

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 4

LEFEL 4

EXAMINATION PERIOD
JANUARY 2026

CYFNOD ARHOLIADAU
IONAWR 2026

MODULE CODE: GOEN4001

MATHEMATICS TEST 1

SOLUTIONS

TIME: 2 HOURS

ANSWER ALL QUESTIONS

Equipment or materials required – calculators & formula book

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QUESTION ONE (35 marks)

a) Find the first derivative of the following functions:

i. $y = 2x^6 - 5x^3 + 4x - 9$ [2]

$$\frac{dy}{dx} = 12x^5 - 15x^2 + 4$$

ii. $y = \frac{4}{x^2} - \sqrt[5]{x} + \frac{3}{e^2}$ [3]

$$y = 4x^{-\frac{1}{2}} - x^{\frac{1}{5}} + 3e^{-2}$$

$$\frac{dy}{dx} = -2x^{-\frac{3}{2}} - \frac{1}{5}x^{-\frac{4}{5}} + 0$$

iii. $y = 4\sin(4x) - 2\cos(5x) + 3\tan\left(\frac{x}{3}\right)$ [3]

$$\frac{dy}{dx} = 16\cos(4x) + 10\sin(5x) + \sec^2\left(\frac{x}{3}\right)$$

iv. $y = 5x^4e^{2x}$ [4]

$$u = 5x^4 \quad \text{and} \quad \frac{du}{dx} = 20x^3$$

$$v = e^{2x} \quad \text{and} \quad \frac{dv}{dx} = 2e^{2x}$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = 5x^4 (2e^{2x}) + e^{2x}(20x^3)$$

$$\frac{dy}{dx} = 10x^4e^{2x} + 20x^3e^{2x}$$

v. $y = \frac{3\cos(3x)}{e^x+5}$ [4]

$$u = 3\cos(3x) \quad \text{and} \quad \frac{du}{dx} = -9\sin(3x)$$

$$v = e^x + 5 \quad \text{and} \quad \frac{dv}{dx} = e^x$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{(v)^2}$$

$$\frac{dy}{dx} = \frac{(e^x + 5)(-9 \sin(3x)) - (3 \cos(3x))(e^x)}{(e^x + 5)^2}$$

$$\frac{dy}{dx} = \frac{(-9 \sin(3x))(e^x + 5) - (3e^x \cos(3x))}{(e^x + 5)^2}$$

vi. $y = \frac{2}{7}(3x^4 + 4x)^3$ [3]

$$\frac{dy}{dx} = \frac{6}{7} (3x^4 + 4x)^2 \times (12x^3 + 4)$$

$$\frac{dy}{dx} = \frac{6}{7} (3x^4 + 4x)^2 (12x^3 + 4)$$

or

$$\text{let } u = 3x^4 + 4x \text{ so } y = \frac{2}{7} u^3$$

$$\frac{dy}{du} = \frac{6}{7} u^2 \quad \text{also} \quad \frac{du}{dx} = 12x^3 + 4$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{6}{7} u^2 \times (12x^3 + 4) = \frac{6}{7} (3x^4 + 4x)^2 \times (12x^3 + 4)$$

$$\frac{dy}{dx} = \frac{6}{7} (3x^4 + 4x)^2 (12x^3 + 4)$$

vii. $y = \frac{1}{5} \sqrt[5]{(5x^4 + \sin(2x))}$ [3]

$$y = \frac{1}{5} (5x^4 + \sin(2x))^{\frac{1}{5}}$$

$$\frac{dy}{dx} = \frac{1}{25} (5x^4 + \sin(2x))^{\frac{-4}{5}} (20x^3 + 2 \cos(2x))$$

or

$$\text{let } u = 5x^4 + \sin(2x) \quad \text{so} \quad y = \frac{1}{5}u^{\frac{1}{5}}$$

$$\text{so} \quad \frac{dy}{du} = \frac{1}{25}u^{-\frac{4}{5}}$$

$$\text{also} \quad \frac{du}{dx} = 20x^3 + 2\cos(2x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{1}{25}u^{-\frac{4}{5}} \times (20x^3 + 2\cos(2x))$$

$$\frac{dy}{dx} = \frac{1}{25}(5x^4 + \sin(2x))^{-\frac{4}{5}} (20x^3 + 2\cos(2x))$$

b) The variable w is related to the variables x and y by the following:

$$w = 3x^4y^3 + 5x^3 + 6y^2 + e^{2x} - \cos(3y)$$

Use Partial Differentiation techniques to find following second order Partial Derivatives,

$$(i) \frac{\partial^2 w}{\partial x^2} \quad (ii) \frac{\partial^2 w}{\partial y^2} \quad (iii) \frac{\partial^2 w}{\partial x \partial y} \quad (iv) \frac{\partial^2 w}{\partial y \partial x}$$

[6]

$$w = 3x^4y^3 + 5x^3 + 6y^2 + e^{2x} - \cos(3y)$$

$$\frac{\partial w}{\partial x} = 12x^3y^3 + 15x^2 + 2e^{2x}$$

[1]

$$\frac{\partial^2 w}{\partial x^2} = 36x^2y^3 + 30x + 4e^{2x}$$

[1]

$$\frac{\partial^2 w}{\partial y \partial x} = 36x^3y^2$$

[1]

$$w = 3x^4y^3 + 5x^3 + 6y^2 + e^{2x} - \cos(3y)$$

$$\frac{\partial w}{\partial y} = 9x^4y^2 + 12y + 3\sin(3y)$$

[1]

$$\frac{\partial^2 w}{\partial y^2} = 18x^4y + 12 + 9\cos(3y)$$

[1]

$$\frac{\partial^2 w}{\partial x \partial y} = 36x^3y^2$$

[1]

c) If $y = 3e^x + x^2$ show that

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 2x^2 - 6x + 2$$

[4]

$$LHS = \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$

$$y = 3e^x + x^2$$

$$\frac{dy}{dx} = 3e^x + 2x$$

[1]

$$\frac{d^2y}{dx^2} = 3e^x + 2$$

[1]

$$LHS = \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y$$

$$LHS = 3e^x + 2 - 3(3e^x + 2x) + 2(3e^x + x^2)$$

$$LHS = 3e^x + 2 - 9e^x - 6x + 6e^x + 2x^2$$

$$LHS = 2x^2 - 6x + 2$$

$$RHS = 2x^2 - 6x + 2$$

$$LHS = RHS$$

[2]

d) Find the 3th derivative of:

$$y = x\sin(x)$$

$$y = x\sin(x)$$

[3]

$$\frac{dy}{dx} = x \cos(x) + \sin(x)$$

$$\frac{d^2y}{dx^2} = -x \sin(x) + \cos(x) + \cos(x)$$

$$\frac{d^3y}{dx^3} = -x \cos(x) - \sin(x) - \sin(x) - \sin(x)$$

$$\frac{d^3y}{dx^3} = -x \cos(x) - 3 \sin(x)$$

QUESTION TWO (33 marks)

a) Express, $\frac{5x+7}{(x+1)(x-3)}$ in partial fractions

[9]

$$\frac{5x+7}{(x+1)(x-3)} = \frac{A}{(x+1)} + \frac{B}{(x-3)}$$

$$\frac{(5x+7)(x+1)(x-3)}{(x+1)(x-3)} = \frac{A(x+1)(x-3)}{(x+1)} + \frac{B(x+1)(x-3)}{(x-3)}$$

$$5x+7 = A(x-3) + B(x+1)$$

[2]

let $x = 3$

$$5x+7 = A(x-3) + B(x+1)$$

$$15+7 = A(3-3) + B(3+1)$$

$$22 = A(0) + B(4)$$

$$22 = 4B$$

$$B = \frac{22}{4} \quad , \quad B = 5.5$$

[4]

let $x = -1$

$$5x+7 = A(x-3) + B(x+1)$$

$$-5+7 = A(-1-3) + B(-1+1)$$

$$2 = A(-4) + B(0)$$

$$2 = -4A$$

$$A = -\frac{2}{4} \quad , \quad B = 0.5$$

[2]

$$\frac{5x + 7}{(x + 1)(x - 3)} = \frac{A}{(x + 1)} + \frac{B}{(x - 3)}$$

$$\frac{5x + 7}{(x + 1)(x - 3)} = \frac{-0.5}{(x + 1)} + \frac{5.5}{(x - 3)}$$

[1]

b) An engineer is designing a triangular truss ABC. The sides are AB = 15m and AC = 10m and the angle at vertex A is 120°. Determine:

1. The length of side BC.
2. The angles at vertices B and C.

[5]

$$AB = 15m \text{ and } AC = 10m$$

Applying the Cosine rule:

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC) \cos A$$

$$BC^2 = 15^2 + 10^2 - 2(15)(10) \cos 120^\circ$$

$$BC^2 = 225 + 100 - 300 \cos 120^\circ$$

$$BC^2 = 225 + 100 - 300 (-0.5)$$

$$BC = \sqrt{475}$$

$$BC = 21.79 \text{ m} \quad [2]$$

Applying the sin rules to find the angle B:

$$\frac{\sin B}{AC} = \frac{\sin A}{BC}$$

$$\frac{\sin B}{10} = \frac{\sin 120^\circ}{21.79}$$

$$\sin B = \frac{\sin 120^\circ}{21.79} \times 10$$

$$\sin B = \frac{0.866}{21.79} \times 10$$

$$\sin B = 0.397$$

$$B = \sin^{-1} 0.397$$

$$B = 23.4^\circ$$

$$\text{angle } B = 23.4^\circ \quad [2]$$

$$\text{angle } C = 180^\circ - A - B$$

$$\text{angle } C = 180^\circ - 120^\circ - 23.4^\circ = 36.6^\circ \quad [1]$$

- c) Factorise $x^2 - 11x + 24$, hence solve the following trigonometric equation for $0^\circ \leq x \leq 360^\circ$:

$$\tan^2 x - 11 \tan x + 24 = 0$$

[6]

$$x^2 - 11x + 24$$

$$(x - 8)(x - 3)$$

[2]

$$(\tan x - 8)(\tan x - 3) = 0$$

$$\tan x - 8 = 0$$

$$\tan x = 8$$

$$x = \tan^{-1} 8$$

$$x = 82.87^\circ \quad \text{or} \quad x = 262.87^\circ$$

[2]

$$\tan x - 3 = 0$$

$$\tan x = 3$$

$$x = \tan^{-1}(3)$$

$$x = 71.57^\circ \quad \text{or} \quad x = 251.57^\circ$$

[2]

- d) A curve $y = x^2 - 4x + 3$ and the line $y = 2x + 1$ intersect at the points M and N. By solving these equations simultaneously, find the co-ordinates of M and N.

[6]

$$y = x^2 - 4x + 3$$

$$y = 2x + 1$$

$$x^2 - 4x + 3 = 2x + 1$$

[1]

$$x^2 - 6x + 2 = 0$$

[1]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-6) \pm \sqrt{(-6)^2 - (4)(1)(2)}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{36 - 8}}{2} = \frac{6 \pm 5.29}{2}$$

$$x = \mathbf{5.645}$$

$$x = \mathbf{0.355}$$

[2]

$$y = 2x + 1$$

$$\text{if } x = \mathbf{5.645} \quad y = 2(\mathbf{5.645}) + 1$$

$$y = \mathbf{12.29}$$

$$\text{if } x = \mathbf{0.355} \quad \text{and } y = 2x + 1$$

$$y = 2(0.355) + 1$$

$$y = \mathbf{1.71}$$

[2]

Solution: Coords are

$$(\mathbf{5.645, 12.29}) \quad \text{and} \quad (\mathbf{0.355, 1.71})$$

e) Solve the following trigonometric equations for $0^\circ \leq x \leq 360^\circ$:

i. $2 \sin x = 0.75$

[2]

$$2 \sin x = 0.75 \quad \text{so} \quad \sin x = \frac{0.75}{2}$$

$$x = \sin^{-1} 0.375$$

$$x = 22^\circ$$

$$x = 180^\circ - 22^\circ$$

$$x = 158^\circ$$

$$\mathbf{x = 22^\circ \quad , \quad x = 158^\circ}$$

ii. $4 \cos x - 1 = 0$

[3]

$$4 \cos x - 1 = 0$$

$$4 \cos x = 1$$

$$4 \cos x = 1 \quad \text{so} \quad \cos x = \frac{1}{4}$$

$$\cos x = 0.25$$

$$x = \cos^{-1} 0.25$$

$$x = 75.5^\circ$$

$$x = 360^\circ - 75.5^\circ$$

$$x = 284.5^\circ$$

$$\mathbf{x = 75.5^\circ \quad , \quad x = 284.5^\circ}$$

iii. $-5 \tan x = 1$

[2]

$$\tan x = -\frac{1}{5}$$

$$x = \tan^{-1}\left(\frac{1}{5}\right)$$

$$x = 11.3^\circ \quad \text{not a solution}$$

$$x = 180^\circ - 11.3^\circ$$

$$\mathbf{x = 168.7^\circ}$$

$$x = 360^\circ - 11.3^\circ$$

$$\mathbf{x = 348.7^\circ}$$

$$\mathbf{x = 168.7^\circ, \quad 348.7^\circ}$$

QUESTION THREE (32 marks)

a) Solve the following equations, using the appropriate method, for x :

i. $\ln(2x - 3) + \ln(3) = \ln(5x + 9)$

[4]

$$\ln(2x - 3) + \ln(3) = \ln(5x + 9)$$

$$\cancel{\ln(2x - 3)} \cdot \cancel{(3)} = \cancel{\ln(5x + 9)}$$

$$(2x - 3)(3) = 5x + 9$$

[2]

$$6x - 9 = 5x + 9$$

$$6x - 5x = 9 + 9$$

$$x = 18$$

[2]

ii. $\log(3x + 2) + \log(x + 1) = \log 15$

[4]

$$\log(3x + 2)(x + 1) = \log 15$$

[1]

$$\cancel{\log(3x + 2)(x + 1)} = \cancel{\log 15}$$

$$(3x + 2)(x + 1) = 15$$

[1]

$$3x^2 + 3x + 2x + 2 = 15$$

$$3x^2 + 5x + 2 = 15$$

$$3x^2 + 5x - 13 = 0$$

[1]

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-13)}}{2(3)} = x = \frac{-5 \pm \sqrt{25 + 156}}{6}$$

$$x = \frac{-5 \pm 13.45}{6}$$

$x = -3.075$ not a solution $x = 1.4$ is the solution

[1]

iii. $4^{(x+3)} \times 2^{(2-x)} = 8^{(x+1)}$ [4]

$$\log 4^{(x+3)} \times 2^{(2-x)} = \log 8^{(x+1)}$$

[1]

$$\log 4^{(x+3)} + \log 2^{(2-x)} = \log 8^{(x+1)}$$

[1]

$$(x+3) \log 4 + (2-x) \log 2 = (x+1) \log 8$$

[1]

$$(x+3) 0.6 + (2-x) 0.3 = (x+1)(0.9)$$

$$0.6x + 1.8 + 0.6 - 0.3x = 0.9x + 0.9$$

$$0.6x - 0.3x - 0.9x = 0.9 - 1.8 - 0.6$$

$$-0.6x = -1.5$$

$$x = \frac{1.5}{1.2}$$

$$x = 2.5$$

[1]

i. $\frac{6x+1}{3x-2} = \frac{4}{5}$ [3]

$$4(3x - 2) = 5(6x + 1)$$

[1]

$$12x - 8 = 30x + 5$$

$$12x - 30x = 5 + 8$$

$$-18x = 13$$

$$x = -\frac{13}{18} = -0.72$$

[2]

$$\text{ii. } \frac{(4x+1)}{5} + \frac{(2x-3)}{3} = \frac{11}{6}$$

[3]

$$30 \left(\frac{(4x+1)}{5} + \frac{(2x-3)}{3} = \frac{11}{6} \right)$$

[1]

$$\frac{30(4x+1)}{5} + \frac{30(2x-3)}{3} = \frac{30(11)}{6}$$

$$6(4x+1) + 10(2x-3) = 5(11)$$

[1]

$$24x + 6 + 20x - 30 = 55$$

$$44x = 79$$

$$x = \frac{79}{44} = 1.8$$

[1]

b) Factorise completely, the following algebraic expressions:

$$\text{i. } P^2 - 13P + 42$$

[2]

$$P^2 - 13P + 42$$

$$(P - 6)(P - 7)$$

$$\text{ii. } 4b^2 - 49 \quad [2]$$

$$4b^2 - 49$$

$$(2b - 7)(2b + 7)$$

$$\text{iii. } mn + 5mp + 3nq + 15pq \quad [2]$$

$$mn + 5mp + 3nq + 15pq$$

$$m(n + 5p) + 3q(n + 5p)$$

$$= (n + 5p)(m + 3q)$$

c) Solve the following quadratic equations by using the factorisation method:

$$\text{i. } 6m^2 + 17m + 12 = 0 \quad [2]$$

$$(3m + 4)(2m + 3) = 0$$

$$3m + 4 = 0 \quad \text{or} \quad 2m + 3 = 0$$

$$3m + 4 = 0, \quad 3m = -4, \quad m = -\frac{4}{3}$$

$$2m + 3 = 0, \quad 2m = -3, \quad m = -\frac{3}{2}$$

[2]

$$\text{ii. } y^2 + 13y + 40 = 0 \quad [2]$$

$$y^2 + 13y + 40 = 0$$

$$(y + 8)(y + 5) = 0$$

$$y + 8 = 0 \quad \therefore y = -8$$

$$y + 5 = 0 \quad \therefore y = -5$$

[2]

d) Solve the following quadratic equation by using the quadratic formula:

$$3w^2 - 2w - 8 = 0$$

[4]

$$w = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$w = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-8)}}{2(3)}$$

[1]

$$w = \frac{2 \pm \sqrt{4 + 96}}{6} = \frac{2 \pm \sqrt{100}}{6}$$

[1]

$$w = \frac{2 \pm 10}{6}$$

$$w = \frac{2 + 10}{6} = 2$$

$$w = \frac{2 - 10}{6} = -1.333$$

[2]

End