

Q 1

a)

i)

$$y = \frac{x^2}{2x+1}$$

$$f = 2x+1$$

$$g = x^2$$

$$y' = (2x+1) \times (2x)$$

$$\frac{(2x+1) \times (2x)}{(2x+1)^2}$$

i)

$$y = \frac{x^2}{2x+1}$$

$$= \frac{2x+1(2x) - x^2(2)}{(2x+1)^2}$$

$$= \frac{4x^2 + 2x - 2x^2}{(2x+1)^2}$$

$$\boxed{= \frac{2x^2 + 2x}{(2x+1)^2}}$$

$$\begin{aligned}
 \text{(ii)} \quad y &= \sqrt{x^2 + 1} \\
 &= \frac{1}{2} \times (x^2 + 1)^{-\frac{1}{2}} + \frac{dy}{dx} (x^2 + 1) \\
 &= \frac{1}{2} + (-\sqrt{x^2 + 1}) \times 2x \\
 &\quad x(-\sqrt{x^2 + 1})
 \end{aligned}$$

$$\boxed{= \frac{dx}{\sqrt{x^2 + 1}}}$$

$$\begin{aligned}
 \text{(iii)} \quad y &= 3x^{\frac{3}{2}} + \frac{3}{x^2} - 6x \\
 &= \frac{9}{2} x^{\frac{1}{2}} + \frac{dy}{dx} 3x^{-2} - 6 \\
 &= \frac{9}{2} \times \frac{1}{2} x^{-\frac{1}{2}} - 6x^{-3} - 6
 \end{aligned}$$

$$\boxed{= \frac{9}{2} \sqrt{x} + \frac{-6}{x^3} - 6}$$

iv) $y = \sec^3 4x$

~~14~~ $3 \sec(4x)^2 \frac{dy}{dx} (\sec(4x))$

~~14~~ $\sec(4x)^2 \sec(4x) \tan(4x)$

$3 \sec(4x)^2 \frac{dy}{dx} (4x)$

$= 3 \sec(4x)^2 \sec(4x) \tan(4x)$

$= 12 \sec(4x)^3 \tan(4x)$

v) $y = (x + \sin 2x)$

$\frac{dy}{dx} \sin(2x) \cdot 2$

(Q02)

(i)

$$\int \frac{2x}{(4+3x^2)} dx$$

$$2 \times \int \frac{1}{u} du$$

$$2 \times \frac{1}{6} \frac{1}{u} du \quad 2 \times \frac{1}{6} \ln|u|$$
$$\boxed{= \frac{1}{3} \ln|4+3x^2|} \quad 2 \times \frac{1}{6} \ln|4+3x^2|$$

(c) $F(x) = \ln(\operatorname{cosec} x - \cot x)$

$$F'(x) = \operatorname{cosec} x$$

Q 3

$$\begin{array}{c|cc} & 1 & i \\ \hline 1 & 1 & i \\ -i & -i & -i^2 \end{array}$$

$$(i) \left(\frac{1+i}{1-i} \right)^3 = \frac{1+i^3}{1-i^3} = \frac{1-i}{1+i}$$

$$\left(\frac{1-i}{1+i} \right) \times \left(\frac{1+i}{1-i} \right) = \frac{1-i^2}{1-i^2}$$

$$\left(\frac{1+i}{1-i} \right)^3 \times \left(\frac{1+i}{1-i} \right) = \left(\frac{1+2i+i^2}{1-i^2} \right)^3$$

$$= \left(\frac{1+2i-1}{1-(-1)} \right)^3 = \left(\frac{2i}{2} \right)^3 = i^3 = -i$$

$$\boxed{= 0-i}$$

$$ii) \frac{3}{(1+i)(2+i)}$$

$$= \frac{3}{1+3i}$$

$$\frac{3(1-3i)}{(1+3i)(1-3i)}$$

$$\frac{3(1-3i)}{10}$$

$$\frac{3-9i}{10}$$

$$\boxed{\frac{3}{10} - \frac{9}{10}i}$$

$$\begin{array}{r|rr} & 1 & i \\ \hline 2 & 2 & 2i \\ i & i & i^2 \end{array}$$

$$= 2 + 3i - 1$$

$$= 1 + 3i$$

$$\begin{array}{r|rr} & 1 & 3i \\ \hline 1 & 1 & 3i \\ -3i & -3i & -9i^2 \end{array}$$

$$-9 \times -1 = 9$$

$$9 + 1 = 10$$

$$\text{iii) } \frac{1+i}{1-i} - (1+2i)(2+2i) + \frac{3-i}{1+i}$$

$$\begin{array}{r|l} & 1 \quad 2i \\ 2 & 2 \quad 4i \\ 2i & 2i \quad 4i^2 \end{array}$$

$$= 2 + 6i + 4i^2$$

$$= 2 + 6i - 4$$

$$= -2 + 6i$$

$$= \frac{1+i}{1-i} - 2 + 6i + \frac{3-i}{1+i}$$

$$\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{1+2i+i^2}{1-i^2}$$

$$= \frac{1+2i-1}{1-(-1)} = \frac{2i}{2} = i$$

$$= i - 2 + 6i + \frac{3-i}{1+i}$$

$$\frac{3-i}{1+i} \times \frac{1-i}{1-i} = \frac{3-6i+i^2-1}{1-i^2}$$

$$= \frac{2-6i-1}{1-(-1)} = \frac{1-6i}{2}$$

$$= \frac{1}{2} - 3i$$

$$\begin{array}{r|l} & 3 \quad -i \\ 3 & 6 \quad -8i \\ -3i & -3i \quad i^2 \\ & 1 \quad -2i \\ & -i \quad -1 \quad i^2 \end{array}$$

iii)

$$i - 2i + 6 + \frac{5}{2} - 3i$$

$$= -4i + \frac{17}{2}$$

$$\boxed{= \frac{17}{2} - 4i}$$

$$\frac{3-i}{1+i} \times \frac{1-i}{1-i} = \frac{2-4i}{2}$$

$$= 1 - 2i$$

$$i - 3i + 6 + 1 - 2i$$

$$= -4i + 7$$

$$\boxed{= 7 - 4i}$$

$$i - (-2 + 6i) + 1 - 2i$$

$$= + + 2 - 6i + 1 - 2i$$

$$\boxed{= 3 - 7i}$$

(iv) $2i(i-1) + (\sqrt{3}-i) + (1+i)(1+i)$

	$2i$
i	$2i^2$
-1	$-2i$

$$= -2i + 2 \times -1$$

$$= -2 - 2i$$

$$(1+i)(1+i)$$

	1	i
1	1	i
i	i	i^2

$$= 0 + 2i$$

$$(-2 - 2i) + \sqrt{3} - i + 2i$$

$$= -2i - i + 2i - 2 + \sqrt{3}$$

$$= -i - 2 + \sqrt{3}$$

$$= (-2 + \sqrt{3}) - i$$

$$\boxed{= (-2 + \sqrt{3}) - i}$$

$$b) (4 + 7i)(3 - 2i)$$

$$\begin{array}{r|rr} & 4 & 7i \\ 3 & 12 & 21i \\ -2i & -8i & -14i^2 \end{array}$$

$$\begin{aligned} & 12 + 21i - 14i^2 \\ & = 12 + 21i(-14 \times -1) \\ & = 26 + 21i \end{aligned}$$

$$R = \sqrt{a^2 + b^2}$$

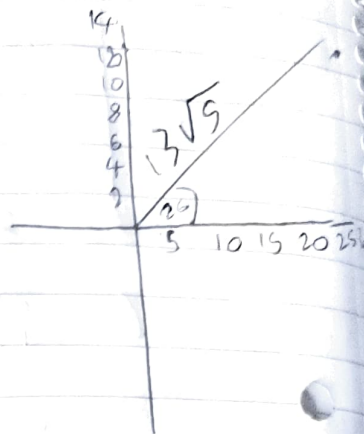
$$R = \sqrt{26^2 + 21^2}$$

$$R = 13\sqrt{5}$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = 26.56^\circ$$

$$\boxed{13\sqrt{5} \angle 26.56^\circ}$$



(C) $1 + 3i$ $1 + i$

$$\frac{1 + 3i}{1 + i}$$

$$\frac{1 + 3i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$= \frac{3i - 3i^2}{1 - i^2}$$

$$= \frac{3i + 3}{2}$$

$$\frac{1 + 3i}{1 + i} \times \frac{1 - i}{1 - i}$$

$$\frac{1(1+i) + 3i(1-i)}{(1+i)(1-i)}$$

$$\frac{1 \times 1 + 1(-i) + 3i \times 1 - 3i(-i)}{(1+i)(1-i)}$$

$$= \frac{1 - i + 3i - 3}{(1+i)(1-i)} = \frac{-2 + 2i}{(1+i)(1-i)}$$

$$= \frac{-2 + 2i}{2}$$

$$\frac{((1 \times 1) + 3 \times 1) + ((3 \times 1) - 1 \times 1)}{1^2 + 1^2}$$

$$= \frac{4 + 2i}{2}$$

$$= \frac{2(2 + i)}{2}$$

$$\boxed{= 2 + i}$$

$$\begin{array}{r|rr} & 1 & 3i \\ \hline 1 & 1 & 3i \\ -i & -1 & -3i^2 \\ \hline & 1 & i \\ -i & -i & -i^2 \end{array}$$

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Maths exam 2

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