

2nd Maths Paper

1. a) (i) $y = \frac{x^2}{2x+1}$ $u = x^2$ $\frac{du}{dx} = 2x$ $v = 2x+1$ $\frac{dv}{dx} = 2$

$$\frac{dy}{dx} = \frac{2x(2x+1) - 2x^2}{(2x+1)^2}$$

$$= \frac{2x^2 + 2x}{(2x+1)^2} = \frac{2x(x+1)}{(2x+1)^2}$$

(ii) $y = \sqrt{x^2+1}$ $u = x^2+1$ $\frac{du}{dx} = 2x$ $y = u^{1/2}$ $\frac{dy}{du} = \frac{1}{2}u^{-1/2}$

$$\frac{dy}{dx} = \frac{x}{\sqrt{x^2+1}}$$

(iii) $y = 3x^{3/2} + 3/x^2 - 6x$ $\frac{dy}{dx} = 9/2 x^{1/2} - 6/x^3 - 6 = 9x^{1/2} - 12/x^3 - 6 = 3(3x^{1/2} - 4x^{-3} - 4)$

(iv) $y = \sec^3 4x$ $u = \sec 4x$ $\frac{du}{dx} = 4 \sec 4x \tan 4x$ $\frac{dy}{dx} = 12 \sec^3 4x \tan 4x$

(v) $y = x + \sin 2x$ $\frac{dy}{dx} = 1 + 2 \cos 2x$

$$(vi) \quad y = \frac{1-10x}{(2x-1)^2} \quad u = 1-10x \quad \frac{du}{dx} = -10 \quad v = (2x-1)^2 \quad \frac{dv}{dx} = 4(2x-1)$$

$$\frac{dy}{dx} = \frac{-10(2x-1)^2 - 4(2x-1)(1-10x)}{(2x-1)^4}$$

~~$$y = \frac{1-10x}{(2x-1)^2}$$~~

b) $y = \frac{1}{x} + \cos 2x$

$$\frac{dy}{dx} = -x^{-2} - 2 \sin 2x$$

$$\frac{d^2y}{dx^2} = 2x^{-3} - 4 \cos 2x$$

$$\frac{d^3y}{dx^3} = -6x^{-4} + 8 \sin 2x$$

$$\frac{d^4y}{dx^4} = 24x^{-5} + 16 \sin 2x$$

$$\frac{d^5y}{dx^5} = -120x^{-6} - 32 \sin 2x$$

c) $f(x) = \ln(\operatorname{cosec} x - \cot x)$ $u = \operatorname{cosec} x - \cot x$ $y = \ln u$ $\frac{dy}{dx} = \frac{1}{u} \frac{du}{dx}$ $\frac{du}{dx} = -\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x$

$$\frac{dy}{dx} = \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$$

$$= \frac{-\cos x + 1}{\sin x}$$

$$= 1 - \frac{\cos x}{\sin x} = 1 - \cot x$$

$$\frac{dy}{dx} = \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x} \quad \text{--- (iv)}$$

$$= \frac{\left(\frac{-\cos x}{\sin^2 x} \right) + \left(\frac{1}{\sin^2 x} \right)}{\left(\frac{1}{\sin x} - \frac{\cos x}{\sin x} \right)}$$

$$= \frac{-\cos x + 1}{\sin^2 x} \cdot \frac{1 - \cos x}{1 - \cos x} = \frac{(1 - \cos x)^2}{\sin^3 x}$$

$$= \frac{1 - 2\cos x + \cos^2 x}{\sin^3 x}$$

$$\begin{aligned}
 2. a) (i) \quad & \int \frac{2x}{(4+3x^2)} dx \quad u = 4+3x^2 \\
 & \frac{du}{dx} = 6x \quad dx = \frac{du}{6x} \\
 & = \int \frac{2x}{u} \frac{du}{6x} \\
 & = \frac{1}{3} \int \frac{1}{u} = \frac{1}{3} \ln u \\
 & = \frac{\ln(4+3x^2)}{3} + c
 \end{aligned}$$

$$\begin{aligned}
 (ii) \quad & \int_0^4 e^{1/2x} dx \quad u = e^{1/2x} \\
 & \frac{du}{dx} = \frac{1}{2} e^{1/2x} \quad dx = \frac{du}{\frac{1}{2}} \\
 & \int_0^4 u^{1/2x} 2du \\
 & = \left[2u^{1/2x} \right]_0^4 = \left[2e^{1/2x} \right]_0^4 \\
 & = (14.78 - 2) \\
 & = 12.78
 \end{aligned}$$

$$(iii) \quad \int_0^4 \frac{5x+13}{(2x+1)(x+4)} = \int_0^4 \frac{A}{2x+1} + \frac{B}{x+4}$$

$$5x+13 = A(x+4) + B(2x+1)$$

$$x = -1/2 \quad 5(-1/2) + 13 = 11/2 = 7/2 A \quad A = 3$$

$$x = -4 \quad 5(-4) + 13 = -7 = -7B \quad B = 1$$

$$\int_0^4 \frac{3}{2x+1} + \frac{1}{x+4}$$

$$= \left[3 \ln(2x+1) + \ln(x+4) \right]_0^4$$

$$= 8.67 - \ln 4 = 7.78$$

$$\begin{aligned}
 \text{b) } f(x) &= \frac{4}{x(1+4\ln x)^2} & u &= 1+4\ln x \\
 & & \frac{du}{dx} &= \frac{4}{x} & dx &= \frac{du \cdot x}{4} \\
 & & & & & \\
 & \int \frac{4}{x u^2} \cdot \frac{du \cdot x}{4} \\
 &= \int \frac{1}{u^2} du = -\frac{1}{u} \\
 &= -\frac{1}{1+4\ln x}
 \end{aligned}$$

$$\begin{aligned}
 \text{c) (i) } \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x-1}} & \quad u = 3e^{2x} \quad \frac{du}{dx} = 6e^{2x} \quad \frac{dv}{dx} = (e^x-1)^{1/2} \\
 & \quad \quad \quad \frac{du}{dx} = 6e^{2x} \quad \quad \quad v = \frac{2(e^x-1)^{3/2}}{3} \\
 \int_{\ln 2}^{\ln 5} \frac{3e^{2x}}{\sqrt{e^x-1}} &= 3e^{2x} \left(\frac{2(e^x-1)}{3} \right) - \int 6e^{2x} \left(\frac{2(e^x-1)}{3} \right) \\
 &= 2e^{2x}(e^x-1) - \int 2e^{2x}(2e^x-2) \quad u = 2e^x \\
 &= e^{2x}(e^x-1) - \int 4e^{3x} - 4e^{2x} \quad \frac{du}{dx} = 2e^x \quad dx = \frac{du}{2e^x} \\
 &= e^{3x} - e^{2x} - \frac{4}{3}e^{3x} + 2e^{2x} \\
 &= -\frac{1}{3}e^{3x} + e^{2x} \\
 &= \left[e^{2x} - \frac{1}{3}e^{3x} \right]_{\ln 2}^{\ln 5} = \left(\cancel{12} - \frac{50}{3} - \frac{4}{3} \right) \\
 &= -\frac{54}{3} = -18
 \end{aligned}$$

$$(ii) \int_1^e \ln x \, dx \quad \begin{array}{l} u = \ln x \\ \frac{du}{dx} = \frac{1}{x} \end{array} \quad \begin{array}{l} \frac{dv}{dx} = 1 \\ v = x \end{array}$$

$$\int_1^e \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx$$

$$= [x \ln x]_1^e - \int_1^e 1 \, dx$$

$$= [x \ln x - x]_1^e$$

$$= e - 1$$

$$= 0 - -1 = 1$$

$$\begin{aligned}
 3. a) (i) \left(\frac{1+i}{1-i} \right)^3 &= \left(\frac{1+2i+i^2}{1-2i+i^2} \right) \left(\frac{1+i}{1-i} \right) \\
 &= \frac{2i}{-2i} = -1 \quad -1 \left(\frac{1+i}{1-i} \right) = \left(\frac{-1-i}{-1+i} \right) \\
 \left(\frac{-1-i}{-1+i} \right) \times \left(\frac{-1-i}{-1-i} \right) &= \frac{1+2i+i^2}{1-i^2} = \frac{2i}{2} \\
 &= i
 \end{aligned}$$

$$\begin{aligned}
 (ii) \frac{3}{(1+i)(2+i)} &= \frac{3}{2+3i+i^2} = \frac{3}{1+3i} \\
 \frac{3}{1+3i} \times \frac{1-3i}{1-3i} &= \frac{3-9i}{1-9i^2} = \frac{3-9i}{10} = \frac{3(1-3i)}{10}
 \end{aligned}$$

$$\begin{aligned}
 (iii) \frac{1+i}{1-i} - \frac{(1+2i)(2+2i)}{1+i} + \frac{3-i}{1+i} \\
 = \frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{1-i^2} = \frac{2i}{2} = i \\
 \frac{(1+2i)(2+2i)}{1+i} = \frac{2+6i+4i^2}{1+i} = \frac{-2+6i}{1+i} \\
 \frac{3-i}{1+i} \times \frac{1-i}{1-i} = \frac{3-4i+i^2}{1-i^2} = \frac{2-4i}{2} = 1-2i \\
 = i - (-2+6i) + (1-2i) \\
 = 1-7i
 \end{aligned}$$

$$\begin{aligned}
 (iv) 2i(i-1) + (\sqrt{3}-i) + (1+i)(\overline{1+i}) \\
 = (2i^2-2i) + (\sqrt{3}-i) + (2) \\
 = (-2-2i) + (\sqrt{3}-i) + 2 \\
 = \sqrt{3}-3i
 \end{aligned}$$

$$\begin{aligned} \text{b) } z &= (4 + 7i)(3 - 2i) = 12 + 13i - 14i^2 \\ &= 26 + 13i \end{aligned}$$

$$|z| = \sqrt{26^2 + 13^2} = 29.07$$

$$\theta = \tan^{-1}\left(\frac{13}{26}\right) = 26.57^\circ$$

$$z = 29.07 \angle 26.57^\circ$$

$$\begin{aligned} \text{c) } (1 + 3i)(1 + i) &= 1 + 4i + 3i^2 \\ &= -2 + 4i \\ &= 2(-1 + 2i) \end{aligned}$$