot 1 starts the work and the second vehicle from Depot 3. The presence of a negative or zero number indicates that there is no transfer point for the first vehicle. In fact, the first vehicle first moves from Depth No. 1 to the customer 6, then 3, then returns to Depth 1. The second vehicle moves from Depot 3 and then goes to the regional depot rd (1), then to the transfer point 2, then to the customer 2, then to the customer 5 and then return to the depot 3.

4.1.2 Initial initialization of the first generation

Most evolutionary algorithms use the randomization method for first-generation solutions but the final results of the problem are affected by the proper search of the algorithm. Therefore, the starting points of the search, in accordance with the problem structure and the parameters of the solving algorithm, will result in the ideal results.

4.1.3 Calculate the objective function

By defining the chromosome, all restrictions are satisfied according to the type of chromosome that is defined, except for the limitation of vehicle capacity and time window. All major customers are covered by one vehicle. Because every vehicle cannot accept both of the major customer and retail customers. (All major or all retail).

In order to correct the problem of carrying customers

6	3	7	2	5	8	1	4
---	---	---	---	---	---	---	---

First, identify and separate the first and last major customer.

6	3	7	2	5
---	---	---	---	---

Then remove the retail customer from it

3	2
---	---

After separation, the vector is as follows

6	7	5
---	---	---

First, we add remaining retail customers to the retail clients that are separated. And these customers are carried by the penultimate vehicle, then we add remaining matrix elements that include major customers and vehicles which will be changed as follows after the modifications.

3	2	1	4	8	6	7	5
---	---	---	---	---	---	---	---

This is called a penalty method, which limitation of vehicle and time window by solving the problem.

4.1.4 Selection mechanism

The NSGA-II algorithm selection mechanism in the crossover and mutation, based on the tournament.

4.1.5 Genetic operator

4.1.5.1 Reproduced

According to the concept of genetic evolution, good parents may produce better offspring.

Therefore, chromosomes with more fitting values are more favourable. As a result, the transfer of best solutions in each generation to the next has great importance. Following this function, PR percent of chromosomes with more fitness in each generation, unchanged transfer to the next generation. This is an elite strategy.

4.1.5.2 Crossover (recombination) operator

The Crossover function is used at the same time on two chromosomes (solves) in each generation and generates a new child (solve) by combining the characteristics of both parent chromosomes. The

purpose of using this function is changing the structure of the genes in the parent chromosome by combining them and creating a child with higher fitness. Since $Pr\%$ of the chromosomes in each generation are allocated to reproduce. The remainder of each generation $((1-Pr) \%$ of the next-generation chromosomes) is produced by a crossover function.

4.1.5.3 Single-point crossover (one point crossover) operator

In this function, a random spot of the parent's chromosome is selected to be split into two pieces. If the number of genes (Dimension components of each solved vector) that corresponds to the presentation method presented in the genetic method is the number of tasks in the problem (n), then there is $n-1$ candidate point for the Crossover. Consequently, the probability of selecting any one point crossover function is $1 / (n-1)$. The first point for the point crossover is between the first and second genes. The second point crossover is between second and third genes in the chromosome up to the end. It is notable that as the number of breakpoints increases, the function of the algorithm reduces, but the problem space is searched more fully (Fattahi and Aram, 2009).

One point crossover in the above example is done in 5 ways. First, randomly, the crossover is performed once in the first row of the chromosome, and the other three rows are fixed. Also, this will be done for the second, third, fourth, and fifth rows, and the entire matrix will crossover once again. For example, the following gene is selected as the first parent of the first row of the example chromosome.

6	3	7	2	5	8	1	4
---	---	---	---	---	---	---	---

The second parent of below gene

3	1	7	4	6	2	8	5
---	---	---	---	---	---	---	---

The new genes are obtained as follows.

3	1	7	2	5	8	1	4
---	---	---	---	---	---	---	---

6	3	7	4	6	2	8	5
---	---	---	---	---	---	---	---

This method can be done for four other rows on the condition that the remaining rows remain constant. But the fifth mode will be that the entire chromosome will be considered as a gene and will be cut into columns, and crossover action will be taken.

4.1.6 Mutation operator

A genetic operator is created to change the structure of genes and a small change at a point in the character code in the chromosome. By executing this operator, a small change in the sequence of

genes results in a new chromosome. The main goal is to use the mutation operator, prevent the loss of the local optimization algorithm and increase the search diversity of the problem's answer space. In other words, the mutation causes the diversity of the population to be preserved and a new genetic structure in the population with random variations of some genes.

In fact, a mutation operator is a simple form of local search. Each chromosome in one generation will not necessarily be under the mutation operator. According to this method, a P_m probability, which is usually better to be small, is defined. If a random chance is created for each chromosome and no mutation occurs, the children will be produced after the Crossover action without any change. If a mutation occurs, one or more parts of the produced chromosome will change. In the forthcoming research, swap method is used for mutation. In the swap two elements is selected from a row.

6	3	7	2	5	8	1	4
---	---	---	---	---	---	---	---

The new row is based on two selected elements and move of them done.

6	8	7	2	5	3	1	4
---	---	---	---	---	---	---	---

The second, third and fourth rows of the matrix are also available. And the fifth way is that the column will be replaced in two chromosomes, as shown below.

6	3	7	2	5	8	1	4
-3	0	-2	1	2	1	-2	-3
-1	-2	0	-3	1	2	-3	-2
1	3	1	2	1	3	2	1

Table 4.2

After the movement will be as follows

6	8	7	2	5	8	1	4
-3	1	-2	1	2	0	-2	-3
-1	2	0	-3	1	-2	-3	-2
1	3	1	2	1	3	2	1

Table 4.3

Stop condition

One of the methods used to stop repeatable algorithms is the number of generations that can be determined by the algorithm going forward for a few generations and then stopped. Another way to determine the best solutions to the generation which, if it repeated k times, the operation ends.

According to the definitions, we can compare the performance of the problem algorithm with concerning nature. The description of the comparison is briefly mentioned in the table below.

Compare the algorithm with nature

Genetic Algorithm	Nature	The desired issue
feasible solution	A member living in the environment	feasible solution
objective function	The degree of compliance with the environment	Minimizing cost and waiting times for customers
Generation	Population of organisms	Solution Matrix Set
use of genetic operators	Evolution of the population to adapt to the environment	The move towards the lowest cost
Optimization issue	Environment around	Minimizing the costs being investigated
Chromosome	Array	Arrangement of all tasks
gene	Specifications and features	Component
Allele	value of each attribute	value of each component
Locus	Gene position on the chromosome	The position of each component in the solution vector

Table 4.4

4.1.7 Creating a sample problem

In order to evaluate the performance of the algorithms presented in this study, ten issues are sampled with the change in the variables ncd, nrd, ntp, nc, nbc nk, according to the following table.

It should be noted that in order to perform sample tests, the algorithms used in this thesis are used and implemented with MATLAB software on a personal laptop with cor i5 2.27 GHz processor and 4 gigabytes of RAM.

The fixed parameters in the ten examples are as follows.

The values of the problem parameters in creating the sample of the test issues

Row	Parameter	Size of each job
1	d_i	$u\sim[10,100]$
2	$I_{gh}:$	$u\sim[1,10]$
3	t_{ij}	$u\sim[20,200]$
4	e_i	$i(1,2)$
5	k_k	$u\sim[200,300]$
6	m_j	$u\sim[1000,2000]$
7	v_j	$u\sim[200,400]$
8	b_k	$u\sim[100,200]$
9	c_k	$u\sim[1000,50000]$
10	f_j	$u\sim[20000,40000]$
11	PT_j	$u\sim[1,2]$
12	t_{je}	$u\sim[2,20]$
13	l_j	0
14	e_j	$u\sim[2000,4000]$

Table 4.5

Example with variable parameters

Name	ncd	nrd	ntp	nc	Nbc	nk
1	2	2	2	6	2	3
2	2	2	2	8	2	3
3	2	2	2	10	3	3
4	3	2	2	12	3	3
5	3	2	2	12	4	3
6	3	2	2	12	4	4
7	3	2	2	14	4	4
8	3	3	3	14	4	4
9	3	3	3	16	4	4
10	3	3	3	18	4	5

Table 4.6

4.1.8 Criteria for evaluating two algorithms

The convergence to Pareto's optimal answers, Providing density and diversity among the set of solutions obtained are two main objectives of each multi-objective evolutionary algorithm. Since these two goals conflict with each other, there isn't any criterion that can decide alone on the algorithm's performance. If it was possible to achieve such a criterion, it would be possible to comment on the superiority of an algorithm to the other algorithm. (Khishtandar, 2012)

Compare the performance of two algorithms

		NSGAI	MOFA
NO1	f1	10	10
	f2	201726255.5	170388431.3
	f3	44	44
	f4	12242881	93825114
	time	2.212322318	2.821574575
NO2	f1	10	10
	f2	749689801	700995166.6
	f3	44	44
	f4	47515852	442081804
	time	2.088540593	2.799537522
NO3	f1	10	3
	f2	1228797613	1006766984
	f3	44	42
	f4	208890833	56030668
	time	2.516378727	3.899595378
NO4	f1	10	4
	f2	1545042825	1226265657
	f3	44	44
	f4	166603825	283822650
	time	2.217340408	3.50624294
NO5	f1	10	9
	f2	1049424935	869044383.1
	f3	44	44
	f4	49470648	210360703
	time	2.166805761	3.120399716
NO6	f1	10	7
	f2	1558154468	1474196620
	f3	44	43.17624798
	f4	710519799	310290501
	time	2.44185397	4.484996232
NO7	f1	10	7
	f2	703632396.3	734945330.6
	f3	44	43.03872201

	f4	52239547	230274295
	time	2.440152726	3.472440069
NO8	f1	10	2
	f2	1140055062	960128456
	f3	44	40
	f4	134773580	257649244
	time	2.379768547	10.93160019
NO9	f1	10	7
	f2	2262875816	1926571221
	f3	44	44
	f4	83878974	718194790
	time	2.569433358	3.684956017
NO10	f1	10	1
	f2	2309260706	1879646689
	f3	44	20
	f4	178150395	0
	time	2.77009924	9.372277485

Table 4.7

In the table above, the variables f_n are as follows:

Define the comparison parameters of the two algorithms

f1	Number of Pareto Front
f2	Distance to ideal
f3	Uniformity
f4	extension
time	Time(second)

Table 4.8

4.1.8.1 Number of Pareto Front

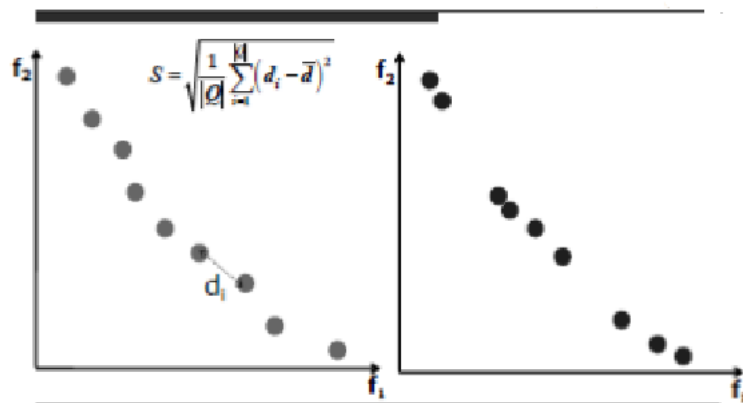
An algorithm that can provide more non-dominated solutions in the Pareto archive is more successful in plotting the real Pareto optimal level and makes the decision-maker faced with more options.

4.1.8.2 Distance from the ideal point

This criterion is used to measure the proximity to the real Pareto level. If the value of this criterion is smaller, the utility of the Pareto set will be higher. The distance to the ideal in both algorithms is almost balanced.

4.1.8.3 Uniformity (Crowding Distance)

In fact, criteria of crowding distance estimates the standard deviation of different values. When the results are uniformly placed side by side. The amount of spacing (**S**) will be small. Therefore, an algorithm that its final non-dominated solutions have Small spacing will be more favourable.



Uniformity (figure 4.1)

As shown in the comparison table, the uniformity of the firefly algorithm (MOFA) is better.

4.1.8.4 Extension

The measurement criterion measures the length of the diameter of the space cube, which is constructed by the end values of the set of non-dominated solutions in the target space. So if this criterion is more extensive, it shows the further expansion of the Pareto's archive. In the extension, the performance of both algorithms is almost balanced, in some cases the firefly (MOFA) and, in some cases, the NSGA-II performance is better.

4.1.8.5 Time to solve

As the comparison table describes the performance of the two algorithms, the time of the algorithm NSGA-II is better. In general, the NSGA-II performance is better than the firefly algorithm (MOFA) in most cases.

4.1.9 Adjust the parameters of the algorithm

The science of statistics is used to process the initial data of the problem in order to achieve optimal planning and scientific decision making and its applications in the fields of statistical quality control, designing experiments, data mining and forecasting. In all research areas, researchers use experiments to get information about a process or a particular system. The experiment is a set of tests that purposefully changes the input variables of a process so that it is possible to observe and identify

possible changes in the output response, and most of the issues in the scientific and engineering fields require the study of the system and the conduct of the experiment.

Designing experiments is a powerful statistical technique for improving the quality and reducing the cost of process design or product design. Significant methods that have already been developed by various researchers for designing experiments and analysing data in quality engineering. There are many similarities between these two approaches. One of these similarities is the applicability of the same testing designs in both approaches. A fundamental difference between the two approaches is that in the classical design of experiments, the primary purpose is recognizing the Mean changes and its adjustment, but in quality engineering, the aim is reducing dispersion at first and then setting the average (Montgomery, 2011).

In the Taguchi method, the factors are divided into two main categories: controllable and noise factors. Noises are the factors that we do not control on them. Aware of the fact that the elimination of noise is not possible or not cost-effective, The Taguchi method seeks to minimize the effect of noise and determine the best level for controllable factors. In addition to determining the best level of each factor, The Taguchi method determines the importance of each factor regarding their main effect on the response variable. The Taguchi method is a robust experiment design that converts the response variable values to a rate called the signal-to-noise (S / N). Generally, the signal word refers to the optimal value (average of the response variable), and the noise refers to the undesirable value (standard deviation). Therefore, the S / N rate refers to the amount of dispersion in the variable. The aim is to minimize S / N rates.

Taguchi's objective function into three groups.

- 1 .The smaller the type so it is better
2. The larger the type so it is better
3. The closer to the nominal value is better

Initial design of the parameters

Level 1	Level 2	Level 3	
0.8	0.9	0.95	Percentage of Crossover
0.1	0.05	0.01	Percentage of Mutation
10	20	50	Population
20	50	100	number of repetitions

Table 4.9

The initial design table of the parameters as shown in Table 4.9, there are four three-level parameters available. It also considers the Crossover action at three levels, which represents the Crossover percentage at the defined levels.

Designing of problem Scenario

scenario	Percentage of Crossover	Percentage of Mutation	Population	number of repetitions
scenario 1	1	1	1	1
scenario 2	1	2	3	2
scenario 3	1	3	2	3
scenario 4	2	1	3	3
scenario 5	2	2	2	1
scenario 6	2	3	1	2
scenario 7	3	1	2	2
scenario 8	3	2	1	3
scenario 9	3	3	3	1

Table 4.10

According to the Taguchi method, instead of $3^4 = 81$, we will have nine different scenarios, in which case the description of the parameters is as above. The following nine scenarios are created with six examples and five times of repetitions for each combination. And the objective function table (signal/noise and **RPD**) is provided.

The objective function obtained from the NSGA-II algorithm

Mean Fitness		scenario 1	scenario 2	scenario 3	scenario 4	scenario 5	scenario 6	scenario 7	scenario 8	scenario 9
	run1	0.03	0.44	0.38	0.88	0.07	0.09	0.18	0.17	0.18
	run2	0.04	0.42	0.34	0.94	0.07	0.08	0.20	0.20	0.19
example1	run3	0.04	0.45	0.37	0.84	0.07	0.08	0.18	0.17	0.19
	run4	0.04	0.43	0.33	0.81	0.07	0.08	0.18	0.16	0.17
	run5	0.03	0.50	0.36	0.89	0.07	0.09	0.19	0.18	0.16
	run1	0.03	0.46	0.35	0.88	0.07	0.09	0.18	0.17	0.17
	run2	0.04	0.43	0.40	0.86	0.08	0.08	0.16	0.19	0.16
example2	run3	0.03	0.47	0.32	0.92	0.07	0.08	0.19	0.17	0.17

	run4	0.03	0.43	0.34	0.98	0.07	0.09	0.20	0.16	0.19
	run5	0.04	0.50	0.32	0.88	0.08	0.09	0.18	0.17	0.18
	run1	0.03	0.42	0.35	0.90	0.08	0.08	0.17	0.19	0.19
	run2	0.04	0.47	0.35	0.88	0.07	0.08	0.17	0.19	0.18
example3	run3	0.03	0.47	0.40	0.81	0.07	0.09	0.16	0.19	0.19
	run4	0.04	0.50	0.38	0.84	0.08	0.09	0.19	0.19	0.17
	run5	0.04	0.41	0.34	0.98	0.07	0.09	0.17	0.16	0.18
	run1	0.03	0.45	0.36	0.83	0.07	0.09	0.19	0.16	0.18
	run2	0.03	0.45	0.35	0.96	0.07	0.08	0.19	0.16	0.18
example4	run3	0.04	0.42	0.36	0.88	0.07	0.08	0.20	0.18	0.19
	run4	0.04	0.43	0.34	0.84	0.08	0.10	0.17	0.20	0.16
	run5	0.04	0.48	0.32	0.92	0.08	0.10	0.17	0.18	0.17
	run1	0.04	0.48	0.32	0.92	0.08	0.10	0.18	0.17	0.18
	run2	0.04	0.44	0.33	0.85	0.08	0.08	0.18	0.17	0.17
example5	run3	0.04	0.49	0.39	0.93	0.08	0.09	0.19	0.18	0.18
	run4	0.04	0.47	0.36	0.95	0.08	0.10	0.18	0.20	0.19
	run5	0.04	0.48	0.37	0.87	0.07	0.08	0.18	0.17	0.18
	run1	0.04	0.47	0.38	0.83	0.07	0.09	0.17	0.19	0.18
	run2	0.03	0.45	0.39	0.94	0.08	0.09	0.18	0.16	0.19
example6	run3	0.04	0.46	0.37	0.94	0.07	0.09	0.16	0.20	0.18
	run4	0.04	0.41	0.33	0.99	0.06	0.09	0.20	0.19	0.17
	run5	0.04	0.46	0.36	0.98	0.07	0.08	0.16	0.19	0.20

Table 4.11

4.1.10 Select of the best factors

In each test run, the value of the objective function obtained according to the Taguchi method should be converted to the signal-to-noise ratio (S / N). In the Taguchi method, since the goal is maximizing, S / N is in the role of rate variable which objective function in each run convert to this rate and According to that decision will be made. And obtained from following equation

$$S/N_l = -10 \log \left(\frac{1}{n} \sum_{i=1}^n \frac{1}{y_i^2} \right)$$

The table shows the algorithm's performance

Mean S/N	-14.41	-3.44	-4.51	-0.49	-11.44	-10.60	-7.44	-7.50	-7.47
	scenario 1	scenario 2	scenario 3	scenario 4	scenario 5	scenario 6	scenario 7	scenario 8	scenario 9
	29.98	2.25	2.66	1.14	15.27	11.02	5.41	6.00	5.71
	26.47	2.38	2.92	1.06	14.44	11.91	5.01	5.12	5.13
	26.74	2.22	2.68	1.20	13.73	12.19	5.59	5.95	5.40
	26.39	2.30	3.03	1.23	13.49	12.09	5.69	6.11	5.96
	29.73	2.01	2.81	1.12	13.97	10.66	5.14	5.43	6.07
	29.23	2.17	2.88	1.14	15.35	11.52	5.64	5.94	5.73
	27.17	2.30	2.52	1.17	13.14	12.08	6.12	5.37	6.14
	30.24	2.14	3.11	1.09	14.35	12.50	5.39	5.98	5.81
	30.23	2.31	2.93	1.02	14.89	10.60	5.09	6.17	5.20
	26.15	2.00	3.11	1.13	13.10	11.72	5.52	5.85	5.67
	28.76	2.41	2.83	1.12	12.56	12.47	5.87	5.36	5.17
	27.25	2.13	2.83	1.14	13.76	12.46	5.83	5.38	5.48
	31.10	2.12	2.52	1.23	14.75	11.54	6.11	5.26	5.22
	28.35	2.01	2.63	1.19	12.61	11.76	5.16	5.24	5.75
	26.08	2.45	2.98	1.02	14.70	10.89	5.72	6.22	5.45
	29.20	2.22	2.77	1.20	14.94	11.28	5.23	6.08	5.41
	29.56	2.22	2.82	1.04	15.24	12.31	5.13	6.08	5.47
	25.48	2.40	2.78	1.13	14.58	12.37	5.09	5.55	5.25
	26.47	2.34	2.92	1.19	13.03	10.21	5.89	5.11	6.19
	25.24	2.07	3.12	1.09	13.24	10.51	5.84	5.44	5.97
	27.30	2.07	3.10	1.09	13.11	10.05	5.41	5.75	5.61

	26.89	2.29	3.05	1.18	12.73	11.90	5.61	5.85	5.85
	25.21	2.04	2.54	1.07	12.82	11.33	5.20	5.67	5.62
	27.70	2.11	2.79	1.05	12.84	10.30	5.57	5.09	5.36
	28.49	2.10	2.72	1.15	15.34	12.05	5.55	5.80	5.69
	25.21	2.11	2.66	1.20	15.02	11.03	5.92	5.20	5.70
	30.06	2.23	2.57	1.07	12.56	10.81	5.42	6.14	5.31
	26.98	2.15	2.72	1.06	13.79	11.62	6.12	5.11	5.56
	25.86	2.44	3.02	1.01	15.40	10.74	5.03	5.20	5.80
	25.22	2.19	2.81	1.02	13.38	12.16	6.11	5.21	5.03

Table 4.12

The results of the algorithm in Table 4-9 show the performance quality of the algorithm.

Table 4-10 shows how to calculate the **RPD** value. M_i is the value of the objective function which is obtained for each run of the algorithm. M_{min} is the smallest value of the objective function, which, for each sample problem, is obtained from solving all three algorithms

Table 4-10 Performance Tolerance of algorithm

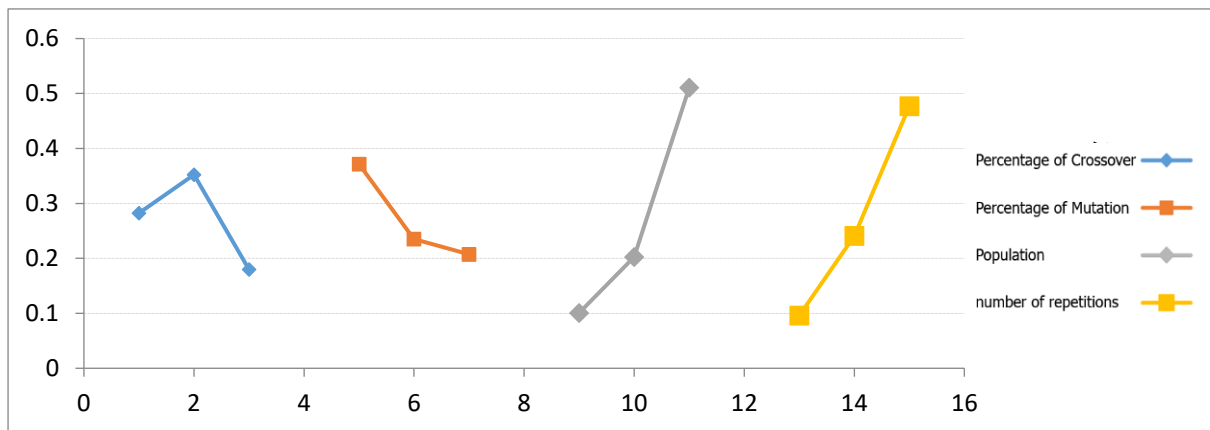
Mean RPD	0.48	0.50	0.50	0.52	0.46	0.55	0.57	0.51	0.53
	scenario 1	scenario 2	Scenario 3	Scenario 4	Scenario 5	Scenario 6	Scenario 7	Scenario 8	Scenario 9
	1.00	0.69	0.00	0.47	1.00	0.26	0.61	0.91	0.66
	0.03	1.00	0.73	0.00	0.56	0.84	0.00	0.00	0.00
	0.11	0.62	0.07	0.82	0.15	1.00	0.87	0.86	0.32
	0.00	0.81	1.00	1.00	0.00	0.94	1.00	1.00	0.90
	0.94	0.00	0.45	0.36	0.30	0.00	0.21	0.35	1.00
	0.78	0.58	0.66	0.79	1.00	0.53	0.58	0.74	0.60
	0.28	0.97	0.00	1.00	0.02	0.81	1.00	0.00	1.00
	1.00	0.48	1.00	0.49	0.60	1.00	0.33	0.79	0.68
	1.00	1.00	0.75	0.00	0.82	0.00	0.00	1.00	0.00
	0.00	0.00	1.00	0.77	0.00	0.63	0.46	0.63	0.54
	0.58	0.91	0.70	0.49	0.00	1.00	0.78	0.14	0.00
	0.27	0.32	0.71	0.60	0.59	1.00	0.74	0.16	0.56
	1.00	0.29	0.00	1.00	1.00	0.44	1.00	0.02	0.10
	0.50	0.00	0.26	0.82	0.03	0.59	0.00	0.00	1.00
	0.00	1.00	1.00	0.00	0.98	0.00	0.63	1.00	0.52
	0.93	0.49	0.00	1.00	0.88	0.54	0.20	1.00	0.19
	1.00	0.50	0.16	0.00	1.00	0.98	0.07	1.00	0.27
	0.06	1.00	0.02	0.60	0.73	1.00	0.00	0.50	0.00
	0.32	0.84	0.46	0.92	0.00	0.00	1.00	0.00	1.00
	0.00	0.00	1.00	0.32	0.11	0.17	0.95	0.38	0.80

	0.66	0.13	1.00	0.28	0.17	0.00	0.52	0.88	0.54
	0.54	1.00	0.92	1.00	0.00	0.94	1.00	1.00	1.00
	0.00	0.00	0.00	0.15	0.04	0.68	0.00	0.78	0.54
	0.78	0.33	0.49	0.00	0.05	0.15	0.91	0.00	0.00
	1.00	0.26	0.37	0.81	1.00	1.00	0.85	0.94	0.68
	0.00	0.00	0.23	1.00	0.89	0.22	0.85	0.10	0.89
	1.00	0.39	0.00	0.36	0.00	0.06	0.41	1.00	0.40
	0.41	0.16	0.37	0.31	0.48	0.65	1.00	0.00	0.72
	0.16	1.00	1.00	0.00	1.00	0.00	0.00	0.10	1.00
	0.00	0.27	0.58	0.10	0.33	1.00	0.99	0.12	0.00

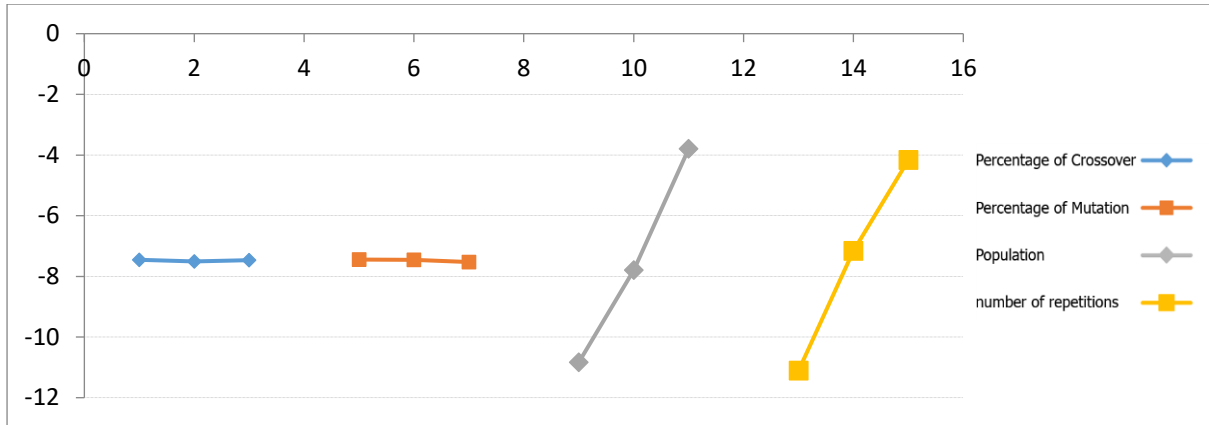
Table 4.13

4.1.11 Graphs

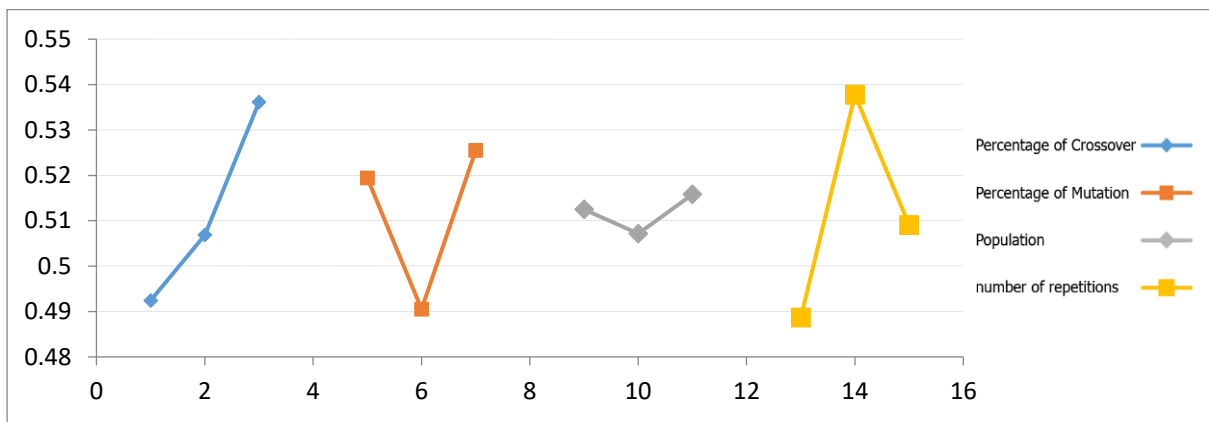
According to the graphs obtained in the Taguchi method, Parameters of percentage crossover, mutation, number of population, and number of repeats are selected for first, second, third and fourth levels respectively.



Graph Objective function (Figure 4.2)



Graph: signal and noise (Figure 4.3)



Graph tolerance algorithm performance (Figure 4.4)

4.2 Result

In this study, we considered a mathematical model for inventory flow between distribution centers and transmission points. The components of the problem were divided into three sections of the location, the general allocation problem, and the problem of vehicle routing, with two objective functions, one of them is seeking to minimize the cost of the payment, and the other to maximize the level of service. Due to the time window added to the problem model, the accuracy of the vehicle's arrival time increases to the customer. Therefore, we need NP-HARD algorithms to investigate and solve the problem which will increase the quality of service businesses.

4.3 Future suggestions

- Designing a two-objective model for the problem of locating - Routing considering scheduling periods in the design of the distribution network

- Modelling and implementing the system by utilizing the simulation process with the maximum real-world likeness
- Consider Fuzzy Time Window with the aim of Customer Satisfaction Survey

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