

Q1)

a) (i) $\int (x^3 + 2x)(x^2 - 4x) dx$

$$\int x^5 - 4x^4 + 2x^3 - 8x^2 dx$$

$$= \frac{x^6}{6} - \frac{4x^5}{5} + \frac{2x^4}{4} - \frac{8x^3}{3} + C$$

||| |||

⑥

~~(i)~~ (ii) $\int (x^4 + 4x)^3 - 2x + 4 \cos 5x dx$

$$\int (x^4 + 4x)(x^4 + 4x)(x^4 + 4x) - 2x + 4 \cos 5x dx$$

$$\int (x^8 + 4x^5 + 4x^5 + 16x^2)(x^4 + 4x) - 2x + 4 \cos 5x dx$$

i.e. $\int (x^8 + 8x^5 + 16x^2)(x^4 + 4x) - 2x + 4 \cos 5x dx$

$$= \int x^{12} + 4x^9 + 8x^9 + 32x^6 + 16x^6 + 64x^3 - 2x + 4 \cos 5x dx$$

$$= \int x^{12} + 12x^9 + 48x^6 + 64x^3 - 2x + 4 \cos 5x dx$$

$$= \frac{x^{13}}{13} + \frac{12x^{10}}{10} + \frac{48x^7}{7} + \frac{64x^4}{4} - \frac{2x^2}{2} + \frac{4}{5} \sin 5x + C$$

||||| ⑥

~~(i)~~ (ii) $\int_0^5 (3x^2 - 1)(x + 5) dx$

$$\int_0^5 3x^3 + 15x^2 - x - 5 dx$$

$$\left[\frac{3x^4}{4} + \frac{15x^3}{3} - \frac{x^2}{2} - 5x \right]$$

$$= \left[\frac{3(s)^4}{4} + s(s)^3 - \frac{(s)^2}{2} - s(s) \right] - [0]$$

$$= [468.75 + 625 - 12.5 - 25] - [0]$$

$$= \underline{\underline{1056.25}}$$

④

$$(iv) \int_0^1 (e^x + 2)(e^{2x} + 1) dx$$

$$\int_0^1 e^{3x} + e^x + 2e^{2x} + 2$$

$$\left[\frac{1}{3}e^{3x} + e^x + e^{2x} + 2x \right]_0^1$$

$$= \left[\frac{1}{3}e^3 + e^1 + e^2 + 2 \right] - \left[\frac{1}{3}e^0 + e^0 + e^0 + 0 \right]$$

$$= [6.70 + 2.72 + 7.39 + 2] - [0.33 + 1 + 1]$$

$$= [18.81] - [2.33]$$

$$= 16.48$$

⑤

$$b)(i) \int \underbrace{5x}_u \cos 2x \, dx$$

$$\uparrow \frac{dv}{dx}$$

$$= \frac{5x}{2} \sin 2x - \int \frac{5}{2} \sin 2x \, dx$$

$$\frac{du}{dx} = 5$$

$$v = \frac{1}{2} \sin 2x$$

$$= \frac{5x}{2} \sin 2x + \frac{5}{4} \cos 2x + C$$

||||| ⑤

$$(ii) \int \underbrace{2x}_{\frac{dv}{dx}} \ln x \, dx$$

$$\uparrow u$$

$$= x^2 \ln x - \int \frac{x^2}{x} dx$$

$$v = x^2 \quad \frac{dv}{dx} = \frac{1}{x}$$

$$= x^2 \ln x - \frac{x^2}{2} + C$$

|||| ④

c) $\int \frac{21x^2 + 4x}{7x^3 + 2x^2 - 1} dx$

$= \ln(7x^3 + 2x^2 - 1) + C$

4 1 (1)

(ii) $\int \frac{5}{7x+2} dx$

$5 \int \frac{1}{7x+2} = \frac{5}{7} \ln(7x+2) + C$

1 (1)

(iii) $\int \frac{x^2}{3x^3+1} dx$

$= \frac{1}{9} \ln(3x^3+1) + C$

1 (1)

d) $\int \frac{\ln x}{x(1+\ln x)^2} dx$

$u = 1 + \ln x \rightarrow u - 1 = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$\int \frac{u-1}{x u^2} x dx$

$x du = dx$

$\int \frac{u}{u^2} - \frac{1}{u^2} du$

$\int \ln u + \frac{1}{u} + C$

$\int \frac{1}{u} - u^{-2} du$

$= \ln(1 + \ln x) + \frac{1}{1 + \ln x} + C$

$\int \ln u + u^{-1} + C$

1 1

1

1

(4)

$$e) \int \frac{1}{2} \sin^5 x \cos x$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$\int \frac{1}{2} u^5 \cancel{\cos x} \frac{du}{\cancel{\cos x}}$$

$$\left(\frac{du}{\cos x} = dx \right)$$

$$= \frac{1}{12} u^6 + C$$

$$= \frac{1}{12} \sin^6 x + C$$

③

$$Q2) \quad z_1 = 2+3j \quad z_2 = 5-2j \quad z_3 = 4+j$$

$$(i) \quad z_1 + z_2$$

$$= 7+j$$

| ①

$$(ii) \quad z_1 - z_3$$

$$= (2+3j) - (4+j)$$

$$= -2+2j$$

| ②

$$(iii) \quad \frac{z_1}{z_2} = \frac{2+3j}{5-2j} \times \frac{5+2j}{5+2j}$$

$$= \frac{10+4j+15j+6j^2}{25+10j-10j-4j^2}$$

$$= \frac{10+19j-6}{25+4}$$

$$= \frac{4}{29} + \frac{19j}{29}$$

| ③

$$(iv) \quad \bar{z}_1 + \bar{z}_3$$

$$= 2-3j + 4-j$$

$$= 6-4j$$

| ④

$$v) (z_1)^2 + z_2 + z_3$$

$$(2+3j)(2+3j) + 5-2j + 4+j$$

$$4+12j+9j^2 + 5-2j + 4+j$$

$$4+11j$$

④

$$b) 14x^2 = -3x - 9$$

$$14x^2 + 3x + 9 = 0$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(14)(9)}}{2(14)}$$

$$x = \frac{-3 \pm \sqrt{-495}}{28}$$

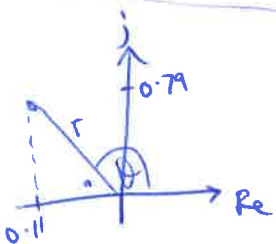
$$x = \frac{-3 \pm 22.25j}{28}$$

$$x = \frac{-3}{28} + \frac{22.25j}{28}$$

$$x = \frac{-3}{28} - \frac{22.25j}{28}$$

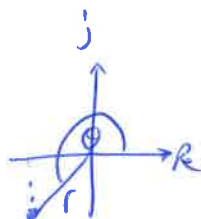
$$x = -0.11 + 0.79j$$

$$x = -0.11 - 0.79j$$



$$r = 0.80$$

$$\theta = 97.93$$

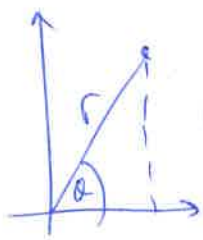


$$r = 0.80$$

$$\theta = 262.07$$

$$0.80(\cos 97.93 + j \sin 97.93) \quad \& \quad 0.80(\cos 262.07 + j \sin 262.07)$$

c) (i) $(6 + 5j)^5$



$$r = 7.81$$

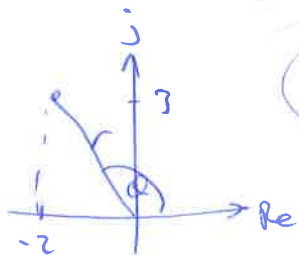
$$\theta = 39.81^\circ$$

$$\text{DMT} = (7.81)^5 (\cos 5 \times 39.81 + j \sin 5 \times 39.81)$$

$$29057.29 (\cos 199.05 + j \sin 199.05)$$

$$= -27465.94 - 9484.10j$$

d) $(-2 + 3j)^7$



$$r = 3.61$$

$$\theta = 123.69^\circ$$

$$\text{DMT: } (3.61)^7 (\cos 7 \times 123.69 + j \sin 7 \times 123.69)$$

$$7990.07 (\cos 865.83 + j \sin 865.83)$$

$$-6610.78 + 4487.62j$$

$$d) 1 - 8j$$

1 ①

$$\begin{aligned} e) & (6+j)(2-5j)(1-j) \\ & (12 - 30j + 2j - 5j^2)(1-j) \\ & (17 - 28j)(1-j) \\ & 17 - 17j - 28j + 28j^2 \end{aligned}$$

$$\boxed{-11 - 45j}$$

1 1

$$r = 46.32$$

$$\theta = 256.26^\circ$$

1

$$\therefore \boxed{46.32 (\cos 256.26 + j \sin 256.26)}$$

1

⑤

Q3)

$$(i) AB = \begin{pmatrix} -5 & 16 & 7 \\ 34 & -19 & 26 \\ -3 & 3 & 3 \end{pmatrix}$$

5
(-1 per incorrect element)

$$(ii) BA = \begin{pmatrix} 23 & 2 & 19 \\ -21 & -27 & 25 \\ -7 & 8 & -17 \end{pmatrix}$$

5
(-1 per incorrect element)

$$(iii) AC = \begin{pmatrix} 38 & 28 \\ 60 & 49 \\ 10 & 7 \end{pmatrix}$$

3
(-1 per incorrect element)

(iv) CA - can't do.

1

$$(v) |A| = 5(10 - 0) + 1(14 + 1) + 6(0 - 5)$$

$$|A| = 35$$

1

1

$$(vi) |B| = 3(-5 - 4) - 1(2 + 12) + 1(2 - 15)$$

$$|B| = -54$$

1

1

$$(vii) A + B = \begin{pmatrix} 8 & 0 & 7 \\ 9 & 0 & 3 \\ -2 & 1 & 3 \end{pmatrix}$$

1

(viii) A + D - can't do.

1

b)

$$9x^2 + 3x = 7$$

$$9x^2 + 3x - 7 = 0$$

$$x = \frac{-3 \pm \sqrt{9^2 - 4(9)(-7)}}{2(9)}$$

$$x = \frac{-3 \pm 18.25}{18}$$

$$x = 0.85$$

$$x = -1.18$$

⑤

$$c) |w| = x(4(x+2)-10) - 3(4(x+2)+4) - 4(20+8)$$

$$= x(4x-2) - 3(4x+12) - 4(28)$$

$$= 4x^2 - 2x - 12x - 36 - 112$$

$$4 = 4x^2 - 14x - 148$$

$$0 = 4x^2 - 14x - 152$$

$$x = \frac{+14 \pm \sqrt{(-14)^2 - 4(4)(-152)}}{2(4)}$$

$$x = 8.16$$

$$x = -4.65$$

⑦

$$d) \quad 5x + 3y + z = 25$$

$$3x - 2y + 3z = 26$$

$$2x + y - 5z = -6$$

$$\begin{pmatrix} 5 & 3 & 1 \\ 3 & -2 & 3 \\ 2 & 1 & -5 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 25 \\ 26 \\ -6 \end{pmatrix}$$

↑
A

$$|A| = 105$$

$$A_1 = \begin{pmatrix} 25 & 3 & 1 \\ 26 & -2 & 3 \\ -6 & 1 & -5 \end{pmatrix}$$

$$|A_1| = 525$$

$$A_2 = \begin{pmatrix} 5 & 25 & 1 \\ 3 & 26 & 3 \\ 2 & -6 & -5 \end{pmatrix}$$

$$|A_2| = -105$$

$$A_3 = \begin{pmatrix} 5 & 3 & 25 \\ 3 & -2 & 26 \\ 2 & 1 & -6 \end{pmatrix}$$

$$|A_3| = 315$$

$$x = \frac{525}{105}$$

$$y = \frac{-105}{105}$$

$$z = \frac{315}{105}$$

$$x = 5$$

$$y = -1$$

$$z = 3$$

