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OPTIMISATION AND SIMULATION TECHNIQUES FOR DESIGNING RACE CAR WINGS



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[Introduction: The Evolution and Importance of Finite Element Analysis](#)

Finite Element Analysis (FEA) is a computational method used to predict how a product will interact with real-world physical forces. These include forces such as stress, vibration, heat, fluid flow, and more. The method divides a large, complex problem into a collection of smaller, simpler elements that are interconnected at points called nodes. The behaviour of each element is accurately described with a set of equations, and the overall behaviour of the entire system is then found by assembling these individual equations together (Bathe, 1996).

FEA is a pivotal tool in a range of fields. Structural engineers, for instance, utilize it to understand how structures such as bridges or buildings will respond to loads and stresses. Automotive and aerospace industries apply it to optimize designs for vehicles and aircraft, improving their efficiency and safety. In the field of biomechanics, FEA helps researchers understand the mechanics of biological systems, aiding in the development of medical implants and prosthetics.

[Understanding FEA Optimization Techniques](#)

Optimization of FEA can dramatically enhance its efficiency, accuracy, and overall utility. Optimization processes help to fine-tune the model by refining parameters, reducing errors, and ensuring that the model accurately represents real-world behaviour as closely as possible.

[Adaptive Meshing](#)

Adaptive meshing, for example, refines the mesh in areas where high stresses or gradients are expected, thus enhancing the precision of results (Babuska and Strouboulis, 2001). This development was a crucial step in the evolution of FEA, facilitating more accurate and efficient simulations.

[Model Reduction Techniques](#)

Model reduction techniques have also proven pivotal in the enhancement of FEA. Techniques such as Proper Orthogonal Decomposition (POD) and Component Mode Synthesis (CMS) simplify models while maintaining essential dynamic characteristics, thereby reducing the computational complexity of large-scale finite element models (Kerschen et al., 2005).

[Historical Progression of FEA Optimization Techniques](#)

The concept of FEA dates back to the work of the mathematician Richard Courant in the 1940s, who developed the initial concepts in the context of solving problems in torsion (Courant, 1943). However, the significant advancement of FEA as a practical tool for engineering design did not occur until the 1970s. The advent of powerful and affordable digital computers marked a turning point. These computational advancements made it possible to break down complex structures into finite elements automatically, rather than the time-consuming task of manual creation. This automation sped up model generation and analysis, marking a significant step forward in the field (Zienkiewicz and Taylor, 2000).

[Application of FEA Optimization in Modern Design Process](#)

The implications of these optimizations for the design process are vast. They allow for the rapid iteration and optimization of designs, early identification of potential issues, reduction in the need for costly physical prototypes, and exploration of a wider range of design alternatives. These benefits can lead to more innovative and effective solutions, reduced design cycles, lower costs, improved product performance, and heightened reliability.

Anticipating the Future: Predicted Developments in FEA Optimization Techniques

Looking to the future, trends in FEA optimization appear to follow the trajectory of broader computational advancements. Continued growth in computational power, including the advent of parallel processing and potential developments in quantum computing, may enable the handling of increasingly complex models and simulations at a faster rate. Furthermore, the intersection of FEA and artificial intelligence opens up a new frontier for optimization. Machine learning techniques can further automate the prediction and optimization process, perhaps leading to 'intelligent' FEA software that can accurately predict optimal parameters with less trial-and-error (Yang et al., 2019).

Moreover, the integration of FEA with digital twin technology, a virtual replica of a physical product that mirrors real-time changes, could offer robust predictive and diagnostic capabilities. The amalgamation of these technologies could be used to predict the performance of a design under various conditions and to monitor and troubleshoot issues in real-time (Rosen et al., 2015).

Conclusion: The Continued Impact of FEA Optimization on Design Efficiency

In conclusion, the value of Finite Element Analysis (FEA) in engineering and design cannot be overstated. As a computational tool that enables the simulation of physical phenomena, FEA has had a transformative impact across various industries, from structural engineering and automotive design to biomechanics and beyond (Hughes, 2012). Its ability to predict how products react to real-world forces, coupled with the power to simulate complex systems, has allowed for unprecedented levels of design refinement and innovation (Zienkiewicz et al., 2005).

The continuous development and optimization of FEA techniques over the years has greatly increased its utility. The development of adaptive meshing and model reduction techniques, for instance, has not only improved the accuracy of FEA simulations but also made them more efficient (Liu, 2013). These advances have helped tackle more complex problems and delivered more reliable solutions, thus helping to streamline the design process (Aitharaju & Ananthasuresh, 2005; Jee & Kim, 2019).

Looking forward, the prospects for FEA optimization techniques appear highly promising. The integration of FEA with other computational techniques, such as parallel processing, quantum computing, machine learning, and AI, is anticipated to further enhance the predictive and optimization capabilities of FEA (Oden et al., 2006). This could transform FEA into an intelligent system that can provide better guidance for design parameters, thereby reducing the amount of trial and error involved in setting up an FEA model.

Moreover, the merging of FEA with digital twin technology offers exciting potential for predictive and diagnostic capabilities (Rosen et al., 2015). This could revolutionize how we monitor and manage the lifecycle of structures and systems, providing powerful tools for predictive maintenance and fault detection.

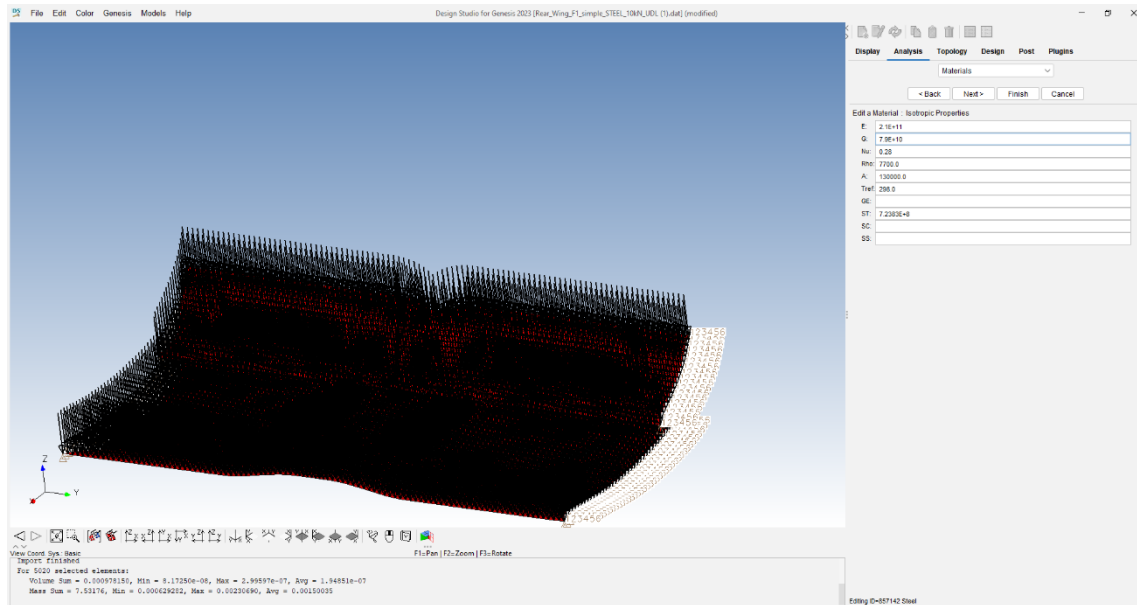
The impact of these advancements on the design process will be profound. They promise to speed up design cycles, improve product performance, reduce costs, and boost reliability. Furthermore, by allowing engineers and designers to anticipate potential problems early in the design process, FEA optimization can minimize the need for costly reworks and physical prototypes, leading to more sustainable practices in design and manufacturing (Bathe, 2006).

While it is important to keep in mind that these developments will require us to address new challenges, such as data privacy, security, and the need for new skills and training, the overall implications of optimized FEA are undeniably positive (Kruse & Fütterer, 2020). Ultimately, these advancements in FEA optimization promise to push the boundaries of what is possible in design and engineering, fostering greater innovation and efficiency across numerous fields (Hughes, 2012; Zienkiewicz et al., 2005).

Therefore, FEA, with its continuous optimization and integration with emerging technologies, stands as a testament to the power of computational design tools (Oden et al., 2006). As we move forward, these evolving capabilities will undoubtedly continue to redefine the landscape of design, driving us toward an era of unprecedented technological innovation and creative solutions.

Design and Layup of a 25-Ply Wing Model in Design Studio

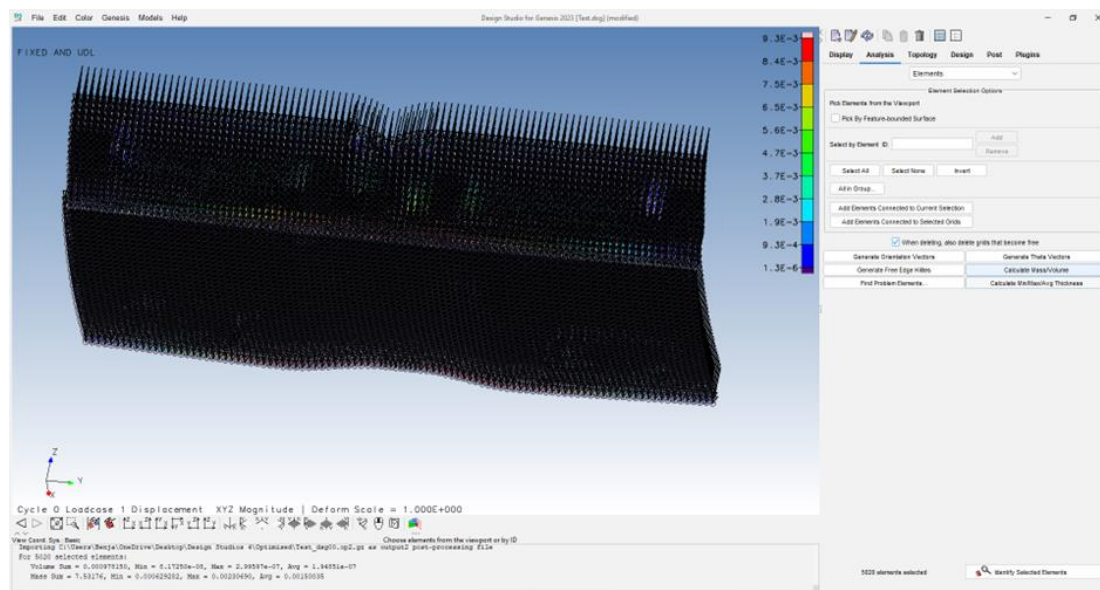
The design of the rear wing was provided, and it was made of Steel with Isotropic Properties:



For the purpose of comparison, research was conducted to understand the Newton force that is imposed upon an F1 rear wing and the deflection requirements. The research used stated the following: “The test demands that the rear wing can deflect no more than 2mm vertically when a 200N load is applied vertically.” (Noble, J., 2020)

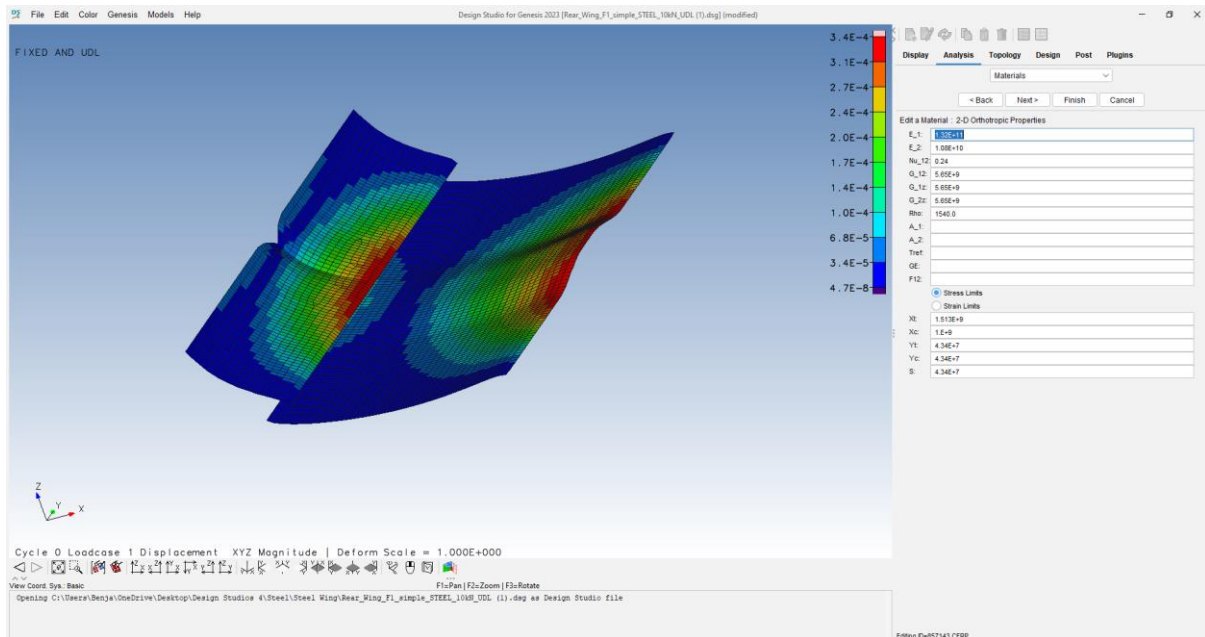
- 200N Load vertically imposed on the rear wing.
- Maximum 2mm Deflection

The steel model of the wing was imposed with 200N of downward force and deflected by 0.34mm, which meets the criteria. However it does not make it the optimum choice due to the 7.5Kg Mass.

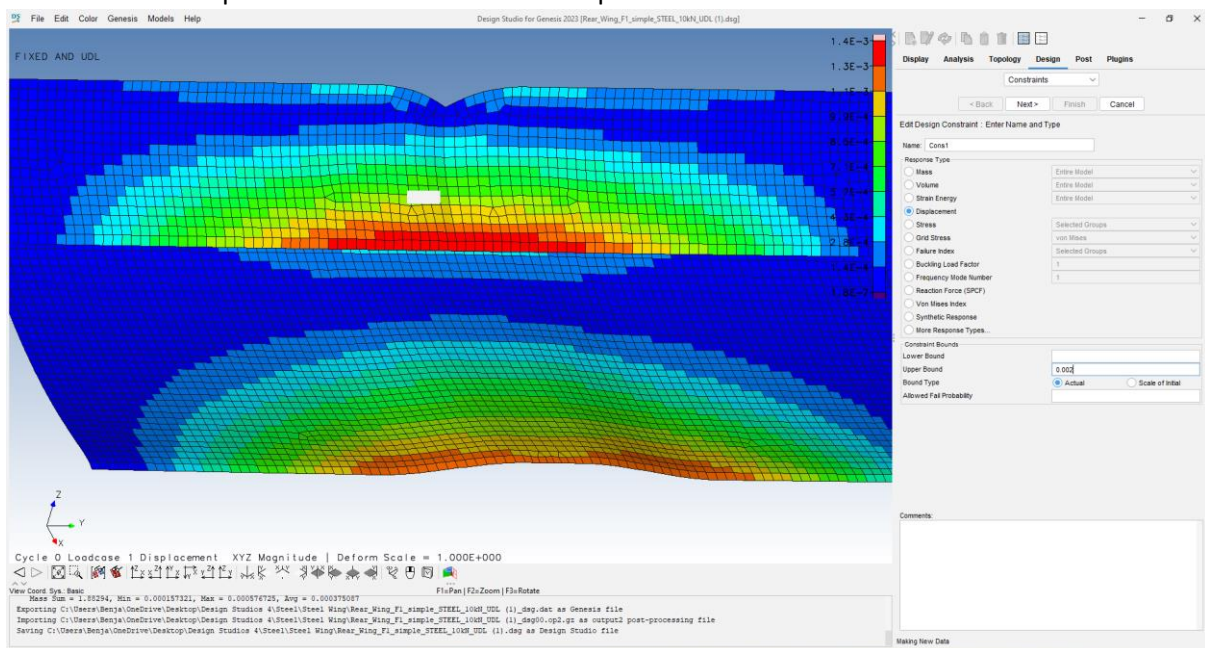


The simulation was conducted by spreading a 200N force across 5263 grids, which equated to 0.038N/Grid.

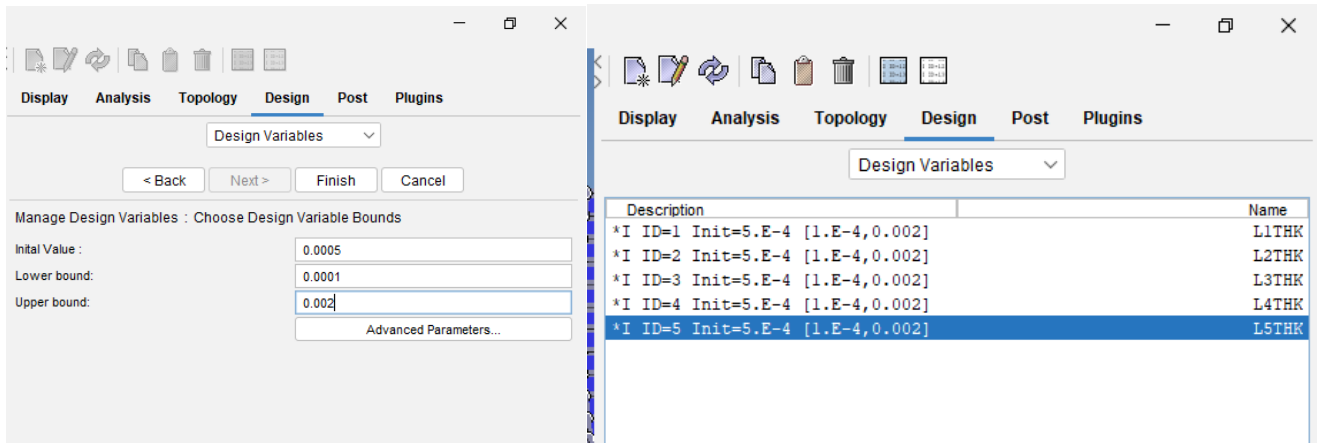
Since the material was steel, there was a need to change its properties to simulate that of Carbon Fibre Reinforced Plastics (CFRP). The properties used were provided in the assignment brief, and a new material with orthotropic properties was created.



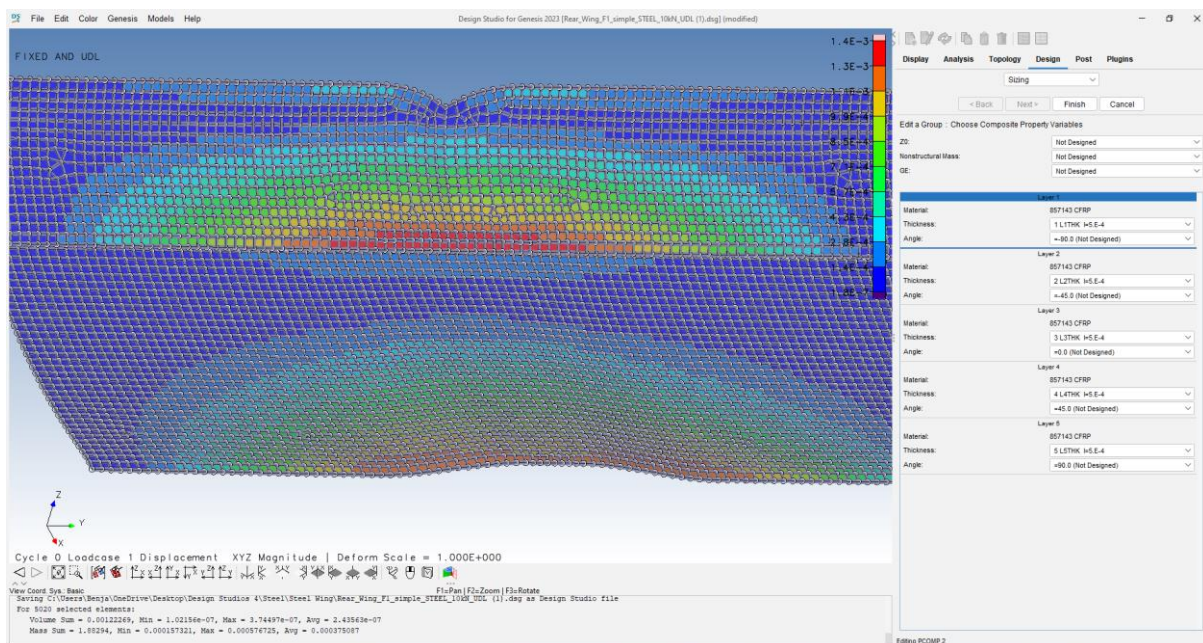
The maximum displacement limit of 2mm was implemented to adhere to the tolerances researched.



The layer design variables were set according to the assignment brief, where 1 Ply equals 0.1mm in thickness. Each layer was assigned its corresponding CFRP Material. The layers consist of 5 Plies = 0.5mm Thickness/Layer.



The Layers were assigned to their corresponding CFRP Material:



The rear wing underwent analysis using both Steel and Composite CFRP. The data provided from those is as follows:

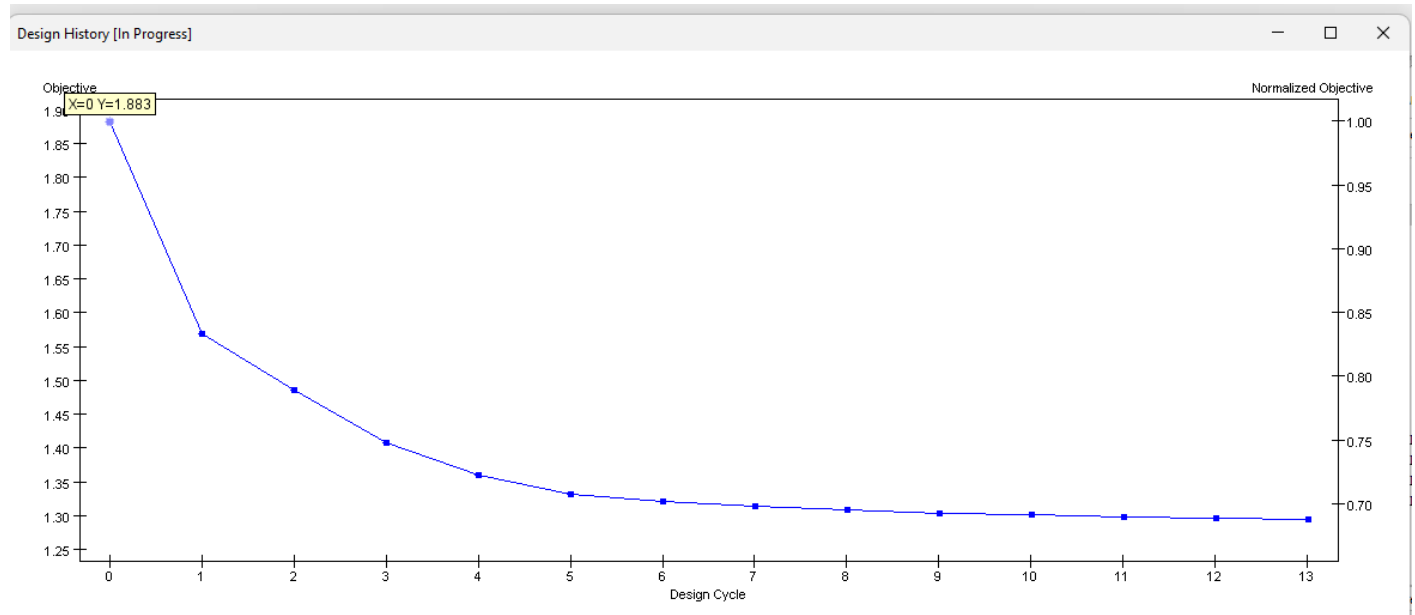
RUN	MATERIAL	MASS	DEFLECTION	LOAD	STIFFNESS	STIFFNESS/WEIGHT
Steel Baseline	Steel	7.53Kg	0.34mm	200N	588N/mm	7.96N ² /mm
Composite Baseline	CFRP	1.88Kg	1.4mm	200N	142N/mm	7.69N ² /mm

The data indicates an improvement from the Steel design. The mass decreased by 5.68Kg, while the deflection decreased by 7.9mm to meet the criteria of 200N Force with <2mm of deflection. However, the design can further be improved by optimizing using the Design Studios Genesis software.

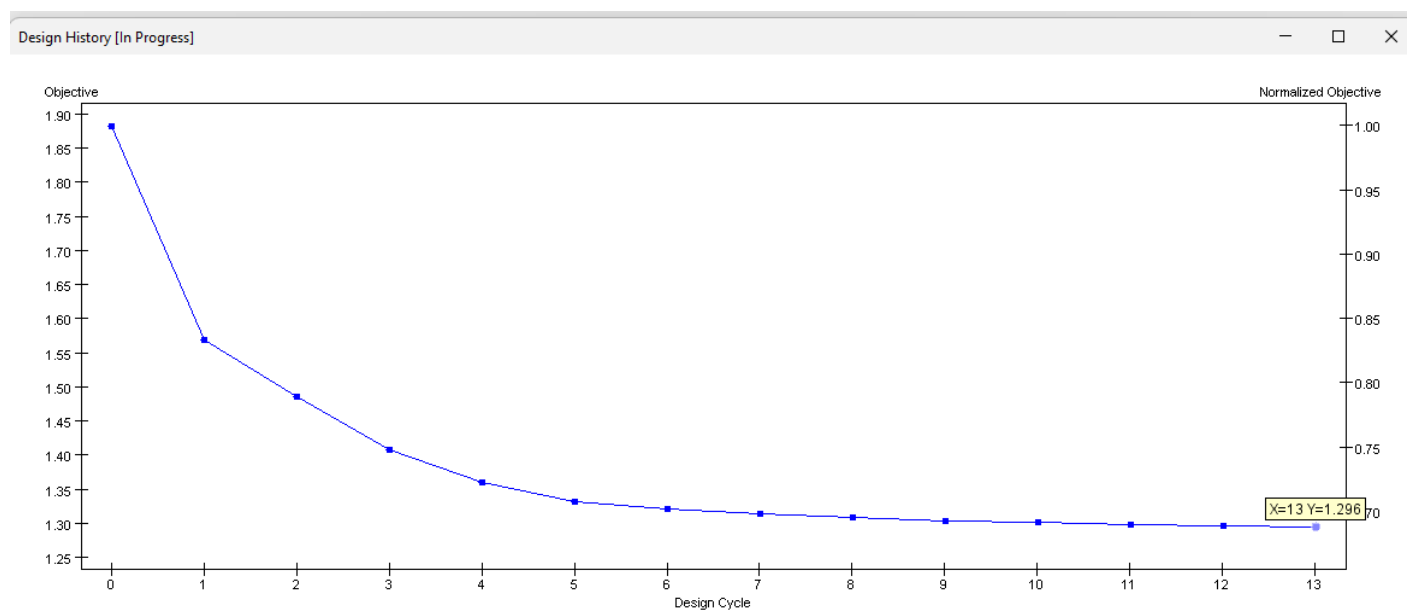
Optimisation of Wing Structure: Minimising Mass with Displacement Constraints

With the prior changes in material from Steel to CFRP an improvement was made. Further progressing this, Design Studios can optimize the structure through Genesis. This involved reducing weight while meeting the 2mm displacement constraints. The mass of the wing started at 1.88Kg as an un-optimised CFRP.

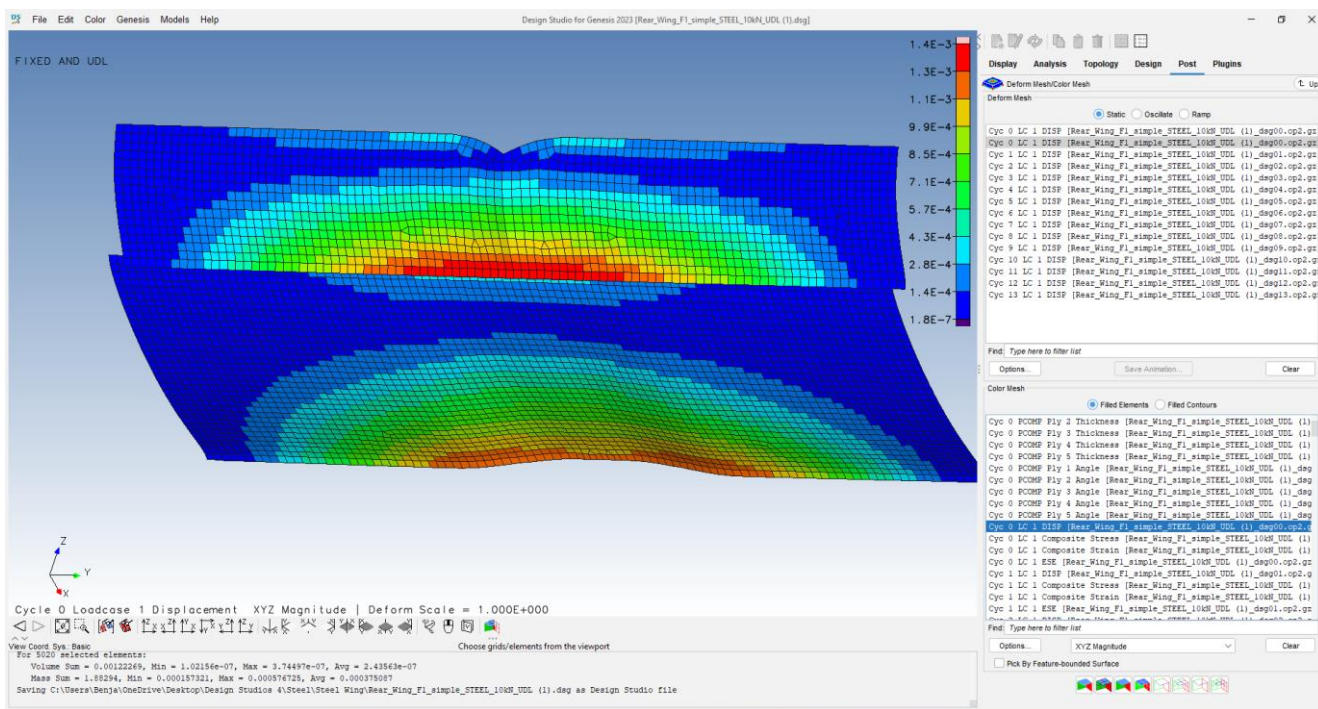
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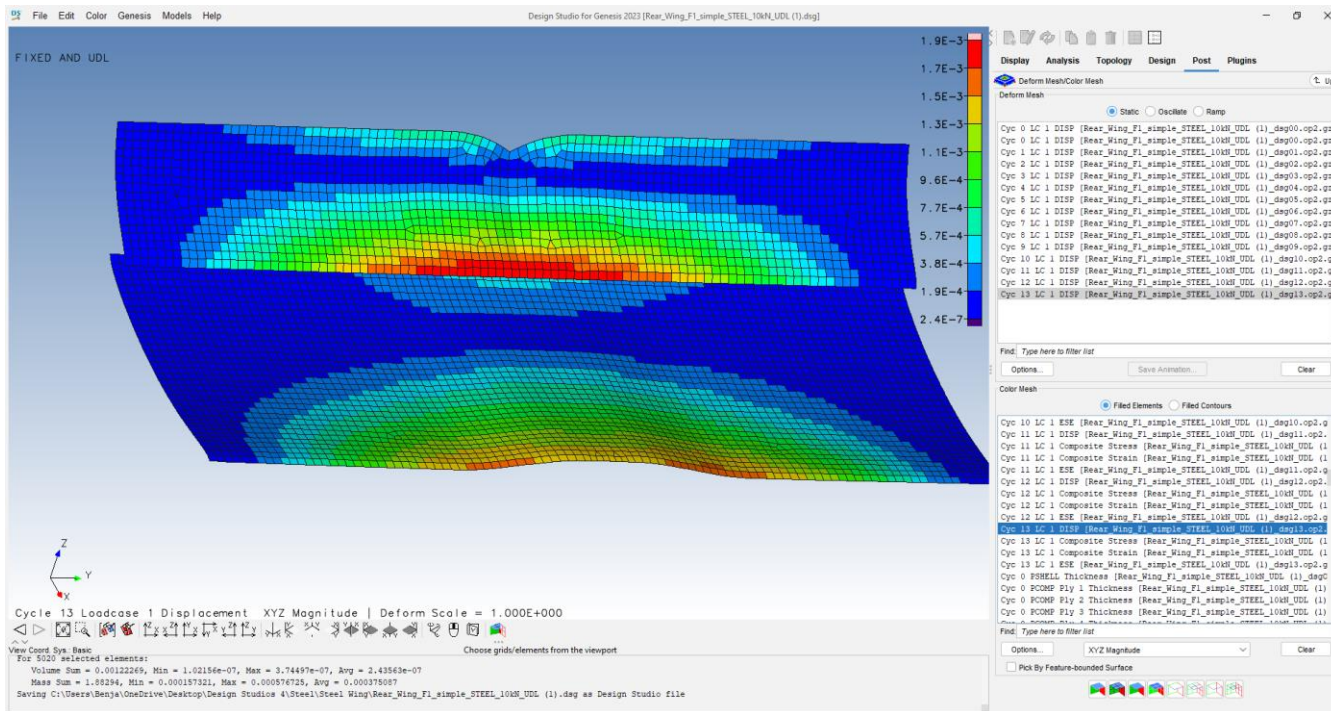
The rear wing optimised over the course of 13 design cycles and reduced to a mass of 1.3Kg.



With this optimisation a change in deflection was also noted between Design Cycle 0 and Design Cycle 13.



The original Deflection of 1.4mm optimized to be 1.9mm of Deflection which is within the 2mm tolerance and achieving optimum stiffness/weight ratio.



The optimized structure was then compared in the table:

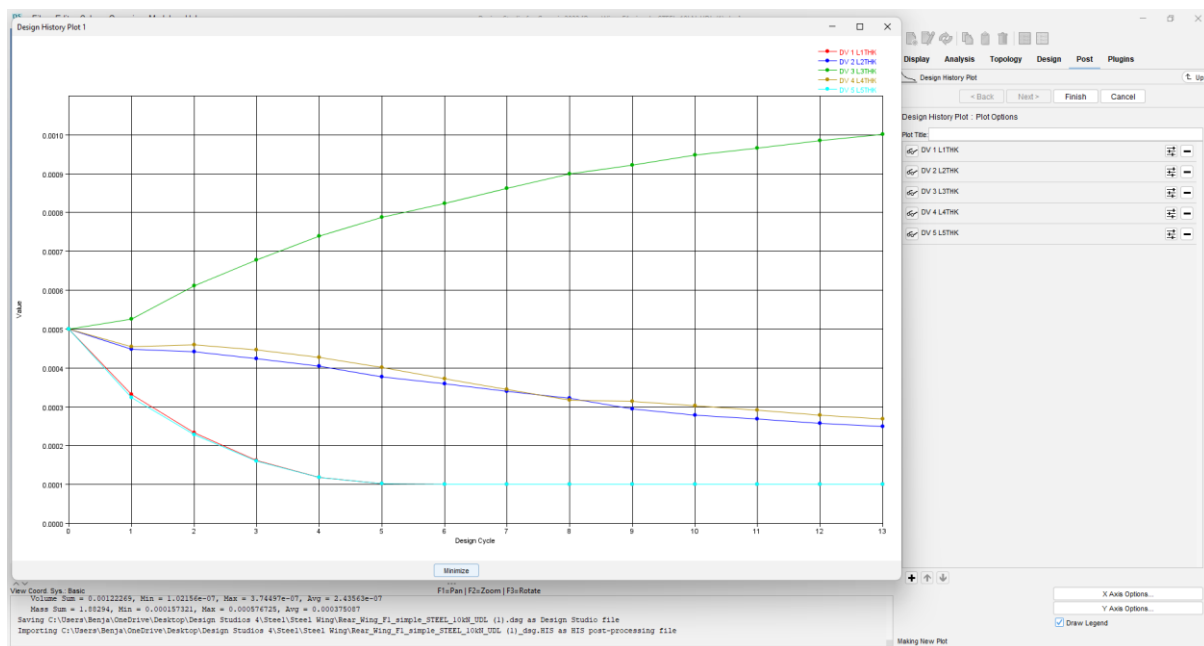
RUN	MATERIAL	MASS	DEFLECTION	LOAD	STIFFNESS	STIFFNESS/WEIGHT
Steel Baseline	Steel	7.53Kg	0.34mm	200N	588N/mm	7.96N ² /mm
Composite Baseline	CFRP	1.88Kg	1.4mm	200N	142N/mm	7.69N ² /mm
Optimized Composite	CFRP	1.3Kg	1.9mm	200N	105N/mm	8.27N ² /mm

The mass of the rear wing was reduced by a further 580g, which allowed for an increase in deflection by 0.5mm, while still maintaining the <2mm deflection limit. A Design History Plot was obtained to observe how the layer thickness evolved due to optimization. Before optimization, each layer was 0.5mm thick, calculated as (1 Ply=0.1mm) x (5 Ply/Layer) = 25 plies.

- Layer 1 was optimized to have a thickness of 0.1mm (1 Ply) at -90°.
- Layer 2 was optimized to have a thickness of 0.297mm (2.97 plies) at -45°.
- Layer 3 was optimized to have a thickness of 1mm (10 plies) at 0°.
- Layer 4 was optimized to have a thickness of 0.2693mm (2.693 plies) at 45°.
- Layer 5 was optimized to have a thickness of 0.1mm (1 Ply) at 90°.

The total number of plies came to 17.663.

This design resulted in a reduction of 7.337 plies to optimize the structure. However, since each ply has a thickness of 0.1mm, the optimization that resulted in 2.97 and 2.693 plies is not manufacturable.



The following the data once optimized on the 13th design cycle with a Mass of 1.296Kg and also the Layer thickness.

DESIGN CYCLE # 13, OBJECTIVE = 1.296210E+00, MAXIMUM VIOLATION = 0.0%.

DESIGN VARIABLE # 1 = 1.001202E-04 L1THK

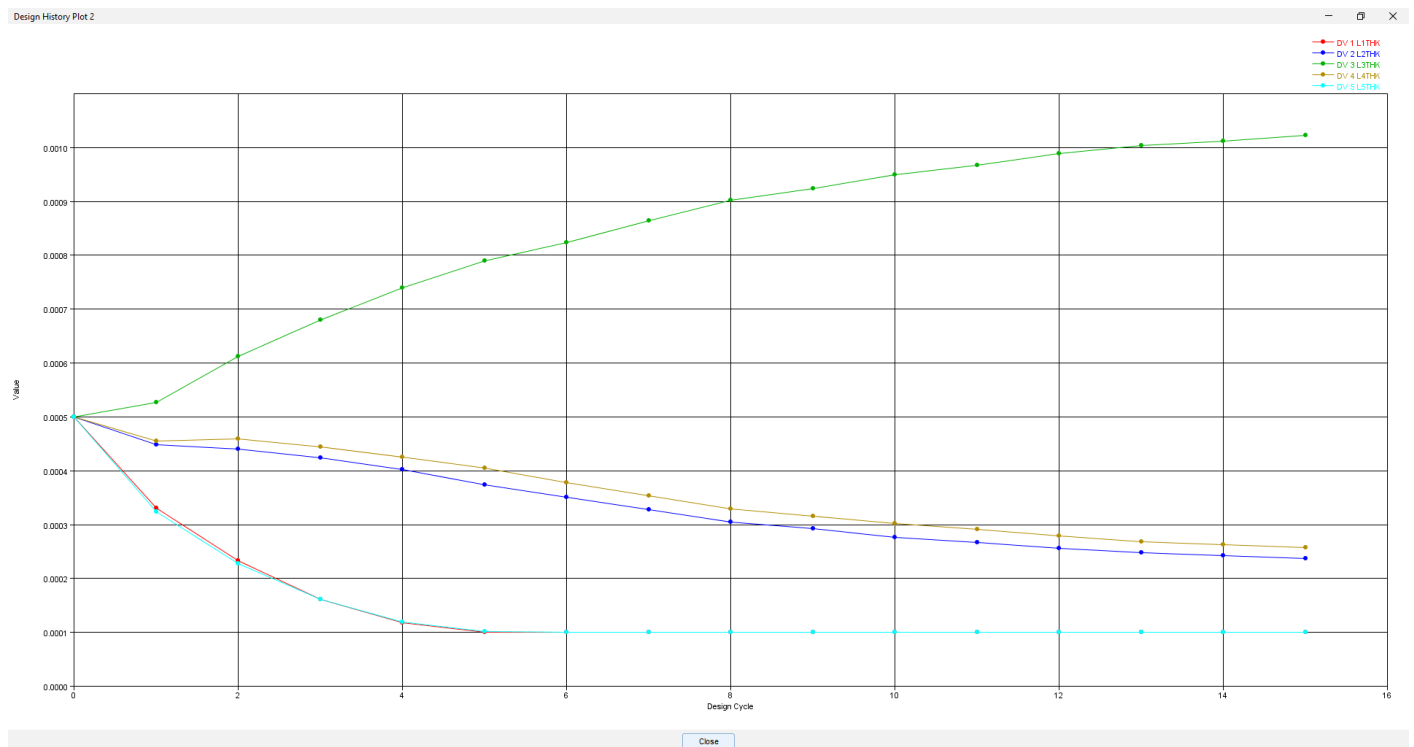
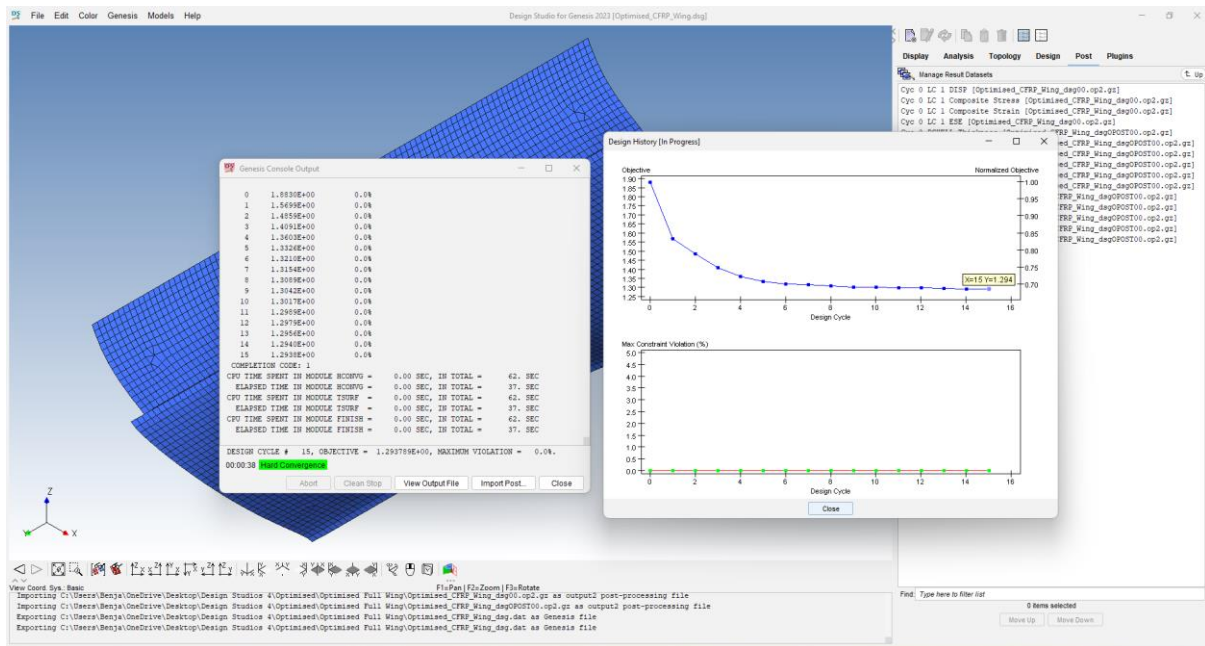
DESIGN VARIABLE # 2 = 2.496518E-04 L2THK

DESIGN VARIABLE # 3 = 1.001758E-03 L3THK

DESIGN VARIABLE # 4 = 2.692918E-04 L4THK

DESIGN VARIABLE # 5 = 1.001233E-04 L5THK

After further optimization, a new set of data was collected. The weight was reduced to 1.294kg, but the displacement remained consistent at 1.9mm.



DESIGN CYCLE # 15, OBJECTIVE = 1.293789E+00, MAXIMUM VIOLATION = 0.0%.

DESIGN VARIABLE # 1 = 1.000000E-04 L1THK

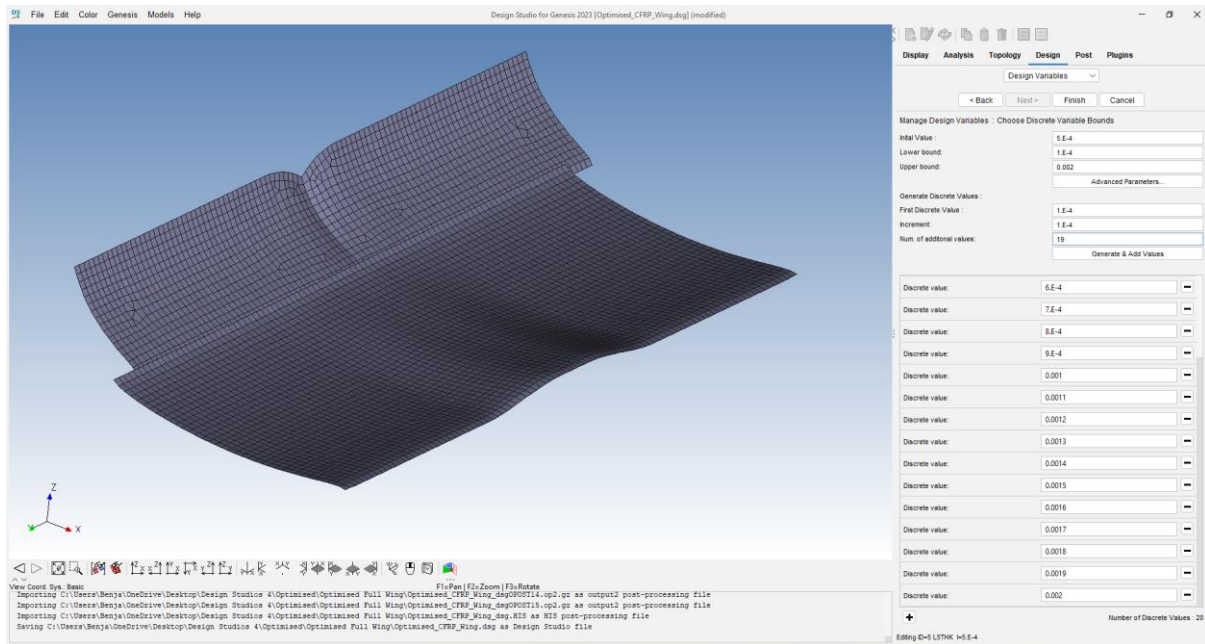
DESIGN VARIABLE # 2 = 2.370798E-04 L2THK

DESIGN VARIABLE # 3 = 1.022774E-03 L3THK

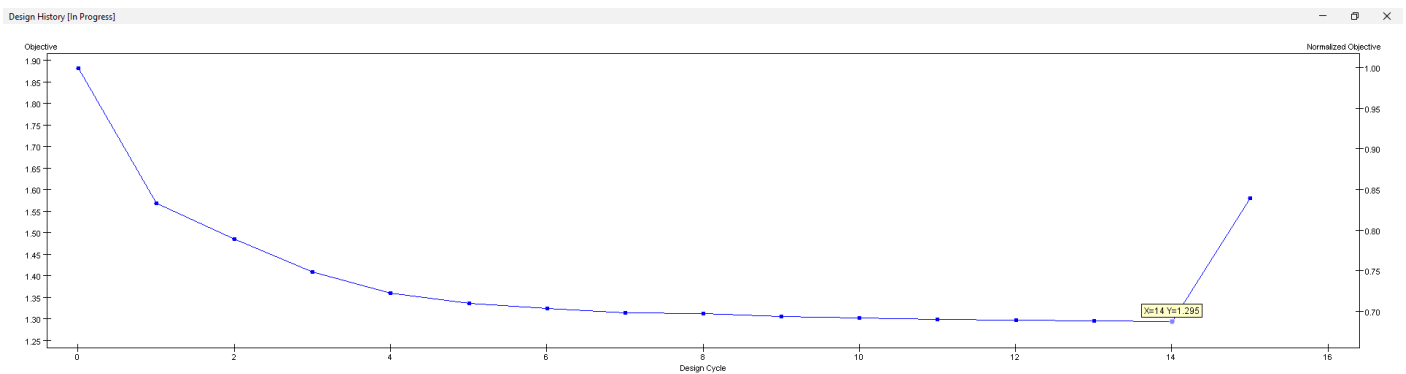
DESIGN VARIABLE # 4 = 2.578774E-04 L4THK

DESIGN VARIABLE # 5 = 1.000000E-04 L5THK

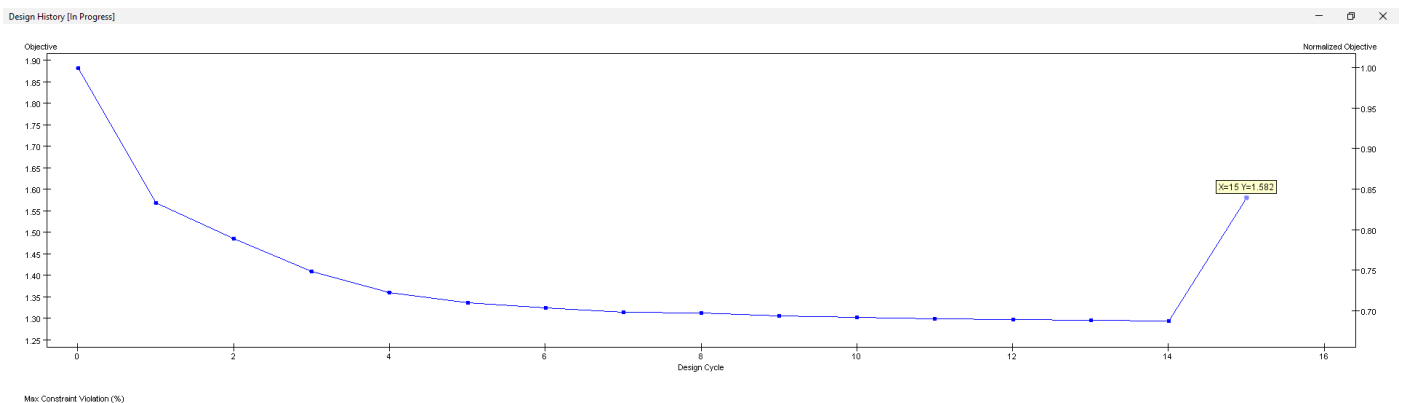
Due to the plies not being manufacturable, discrete values were set to define those variables. They allow up to 2mm in discrete values to ensure manufacturability of the design. The discrete values increment from 0.1mm to 2mm by 0.1mm at a time to reach the 2mm.



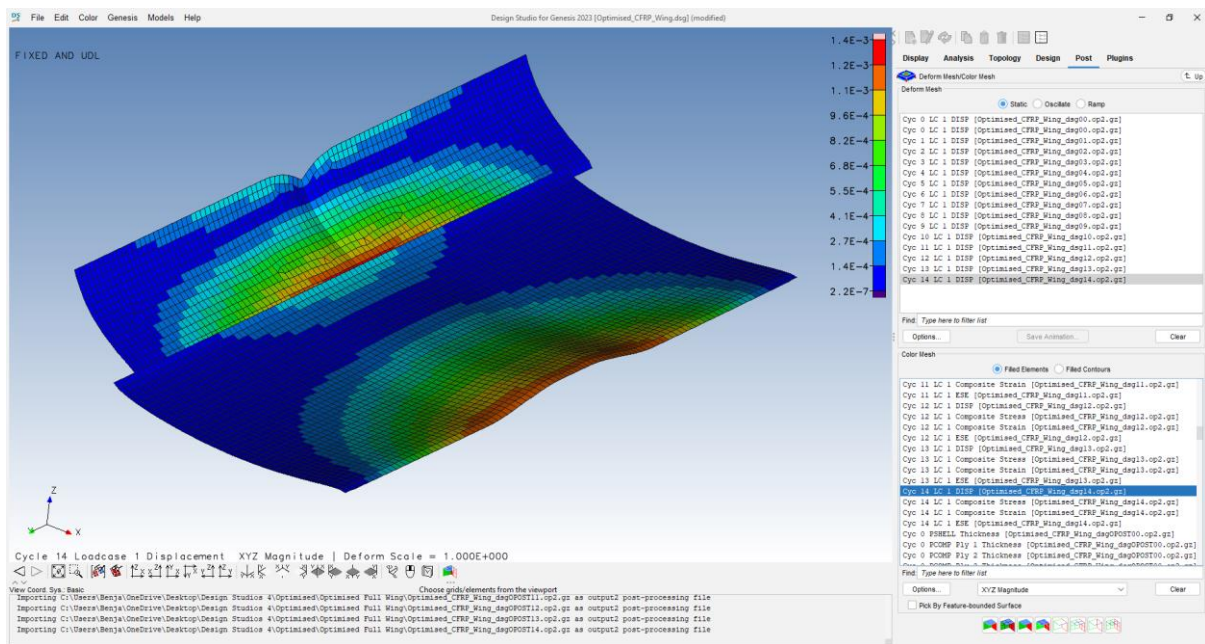
From the data, it is evident that the design was optimized to its lowest mass. In the final plot, the discrete design variables increased the mass to ensure the plies remained within the specified limit.



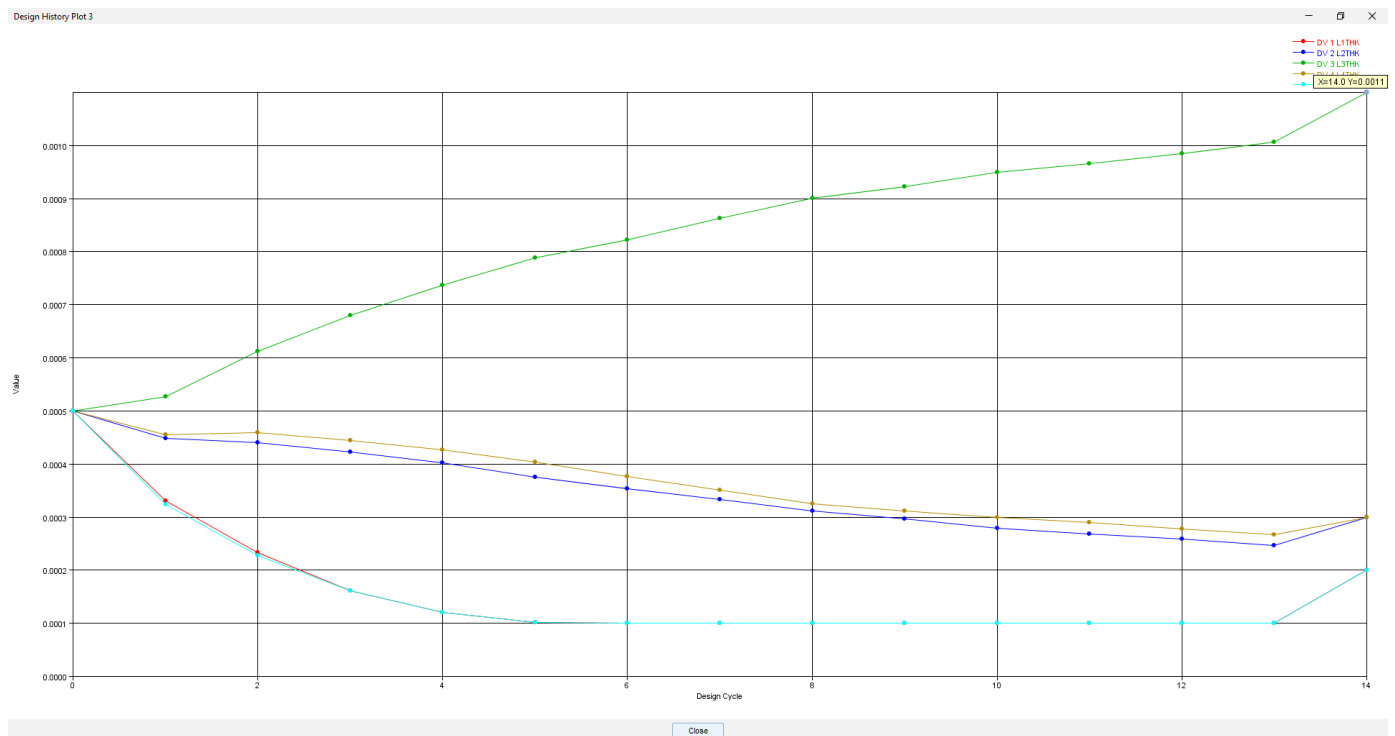
Because of this, there is an increase from 1.294kg up to 1.582kg to meet manufacturability.



Deflection for a Manufacturable Design – 1.4mm Deflection which is 0.5mm less than the ideal optimisation. Mass 1.582kg.



The design now meets the manufacturability constraints to be created.



DESIGN CYCLE # 0, OBJECTIVE = 1.882991E+00, MAXIMUM VIOLATION = 0.0%.

DESIGN VARIABLE # 1 = 5.000000E-04 L1THK

DESIGN VARIABLE # 2 = 5.000000E-04 L2THK

DESIGN VARIABLE # 3 = 5.000000E-04 L3THK

DESIGN VARIABLE # 4 = 5.000000E-04 L4THK

DESIGN VARIABLE # 5 = 5.000000E-04 L5THK

DESIGN CYCLE # 14, OBJECTIVE = 1.581713E+00, MAXIMUM VIOLATION = 0.0%.

DESIGN VARIABLE # 1 = 2.000000E-04 L1THK

DESIGN VARIABLE # 2 = 3.000000E-04 L2THK

DESIGN VARIABLE # 3 = 1.100000E-03 L3THK

DESIGN VARIABLE # 4 = 3.000000E-04 L4THK

DESIGN VARIABLE # 5 = 2.000000E-04 L5THK

The Design to be manufacturable optimised to have 21 Ply's = 2.1mm thickness with 1.4mm of deflection and a mass of 1.582Kg. With the optimized manufacturable data the table is as follows:

RUN	MATERIAL	MASS	DEFLECTION	LOAD	STIFFNESS	STIFFNESS/WEIGHT
Steel Baseline	Steel	7.53Kg	0.34mm	200N	588N/mm	7.96N ² /mm
Composite Baseline	CFRP	1.88Kg	1.4mm	200N	142N/mm	7.69N ² /mm
Optimized Composite	CFRP	1.3Kg	1.9mm	200N	105N/mm	8.27N ² /mm
Optimized Manufacturable Composite	CFRP	1.58Kg	1.4mm	200N	142N/mm	9.15N ² /mm

The optimised manufacturable CFRP is 1.58Kg with a deflection of 1.4mm. The Ply's are set out in the following angles:

- Layer 1 – 2 Plies at -90
- Layer 2 – 3 Plies at -45
- Layer 3 - 11 Plies at 0
- Layer 4 – 3 Plies at 45
- Layer 5 – 2 Plies at 90

Although an increase was seen in mass for the final part the Stiffness/Weight increased compared to the Optimized Composite. While this can be seen as a benefit it is also a downside. In Formula 1 competition, minimizing weight is crucial to optimize performance. A lighter vehicle benefits from quicker acceleration, enhanced braking capabilities, and sharper cornering precision. When engine power remains consistent, reducing the car's weight by even a few kilograms can lead to significant improvements in lap times, potentially elevating a car from a mid-grid position to pole. By harnessing materials like carbon fibre composites and innovative metal alloys, as well as through judicious component positioning, teams can achieve weight savings without compromising the vehicle's safety or structural soundness. Furthermore, how weight is distributed is instrumental in influencing the vehicle's stability and manoeuvrability, ensuring tires maintain optimal traction. Tactics to decrease weight in F1 vehicles are pivotal for both swiftness and gaining a competitive edge on the track. (Nevey, S. 2002)

Experimentation for F1 Rear Wing Prototype

1. Objectives:

- Purpose:
 - Measure the rear wing's actual deflection when subjected to specific forces.
 - Validate simulation results from Design Studios Genesis.
- Discussion:
 - Validating simulation results is essential as relying solely on software can lead to oversights. It ensures the wing's performance is not only efficient on paper but also in real-world scenarios, maintaining F1 car stability and efficiency.

2. Prototype Creation:

- Procedure:
 - Fabricate the wing using the specified number of plies and their orientations as per the simulation.
- Discussion:
 - Replicating the design from the simulation is crucial. Even minor deviations in ply orientation or number can lead to different structural behaviours, affecting the comparison's validity.

3. Test Setup:

- Procedure:
 - Implement tests in a controlled environment like a wind tunnel or static test rig.
 - Ensure the wing's orientation corresponds with the force direction (vertical or horizontal) and the magnitude (200N or 500N).
- Discussion:
 - Using a controlled environment eliminates inconsistencies from external variables. The static test rig, for instance, helps in isolating the wing's behaviour from other factors, providing a direct comparison with the simulation.

4. Instrumentation:

- Procedure:
 - Strategically place strain gauges on the wing to capture localized strain.
 - Use high-resolution displacement sensors like LVDTs at loading points.
 - Prioritize sensor calibration to ensure accurate readings.
- Discussion:
 - Strain gauges are chosen because they provide insights into localized stress and potential weak points. Meanwhile, LVDTs offer precise deflection measurements, directly comparing with simulation predictions.

5. Testing:

- Procedure:
 - Secure and stabilize the wing, gradually apply loads, record strain and deflection data, and note any permanent changes post-test.
 - Conduct individual tests for each wing.
- Discussion:
 - The emphasis on stabilization and gradual load application ensures that the wing's response is purely due to the applied loads and not external dynamics. Individual wing tests help in identifying any inconsistencies between the wings, ensuring comprehensive validation.

6. Data Acquisition:

- Procedure:
 - Log sensor data in a computerized system, with timestamps facilitating analysis.
 - Use graphs for an intuitive comparison with simulation results.
- Discussion:
 - Automated logging systems reduce errors and streamline the comparison process. Graphical representations highlight discrepancies, enabling swift corrective actions or insights.

7. Comparison with Simulation:

- Procedure:
 - Overlay experimental and simulation data, calculating percentage differences for strain and deflection.
 - Deep dive into significant variance areas.
- Discussion:
 - Overlapping real and simulated data provides a visual measure of accuracy. Discrepancies can emerge from multiple sources – be it simulation assumptions, test environment variations, or manufacturing nuances. Understanding these differences can inform future design refinements.

8. Considerations:

- Points:
 - Monitor environmental conditions; smoothly apply loads; consider multiple test iterations.
- Discussion:
 - CFRP's properties can vary with temperature and humidity, so monitoring and possibly controlling these factors is pivotal. Repeated tests ensure reliability, helping rule out anomalies or outliers.

9. Conclusion & Recommendations:

- Summary:
 - Collate findings, emphasizing matches and divergences between simulation and experiment.
- Discussion:
 - Acceptable discrepancies validate the simulation's reliability. However, significant variances necessitate revisiting simulation parameters, prototype evaluation, or both. Recommendations could involve design tweaks, material considerations, or altered simulation assumptions based on experimental insights.

Comprehensive Analysis and Correlation of F1 Rear Wing Design to Technical Literature and Industry Standards

The F1 rear wing is a critical component of a Formula 1 car, playing a fundamental role in the aerodynamic performance of the vehicle. It influences the car's top speed, stability, and the ability to follow other cars closely.

To understand the F1 rear wing, it's important to consult various technical literature and standards:

1. **Fédération Internationale de l'Automobile (FIA) Technical Regulations:** The FIA, which is the governing body for motorsport worldwide, has a set of technical regulations for Formula 1. The rear wing's dimensions, materials, location, and many other aspects are meticulously defined within these regulations to ensure a level playing field and safety.
2. **Aerodynamics of Road Vehicles:** This comprehensive text covers the principles of aerodynamics as they apply to road vehicles, including race cars. While not F1-specific, it provides foundational knowledge on the subject.
3. **"Race Car Aerodynamics: Designing for Speed" by Joseph Katz:** This book is another essential reference for understanding racing car aerodynamics, and it delves into the principles behind wing design and the interactions of various aerodynamic components.
4. **Computational Fluid Dynamics (CFD) Journals:** Many technical articles in journals cover the specifics of flow over racing cars, including the intricacies of the rear wing. These are more advanced, providing insights into innovative research and methodologies for optimizing wing design.
5. **Technical Papers from the Society of Automotive Engineers (SAE):** The SAE has a range of technical papers discussing innovations, experiments, and findings related to the automotive field, including Formula 1 aerodynamics.
6. **Formula 1 Team Technical Publications:** Occasionally, F1 teams release technical documents or presentations that discuss their aerodynamic philosophies or provide insights into their design processes. These are valuable for understanding specific design rationales and innovations.
7. **Patent Databases:** F1 teams sometimes patent innovative design elements, and these patents can be a wealth of information. While they can be technical and dense, they often contain diagrams and detailed descriptions.
8. **"The Science of F1 Design" by David Tremayne:** This book delves into the technicalities of F1 car design, touching on the importance and evolution of the rear wing.
9. **Wind Tunnel Testing Literature:** Wind tunnels are extensively used in the development of F1 car components, including the rear wing. Reading up on the methodologies and principles behind wind tunnel testing can provide a better understanding of how the rear wing's design is validated and refined.
10. **DRS (Drag Reduction System) Literature:** The DRS is an adjustable flap on the rear wing, introduced by the FIA to reduce drag and thereby increase speed on specific parts of the track. Understanding DRS and its implications requires a deep dive into both FIA regulations and aerodynamics literature.

When designing or analyzing the F1 rear wing, it's crucial to have a comprehensive grasp of the regulatory environment, the scientific principles at play, and the state-of-the-art methodologies employed in the sport. The above literature and standards provide a roadmap to attain this understanding.

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