



PRIFYSGOL CYMRU
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LOCATION SWANSEA

LLEOLIAD ABERTAWE

LEVEL 4

LEFEL 4

**EXAMINATION PERIOD
MAY 2017**

**CYFNOD ARHOLIADAU
MAI 2017**

**MODULE CODE: M4X00391/M1X8514
MATHEMATICS TEST 2**

TIME: 1 ½ HOURS

ANSWER TWO QUESTION OUT OF THREE

Equipment or materials required –

Calculators allowed

Formula booklet provided

1. (a) If $z_1 = \frac{2-j}{2+j}$ and $z_2 = \frac{2j-1}{1-j}$, express z_1 and z_2 in $a + bj$ form.

Sketch on an Argand Diagram showing the points P and Q representing the complex numbers

- (i) $5z_1 + 2z_2$ (ii) $5z_1 - 2z_2$ respectively. (15 marks)

- (b) If $p = 2 + j$, $q = 3 - 5j$ and $r = -2 - j$

find the following, expressing your results in Rectangular Form :

(i) $3p - 4q - 5r$

(ii) $p^2 + q^2 + r^2$

(iii) pqr (10 marks)

- (c) Show that the roots of the quadratic equation

$$x^2 - 4x + 13 = 0 \text{ are } 2 + 3j \text{ and } 2 - 3j.$$

Further, express these solutions in Polar Form. (9 marks)

- (d) Use De Moivre's Theorem to find $(2 - j)^6$ (6 marks)

2. (a) Evaluate the following, giving your answers correct to TWO decimal places.

(i) $\int_{-1}^2 x^2(x-3) dx$

(ii) $\int_2^3 \frac{(2x-1)}{x^2} dx$

(iii) $\int_1^4 (\sqrt{x} - 3)(\sqrt{x} + 5) dx$ (16 Marks)

- (b) Using the substitution, let $u = 4x^2 + 3$ find

$$\int 3x(4x^2 + 3)^5 dx$$

Expressing your result in its simplest form. (6 Marks)

(c) $\int \frac{3x^2+17x+32}{x(x+4)^2} dx$ (10 Marks)

- (d) Use Integration 'by Parts' to find

$$\int x \cdot \sec^2 x dx$$
 (8 Marks)

3. (a) If $A = \begin{pmatrix} 2 & 4 \\ 1 & -3 \\ 8 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -3 & 4 \\ 2 & 0 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -2 & 5 \\ 3 & -4 & -1 \\ 6 & -1 & -2 \end{pmatrix}$

Find the following, if possible :

- (i) The Determinant of A
- (ii) The Determinant of B
- (iii) The Determinant of C
- (iv) AB
- (v) AC
- (vi) CA
- (vii) A^2
- (viii) C^2

(13 marks)

(b) If $2 \begin{vmatrix} x & 3 & 4 \\ -1 & 2 & 8 \\ 0 & 4 & 3 \end{vmatrix} = x^2$,

Show that this equation may be expressed as $x^2 + 52x + 14 = 0$.

Hence, find the two values of x giving each answer correct to ONE significant figure.

(10 marks)

(c) Find the Inverse of the following matrix

$$\begin{pmatrix} 3 & -1 & 0 \\ 2 & -4 & 8 \\ 6 & 2 & -3 \end{pmatrix}$$

(9 marks)

(d) A pulley system gives the following linear equations :

$$a_1 + a_2 = 0$$

$$2a_1 + T = 20$$

$$5a_2 + T = 50$$

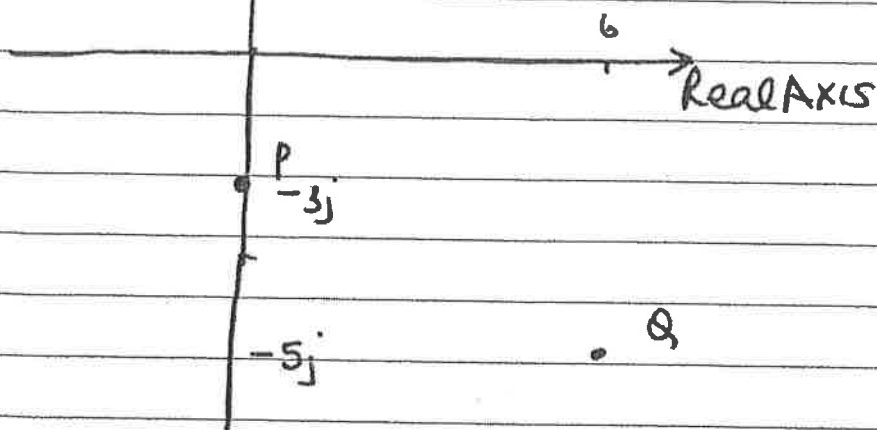
where a_1 , a_2 represent acceleration and T represents tension in the rope.

Using Cramer's Rule, determine a_1 , a_2 and T .

(8 marks)

(i)	$z_1 = \frac{(2-j)(2-j)}{(2+j)(2-j)}$				1
	$\frac{4 - 2j - 2j + j^2}{4 - j^2}$				
	$\frac{3 - 4j}{5}$				
	$\underline{\underline{\frac{3}{5} - \frac{4}{5}j}}$				1 1
	$z_2 = \frac{(2j-1)(1+j)}{(1-j)(1+j)}$				1
	$\frac{2j + 2j^2 - 1 - j}{1 - j^2}$				
	$\frac{-3 + j}{2}$				
	$\underline{\underline{-\frac{3}{2} + \frac{j}{2}}}$				1 1
(ii)	$5z_1 + 2z_2$				1
	$5\left(\frac{3}{5} - \frac{4}{5}j\right) + 2\left(-\frac{3}{2} + \frac{j}{2}\right)$				
	$\begin{array}{r} 3 - 4j - 3 + j \\ 0 - 3j \end{array}$	(P)			1 1
(iii)	$5z_1 - 2z_2$				
	$5\left(\frac{3}{5} - \frac{4}{5}j\right) - 2\left(-\frac{3}{2} + \frac{j}{2}\right)$				1
	$\begin{array}{r} 3 - 4j + 3 - j \\ 6 - 5j \end{array}$	(Q)			1 1

Axes labelled
Correctly. 1



Correct position
of P 1

Correct position
of Q 1

15

(b) $P = 2 + j$ $q = 3 - 5j$ $r = -2 - j$

(i) $3P - 4q - 5r$

$$3(2+j) - 4(3-5j) - 5(-2-j)$$

$$6 + 3j - 12 + 20j + 10 + 5j$$

$$\underline{4 + 28j}$$

1 1
(2)

(ii) $P^2 + q^2 + r^2$

$$(2+j)(2+j) + (3-5j)(3-5j) + (-2-j)(-2-j)$$

$$4 + 4j + j^2 + 9 - 15j - 15j + 25j^2$$

$$+ 4 + 2j + 2j + j^2$$

$$3 + 4j \quad -16 - 30j + 3 + 4j$$

$$-10 - 22j$$

1 1 1

1 1

(5)

(iii) Pqr

$$(2+j)(3-5j)(-2-j)$$

$$(6 - 10j + 3j - 5j^2)(-2-j)$$

$$(11 - 7j)(-2-j)$$

$$-22 - 11j + 14j + 7j^2$$

$$-29 + 3j$$

1

1 1 (3)

$$(iii) \quad x^2 - 4x + 13 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(13)}}{2}$$

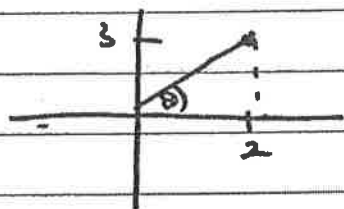
$$x = \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$x = \frac{4 \pm \sqrt{-36}}{2}$$

$$x = \frac{4 \pm 6j}{2}$$

$$x = 2 \pm 3j \quad \text{hence proved.}$$

$$|2+3j| = \sqrt{2^2 + 3^2} = \sqrt{13}$$



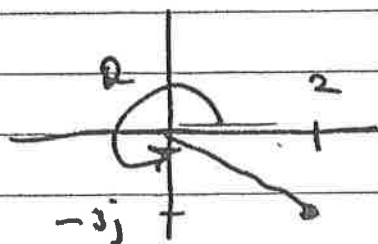
$$\tan \theta = \frac{3}{2}$$

$$\theta = \tan^{-1} 1.5$$

$$\theta = 56.31^\circ$$

$$\sqrt{13} (\cos 56.31^\circ + j \sin 56.31^\circ) \quad (\text{or } \sqrt{13} \angle 56.31^\circ)$$

$$|2-3j| = \sqrt{2^2 + (-3)^2} = \sqrt{13}$$



$$\therefore \theta = 360 - 56.31^\circ$$

$$\theta = 303.69^\circ \quad (\text{or } -56.31^\circ)$$

$$\sqrt{13} (\cos 303.69^\circ + j \sin 303.69^\circ)$$

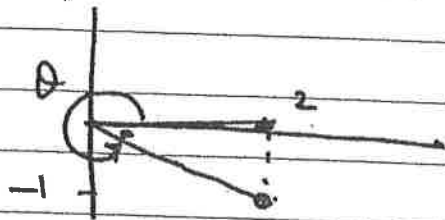
$$\text{or } (\sqrt{13} \angle 303.69^\circ)$$

(iv) $(2-j)^6$

Finally, expressing $2-j$ in polar form.

$$|2-j| = \sqrt{2^2 + (-1)^2}$$

$$= \underline{\underline{\sqrt{5}}}$$



$$\tan \theta = \frac{1}{2}$$

$$\theta = \tan^{-1} \frac{1}{2}$$

$$26.57^\circ$$

$$\therefore \theta = 360 - 26.57$$

$$\theta = 333.43^\circ$$

$\therefore (2-j)$ in polar form is $\sqrt{5} \angle 333.43^\circ$

or $\sqrt{5} (\cos 333.43^\circ + j \sin 333.43^\circ)$

$\therefore (2-j)^6$ is $(\sqrt{5})^6 (\cos 333.43 + j \sin 333.43)^6$

Applying De M.

$$125 (\cos 6 \times 333.43 + j \sin 6 \times 333.43)$$

$$125 (\cos 2000.6 + j \sin 2000.6)$$

$$125 (-0.936 + -0.352j)$$

$$-117$$

$$-44.98j$$

(Correct answer $-117 - 44.98j$)

Q2

(a) (i)

$$\int_{-1}^2 x^2(x-3) dx$$

$$\int_{-1}^2 x^3 - 3x^2 dx$$

$$\frac{x^4}{4} - \frac{3x^3}{3}$$

$$\left[\frac{(2)^4}{4} - (2)^3 \right] - \left[\frac{(-1)^4}{4} - (-1)^3 \right]$$

$$[4 - 8] - [0.25 + 1]$$

$$[-4] - [1.25]$$

$$= \underline{\underline{-5.25}} \quad (2 \text{ dec places})$$

(ii)

$$\int_2^3 \frac{(2x-1)}{x^2} dx$$

$$\int_2^3 x^{-2}(2x-1) dx$$

$$\int_2^3 2x^{-1} - x^{-2} dx$$

$$2 \ln x + x^{-1}$$

$$\left[2 \ln 3 + \frac{1}{3} \right] - \left[2 \ln 2 + \frac{1}{2} \right]$$

$$[2.5306] - [1.8863] = \underline{\underline{0.64}} \quad (2 \text{ dec places})$$

$$Q1 (a) (iii) \int (\sqrt{x}-3)(\sqrt{x}+5) dx$$

$$\int_1^4 x + 5\sqrt{x} - 3\sqrt{x} - 15 dx$$

$$\int_1^4 x + 2\sqrt{x} - 15 dx$$

$$\text{i.e.} \int_1^4 x + 2x^{1/2} - 15 dx$$

$$\left[\frac{x^2}{2} + \frac{2x^{3/2}}{3/2} - 15x \right]_1^4$$

$$\left[\frac{x^2}{2} + \frac{4}{3} \sqrt{x^3} - 15x \right]_1^4$$

$$\left[\frac{16}{2} + \frac{4}{3} \sqrt{64} - 60 \right] - \left[\frac{1}{2} + \frac{4}{3} - 15 \right]$$

$$\left[8 + 10.6667 - 60 \right] - \left[0.5 + 1.3333 - 15 \right]$$

$$\left[-41.3333 \right] - \left[-13.1667 \right]$$

$$= -28.1666$$

$$= \underline{\underline{-28.17}} \text{ (2 decimal places)}$$

1
(6 marks)

16

(b)

$$\int 3x(4x^2+3) dx$$

$$\text{Let } u = 4x^2 + 3$$

$$\frac{du}{dx} = 8x$$

$$\therefore \frac{du}{8x} = dx$$

$$\int \cancel{3x} u^5 \cdot \frac{du}{\cancel{8x}}$$

$$\frac{3}{8} \frac{u^6}{6} + C$$

$$\frac{3}{48} (4x^2+3)^6 + C$$

Simplifying to give

$$\text{i.e. } \frac{1}{16} (4x^2+3)^6 + C$$

($\frac{1}{16}$ required
or 0.0625)

$$(c) \int \frac{3x^2+17x+32}{x(x+4)^2} dx$$

Partial fractions needed.

$$\frac{A}{x} + \frac{B}{(x+4)} + \frac{C}{(x+4)^2} = \frac{3x^2+17x+32}{x(x+4)^2}$$

x by $x(x+4)^2$

$$A(x+4)^2 + Bx(x+4) + Cx = 3x^2+17x+32$$

Put $x=0$

$$16A = 32$$

$$\underline{A = 2}$$

$$124 \quad x = -4$$

$$-4C = 3(-4)^2 + 17(-4) + 32$$

$$-4C = 48 - 68 + 32$$

$$-4C = 12$$

$$\therefore \underline{C = -3}$$

Put $x = 1$ (or any other suitable no.)

$$25A + 5B + C = 3 + 17 + 32$$

$$50 + 5B - 3 = 52$$

$$5B = 52 - 50$$

$$\underline{B = 1}$$

So PF's are $\frac{2}{x} + \frac{1}{(x+4)} - \frac{3}{(x+4)^2}$

$$\int \frac{2}{x} dx = 2 \ln x$$

$$\int \frac{1}{(x+4)} dx = \ln(x+4)$$

$$\int \frac{3}{(x+4)^2} dx = -\frac{3}{(x+4)}$$

So answer $2 \ln x + \ln(x+4) + \frac{3}{(x+4)} + \textcircled{C}$

10 marks

(91)

$$\int x \sec x \, dx.$$

$$\text{Let } u = x$$

$$\therefore \frac{du}{dx} = 1$$

$$\text{Let } \frac{dv}{dx} = \sec^2 x$$

$$\therefore v = \tan x$$

Sub in 'Parts' formulae.

$$\int x \sec^2 x \, dx = x \tan x - \int \tan x \cdot 1 \, dx$$

Writing $\tan x$ as

$$x \tan x - \int \frac{\sin x}{\cos x} \, dx$$

$$x \tan x - \left[-\ln(\cos x) \right] + c$$

$$x \tan x + \ln(\cos x) + c$$

8.

$$(ii) \begin{vmatrix} -3 & 4 \\ 2 & 0 \end{vmatrix}$$

$$= 0 - 8$$

$$= \underline{\underline{-8}}$$

1

$$(iii) \begin{vmatrix} 1 & -2 & 5 \\ 3 & -4 & -1 \\ 6 & -1 & -2 \end{vmatrix}$$

$$= 1(8 - 1) + 2(-6 + 6) + 5(-3 + 24)$$

$$1(7) + 2(0) + 5(21)$$

$$7 + 0 + 105$$

$$\underline{112}$$

1

(iv) AB

$$\begin{pmatrix} 2 & 4 \\ 1 & -3 \\ 8 & 2 \end{pmatrix} \begin{pmatrix} -3 & 4 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 8 \\ -9 & +4 \\ -20 & 32 \end{pmatrix}$$

3×2 2×2
Yes

2

(-1 for each wrong to zero!)

(v) AC NOT POSSIBLE

1

(vi) CA ~~NOT POSSIBLE~~

$$\begin{pmatrix} 1 & -2 & 5 \\ 3 & -4 & -1 \\ 6 & -1 & -2 \end{pmatrix} \begin{pmatrix} 2 & 4 \\ 1 & -3 \\ 8 & 2 \end{pmatrix} = \begin{pmatrix} 40 & 20 \\ -6 & 22 \\ -5 & 23 \end{pmatrix}$$

3×3 3×2
Yes

2

(-1 for each wrong to zero!)

(VII)	A	NOT POSSIBLE	1
(VIII)	$\begin{pmatrix} 1 & -2 & 5 \\ 3 & -4 & -1 \\ 6 & -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & -2 & 5 \\ 3 & -4 & -1 \\ 6 & -1 & -2 \end{pmatrix}$ $= \begin{pmatrix} 25 & 1 & -3 \\ -15 & 11 & 21 \\ -9 & -6 & 35 \end{pmatrix}$		3 (-1 for each wrong down to zero!)
			13

(b)
$$2 \begin{vmatrix} x & 3 & 4 \\ -1 & 2 & 8 \\ 0 & 4 & 3 \end{vmatrix} = x^2$$

$$2 [x(6-32) - 3(-3-0) + 4(-4-0)] = x^2 \quad ||1$$

$$2 [-26x + 9 - 16] = x^2$$

$$2 [-26x - 7] = x^2$$

$$-52x - 14 = x^2$$

i.e.
$$x^2 + 52x + 14 = 0$$

Solving
$$x = \frac{-52 \pm \sqrt{(52)^2 - 4(1)(14)}}{2}$$

$$x = \frac{-52 \pm \sqrt{2704 - 56}}{2}$$

$$x = \frac{-52 \pm \sqrt{2648}}{2}$$

$$x = \frac{-52 \pm 51.4587}{2}$$

so
$$x = -0.3 \quad (1 \text{ sig fig})$$

$$x = -50 \quad (1 \text{ sig fig})$$

$$\begin{pmatrix} 2 & -4 & 8 \\ 6 & 2 & -3 \end{pmatrix}$$

$$|A| = 3 \begin{pmatrix} 12-16 \\ 3(-4) \end{pmatrix} + 1 \begin{pmatrix} -6-48 \\ +1(-54) \end{pmatrix} + 0 \begin{pmatrix} 4+24 \\ +0(28) \end{pmatrix}$$

$$= -66$$

Cofactor Matrix.

$$\begin{pmatrix} +4 & +54 & +28 \\ -3 & -9 & -12 \\ +8 & -24 & -10 \end{pmatrix}$$

5
(-1 for each element. down to zero!)

Transposing (there)

$$\begin{pmatrix} -4 & -3 & -8 \\ 54 & -9 & -24 \\ 28 & -12 & -10 \end{pmatrix}$$

∴ Inverse is

$$\begin{pmatrix} -4 & -3 & -8 \\ 54 & -9 & -24 \\ 28 & -12 & -10 \end{pmatrix}$$

$$-66$$

9

(d)

$$\begin{aligned} a_1 + a_2 + 0 &= 0 \\ 2a_1 + T &= 20 \\ 5a_2 + T &= 50 \end{aligned}$$

In Matrix form

$$\begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 5 & 1 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ T \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 50 \end{pmatrix}$$

$$\text{So Coef. Matrix } A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 5 & 1 \end{pmatrix}$$

$$\begin{aligned} |A| &= 1(0-5) - 1(2-0) + 0(10-0) \\ &= -5 - 2 \\ &= \underline{\underline{-7}} \end{aligned}$$

Modified Coef. Matrices.

$$A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 20 & 0 & 1 \\ 50 & 5 & 1 \end{pmatrix}$$

$$\begin{aligned} |A_1| &= 0(0-5) - 1(\overline{20-50}) + 0(100-0) \\ &\therefore |A_1| = \underline{\underline{30}} \end{aligned}$$

$$A_2 = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 20 & 1 \\ 0 & 50 & 1 \end{pmatrix}$$

$$\begin{aligned} |A_2| &= 1(20-50) - 0(\text{---}) + 0(\text{---}) \\ &= \underline{\underline{-30}} \end{aligned}$$

$$A_3 = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & 20 \\ 0 & 5 & 50 \end{pmatrix}$$

$$\begin{aligned} |A_3| &= 1(0-100) - 1(100-0) + 0(\text{---}) \\ &\quad \underline{\underline{-200}} \end{aligned}$$

$$\therefore a_1 = \frac{30}{-7} = -\frac{30}{7} \quad a_2 = \frac{-30}{-7} = \frac{30}{7} \quad |||$$

-200

