

University of Wales Trinity Saint David
School of Engineering

Advanced Engine Design
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Samuel Treleaven
1702453
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Con-Rod Acceleration, Forces, Displacements

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Appendices

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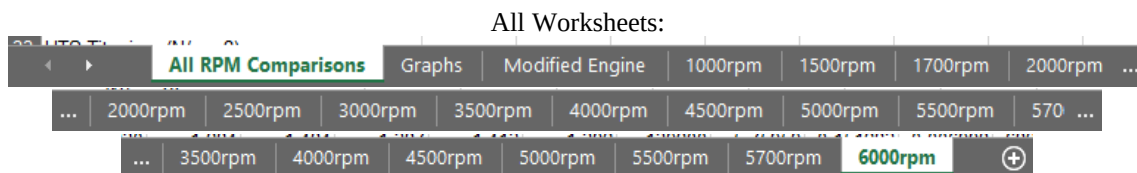
Con-Rod

Acceleration, Forces, Displacements

Introduction

All the calculations and analysis are comparing the information calculated for a Ford Sigma 1.6L production engine and for a modified engine which simulates a turbo being added as well as a higher RPM range.

To see all the calculations carried out for each RPM, see Appendix A (and Appendix B for the formulas used). I repeated this for RPM values 1000, 1500, 1700, 2000, 2500, 3000, 3500, 4000, 4500, 5000, 5500, 2700, 6000. This was also repeated for the modified engine. Included in these were calculations for side force and multiple frictional forces as well as multiple work done calculations.



The production engine calculations are in the individual RPM sheets
and the modified engine calculations are in their own sheet.

A. Peak Gas Forces

Peak gas forces were calculated using $F_{gas} = pressure (N/m^2) \times \pi \times \frac{[bore (m)]^2}{4}$.

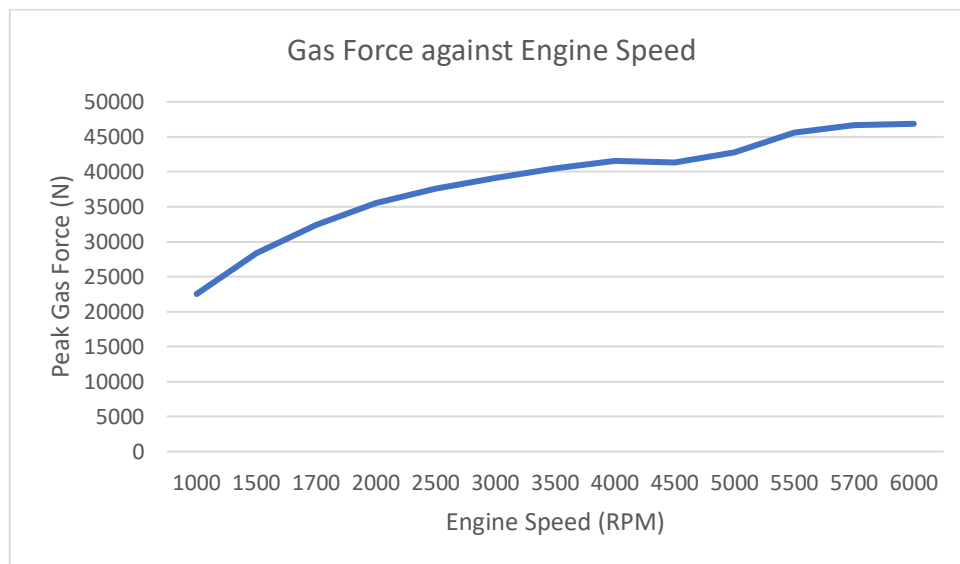
The peak gas force occurred at around 28 degrees after TDC for each RPM value. As the RPM increased so did the peak gas force.

1. Production Engine

Figure 1 shows the calculated Peak Gas Force at each RPM. The Peak Gas Force for this engine is 46862.91N at 6000rpm.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1								Standard						
2		1000	1500	1700	2000	2500	3000	3500	4000	4500	5000	5500	5700	6000
3	Peak Gas Force (N)	22526.6	28363.02	32397.71	35531.47	37575.47	39111.04	40468.92	41573.15	41324.14	42759.72	45579.77	46690.86	46862.91

Figure 1



Graph 1

2. Modified Engine

Figure 2 shows the calculated Peak Gas Force at each RPM for the modified engine. As you can see, the Peak Gas Force is 53418.82N at 8000 RPM

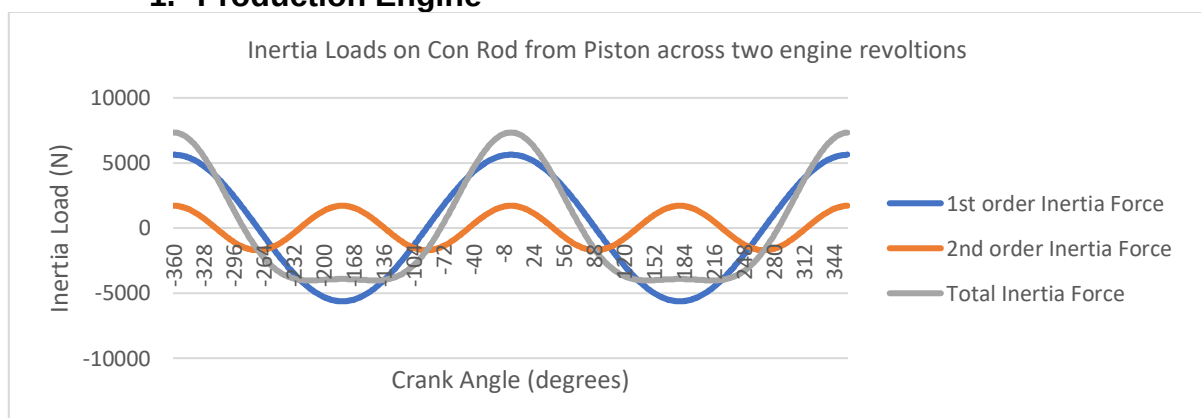
	A	P	Q	R	S	T	U
1					Modified RPM (x10 ³)		
2		6	6.5	7	7.5	7.7	8
3	Peak Gas Force (N)	49801.79	50808.17	51739.93	52607.38	52938.26	53418.82

Figure 2

B. Inertia Loads

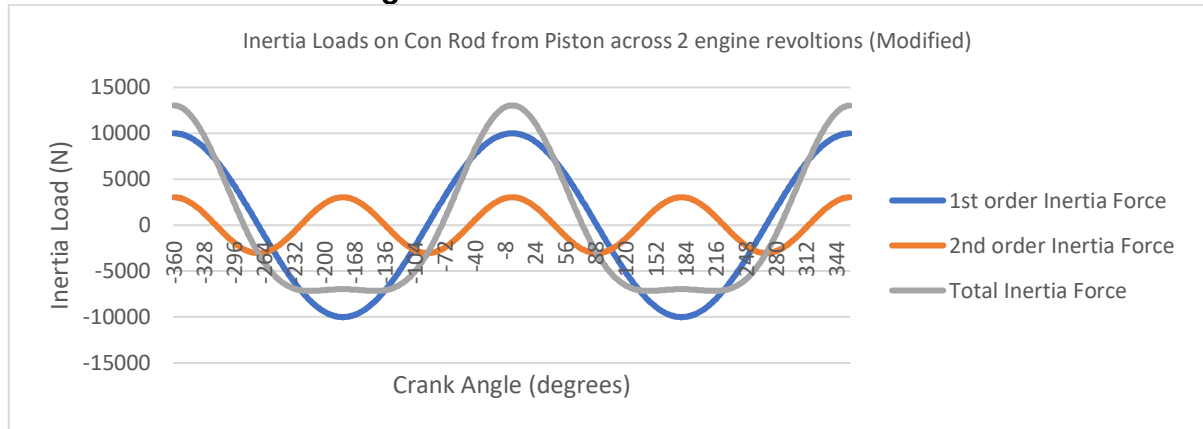
Graphs 2 and 3 show the load applied to the con rod for two full revolutions each. The peaks of the total inertia force is at Top Dead Centre (TDC). The total inertia force is the two inertia forces added together to get a more accurate approximation of what the inertia is. The 1st order term is taken from the acceleration of the crank pin.

1. Production Engine

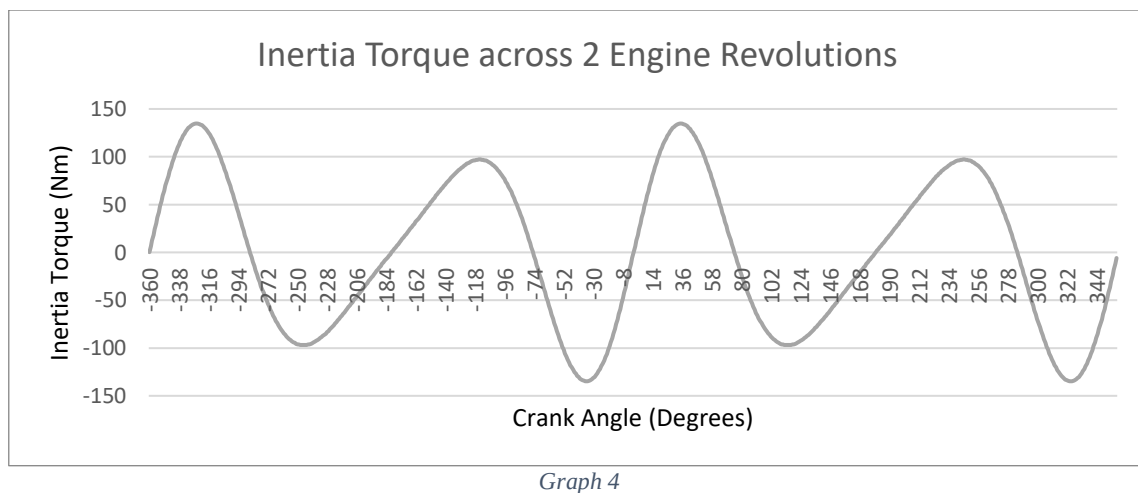


Graph 2

2. Modified Engine



Further investigating into the two engine revolutions results in the following graph:



Graph 4 shows that the amount above the x-axis is approximately equal to the amount below which confirms the equation as being correct as the total inertia should equal zero over this cycle. This is also shown in the equation of the SUM of all the inertia torques which on all the sheets is approximately 1×10^{-15} which is approximately 0.

C. Highest Net Compressive Force

This was calculated using the following formula, the results are to aid the calculations in section E.

$$F_{rod} = \frac{F_{net}}{\cos \phi}$$

$$F_{net} = F_{gas} - F_{inertia}$$

$$\phi = \text{con rod angle}$$

$\cos(\phi)$ is used to resolve the force in the direction of the con rod.

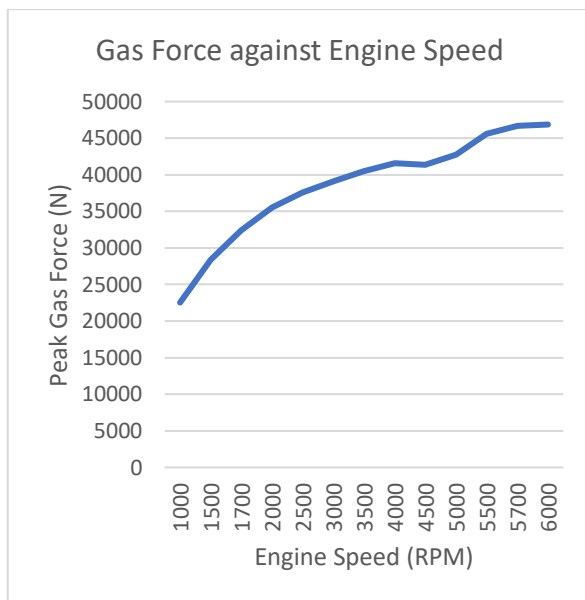
1. Production Engine

The highest calculated force was 40953.84N at 5700 RPM.

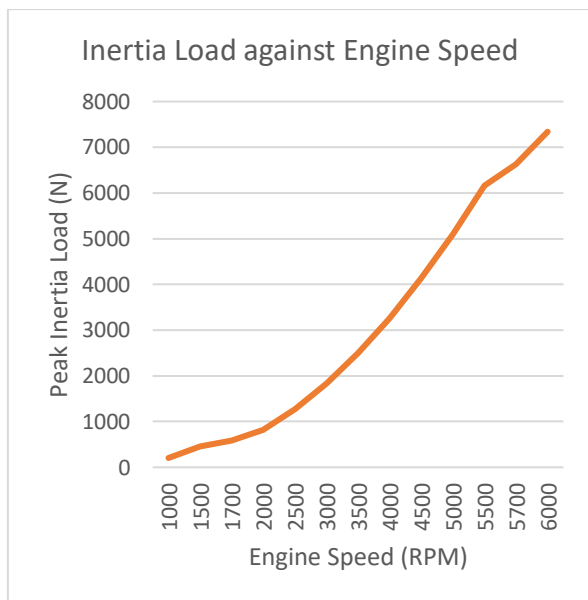
	A	K	L	M	N
1			Standard		
2		5000	5500	5700	6000
6	Peak Net Compressive Force on Rod (N)		40333.04	40953.84	40385.44

Figure 3

The reason this result was at 5700RPM and not at the 6000RPM, as we saw for the peak gas pressures, is due to the increase in inertia being larger than the increase in gas pressure between 5700RPM to 6000RPM as shown in graphs 5 and 1.



Graph 1



Graph 5

2. Modified Engine

Similarly to the production engine, the inertia increases at a greater rate than the gas force does. Therefore the peak force is was 43253.59N at 6000RPM.

	A	P	Q	R	S	T	U
1				Modified RPM (x10^3)			
2		6	6.5	7	7.5	7.7	8
6	Peak Net Compressive Force on Rod (N)	43253.59	43089.45	42756.36	42264.69	42025.54	41622.77

Figure 4

The compressive force is larger for the modified engine than the production engine because the modified engine has a higher gas pressure.

D. Highest Net Tensile Load

The Highest Net Tensile Load on the con rod was taken from the total inertia load at TDC on the exhaust stroke.

1. Production Engine

In figure 5 there's an obvious positive correlation between the RPM and the calculated peak net tensile load.

	A	K	L	M	N
1			Standard		
2		5000	5500	5700	6000
7	Peak Net Tensile Load on Rod (N)		6167.785	6624.507	7340.174

Figure 5

This positive correlation continues infinitely because of the nature of the relationship between peak inertia and RPM can be shown by the following formula.

$$\frac{\text{inertia force @ RPM}_1}{\text{inertia force @ RPM}_2} = \frac{\text{RPM}_1^2}{\text{RPM}_2^2}$$

2. Modified Engine

Figure 6 shows the same positive correlation with the peak occurring at the highest calculated RPM of 8000RPM, with 13049.2N.

	A	P	Q	R	S	T	U
1				Modified RPM (x10^3)			
2		6	6.5	7	7.5	7.7	8
7	Peak Net Tensile Load on Rod (N)	7340.174	8614.51	9990.792	11469.02	12088.86	13049.2

Figure 6

It is noticed that the highest tensile load is much larger on the modified engine than the production engine due to the fact the modified engine can go to a higher RPM, therefore, experiences much higher inertia loads.

E. Maximum Compression and Tensile Stresses

The maximum compression and tensile stresses in the con rod were calculated using the following formula.

$$\sigma = \frac{F}{A}$$

F is the maximum compressive force and tensile force respectively

A is the cross-sectional area of the con rod, which is 195mm²

Both the maximum compressive stress and the maximum tensile stress are at the same RPM as their respective max forces. These stress' are required to calculate their static safety factors in section F.

1. Production Engine

	A	L	M	N
1		Standard		
2		5500	5700	6000
10	For Standard Con Rod			
11	Maximum Compression Stress (N/mm ²)		210.0197	207.1048
12	Maximum Tensile Stress (N/mm ²)		33.97183	37.64192

Figure 7

2. Modified Engine

	A	P	Q	R	S	T	U
1		Modified RPM (x10 ³)					
2		6	6.5	7	7.5	7.7	8
10	For Standard Con Rod						
11	Maximum Compression Stress (N/mm ²)	221.8133	220.9715	219.2634	216.742	215.5156	213.4501
12	Maximum Tensile Stress (N/mm ²)	37.64192	44.17697	51.23483	58.8155	61.99415	66.91896

Figure 8

F. Nominal Static Safety Factors

The ultimate tensile strength, UTS, for the Ford con rod is 750N/mm².

$$\text{Nominal Static Safety Factor} = \frac{UTS}{\sigma_{max}}$$

1. Production Engine

Figure 9 shows that the static safety factor is approximately 3.57. This figure meets the expectation of it being at least 2 for it to have a long working life (Ramesh, et al., 2017).

	A	L	M	N
1		Standard		
2		5500	5700	6000
10	For Standard Con Rod			
13	Nominal Static Safety Factor Compressive		3.571094	3.621354
14	Nominal Static Safety Factor Tensile		22.07711	19.9246

Figure 9

2. Modified Engine

The static safety factor is slightly less for the modified engine due to the increase of both engine speed and power. As the figure is larger than 2, the con rod won't break (Ramesh, et al., 2017).

	A	P	Q	R	S	T	U
1		Modified RPM (x10 ³)					
2		6	6.5	7	7.5	7.7	8
10	For Standard Con Rod						
13	Nominal Static Safety Factor Compressive	3.381223	3.394103	3.420543	3.460335	3.480027	3.513702
14	Nominal Static Safety Factor Tensile	19.9246	16.97717	14.63848	12.75174	12.09792	11.20759

Figure 10

G. Maximum Compression Distance & Stretch

The formula used is the same for the distance and stretch, but use different values of stress.

$$\frac{\sigma_{max} \times L}{E}$$

L = Length of Con Rod = 135mm
E = Youngs Modulus = $170 \times 10^3 \text{N/mm}^2$ for forged steel

These figures show that the maximum compression for a production engine is approximately 0.1655mm. This is less than the modified engines result of 0.1748mm. this relationship is also true for the stretch with the production engines being much smaller at 0.0297mm to the modified's 0.0527mm.

1. Production Engine

1	A	L	M	N
2		5500	5700	6000
10	For Standard Con Rod			
13	Maximum Compression Distance (mm)		0.165545	0.163247
14	Maximum Stretch (mm)		0.026778	0.029671

Figure 11

2. Modified Engine

1	A	P	Q	R	S	T	U
2		6	6.5	7	7.5	7.7	8
10	For Standard Con Rod						
13	Maximum Compression Distance (mm)	0.174841	0.174178	0.172831	0.170844	0.169877	0.168249
14	Maximum Stretch (mm)	0.029671	0.034822	0.040385	0.04636	0.048866	0.052748

Figure 12

H. Maximum Compression Distance & Stretch for Titanium

1	A	P	Q	R	S	T	U
2		6	6.5	7	7.5	7.7	8
18	For Titanium Con Rod						
19	Maximum Compression Distance	0.270208882	0.269183488	0.267103	0.264031	0.262537	0.260021
20	Maximum Stretch	0.0458547	0.053815585	0.062413	0.071648	0.07552	0.081519

Figure 13

The stretch and compression for the titanium con rod is much larger than the standard forged steel con rod. However, these values are still very small so won't significantly effect the operating clearances.

Bibliography

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Appendices

Appendix A – Headers of 1000RPM Worksheet in the Excel Workbook

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1	Crank Angle	CYLPR1	CYLPR2	CYLPR3	CYLPR4	AVRG P	AVRG P	Crank Angle	Rod Angle	Displacement	Gas Force	1st order Inertia Force	2nd order Inertia Force	Total Inertia Force	Net Force
2	deg	bar	bar	bar	bar	N/m2	N/m2	Rad	rad	m	N	N	N	N	N
3		-360	0.886	1.053	0.994	0.931	0.966	-6.283185307	7.44232E-17	0	473.501316	156.3924354	47.50128448	203.8937199	269.6075961
4		-359	0.886	1.065	0.994	0.944	0.97225	-6.265732015	0.005300818	8.08144E-06	476.5648597	156.3686161	47.47234798	203.840964	272.7238956

	P	Q	R	S	T	U	V	W	X	Y	Z
1	Compressive Force on Rod	Side Force	Skirt Friction Force	Skirt Work Done	Radial Force	Ring Friction Force	Ring Work Done	Instantaneous Torque	Gas Torque		Inertia Torque
2	N	N	N	J	N	N	J	Nm	Nm		Nm
3		269.6075961	6.60619E-14	6.60619E-15	1.92355E-06	28.76970021	2.876970021	2.33253E-05	2.68872E-15	4.72209E-15	2.03337E-15
4		272.7277273	4.760413306	0.476041331	1.73814E-05	28.95583958	2.895583958	7.02976E-05	0.252549254	0.441304963	0.188759258

	AA	AC	AD
1	Average Gas Force per degree	Volume	Work Done
2		m3	
3	0.003838949	0	0.003838949
4	0.011569815	3.96125E-08	0.011569815

	Q	R	S	T	U	V	W	X	Y	Z
1	Side Force	Skirt Friction Force	Skirt Work Done	Radial Force	Ring Friction Force	Ring Work Done	Instantaneous Torque	Gas Torque		Inertia Torque
2	N	N	J	N	N	J	Nm	Nm		Nm
3	6.60619E-14	6.60619E-15	1.92355E-06	28.76970021	2.876970021	2.33253E-05	2.68872E-15	4.72209E-15		2.03337E-15
4	4.760413306	0.476041331	1.73814E-05	28.95583958	2.895583958	7.02976E-05	0.252549254	0.441304963		0.188759258
725										
726	Total WD in 2 Revs			4.15103E+15	J/cylinder	SUM of Gas Torque		25370.82269	Nm	-6.3283E-15
727				3.45919E+16	J/s/cylinder	Mean Gas Torque per Cylinder		35.23725374	Nm	-8.7893E-18
728						Mean Gas Torque for Engine		140.949015	Nm	

	AA	AC	AD	AE
1	Average Gas Force per degree	Volume	Work Done	
2		m3		
3	0.003838949	0	0.00383895	
4	0.011569815	3.9613E-08	0.01156981	
725				
726	Nm		Total WD	884.245881 J
727	Nm		IMEP	2216177.68 Pa
728				

Appendix B – Headers of 1000RPM Worksheet in the Excel Workbook, in Formula View

	A	B	C	D	E	F	G	H	I	J
1	Crank Angle	CYLPR1	CYLPR2	CYLPR3	CYLPR4	AVRG P	AVRG P	Crank Angle	Rod Angle	Displacement
2	deg	bar	bar	bar	bar	N/m2	N/m2	Rad	rad	m
3	-360	0.886	1.053	0.994	0.931	=B3+C3+D3+E3)/4	=F3*10^5	=A3*PI()/180	=SIN((0.0407/0.134)*SIN(H3))	=0.0407+0.134-0.0407*COS(H3)-0.134*(1-0.5*((0.0407^2)/(0.134^2))*SIN(H3)^2)
4	-359	0.886	1.065	0.994	0.944	=B4+C4+D4+E4)/4	=F4*10^5	=A4*PI()/180	=SIN((0.0407/0.134)*SIN(H4))	=0.0407+0.134-0.0407*COS(H4)-0.134*(1-0.5*((0.0407^2)/(0.134^2))*SIN(H4)^2)

	K	L	M	N	O	P	Q	R	S
1	Gas Force	1st order Inertia Force	2nd order Inertia Force	Total Inertia Force	Compressive	Side Force	Skirt Fric	Skirt Work Done	
2	N	N	N	N	N	N	N	J	
3	=G3*PI()/0.079^2)/4	=0.3504*0.0407*(2*PI()/1000/60)^2*COS(H3)	=0.3504*(0.0407/0.134)*0.0407*(2*PI()/1000/60)^2*COS(2*H3)	=L3+M3	=K3-N3	=O3/COS(I3)	=O3*TAN(H3)	=0.1*Q3	=((R3+R4)/2)*ABS(J4-J3)
4	=G4*PI()/0.079^2)/4	=0.3504*0.0407*(2*PI()/1000/60)^2*COS(H4)	=0.3504*(0.0407/0.134)*0.0407*(2*PI()/1000/60)^2*COS(2*H4)	=L4+M4	=K4-N4	=O4/COS(I4)	=O4*TAN(H4)	=0.1*Q4	=((R4+R5)/2)*ABS(J5-J4)

	T	U	V	W	X	Y	Z	AA	AC	AD
1	Radial Force	Ring Fric	Ring Work Done	Instantaneous Torque	Gas Torque		Inertia Torque	Average Gas Force per degree	Volume	Work Done
2	N	N	J	Nm	Nm		Nm		m3	
3	=G3*PI()/0.079*0.001=0.1*T3	=((U3+U4)/2)*ABS(J4-J3)	=P3*0.0407*SIN(H3+I3)	=K3*0.0407*SIN(H3+I3)	=N3*0.0407*SIN(H3+I3)	=N3*0.0407*SIN(H3+I3)	=N3*0.0407*SIN(H3+I3)	=((K3+K4)/2)*ABS(J4-J3)	=J3*PI()/0.079^2)/4	=((G3+G4)/2)*ABS(AC4-AC3)
4	=G4*PI()/0.079*0.001=0.1*T4	=((U4+U5)/2)*ABS(J5-J4)	=P4*0.0407*SIN(H4+I4)	=K4*0.0407*SIN(H4+I4)	=N4*0.0407*SIN(H4+I4)	=N4*0.0407*SIN(H4+I4)	=N4*0.0407*SIN(H4+I4)	=((K4+K5)/2)*ABS(J5-J4)	=J4*PI()/0.079^2)/4	=((G4+G5)/2)*ABS(AC5-AC4)

	Q	R	S	T	U	V	W	X
1	Side Force	Skirt Friction Force	Skirt Work Done	Radial Force	Ring Friction Force	Ring Work Done	Instantaneous Torque	Gas Torque
2	N	N	J	N	N	J	Nm	Nm
3	=O3*TAN(H3)	=0.1*Q3	=((R3+R4)/2)*ABS(J4-J3)	=G3*PI()/0.079*0.0012	=0.1*T3	=((U3+U4)/2)*ABS(J4-J3)	=P3*0.0407*SIN(H3+I3)	=K3*0.0407*SIN(H3+I3)
4	=O4*TAN(H4)	=0.1*Q4	=((R4+R5)/2)*ABS(J5-J4)	=G4*PI()/0.079*0.0012	=0.1*T4	=((U4+U5)/2)*ABS(J5-J4)	=P4*0.0407*SIN(H4+I4)	=K4*0.0407*SIN(H4+I4)
725								
726			=SUM(S3:S722)	J/cylinder			SUM of Gas Torque	=SUM(X3:X722)
727			=S726*1000/120	J/s/cylinder			Mean Gas Torque per Cylinder	=X726/720
728							Mean Gas Torque for Engine	=X727*4

	Y	Z	AA	AB	AC	AD	AE
1		Inertia Torque	Average Gas Force per degree		Volume	Work Done	
2		Nm			m3		
3		=N3*0.0407*SIN(H3+I3)	=((K3+K4)/2)*ABS(J4-J3)		=J3*PI()/0.079^2)/4	=((G3+G4)/2)*ABS(AC4-AC3)	
4		=N4*0.0407*SIN(H4+I4)	=((K4+K5)/2)*ABS(J5-J4)		=J4*PI()/0.079^2)/4	=((G4+G5)/2)*ABS(AC5-AC4)	
725							
726	Nm	=SUM(Z3:Z722)	Nm		Total WD	=SUM(AD3:AD722)	J
727	Nm	=Z726/720	Nm		IMEP	=AD726/(0.0814*PI()*0.079 Pa	
728	Nm						

Appendix C – Main Worksheet

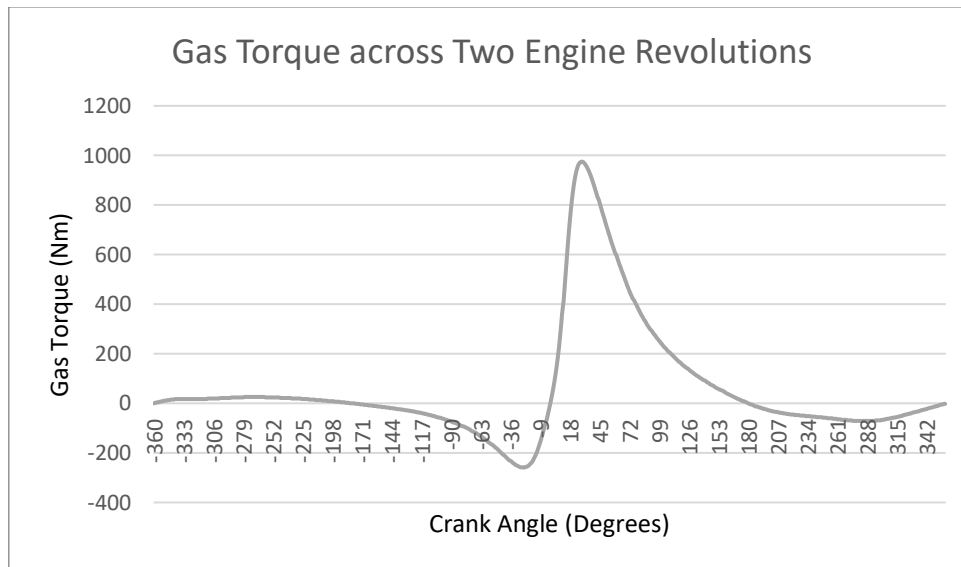
	A	B	C	D	E	F	G	H
1								Standard
2		1000	1500	1700	2000	2500	3000	3500
3	Peak Gas Force (N)	22526.6	28363.02	32397.71	35531.47	37575.47	39111.04	40468.92
4	Inertia Load at TDC Exhaust (N)	203.8937	458.7609	589.2529	815.5749	1274.336	1835.043	2497.698
5								
6	Peak Net Compressive Force on Rod (N)							
7	Peak Net Tensile Load on Rod (N)							
8								
9								
10	For Standard Con Rod							
11	Maximum Compression Stress (N/mm ²)							
12	Maximum Tensile Stress (N/mm ²)							
13	Nominal Static Safety Factor Compressive							
14	Nominal Static Safety Factor Tensile							
15	Maximum Compression Distance (mm)							
16	Maximum Stretch (mm)							
17								

	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1		Standard										Modified RPM (x10 ³)			
2	3000	3500	4000	4500	5000	5500	5700	6000		6	6.5	7	7.5	7.7	8
3	39111.04	40468.92	41573.15	41324.14	42759.72	45579.77	46690.86	46862.91		49801.8	50808.2	51739.9	52607.4	52938.3	53418.8
4	1835.043	2497.698	3262.3	4128.848	5097.343	6167.785	6624.507	7340.174		7340.17	8614.51	9990.79	11469	12088.9	13049.2
5									Inertia at p	6738.03	7907.82	9171.2	10528.2	11097.2	11978.7
6						40333.04	40953.84	40385.44		43253.6	43089.4	42756.4	42264.7	42025.5	41622.8
7						6167.785	6624.507	7340.174		7340.17	8614.51	9990.79	11469	12088.9	13049.2
8															
9															
10															
11						210.0197	207.1048			221.813	220.972	219.263	216.742	215.516	213.45
12						33.97183	37.64192			37.6419	44.177	51.2348	58.8155	61.9941	66.919
13						3.571094	3.621354			3.38122	3.3941	3.42054	3.46034	3.48003	3.5137
14						22.07711	19.9246			19.9246	16.9772	14.6385	12.7517	12.0979	11.2076
15						0.165545	0.163247			0.17484	0.17418	0.17283	0.17084	0.16988	0.16825
16						0.026778	0.029671			0.02967	0.03482	0.04039	0.04636	0.04887	0.05275
17															

Appendix D – Table of Values from Excel

	A	B
26	Bore (mm)	79
27	Con Rod Assembly (kg)	0.4064
28	Piston Assembly (kg)	0.3504
29	Con Rod Length (mm)	134
30	Con Rod Cross Area (mm ²)	195
31	UTS forged steel (N/mm ²)	750
32	Crank Radius (mm)	40.7
33	Piston Ring Thickness (mm)	1.2
34	Youngs Modulus forged steel (N/mm ²)	170x10 ³
35	UTS Titanium (N/mm ²)	950
36	Youngs Modulus Titanium (N/mm ²)	110x10 ³

Appendix E – Gas Torque across Two Engine Revolutions



Graph 6

Appendix F – Buckling Formula

$$Buckling = \frac{\pi^2 \times E \times I}{L^2}$$

Where E is the Youngs Modulus, I is the second moment of area and L is the length of the con rod.

$$I = \frac{B \times D^3}{12}$$

Where B and D are the base and Depth of the con rod respectively.