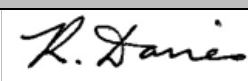


EXAMINATION INTERNAL VERIFICATION FORM (2017-2018)

MODULE :	PSDE5008 Further Engineering Mathematics	LEVEL:	5
PROGRAMME :	HNC Electrical and Electronic Engineering	TERM OF EXAM: (SPRING OR SUMMER)	n/a
LEARNING OUTCOME(S) BEING ASSESSED:	MILO1, MILO 2 (partially covered)	EXAM PAPER:	Main Class Test 1
		EXAMINATION AUTHOR :	A. Thomas

VERSION:	DATE SUBMITTED FOR VERIFICATION:	INTERNAL VERIFIER COMMENTS (Approval or detail of amendments required):	DATE:
1.	15-11-17	A good class test which conforms to the assessment pattern and tests the module learning outcomes appropriately.	22-11-17
2.			

INTERNAL VERIFIER PLEASE TICK TO INDICATE THAT THE EXAMINATION:	
✓	Conforms to the assessment pattern described in the module syllabus
✓	Tests the module learning outcome(s) appropriately

SIGNATURES OF FINAL APPROVAL:	SIGNATURE	DATE
INTERNAL VERIFIER: I confirm that this version of the exam paper is ready to be sent to the external examiner		22-11-17
EXAM AUTHOR: I confirm that internal verifier and external examiner approval have been received and that this version of the exam paper is ready for printing.		

HNC Electrical and Electronic Engineering
PSDE5008 Further Engineering Mathematics
Level 5, Year 2

Class Test 1 (2017-18)

This is a closed book test, worth **45%** of the module mark and addresses Module Intended Learning Outcomes 1 & 2.

Date of test =

Time allowed = 2 hours

Answer both questions

1. Derive the differential equation for an RL circuit, supplied by a source E which is switched on at $t = 0$, given:

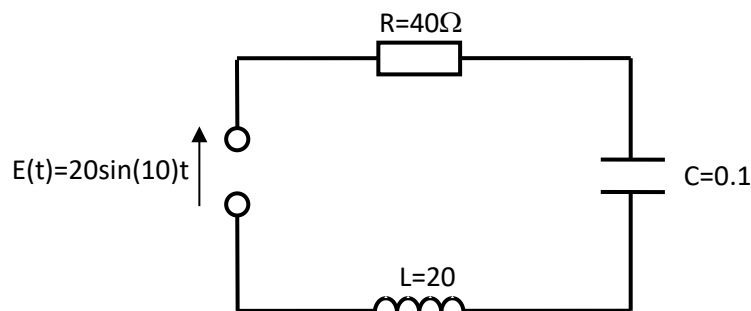
$$R = 40\Omega, L = 0.5 H, E = 30V \text{ and } I(0) = 0$$

(5 Marks)

Hence use the integrating factor method to find the current in the circuit.

(15 Marks)

2. By considering Kirchoff's voltage law, show that the differential equation for the current $I(t)$ in the RLC circuit supplied by a voltage source $E(t)$ switched on at $t = 0$



with $I(0) = 0$ and $I'(0) = 0$, is given by

$$I'' + 2I' + 0.5I = 10\cos(10t)$$

Solve this equation in order to determine the current in the circuit, and hence find the voltage drop across the resistor.

(30 Marks)

PSDE5008 Test 1

Formulae

Integrating Factor for the Solution of First order Differential Equations

$$\frac{dy}{dx} + Py = Q$$

$$ye^{\int P dx} = \int e^{\int P dx} Q dx$$

Second order differential equations with constant coefficients

Where,

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0$$

auxiliary equation:

$$am^2 + bm + c = 0$$

with roots:

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Case 1 (real and different):

$$y = Ae^{m_1 x} + Be^{m_2 x}$$

Case 2 (real and equal):

$$y = e^{m_1 x}(A + Bx)$$

Case 3 (Complex roots):

$$y = e^{ax}(A \cos \beta x + B \sin \beta x)$$

Where,

$$a \frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = r(x)$$

the solution has the form:

$$y = y_c + y_p$$

Where y_p is obtained from making the suggested substitution below:

$r(x)$	Trial Solution
$a \cos kx$	$C \sin kx + D \cos kx$
$a \sin kx$	$C \sin kx + D \cos kx$
ae^{kx}	Ce^{kx}

RLC Circuit

$$E = RI + L \frac{dI}{dt} + \frac{1}{C} \int I dt$$

Standard derivatives

y	$\frac{dy}{dx}$
ax^n	nax^{n-1}
$a \sin(kx)$	$ka \cos(kx)$
$a \cos(kx)$	$-ka \sin(kx)$
ae^{kx}	kae^{kx}
$a \ln x$	$\frac{a}{x}$

Standard integrals

$\int ax^n dx$	$\frac{ax^{n+1}}{n+1} + c$ (except when $n = -1$)
$\int \cos ax dx$	$\frac{1}{a} \sin ax + c$
$\int \sin ax dx$	$-\frac{1}{a} \cos ax + c$
$\int e^{ax} dx$	$\frac{1}{a} e^{ax} + c$
$\int \frac{1}{x} dx$ or $\int x^{-1} dx$	$\ln x + c$
$\int \frac{1}{ax+b} dx$	$\frac{1}{a} \ln(ax+b) + c$

Test 1 Solutions/Mark Scheme

				Qu total
1	$R = 40 \quad L = 0.5 \quad E = 30$ $L \frac{dI}{dt} + RI = E$ $0.5 \frac{dI}{dt} + 40 I = 30$ $\frac{dI}{dt} + 80 I = 60$ $\int P dt = \int 80 dt = 80t$ $\therefore i.f = e^{80t}$ $Ie^{80t} = \int e^{80t} \cdot 60 dt$ $= \int 60 e^{80t} dt$ $Ie^{80t} = 0.75 e^{80t} + C$ When $I = 0 \quad t = 0$ $0 = 0.75 e^0 + C$ $C = -0.75$ $\therefore Ie^{80t} = 0.75 e^{80t} - 0.75$ or $I = 0.75 - 0.75 e^{-80t}$	2	3	5
		2		
		2		
		2		
		2		
		2		
		2		
		2		
		3	15	20

Test 1 Solutions/Mark Scheme

					Qu total
2	$L \frac{dI}{dt} + RI + \frac{1}{C} \int I dt = E(t)$				
	$L \frac{d^2 I}{dt^2} + R \frac{dI}{dt} + \frac{1}{C} I = \frac{dE}{dt}$	1			
	Since $E(t) = 20 \sin 10t$				
	$\frac{dE}{dt} = 200 \cos 10t$	1			
	$20 \frac{d^2 I}{dt^2} + 40 \frac{dI}{dt} + \frac{1}{0.1} I = 200 \cos 10t$	1			
	$\frac{d^2 I}{dt^2} + 2 \frac{dI}{dt} + 0.5 I = 10 \cos 10t$	2			
	Find CF				
	$M^2 + 2M + 0.5 = 0$				
	$M = \frac{-2 \pm \sqrt{2^2 - 4(1)(0.5)}}{2(1)}$				
	$= \frac{-2 \pm \sqrt{2}}{2}$				
	$= -0.293 \text{ or } -1.707$	2			
	real and different				
	$I_c = A e^{-0.293t} + B e^{-1.707t}$	2			
	$I_p = A \sin 10t + B \cos 10t$	1			
	$I'_p = 10A \cos 10t - 10B \sin 10t$	2			
	$I''_p = -100A \sin 10t - 100B \cos 10t$	2			

Test 1 Solutions/Mark Scheme

				Qu total
Sub.	$(-100 A \sin 10t - 100 B \cos 10t) +$ $2(10 A \cos 10t - 10 B \sin 10t) +$ $0.5(A \sin 10t + B \cos 10t) = 10 \cos 10t$ $= -100 A \sin 10t - 100 B \cos 10t$ $+ 20 A \cos 10t - 20 B \sin 10t$ $+ 0.5 A \sin 10t + 0.5 B \cos 10t$ $= 10 \cos 10t$			
Taking sines:	$-100 A - 20 B + 0.5 A = 0$ $-99.5 A - 20 B = 0$ $A = \frac{20 B}{-99.5} = -0.201 B$			
Taking cosines:	$-100 B + 20 A + 0.5 B = 10$ $-100 B + 20(-0.201 B) + 0.5 B = 10$ $-103.52 B = 10$ $B = \frac{10}{-103.52} = -0.097$ $\therefore A = -0.201(-0.097)$ $= 0.019$			
∴ GS.	$I = A e^{-0.293t} + B e^{-1.707t}$ $+ 0.019 \sin 10t - 0.097 \cos 10t$ $I(0) = 0$			

Test 1 Solutions/Mark Scheme

			Qu total
$0 = Ae^0 + Be^0 + 0.019 \sin 0 - 0.097 \cos 0$	1		
$0 = A + B - 0.097$ ①	1		
$\frac{dI}{dt} = -0.293 Ae^{-0.293t} - 1.707 Be^{-1.707t}$ $+ 0.019 \cos 10t + 0.097 \sin 10t$	2		
$I'(0) = 0$			
$0 = -0.293 Ae^0 - 1.707 Be^0 +$ $0.019 \cos 0 + 0.097 \sin 0$	1		
$0 = -0.293A - 1.707B + 0.019$ ②	1		
① $\times 0.293$			
$0 = 0.293A + 0.293B - 0.028$			
$0 = -1.414B - 0.009$			
$1.414B = -0.009$			
$B = -0.005$	1		
$A - 0.005 = 0.097$			
$A = 0.102$	1		
$\therefore I = 0.102e^{-0.293t} - 0.005e^{-1.707t}$ $+ 0.019 \sin 10t - 0.097 \cos 10t$	1		
$V = IR$			
$V = 40[0.102e^{-0.293t} - 0.005e^{-1.707t}$ $+ 0.019 \sin 10t - 0.097 \cos 10t]$	1		
			30

50