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Influence of the mushroomed projectile's head geometry on long-rod penetration

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ABSTRACT

Self-sharpening has been observed in experiments and simulations of long-rod penetration. Penetration performances are influenced by different nose shapes occurred in the penetration of long rods made by different materials. In this paper, to model this nose shape effect, the resistance of the target in the standard Aleksievskii-Tate model is replaced by the resistance determined by the spherical cavity expansion approximation for general nose shapes. The function of nose shape is constructed to establish a modified model of long-rod penetration, in which the nose geometry during the long rod penetration is represented by two main parameters, i.e., the diameter ratio of rod nose and shank η and the nose shape factor N^* . The influences of η and N^* on long-rod penetration and their mechanisms are analyzed. The penetration performance decreases with an increasing diameter ratio η , whose physical basis is the increased radial flow of rod material around the nose, which in turn, leads to the loss of kinetic energy due to larger cavity formed in the targets. The nose shape factor N^* reflects the sharpness of nose shapes, and a smaller value of N^* corresponds to a sharper nose. N^* obviously affects the ballistic performance of long-rod penetration, however, depending on the initial impact velocity. In general, the influence of N^* is relatively smaller than that of η .

1. Introduction

Because of profound background in military application, long-rod penetration has been studied by experiments, simulations, and theories since 1960s. Among them, theoretical analysis is conducted based on the awareness of results and phenomena in experiments and simulations. By abstraction and appropriation, physical models with simple mathematical expressions are established. These theoretical models can reflect the typical physical characteristics observed in experiments and simulations and have prediction abilities, which make them very important in understanding the penetration mechanism.

Theoretical models of long-rod penetration are developed from hydrodynamic to Aleksievskii-Tate model and then to more complex ones. Aleksievskii [1] and Tate [2,3] formulated the classic Aleksievskii-Tate model by adding rod and target strength to Bernoulli equation to modify it. It has become the most successful theoretical model because the typical physical characteristics of long-rod penetration such as deceleration and erosion are reflected by simple expressions and experimental data are reasonably explained.

Based on Aleksievskii-Tate model, several modified theoretical models were formulated subsequently. Rosenberg-Marmor-Mayseless model [4] briefly considered the effect of nose shapes by introducing the equivalent sectional area. Walker-Anderson model [5] considered the first/transient phase and the range of plastic zone, and it is consistent with the real penetration process as the resistance varies with the variation of penetration velocity. Zhang-Huang model [6] changed the static cavity expansion model in Rosenberg-Marmor-Mayseless model into the dynamic cavity expansion model to determine the target resistance. In Lan-Wen model [7,8], different response areas in the target are divided, and the velocity field is hypothesized thus the same variation law of the target resistance as simulation results can be reflected. Kong et al. [9] established a model of long rods penetrating metal and concrete targets, in which the viscosity of targets is considered by adding the first power of velocity into target resistance to more accurately describe the stress states of concrete and ceramic targets.

In long-rod penetration, the rod may deform into different nose shapes during the penetration process and thus the penetration performance will be affected. Magness and Farrand [10] found in their

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experiments that the penetration performance of depleted uranium (DU) rods was 10% higher than that of tungsten heavy alloy (WHA) rods (its density and strength are very close to DU) against armor steel targets at velocities around 1.2–1.9 km/s. The essence of the difference in their penetration performances is the different nose shapes during the penetration process because of their different material properties. The adiabatic shearing property of DU makes the nose of DU rods sharper than that of WHA rods. Rosenberg and Dekel [11] found in their experiments that the titanium alloy (Ti6Al4V) had the same adiabatic shearing property as DU, thus titanium alloy rods also kept sharp nose during the penetration process. In addition, in penetration tests conducted by Rong et al. [12] and Chen et al. [13], other rod materials such as tungsten fiber/metal glass matrix (WF/MG) composite were found to have the similar “self-sharpening” behavior. Based on simulation and theoretical analysis, Li et al. [14] suggested that the “self-sharpening” effect would lower the resistance of penetrator. Thus, the WF/MG composite rods had better penetration performance than WHA rods.

As can be observed from above experimental and simulation results that blunt (mushroom shape) noses may occur for rods made of traditional materials such as WHA (as an example in Fig. 1(a)(b)), while rods made of DU and WF/MG composite keep sharp nose during the penetration process (as an example in Fig. 1(c)(d)(e)). The influence of the nose shape of deformable long rods were discussed by Rosenberg and Dekel [15], based on their experimental and numerical data [11]. However, different nose shapes during the long-rod penetration process are failed to be considered in developed theoretical models of long-rod penetration [4–9].

In this paper, a modified model of long-rod penetration is established by constructing the nose-shape function and determining the resistances of rods with different nose shapes during the penetration process. Based on the present model, the analysis of examples is conducted to study the effect of nose shape on the penetration performance.

2. Model construction

2.1. Basic idea

According to a summary of penetration tests performed by Christman and Gehring [16], long-rod penetration process can be divided into four phases: transient, primary, secondary, and recovery. Among them, the primary phase is the most important one and is the focus of theoretical analysis. Therefore, the modified theoretical model established in the present paper aims at the primary phase.

During the primary phase, because the pressure at the target-rod interface is much higher than the material strength, local materials of the rod and target erode seriously and deform in the semi-hydrodynamic

mode. There is a thin fluid region near the interface. In contrast, the majority of rod and target is assumed to be non-deformed far away from the interface. The rod length decreases as the erosion continues during the penetration process. Above features have been described in mathematical formulas by 1D Alekseevskii-Tate model.

It should be noted that the primary phase of the long-rod penetration is a quasi-steady state, described by an approximately constant pressure level, tail velocity v and penetration velocity u . In this phase, the rod mass erodes at a quasi-steady rate because the decrease rate of rod length $v - u$ is almost constant. As the erosion takes place only at the nose tip, to maintain a quasi-steady erosion rate, the nose shape of the projectile can be simplified as stationary during the penetration.

The modified model established in the present paper assumes the nose shape unchanged during the primary phase of the long-rod penetration. A hypothetical nose shape of a long rod during the penetration is shown in Fig. 2. Two dimensionless parameters, i.e., the diameter ratio of rod nose and shank η and the nose shape factor N^* , are introduced for the geometric description of nose shape.

Referring to the geometric description of the nose shape of a non-deformable projectile introduced by Chen and Li [17], a generatrix function $y = y(x)$ represents an arbitrary nose shape of a long rod, as shown in Fig. 2. In which, the length of the rod nose (along x -direction) is h , and the maximum diameter (along y -direction) is $2R$.

As can be observed from Fig. 2, the maximum radius of the nose R is greater than that of the rod shank (tail) r , which makes the nose like a

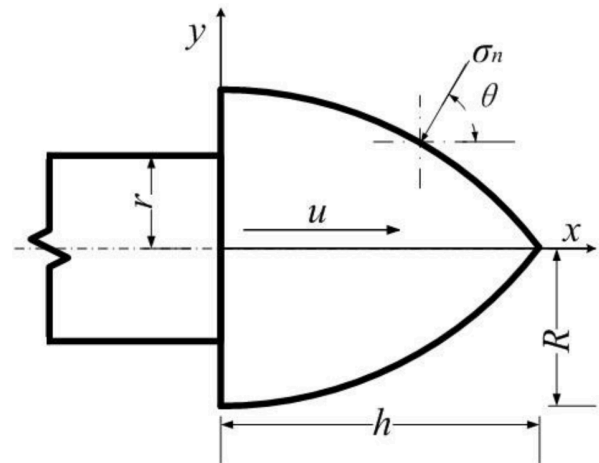


Fig. 2. A hypothetical nose shape of a long rod during the penetration process.

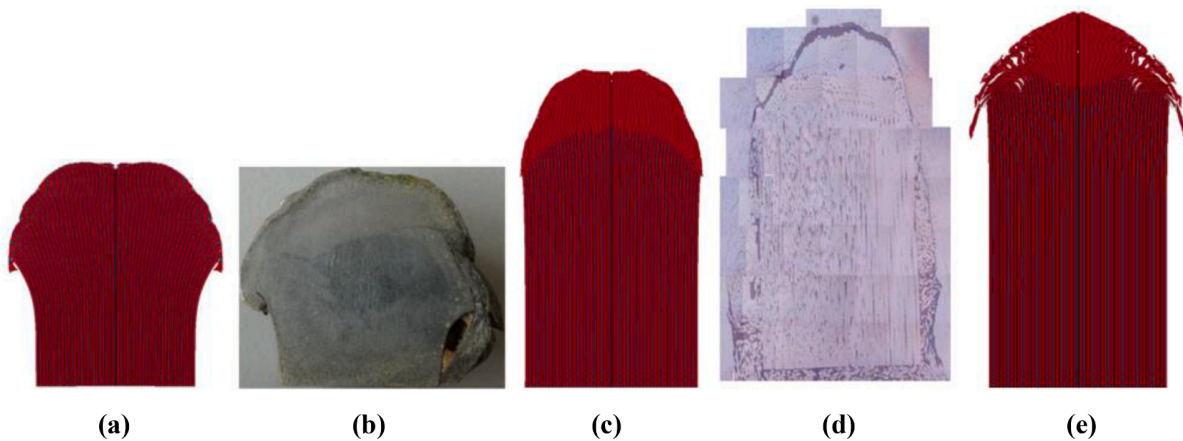


Fig. 1. The residual rods after penetration into Q235 steel targets [14]: (a) Simulated WHA rod, $V_0 = 1300$ m/s, (b) Real WHA rod, $V_0 = 1300$ m/s, (c) Simulated composite rod, $V_0 = 1313$ m/s, (d) Real composite rod, $V_0 = 1313$ m/s, (e) Simulated composite rod, $V_0 = 1766$ m/s.

mushroom. A dimensionless parameter η is defined as the diameter ratio of rod nose and shank:

$$\eta = R / r \quad (1)$$

In other models considering the nose shapes, such as Rosenberg-Marmor-Mayseless model [4] and Zhang & Huang model [6], the value of η is determined as $\eta = \sqrt{2}$. The expression of η is theoretically derived from the law of conservations of mass, momentum and energy by Wen et al. [18]. Based on this complex expression, the value of η is determined by the material parameters such as densities and strengths of rod and target, the impact velocity of rod and the resistance of target.

It should be noted that the maximum radius of the nose R defined in the present model is different from the radius of the final crater. According to the two-step cratering model established by Lee and Bless [19], during the long-rod penetration process, as the material flows at the target-rod interface, a mushroom nose shape is formed in the rod, while a crater is formed in the target. Due to the inertial motion of the crater, the radius of the final crater will be obviously greater than the maximum radius of the nose R . Experiments [20,21] and numerical simulations [22] had indicated that the crater radius is generally several times of the rod radius r . However, as the penetration depth (the crater size along the direction of penetration velocity) is our main interest, we will not consider the inertial motion of the crater. Thus, the radius of the final crater is not involved in the present model.

At any position around the nose, the normal compressive stress σ_n can be expressed as the sum of the static resistance and the dynamic resistance. The static resistance is decided by the constant effective yield strength of the target material in uniaxial stress σ_{yt} . While the dynamic resistance is proportional to the square of the normal cavity expansion velocity u_n .

$$\sigma_n = A\sigma_{yt} + B\rho_t u_n^2 \quad (2)$$

where ρ_t is the constant density of the target material, A and B are two undetermined parameters which will be discussed later. The normal cavity expansion velocity u_n is the normal component of the penetration velocity u

$$u_n = u \cos \theta \quad (3)$$

Meanwhile, the frictional resistance between the rod nose and the target is ignored, that is, the tangential stress $\tau_n = 0$.

Applying surface integral to the whole rod nose, the resistance along the penetration direction can be expressed as:

$$F_x = \pi R^2 (A\sigma_{yt} + B\rho_t u^2 N^*) \quad (4)$$

Therefore, the nose shape factor N^* introduced in the present model is a geometric term associated with the dynamic term in a spherical cavity expansion model, which can be expressed as:

$$N^* = -\frac{2}{R^2} \int_0^h \frac{yy'^3}{1+y'^2} dx \quad (5)$$

In general, the nose shape of the rod during the penetration process is described by the diameter ratio of rod nose and shank η and the nose shape factor N^* . Therefore, these two dimensionless parameters can reflect the 2D characteristics of long-rod penetration and is the core of the present model. That is the distinct difference from the previous models. Therein, based on the geometric description of the nose shape, the penetration resistance can be further determined, and the modified model will be established.

2.2. Governing equations

In non-deformable projectile penetration, according to the dynamic cavity expansion theory [23–25], the target resistance can be expressed as the sum of the static resistance and the dynamic resistance. The

former is decided by the target strength while the latter is proportional to the square of the radial cavity expansion velocity. Chen et al. [26] suggested the general form of the target resistance in non-deformable projectile penetration, in which the target resistance was composed of a target strength term and an inertia term (related to the square of velocity) if the effect of viscosity and strain rate was ignored.

In fact, in the stress balance equation of Alekseevskii-Tate model of long-rod penetration, the target resistance is also the sum of a constant term and a term proportional to the square of velocity. Therefore, in the present model, the resistance at the nose tip can be assumed to have a similar form.

Based on the assumption of the unchanged nose shape during the primary phase of the long-rod penetration, replacing resistance of the target with a cavity expansion model for the rigid nose penetrating a target (Eq. (4)), the stress balance relationship can be expressed as $\pi R^2 (A\sigma_{yt} + B\rho_t u^2 N^*) = \pi r^2 \left(\frac{1}{2} \rho_p (v - u)^2 + Y_p \right)$, which can be simplified as:

$$\eta^2 (A\sigma_{yt} + B\rho_t u^2 N^*) = \frac{1}{2} \rho_p (v - u)^2 + Y_p \quad (6)$$

where ρ_p is the constant density of the projectile and Y_p is the constant effective yield strength of the projectile material in uniaxial stress.

The left side of Eq. (6) is the resistance from the target to the nose, by integrating the stress at the whole rod nose surface, according to Eq. (4). Thus, there is a maximum radius of the nose R at the left side. The right side of Eq. (6) is the pressure from the non-deformed part to the nose. Thus, the action area is the sectional area of the rod πr^2 . Eq. (6) expresses the balance of force at both sides of the interface between the non-deformed part and the nose (mushroom part) of the rod, i.e., the cross section at $x=0$ in Fig. 2. There is no change in momentum of the mass in the projectile's nose. The discrepancy between action areas at two sides of the equation is due to the underlying assumption that the rod nose is in the plastic flow zone.

It should be noted that the force balance equation given by Eq. (6) is based on the assumption that the deformation of the projectile's nose is ignored and mushroomed head penetrates as the rigid body with constant mass. However, the initial head deforms into the mushroomed head at the first phase (transient phase), and the mushroomed head erodes at secondary penetration phase. Therefore, the assumption is only valid at the primary penetration phase, which is the limitation of the present model.

According to Newton's second law, the equation of deceleration can be expressed as

$$(M - m) \frac{dv}{dt} = -\pi r^2 Y_p \quad (7)$$

where $m = \pi r^2 l \rho_p$ is the eroded rod mass, and l is the erosion length of the rod.

The erosion rate of the rod mass is

$$\frac{dm}{dt} = \pi r^2 \rho_p (v - u) \quad (8)$$

Because of slow variations of the tail velocity v and the penetration velocity u in the primary phase of long-rod penetration, the erosion rate of the rod mass is hardly changed. Besides, the equation can be simplified to get the erosion rate of the rod length.

$$\frac{dl}{dt} = v - u \quad (9)$$

In addition, the penetration can be expressed as

$$\frac{dp}{dt} = u \quad (10)$$

Eqs. (6)–(10) are governing equations of the modified model of long-rod penetration, because the variation of tail velocity, penetration

velocity, eroded rod length (mass), and penetration depth with time can be calculated from them. The deceleration equation (Eq. (7)), the rod erosion equation (Eq. (9)) and the penetration depth equation (Eq. (10)) are based on the standard equations in Alekseevskii-Tate model for the projectile's tail. By introducing two nose shape characteristic parameters N^* and η , the force balance equation in Alekseevskii-Tate model is modified into Eq. (6) under following assumptions:

- The deformation phase of the projectile's nose is ignored and the mushroomed head is considered to be a rigid body with constant mass.
- The resistance of the target material in Tate's model is replaced by a spherical cavity expansion expression for the rigid mushroomed head.

Based on the first assumption, the motion of the projectile's nose, as determined by Eq (6), is independent of the erosion of its tail. However, the erosion of its tail, as determined by Eq (9), depends on the nose velocity u (penetration velocity) and the tail velocity v through Eqs (6) and (7).

2.3. Typical nose shapes and their nose shape factors

In this section, five hypothetical typical nose shapes of long rods during the penetration process are shown in Fig. 3. The expressions of their corresponding nose shape factors N^* are given.

A dimensionless parameter $\psi = S/2R$ introduced by Chen and Li

[17] is used for reference to describe the sharpness of the nose shape, where S is the radius of curvature of the nose shape. Specially, ψ is the CRH (caliber-radius-head) for ogive nose.

The nose shape factor of a spherical blunt nose such as Fig. 3(a) can be calculated from Eq. (1)

$$N^* = 1 - \frac{1}{8\psi^2}, \quad 1/2 \leq N^* \leq 1 \quad (11)$$

For a given maximum radius R , with the decrease of the radius of curvature S , ψ decreases, and the nose shape factor N^* decreases, and vice versa.

Specially, when the radius of curvature of the nose shape reaches the minimum, that is $S = R = h$, thus $\psi = 1/2$ and $N^* = 1/2$, corresponding to the semi-spherical nose shown in Fig. 3(b); when $S \rightarrow \infty$, then $\psi \rightarrow \infty$, thus $N^* = 1$, corresponding to the flat nose shown in Fig. 3(c).

The nose shape factor of an ogive nose such as Fig. 3(d) can be calculated from Eq. (1)

$$N^* = \frac{1}{3\psi} - \frac{1}{24\psi^2}, \quad 0 \leq N^* \leq 1/2 \quad (12)$$

Likewise, for a given maximum radius R , with the decrease of the radius of curvature S , ψ decreases and its minimum is $1/2$. After derivation to the above equation we can get that the nose shape factor N^* decreases with increasing ψ when $\psi \geq 1/2$. Thus, N^* decreases with increasing S , which is opposite to the spherical blunt nose. Specially, when $\psi = 1/2$, $N^* = 1/2$, corresponding to the semi-spherical nose.

For a conical nose such as Fig. 3(e), S should be redefined as the rod

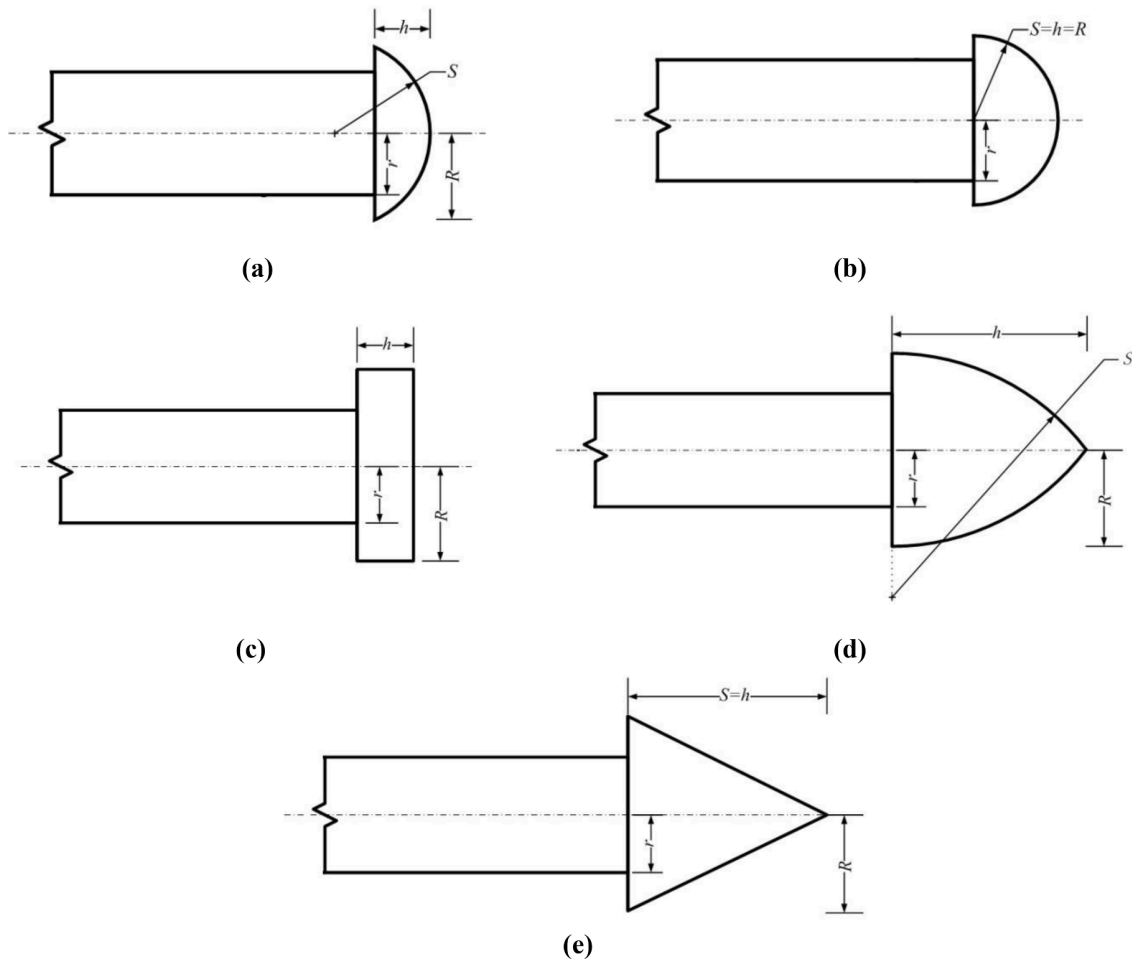


Fig. 3. Five hypothetical typical nose shapes of long rods.

length along the penetration direction, that is, $S = h$. Bring it in the dimensionless parameter $\psi = S/2R$, the nose shape factor can be calculated from Eq. (1)

$$N^* = \frac{1}{1 + 4\psi^2}, 0 \leq N^* \leq 1 \quad (13)$$

For a given maximum radius R , with the decrease of the radius of curvature S , ψ decreases and the nose shape factor N^* increases. Specially, when $S = 0$, then $\psi = 0$, and thus $N^* = 1$, corresponding to the flat nose.

Comparing five nose shapes in Fig. 3 and their corresponding nose shape factor expressions in Eqs. (11)–(13), it is found that the nose shape factor N^* reflects the sharpness of all the nose shapes. The value of N^* decreases with increasing the sharpness, that is, N^* is smaller when the nose shape is sharper, and vice versa. Specially, $N_{\min}^* = 0$ is corresponding to an extremely sharp conical or an ogive nose, and $N_{\max}^* = 1$ is corresponding to a flat nose. In summary, the nose shape factor N^* is the characteristic parameter which reflects the sharpness of nose shapes.

2.4. Parameter A and B's values

When the nose shape is not considered, we can get $N^* = 1$ and $R = r(\eta = 1)$, and then the present model will retrograde to the 1D model. In (1D) Alekseevskii-Tate model, the stress balance relationship at the target-rod interface is expressed as

$$\frac{1}{2}\rho_p(v-u)^2 + Y_p = \frac{1}{2}\rho_t u^2 + R_t \quad (14)$$

Comparing the above equation with Eq. (6) (combining with $N^* = 1$ and $\eta = 1$), we can get

$$A\sigma_{yt} + B\rho_t u^2 = R_t + \frac{1}{2}\rho_t u^2 \quad (15)$$

For general nose shapes, $N^* < 1$, thus Eq. (15) can be correspondingly expressed as the expression of the target resistance R_t .

$$R_t = A\sigma_{yt} + \rho_t u^2 \left(BN^* - \frac{1}{2} \right) \quad (16)$$

The above equation illustrates that the target resistance decreases with increasing sharpness of the rod nose (the decrease of N^*), which fits the physical fact. In addition, R_t decreases with increasing the penetration velocity, which is corresponding to the phenomenon observed in long-rod penetration tests. Therefore, the values of A and B in the present model are suggested as:

$$A = \frac{R_t}{\sigma_{yt}} = \frac{2}{3} \left(1 + \ln \frac{2G}{\sigma_{yt}} \right), B = \frac{1}{2} \quad (17)$$

Up to now, based on the assumption of the unchanged nose shape during the primary phase of the long-rod penetration, the target resistance is determined by the geometric description of the nose shape, thus the modified model of long-rod penetration is established. The governing equations of this model are given by Eqs. (6)–(10). Two dimensionless parameters N^* and η which reflect the 2D characteristics of long-rod penetration are defined by Eq. (1) and Eq. (2), respectively. The values of parameters A and B are suggested by Eq. (17). Therefore, the model can be solved.

3. Model solving

Because of the similarity between the present model and Alekseevskii-Tate model (seen in Section 2.2), the track of solving Alekseevskii-Tate model applied by Jiao and Chen [27] will be followed to solve the governing equations of the new model and get the variations of tail velocity, penetration velocity, rod length (mass) and penetration depth with time.

The relationship between penetration velocity u and tail velocity v

can be derived from Eq. (6)

$$u = \frac{v - \sqrt{2BN^*\mu^2\eta^2v^2 + 2(1 - 2BN^*\mu^2\eta^2)\left(\frac{Y_p - A\sigma_{yt}\eta^2}{\rho_p}\right)}}{1 - 2BN^*\mu^2\eta^2} \quad (18)$$

where $\mu = \sqrt{\rho_t/\rho_p}$ is the density ratio of rods and targets. Eq. (18) can be simplified as

$$u = \frac{v - \sqrt{\lambda^2v^2 + (1 - \lambda^2)V_c^2}}{1 - \lambda^2} \quad (19)$$

where

$$\lambda = \mu\eta\sqrt{2BN^*}, V_c = \sqrt{\frac{2|Y_p - A\sigma_{yt}\eta^2|}{\rho_p}} \quad (20)$$

where λ is the modified density ratio, and V_c is the critical tail velocity, i.e., the minimum initial impact velocity for penetrating the target.

Combining Eqs. (7)–(9) and substituting variables, it can be derived that

$$(M - \pi r^2 l \rho_p)(v - u) \frac{dv}{dl} = -\pi r^2 Y_p \quad (21)$$

Integrating both sides of Eq. (21) we can get

$$\int_{v_0}^v (v - u) dv = \frac{Y_p}{\rho_p} \ln \left(1 - \frac{\pi r^2 l \rho_p}{M} \right) \quad (22)$$

Introducing the relative tail velocity $v_* = v/V_0$ and the relative critical tail velocity $V_{c*} = V_c/V_0$, and substituting Eq. (19) into Eq. (22) and integrating, the relationship between the eroded length of rod l and the relative critical tail velocity v_* can be obtained as

$$l = \frac{M}{\pi r^2 \rho_p} \left\{ 1 - \exp \left[\frac{\Phi_{jp}}{2\bar{\lambda}} \left(-\lambda(v_*^2 - 1) + v_* \sqrt{v_*^2 + \bar{V}_{c*}^2} - \sqrt{1 + \bar{V}_{c*}^2} + \bar{V}_{c*}^2 \ln \frac{v_* + \sqrt{v_*^2 + \bar{V}_{c*}^2}}{1 + \sqrt{1 + \bar{V}_{c*}^2}} \right) \right] \right\} \quad (23)$$

where $\Phi_{jp} = \frac{\rho_p V_0^2}{Y_p}$ is the Johnson damage number. $\bar{\lambda}$ and $\bar{V}_{c*}$ are two dimensionless parameters:

$$\bar{\lambda} = (1 - \lambda^2) / \lambda, \bar{V}_{c*} = V_{c*} \sqrt{\bar{\lambda}} / \lambda \quad (24)$$

The relationship between the residual rod mass m and v_* can be obtained from Eq. (23)

$$m = M \exp \left[\frac{\Phi_{jp}}{2\bar{\lambda}} \left(-\lambda(v_*^2 - 1) + v_* \sqrt{v_*^2 + \bar{V}_{c*}^2} - \sqrt{1 + \bar{V}_{c*}^2} + \bar{V}_{c*}^2 \ln \frac{v_* + \sqrt{v_*^2 + \bar{V}_{c*}^2}}{1 + \sqrt{1 + \bar{V}_{c*}^2}} \right) \right] \quad (25)$$

Substituting the above equation into the deceleration equation (Eq. (7)), it can be obtained

$$dt = -\frac{MV_0}{\pi r^2 \rho_p} \exp \left[\frac{\Phi_{Jp}}{2\lambda} \left(-\lambda(v_*^2 - 1) + v_* \sqrt{v_*^2 + \bar{V}_{c*}^2} - \sqrt{1 + \bar{V}_{c*}^2} + \bar{V}_{c*} \ln \frac{v_* + \sqrt{v_*^2 + \bar{V}_{c*}^2}}{1 + \sqrt{1 + \bar{V}_{c*}^2}} \right) \right] dv_* \quad (26)$$

Integrating both sides of Eq. (26), the variation of v_* with time t is obtained

$$t = \frac{MV_0}{\pi r^2 Y_p} \left(1 + \sqrt{1 + \bar{V}_{c*}^2} \right)^{-\frac{\Phi_{Jp} \bar{V}_{c*}^2}{2\lambda}} \exp \left[\frac{\Phi_{Jp}}{2\lambda} \left(\lambda - \sqrt{1 + \bar{V}_{c*}^2} \right) \right] \cdot \int_{v_*}^1 \left(v_* + \sqrt{v_*^2 + \bar{V}_{c*}^2} \right)^{-\frac{\Phi_{Jp} \bar{V}_{c*}^2}{2\lambda}} \exp \left[\frac{\Phi_{Jp}}{2\lambda} \left(-\lambda v_*^2 + v_* \sqrt{v_*^2 + \bar{V}_{c*}^2} \right) \right] dv_* \quad (27)$$

There is no algebraic expression of Eq. (27), thus the variation of the relative tail velocity v_* with time cannot be algebraically solved. Alternatively, using software such as MATLAB, Eq. (27) can be numerically solved and v_* (v) can be calculated correspondingly to a certain value of t .

Combining the above numerical solution with Eq. (19), the variation of penetration velocity u with time t is further obtained. Then the variation of penetration depth can be obtained by applying Eq. (10).

Combining Eq. (27) with Eq. (23), or integrating Eq. (9) and substituting u and v versus t , the variation of eroded length of a rod l is obtained. Similarly, the variation of residual rod mass can be obtained by combining Eq. (27) with Eq. (25).

In summary, the above solving process is similar with that in solving Alekseevskii-Tate model [27]. However, because the diameter ratio η and the nose shape factor N^* are introduced in the new model, the density ratio μ should be modified as λ , and the critical tail velocity V_c is changed correspondingly. N^* only affects λ while N^* is included in both λ and V_c .

4. The influence of nose shape during the long-rod penetration process

In the present model, the nose shape of the rod is described by the diameter ratio η and the nose shape factor N^* . Thus, these two parameters reflect the 2D characteristics of long-rod penetration. Therefore, the effect of nose shape on the long-rod penetration is analyzed by discussing these two characteristic parameters.

The discussion in this section is based on the case shown in Anderson and Walker's [28] work. In this case, tungsten alloy long rods ($L/D=10$) penetrate 4340 steel targets, and corresponding parameters are listed in Table 1. The nose shape of rod during the penetration process is not considered in the original work. That is, the nose is assumed to be the same as other part of the rod, thus $\eta=1$ and $N^*=1$. To discuss the influence of nose shape, values of η and N^* need to be changed to design different cases. The values of parameter A and B have been suggested by Eq. (17), while other parameters are listed in Table 1.

It is worth to note that, when $\eta=1$ and $N^*=1$ in the present model, which means the rod nose is not considered, Eqs. (6)-(10) will reduce to the governing equations of (1D) Alekseevskii-Tate model. In that case, the calculation result of the present model for Anderson and Walker's [28] case is the same as the calculation result of Alekseevskii-Tate model

for the same case [27].

4.1. The influence of the diameter ratio η

Combining with Anderson and Walker's [28] case, the influence of the diameter ratio η will be discussed. For control variables, let the nose shape factor $N^*=1$ (flat nose), η differs from 1 to $\sqrt{2}$, $\sqrt{3}$ and 2. Firstly, applying the solution in Section 3 and using software such as MATLAB, for rods with different diameter ratios, the variations of tail velocity, penetration velocity, residual rod length (mass), and penetration depth with time are obtained. Secondly, the influence of η on residual rod length and penetration depth will be analyzed under different initial impact velocities.

As can be observed from the velocity-time curves in Fig. 4, with the increase of η , the penetration velocity and the penetration time dramatically decrease. Correspondingly, the penetration depth decreases with increasing η , as shown in Fig. 5. In fact, if checking the new model, the increase of η leads to the decrease of initial penetration velocity (obtained by substituting $v=V_0$ into Eq. (19)), thus the penetration performance is weakened. The physical essence is that the increase of the radial flowing of rod material will lead to the loss of kinetic energy due to larger cavity formed in the targets and the decrease of the penetration performance. Therefore, the diameter ratio η represents the extent of the radial flowing of rod material.

In particular, curves of tail velocity in Fig. 4 are in excellent agreement with each other nearby the initial time. Variations of tail velocities of rods with different diameter ratios are almost the same, and thus the decelerations of tail velocities are extremely similar. According to the analysis of deceleration of long-rod penetration based on (1D) Alekseevskii-Tate model by Jiao and Chen [29], the initial deceleration ($dv/dt|_{t=0}$) and the deceleration index (α) are both related to strength and density of the rod. Meanwhile, $dv/dt|_{t=0}$ is independent with initial impact velocity and target material, while α depends on them and rod length. However, for cases with same rod length, initial impact velocity and materials of rod and target, their initial decelerations and the deceleration indexes calculated from the new model are different after considering different diameter ratios. Therefore, it can be concluded that the 2D effect given by different diameter ratios will distinctly affect the deceleration index of long-rod penetration.

Fig. 6 illustrates the variation of residual length of rods with different diameter ratios during the penetration process. It should be emphasized that, because the present model assumes the nose shape unchanged during the penetration process, the erosion takes place only at the nose tip and the cross-sectional area of rods keeps constant. Therefore, the variation of eroded rod mass is similar to the variation of eroded rod

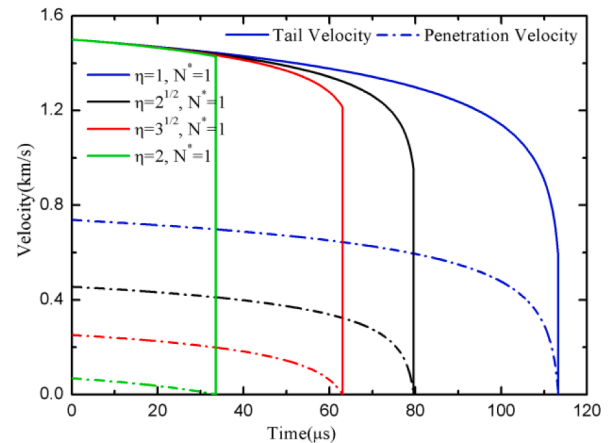


Fig. 4. The velocity-time curves of long-rod penetration with different diameter ratios.

Table 1
Corresponding parameters in Anderson and Walker's [28] case.

V_0 (km/s)	L (mm)	ρ_p (g/cm ³)	ρ_t (g/cm ³)	Y_p (GPa)	R_t (GPa)	σ_{yt} (GPa)
1.5	81.7	17.4	7.8	2.0	4.94	0.735

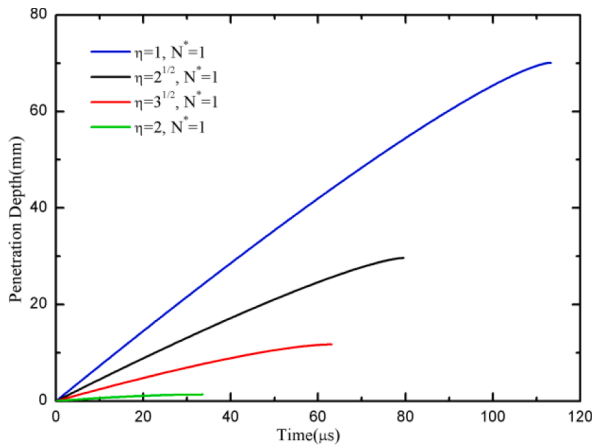


Fig. 5. The penetration depth-time curves of long-rod penetration with different diameter ratios.

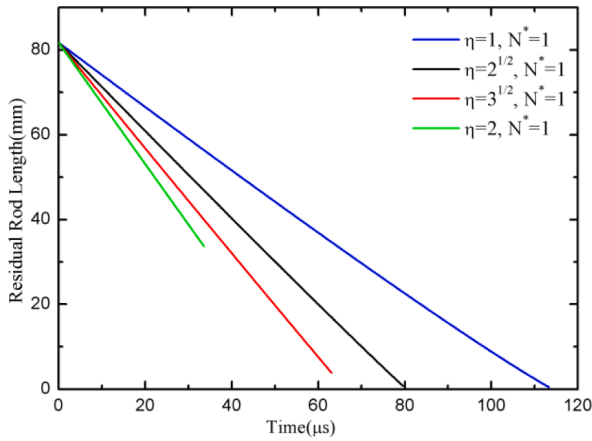


Fig. 6. The residual rod length-time curves of long-rod penetration with different diameter ratios.

length. In the present paper, the variation of rod erosion is studied by analyzing the eroded rod length.

As shown in Fig. 6, the erosion rate of long rods increases with the increase of diameter ratios, which is mainly caused by the decrease of initial penetration velocity. It is notable that the final residual rod length L_r increases significantly with the increase of η . Particularly, for a large diameter ratio rod $\eta = 2$, the final residual rod length L_r is 41.25% of the initial rod length L , which is significantly different from almost totally erosion in typical long-rod penetration.

In fact, in our former analysis based on (1D) Alekseevskii-Tate model [27], the final residual rod length is found to increase with the increase of μ at low speed. The final residual rod length may be relatively long when the target density is larger than the rod density (that is, $\mu > 1$ and $\bar{\mu} > 1$ and $\bar{\lambda} < 0$, which will also cause the same phenomenon. However, with the increase of initial impact velocity, the final residual length of the rod with a large diameter ratio will dramatically decrease.

Fig. 7 illustrates the variation of dimensionless final residual rod length (ratio of the final residual rod length L_r and the initial rod length L) with the initial impact velocity. With increase the initial impact velocity, L_r/L drops rapidly and then approaches to 0. Therefore, within the typical experimental velocity range (from 1.5 km/s to 2.0 km/s), there is a significant difference in L_r/L for rods with different diameter ratios. For example, at an initial impact velocity of 1.5 km/s, rods of $\eta = 1$ and $\eta = \sqrt{2}$ completely erode ($L_r/L = 0$), while L_r/L of rods of $\eta =$

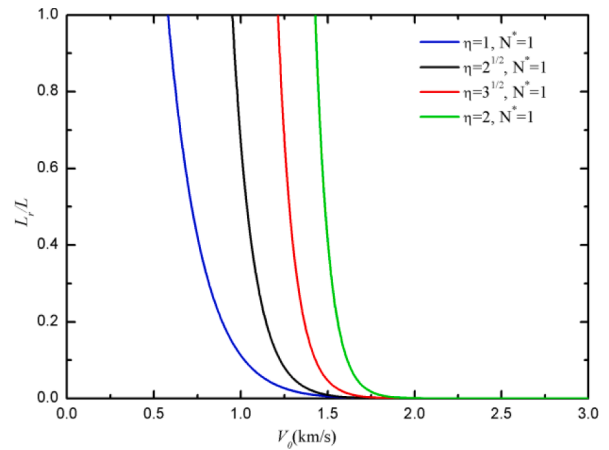


Fig. 7. The dimensionless final residual rod length-initial impact velocity curves of long-rod penetration with different diameter ratios.

$\sqrt{3}$ and $\eta = 2$ are 0.047 and 0.414, respectively. When the initial impact velocity is higher than 2 km/s, the final residual rod length L_r of rods with different diameter ratios approaches to 0, i.e., rods are almost completely eroded.

The relationship between the dimensionless penetration depth in the primary phase and the initial impact velocity is depicted by Fig. 8. As can be observed from the figure, the critical tail velocity $V_c =$

$\sqrt{2|Y_p - A\sigma_{yt}\eta^2|/\rho_p}$ increases with the increase of the diameter ratio η , which is corresponding to the rightward moving of start points of curves. The physical essence of this characteristic is that the lowest kinetic energy for penetration must increase with the increase of the radial flowing of rod material.

All penetration depth curves in Fig. 8 slope gently at high speed, and the limit of penetration depth decreases significantly with the increase of the diameter ratio η . When $\eta = 1$, the curve approaches the hydrodynamic limit $1/\mu$; when $\eta > 1$, curves are far lower than the hydrodynamic limit.

In summary, long-rod penetration is deeply influenced by the diameter ratio η during the penetration process. On the one hand, the radial flowing of rod material will lead to the loss of kinetic energy, which is the reason for the decreasing penetration performance. On the other hand, the lowest kinetic energy for penetration increases with the increase of the radial flowing of rod material. Therefore, at the same initial impact velocity, the initial penetration velocity and the

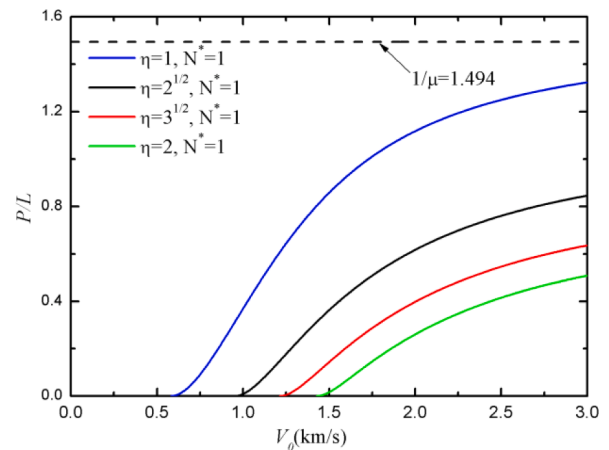


Fig. 8. The dimensionless penetration depth-initial impact velocity curves of long-rod penetration with different diameter ratios.

penetration time decrease with the increase of the diameter ratio, which leads to the significantly decrease of penetration depth.

4.2. The influence of the nose shape factor N^*

In the present model, the nose shape factor N^* is another kernel which reflects the 2D characteristic of long-rod penetration. According to the analysis in Section 2.2, the sharpness of nose shapes is reflected by N^* . The value of N^* is smaller when the nose shape is sharper, and vice versa. The influence of the nose shape factor will be discussed, combining with Anderson and Walker's [28] case.

For control variables, the value of the diameter ratio η should be determined. In fact, $\eta = \sqrt{2}$ is hypothesized in Rosenberg-Marmor-Mayseless model [4] and Zhang-Huang model [6], in which the 2D effect of long-rod penetration is briefly considered. Referring to above models, the value of the diameter ratio is empirically taken as $\eta = \sqrt{2}$ in this section.

To analyze the influence of the nose shape factor, five different nose shapes are hypothesized. They are flat nose, spherical blunt nose, semi-spherical nose, ogive nose and conical nose, respectively. Corresponding geometric parameters of these noses are listed in Table 2.

Applying the solution in Section 3, variations of tail velocity, penetration velocity, residual rod length (mass) and penetration depth with time are obtained. As can be observed from the velocity-time curves in Fig. 9, with the decrease of N^* , the penetration velocity and the penetration time slightly increase. Correspondingly, the penetration depth increases with the decrease of N^* , as shown in Fig. 10. It is worth noting that, the penetration depth increases from 29.62mm to 30.26mm, 32.51mm, 34.07mm and 34.41mm as N^* decreases from 1 to 7/8, 1/2, 7/24 and 1/4, respectively for $\eta = \sqrt{2}$ in Fig. 10; while in Fig. 5, for $N^* = 1$, the penetration depth decreases from 70.08mm to 29.62mm, 11.72mm and 1.33mm as η varies from 1 to $\sqrt{2}$, $\sqrt{3}$ and 2, respectively. Therefore, the influence of the nose shape factor on penetration velocity, penetration time and penetration depth is evidently smaller than that of the diameter ratio, which is the possible reason for only introducing η to reflect the effect of nose shape in the previous models.

Furthermore, curves of tail velocity in Fig. 9 are in excellent agreement with each other around the origin region, which illustrates that variations of the tail velocity of rods with different nose shape factors are almost the same, i.e., initial decelerations of tail velocities are the same. However, because of the slight difference in the penetration time, deviations among these curves occur only at the terminal region of the penetration process, corresponding to the different deceleration indexes. The penetration time and the deceleration index increase with the increase of the sharpness of the rod nose (i.e., the decrease of N^*).

The variation of residual length of rods with different nose shape factors during the penetration process is shown in Fig. 11. All these curves are in excellent agreement with each other, which illustrates the influence of nose shape factors on the erosion rate is relatively small. In addition, as can be observed from the figure, the erosion rate of the rod length slightly decreases with increasing sharpness of the rod nose (the decrease of N^*).

At the end of this section, the effect of nose shape factor on penetration performance at different initial impact velocities is further

Table 2

Geometric parameters of hypothetical nose shapes.

Nose shapes	η	S/R	h/R	Ψ	N^*
Flat nose	$\sqrt{2}$	—	—	—	1
Spherical blunt nose	$\sqrt{2}$	2	$\sqrt{3}/3$	1	7/8
Semi-spherical nose	$\sqrt{2}$	1	1	1/2	1/2
Ogive nose	$\sqrt{2}$	2	$\sqrt{3}$	1	7/24
Conical nose	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{3}$	$\sqrt{3}/2$	1/4

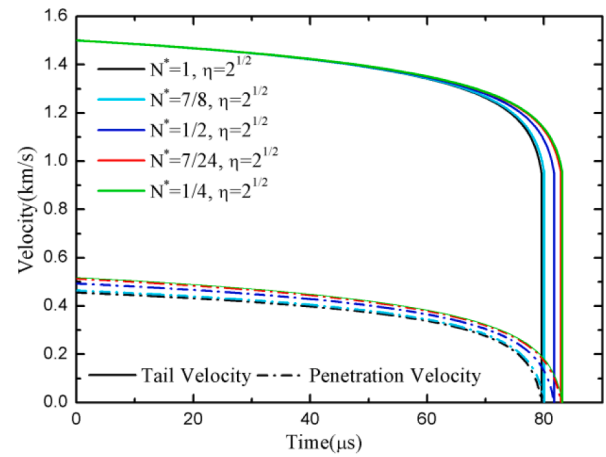


Fig. 9. The velocity-time curves of long-rod penetration with different nose shape factors.

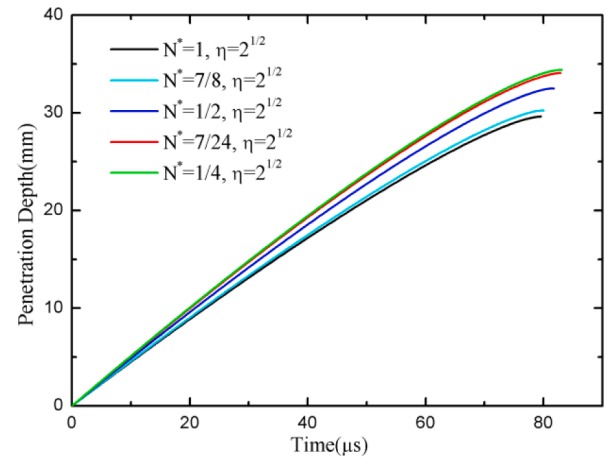


Fig. 10. The penetration depth-time curves of long-rod penetration with different nose shape factors.

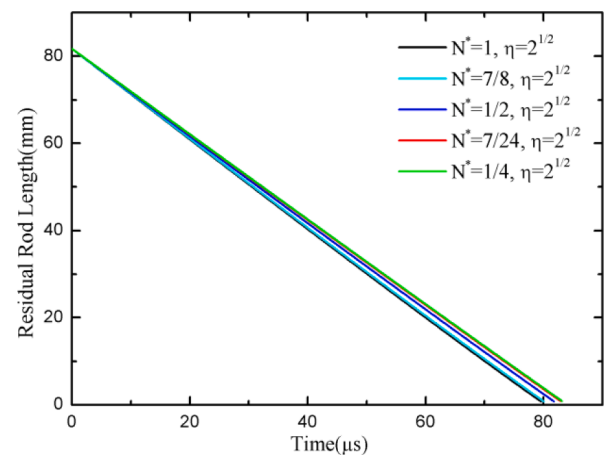


Fig. 11. The residual rod length-time curves of long-rod penetration with different nose shape factors.

analyzed. As can be observed from dimensionless penetration depth curves shown in Fig. 12, penetration performances of rods with different diameter ratios are almost the same at low velocity (lower than 1.5 km/

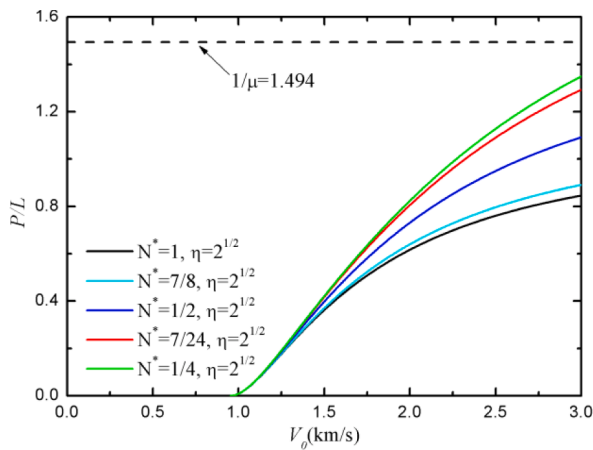


Fig. 12. The dimensionless penetration depth-initial impact velocity curves of long-rod penetration with different diameter ratios.

s). While penetration performances increase with the increase of the sharpness of the rod nose at high speed (higher than 1.5 km/s). With the decrease of diameter ratio N^* , the typical S-shaped dimensionless penetration depth curve elongates toward the upper right, i.e., the limit of the penetration performance will occur in higher velocities. Specially, at a velocity of 3.0 km/s, the dimensionless penetration depth curve of flat nose rod ($N^* = 1$) tend to be slow, i.e., the penetration performance is close to its limit. However, the curves of ogive nose rod ($N^* = 7/24$) and conical nose rod ($N^* = 1/4$) are quite steep, i.e. their penetration performance will increase evidently at higher velocity.

To sum up, the nose shape factor N^* reflects the sharpness of the nose shape of long rods during the penetration process in the present model. Referring to Anderson and Walker's case [28], the penetration velocity and the penetration time increase slightly and the penetration depth correspondingly increases with the decrease of N^* . However, the influence of N^* on the erosion rate and the final residual rod length is negligibly small. In addition, the influence of the nose shape factor significantly depends on the initial impact velocity: the penetration performances of rods with different nose shape factors are almost the same at lower impact velocity, while it increases with the increase of the sharpness of the rod nose at high speed.

In the present model, the 2D characteristic of long-rod penetration is reflect by two characteristic parameters, which describe the effect of the nose shape on the long-rod penetration. Through the case analysis in the present section, the extent of their influence is significantly different—the influence of the diameter ratio η is greater than that of the nose shape factor N^* .

The value of η is empirically determined as $\eta = \sqrt{2}$ in previous models considering the nose shapes, such as Rosenberg-Marmor-Mayseless model [4] and Zhang & Huang model [6]. However, the value of η is determined by the material parameters such as densities and strengths of rod and target, the impact velocity of rod and the resistance of target, based on the expression of η theoretically derived from the law of conservations of mass, momentum and energy by Wen et al. [18].

5. The 2D effect of long-rod penetration

In this section, the 2D effect of long-rod penetration will be analyzed by the modified model established in the present paper, combined with experimental and simulation results.

As mentioned in the introduction, WF/MG composite has the similar “self-sharpening” behavior as DU. Therefore, during the penetration process, WF/MG composite rods keep sharp nose while WHA rods have blunt (mushroom) nose. This characteristic is studied in experiments conducted by Rong et al. [12] and Chen et al. [13]. Through

corresponding simulation, Li et al. [14] analyzed the mechanism of the “self-sharpening” effect and its influence factors.

The experiment of WHA rods and WF/MG composite rods penetrating Q235 steel targets [13] and corresponding numerical simulation [14] will be used to analyze the 2D effect of long-rod penetration. In experiments and simulations, the reinforced tungsten fibers in WF/MG composite rods are 300 μm in diameter and about 80% in volume fraction. Thus, the WF/MG composite rod has a similar density as that of WHA rod. The thickness of target is 250 mm, the length and diameter of the rods are 206 mm and 19 mm, respectively. Other parameters and results of experiments and simulations are listed in Table 3.

Two experimental results and one simulation result are chosen from the nose shapes of residual rods shown in Fig. 1. Curve fitting is applied to these rod noses, as shown in Fig. 13. Corresponding geometry parameters are listed in Table 4. It should be noted that nose shapes during the penetration process are assumed to be similar to those of residual rods. Therefore, nose shapes during the penetration process are approximately presented by nose shapes of residual rods (a) and (b) from experiments at 1.3 km/s. However, the nose shape of the residual rod (c) from the simulation at 1.766 km/s is used because the WF/MG composite rod penetrated through the target at this speed and the residual rod had not been measured.

As can be observed from Fig 13, there is a semi-spherical nose for WHA rod (a) and ogive noses for WF/MG composite rods (b)(c). At an impact velocity of 1.3 km/s, the difference of nose shapes between (a) and (b) is mainly caused by rod materials. Comparing nose shape (b) with (c), it is found that the diameter ratio η of WF/MG composite rods is greater for higher impact velocity. Combined with geometry parameters of nose shapes and material parameters of rods and targets ($A=6.851$, $B=0.5$), the present model can be applied for the theoretical analysis. Fig 14–16 illustrate the penetration performances of rods with different (fitting) nose shapes from experiments and simulations. It should be noted that the present model is applicable for semi-infinite targets, thus the effect of moderately thickness targets such as the bulge on the back of targets is not considered. However, penetration depths approach or exceed the thickness of target in experiments and simulations. Particularly, the bulge on the back of targets and penetration through targets are observed in experiments and simulations of WF/MG composite rods penetrating Q235 steel targets. Therefore, the calculation stops when the penetration depth reaches 250mm for WF/MG composite rods with nose shapes (b) and (c).

Bringing the fitting nose shapes into the present model, the model predictions can briefly reflect the variation of the tail velocity, as shown in Fig 14. Therefore, the above method is feasible for analyzing the 2D effect of long-rod penetration. The difference between the model predictions and the simulation results mostly exists at the late stage of the penetration process, which illustrates that the typical effect of moderately thick targets has not been considered in the new theoretical model for semi-infinite targets. Meanwhile, comparing the results of WF/MG and WHA rods penetrating Q235 steel targets, it can be seen that the above difference decreases with the increasing initial impact velocity.

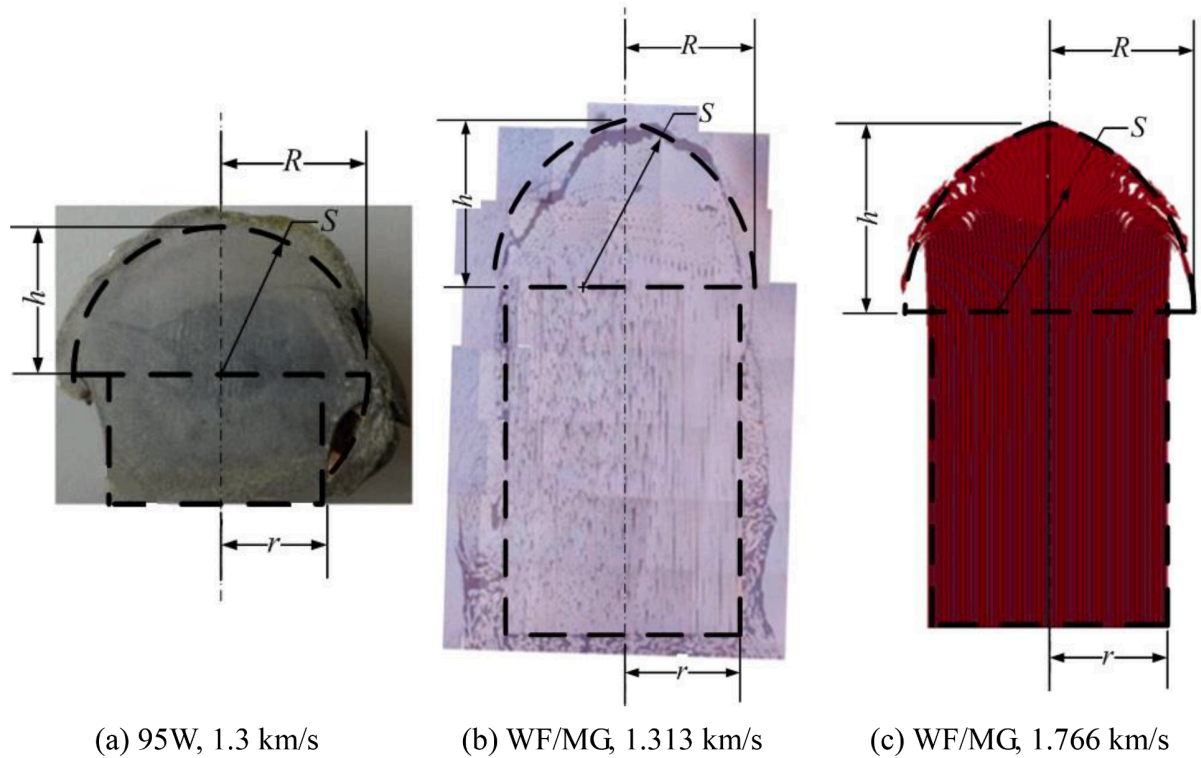
In Fig 14, the initial penetration velocity of WF/MG composite rod (0.823 km/s) is evidently higher than that of WHA rod (0.695 km/s) at the initial impact velocity of 1.3 km/s, corresponding to the greater diameter ratio η of nose shape (b) than that of nose shape (a). Therein the rod with nose shape (b) decelerates slower than that with nose shape (c), and the value of the deceleration index α [29] is smaller. Meanwhile, the duration time of WF/MG composite rod penetration is shorter than that of WHA rod because of the higher penetration velocity.

As can be observed from Fig 15, penetration performances of WHA rod and WF/MG composite rod are significantly different at the same impact velocity. The penetration performance of WF/MG composite rod is higher than that of WHA rod, which is represented by steeper slope of the curve of the former rod than the later one. Because the densities and strengths of these two materials are very close, the difference in penetration performances is mainly caused by their different nose shapes

Table 3

Parameters and results of penetration experiments/simulation.

Target material	ρ_t (g/cm ³)	σ_{yt} (GPa)	Rod material	ρ_p (g/cm ³)	Y_p (GPa)	V_0 (km/s)	Penetration depth (mm)	Residual rod length (mm)
Q235	7.8	0.235	95W	17.45	1.650	1.300	215	25
			WF/MG	17.0	1.650	1.313	250*	40
						1.766	Penetrate through	39.9 (simulation)

**Fig. 13.** Curve fitting of noses of residual rods.**Table 4**

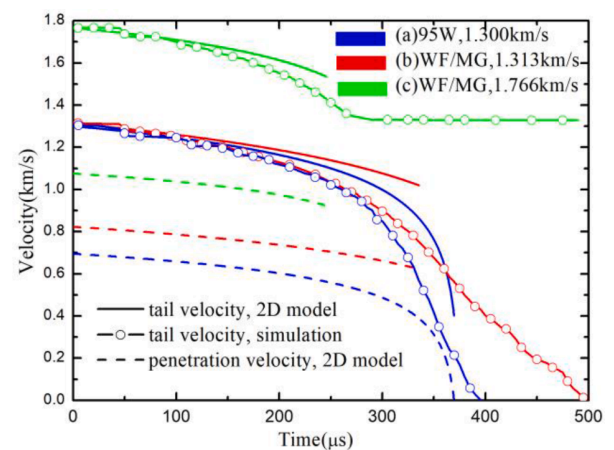
Geometry parameters of nose shapes.

No.	r (mm)	R (mm)	S (mm)	h (mm)	η	ψ	N^*
(a)	9.5	13.091	13.091	13.091	1.378	0.5	0.5
(b)	9.5	10.706	14.177	13.754	1.127	0.673	0.492
(c)	9.5	11.628	15.770	15.256	1.224	0.678	0.488

during the penetration process.

The penetration depth calculated from the present model is compared with the corresponding experimental/simulation results in Table 5. The penetration depth of WHA rod (with nose shape (a)) calculated from the 2D model is about 211 mm, very slightly lower than the experimental result (215 mm). However, for WF/MG composite rods with nose shapes (b) and (c), it is meaningless to compare their penetration depth because the calculation stops when the penetration depth reaches 250 mm. The small difference between the model prediction and experimental result of WHA rod shows the excellent prediction ability of the 2D model in the penetration depth.

The discrepancy between the model predictions and the test data of residual rod lengths is greater than that of penetration depths. The inconsistency may be caused by many factors such as assumptions made, choice of physical properties of both projectile and target. Basically, the assumptions of the present model and the variation of the target resistance during the penetration process are two possible explanations for the discrepancy.

**Fig. 14.** Velocity-time curves for penetration of rods with different shapes.

For WF/MG composite rod, the effects of moderate thickness on the targets, such as the bulge on the back of targets and direct perforation through targets observed in experiments are ignored, because of the assumption of semi-infinite targets made by the present model. Besides, the target resistance varies when the rod approaches the edge of the target.

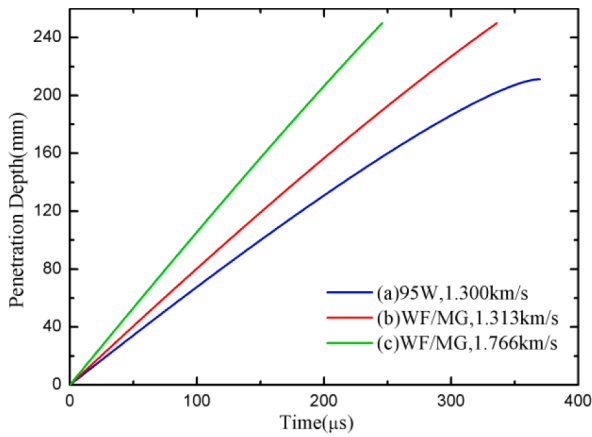


Fig. 15. Penetration depth-time curves for penetration of rods with different shapes.

Table 5

Comparison between the experimental/simulation results and calculated results.

No.	Penetration depth (mm)		Residual rod length (mm)	
	Experiments/ simulation	2D model calculated	Experiments/ simulation	2D model calculated
(a)	215	211	25	2
(b)	250	250	40	54
(c)	Penetrate through	250	39.9	43.6

For WHA rod, the target resistance varies as the penetration velocity decreases during the penetration process. The resistance in metal targets generally decreases with the deceleration of the penetration velocity below 2 km/s, although the variation of the target resistance depends on physical properties of rod and target materials and penetration velocities [7,30,31]. Therefore, the late stage of penetration process no longer belongs to the primary phase (the quasi-steady state), which is the scope of application of this model.

To achieve better agreements between the model predictions and the test data, a more complex 2D model considering the effects of moderate thickness on the targets and the variation of the target resistance should be further developed. With reference to newly developed unified theory for long rod penetration [32], the penetration process should be divided into different phases in the model.

As for the variation of residual rod length shown in Fig. 16, the WHA rod erodes slower than WF/MG composite rod at the same impact

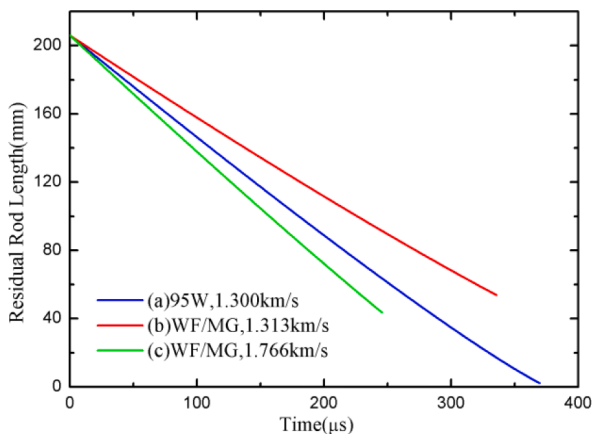


Fig. 16. Residual rod length-time curves for penetration of rods with different shapes.

velocity. Likewise, it is caused by the smaller diameter ratio η of WF/MG composite rod than that of WHA rod. Meanwhile, the erosion rate of WF/MG composite rod is greater at higher impact velocity. In fact, the loss of kinetic energy consists of the erosion and the deceleration, and the erosion rate reflects the loss rate of kinetic energy caused by erosion. The diameter ratio of nose shape (c) is greater than that of nose shape (b), which illustrates that the radial flowing of rod material increases and the loss of kinetic energy accelerates with the increase of impact velocity.

The role of nose profile in long-rod penetration were discussed by Rosenberg and Dekel [11] through a series of numerical simulations and several perforation experiments. In their experiments, residual long rods (after perforation of finite thickness targets) were imaged by flash X-ray and softly recovered using sand boxes. Different nose shapes of these rods were observed and relevant data of residual velocities and residual rod lengths were obtained. Meanwhile, a series experimental data of penetration depths of long rods penetrating infinite targets were also obtained by Rosenberg et al. [33]. Here we compare these results with the model prediction.

In perforation experiments conducted by Rosenberg and Dekel [11], steel (RHA) and aluminum (2024T351) targets with 40 mm in thickness (H) were perforated by long rods made by tungsten alloy, copper and titanium alloy, respectively. The nose shapes of these rods are listed in Table 6, in which the geometry parameters of the nose shapes are obtained by curve fitting.

The densities and strengths of rod and target materials are listed in Table 7 and 8. It should be noted that the target resistance R_t is calculated from Eq. (17). As suggested by Rosenberg et al. [4], the target resistance of RHA is 2.39 GPa. The constant effective yield strength of the target material σ_{yt} for 2024T351 targets is 0.20 GPa. The constant effective yield strength of the projectile material Y_p for WA rods is 1.65 GPa, suggested by Chen et al. [13] and Li et al. [14]. Y_p for Copper and Ti6Al4V rods are 0.64 and 1.56 GPa, respectively.

The experimental results of WA rods penetrating infinite RHA targets [33], and Copper and Ti6Al4V rods perforating Al2024-T351 targets [11] and the relevant model predictions are compared in Tables 7 and 8, respectively. The nose shapes of the residual rods observed in perforation experiments [11] are approximately assumed to be the nose shapes during the penetration process in infinite targets. In addition, Tables 7 and 8 both illustrate the differences between the model predictions and the relevant results in penetration/perforation experiments [11].

The dimensionless penetration depths predicted by the present model are slightly smaller than the experimental results [33]. The residual velocities and the dimensionless final residual rod lengths predicted by the present model are higher than the experimental results [11]. Above conclusions are similarly drawn in the comparisons between the model predictions and the relevant experiments [13] and corresponding numerical simulation [14]. Therefore, the physical soundness and accuracy of the model is proved again.

In conclusion, the modified model established in the present paper can preliminarily explain the 2D effect of long-rod penetration observed in experiments and simulations. The deviation of model prediction from experimental and simulation results are mainly caused by the assumption of semi-infinite targets and the constant target resistance of the present model. In addition, the difference between the assumed nose shape and the real one during the penetration process also affects the discrepancy. Because of the difficulty of the experimental observation of the real nose shape during the penetration process, the simulation method will be used to find more evidences for the 2D effect of long-rod penetration in subsequent analysis.

6. Conclusion

By constructing the function of nose shapes and replacing the resistance of the target by the resistance determined by the spherical cavity expansion approximation for general nose shapes, a modified theoretical

Table 6

Five nose shapes of long rods and their geometry parameters.

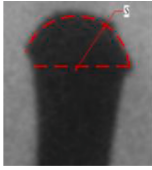
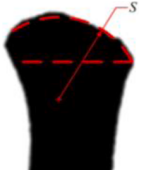
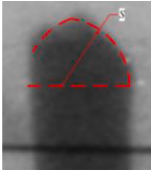
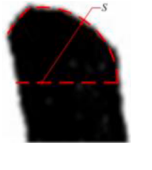
No.	T1	T2	T3	T4	T5
Velocity(km/s)	1.310	1.204	1.191	1.320	1.150
Rod Material	WA		Copper	Ti6Al4V	
Nose profile					
Nose shape	Partially mushroom		Mushroom	Sharp, chisel-like	
r (mm)	4.5				
R (mm)	6.19	6.23	6.18	4.71	4.56
S (mm)	6.49	7.85	6.18	6.51	6.74
h (mm)	4.54	3.07	6.20	6.26	6.38
H	1.38	1.38	1.37	1.05	1.01
Ψ	0.52	0.63	0.5	0.69	0.74
N^*	0.55	0.69	0.5	0.37	0.40

Table 7Experimental results and model predictions for $L=120$ mm WA rods penetrating infinite RHA targets.

No.	V_0 (km/s)	Target material	ρ_t (g/cm ³)	E (GPa)	σ_{yt} (GPa)	R_t (GPa)	Rod material	ρ_p (g/cm ³)	Y_p (GPa)	η	N^*	P/L - (T)	P/L - (M)
T-1	1.310	RHA	7.85	210	0.37	2.39	WA	17.75	1.65	1.38	0.55	0.750	0.744
T-2	1.204									1.38	0.69	0.625	0.617

Table 8Experimental results and model predictions for $L=90$ mm rods perforating $H=40$ mm Al2024-T351 targets.

No.	V_0 (km/s)	Target material	ρ_t (g/cm ³)	E (GPa)	σ_{yt} (GPa)	R_t (GPa)	Rod material	ρ_p (g/cm ³)	Y_p (GPa)	η	N^*	$V_R(T)$ (km/s)	$V_R(M)$ (km/s)	L_R/L (T)	L_R/L (M)
T-3	1.191	Al2024-T351	2.79	79	0.20	1.22	Copper	8.90	0.64	1.37	0.50	1.024	1.090	0.38	0.39
T-4	1.320						Ti6Al4V	4.45	1.56	1.05	0.37	0.953	1.062	0.63	0.70
T-5	1.150								1.56	1.01	0.40	0.592	0.757	0.55	0.60

model of long-rod penetration is established in the present paper. Two characteristic parameters, i.e., the diameter ratio of rod nose and shank η and the nose shape factor N^* , are proposed to describe the effect of the nose shape on the long-rod penetration. Through the case study based on the present model, we declare the following conclusions:

- 1 The effect of the diameter ratio η is relatively great. The penetration performance decreases with increasing the diameter ratio, and the increase of the radial flowing of rod material leads to the loss of kinetic energy.
- 2 The nose shape factor N^* reflects the sharpness of nose shapes, which affects the ballistic performance of long-rod penetration, however, depending on the initial impact velocity. In general, the influence of N^* is relatively smaller than that of η .

Finally, experimental and simulation results are used to analyze the 2D effect of long-rod penetration, and the practicality of the present model is verified. This model provides a practical method to theoretically study the 2D effect of long-rod penetration. However, this model needs to be further improved. Particularly, relationships between nose shape parameters (η and N^*) and material parameters of rod and target and penetration velocity should be determined by combining with experimental and simulation data. Verification experiments and numerical simulations should be further conducted.

Credit Author Statement

Chen Xiaowei confirm he is a full-time Professor of Advanced Research Institute of Multidisciplinary Science, Beijing Institute of

Technology in China. He represent all the authors to agree this submission for possible publication in IJIE.

Declaration of competing interest

We declare that this paper has no any conflict of interest with anyone or any organization.

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