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Contents

1.	Introduction.....	1
2.	Research.....	1
3.	Methodology.....	2
	Gas Loads.....	3
	Piston travel.....	3
	Piston Velocity.....	6
	Piston acceleration.....	6
	Inertia Loads.....	7
	Highest Net forces on connecting rod.....	8
	Stresses.....	8
	Safety factors.....	9
	Compression and Stretch of Rod.....	9
	Side forces.....	10
	Radial force acting on the rings.....	11
	Side and Radial frictional forces.....	11
	Work done.....	12
	Torque.....	12
	Mean effective pressures.....	13
4.	Analysis.....	14
	Gas Force.....	14
	Inertia forces.....	15
	Highest Net compressive force.....	17
	Highest Net tensile force.....	18
	Maximum compressive stress.....	19
	Maximum tensile stress.....	21
	Static safety factors.....	21
	Maximum compression and stretch.....	23
	Titanium [Ti-6Al-4V].....	24

5. Conclusion.....	27
6. References	28
7. Appendix 1. (Graphs).....	29

Table of Figures

Figure 1 Trigonometric depiction of piston movement. [5]	4
Figure 2 Peak gas force per RPM (Standard, modified bore and modified stroke)	14
Figure 3 Standard engine inertia forces.	15
Figure 4 Inertia forces acting upon the rod for three RPMs on each engine.	16
Figure 5 Different order Inertia forces.....	16
Figure 6 Peak Compressive Forces on rod for each engine.....	17
Figure 7 Peak gas torque figures for each engine.	18
Figure 8 Peak Tensile Forces.	19
Figure 9 Peak compressive stresses.....	20
Figure 10 Peak tensile stresses.	21
Figure 11 Compressive safety factor.....	22
Figure 12 Tensile safety factor.	22
Figure 13 Peak stretch (mm).	23
Figure 14 Peak compression (mm)	23
Figure 15 Peak compressive stresses for titanium rod.	24
Figure 16 Peak tensile forces on titanium rod.	25
Figure 17 Compressive safety factors of Titanium.....	25
Figure 18 Tensile safety Factors of Titanium.....	26
Figure 19 Compression movement of the titanium rod for each engine.....	26
Figure 20 Stretch of the titanium rod for each engine.	27

Appendix 1.

Figure 21 Standard engine piston acceleration.	29
Figure 22 Gas torque from the standard engine.....	29
Figure 23 Inertial torque from the standard engine.....	30
Figure 24 Total torque from the standard engine from one complete cycle.	30
Figure 25 Net forces on standard engine.	30
Figure 26 Peak side forces on each engine across the RPM range.	31
Figure 27 Peak frictional side forces on each engine.	31
Figure 28 Peak radial forces for each engine.	31

Figure 29 Peak radial frictional forces for each engine.	32
Figure 30 Indicated, Brake and Frictional mean effective pressures on standard engine across the RPM range.	32
Figure 31 Mechanical efficiency of the standard engine.	32

Table of tables

Table 1 Values of Variables used (Standard engine).....	2
Table 2 Peak gas pressures across RPM for Standard and the modified engines.....	14

1. Introduction.

This technical report has been written to show the correlation between real cylinder pressure data and how using kinematic equations, we're able to retrieve important reliability and safety factor resultants. The main concern for the study was the nominal safety factors for compressive and tensile loads on the piston connecting rod, because "the connecting rod is considered as key component in terms of the structural durability and efficiency" [1] within an internal combustion engine. Hence the reason, "It must be capable of transmitting axial tension, axial compression, and bending stresses caused by the thrust and pull on the piston, and by centrifugal force without bending or twisting" [2].

Furthermore, because in most cases the connecting rod cross-sectional area isn't uniform along the shank, this causes issues regarding mathematical modelling. Although the difficulties are primarily when looking at the buckling potential of the rod, because of the pin-pin joints at either end being that of a large radius, the buckling constant "k" used in the Euler equations shouldn't be assumed to remain as so. Also, it's difficult to definitively specify an effective length for front to rear buckling [1].

Because of these constant high-load stresses in both a compressive and tensile form, the cross-sectional area of the rod shank being too "small" is the components most likely reason for failure. Especially when larger mean effective cylinder pressures (MEP) are desired, in order to return a higher torque and power output.

Therefore, this study was conducted whilst assuming the rod as an ideal beam with a uniform cross-sectional area, with respect to the smallest area represented by the genuine Ford™ connecting rod. Focusing around connecting rod yield strength and tensile strength limits and the reaction to said stresses for both powder forged steel and titanium (Ti-6Al-4V).

2. Research.

The real pressure data was collected by Ford Motor Company™ and it is representative of their 1600cc EcoBoost™ turbocharged gasoline direct injection engine (GTDI) which is known among the automotive world as a good platform for racing or high-power applications. For example, it's the approved engine in FIA British Formula 4 Championship (118Kw), M-Sport Ford™ World Rally Championship cars (280Kw at 6000RPM) and chosen by many for use in open series sprints, rally's and various track applications.

Ford themselves had also used the unit across the board on their OEM vehicles (Fiesta, Focus, Mondeo), along with their old subsidiary company Volvo™ (V40, V60, V70, V80). The direct power outputs from the OEM units are set as 120PS, 150PS, 160PS and 180PS with through the variants. But there is certain cases where the stock power is better represented as 130Kw and 270Nm in an “overboost” condition [3].

Therefore, with this being the case Ford would have had to make this an ideal target power and torque when originally creating the specification of the connecting rod, that should have a safety factor of around 2 - 3 at minimum regarding their tensile and compressive stresses.

3. Methodology.

Standard values	
Bore (m)	0.079
Stroke (m)	0.0814
Displacement (cc)	1596
C:R	10
ConRod Length (m)	0.134
Frictional Coe.	0.1
Rod Shank (m²)	0.00013958
UTS (N/mm²)	750
Youngs Modulus (Gpa)	200
Density (Kg/m³)	7600
Volume of Rod (m³)	4.8842E-05
YS (N/mm²)	520
Masses	
Conrod	267.4
Conrod Cap	103.8
Conrod Bolts	35.19
Conrod Assembly	0.40639
Gudgeon	80.22
Piston	257.7
Piston Ring	15.4
Clips	0.77
Assembly	0.35409
	0.35409

Table 1 Values of Variables used (Standard engine).

Gas Loads.

Starting point for the mathematical analysis was working out the G_f acting inside the cylinder because this is the most important factor for subsequent equations and part of the fundamental principles of which an ICE is based.

The data given had shown the real cylinder pressure data across the four cylinders of the engine. But due to the small differences in each of them, the average pressure across all the cylinders per each degree was taken and assumed as a close enough representation of how the pressure would act. Along with the assumption that each of the cylinders is symmetrical with the same responses to the combustion process.

The equation for force is simple and is shown as; $F = P \times A$.

Where F is the force of gas due to combustion in newtons, P is the pressure in the cylinder in Pascals and A is the area it's acting on, which in this case is the top of the piston and small area covered by the piston compression rings. So, the area is identified as; $\frac{\pi \cdot B^2}{4}$ with B being the size of bore in metres, which for the 1600cc is 0.079m. Therefore, the torque produced from this can be calculated through;

$$T_{Gas} = F_{Gas} \cdot r \cdot \sin(\theta) \cdot [1 + (r \cdot L \cdot \cos(\theta))] \quad [4]$$

Piston travel.

The piston travel is to be used for the calculation of work done further on in the study.

To calculate the travel the equation can be shown as;

$$S = r + L - r \cdot \cos(\theta) - L(1 - \frac{r^2}{L^2} \cdot \sin^2(\theta))^{\frac{1}{2}}$$

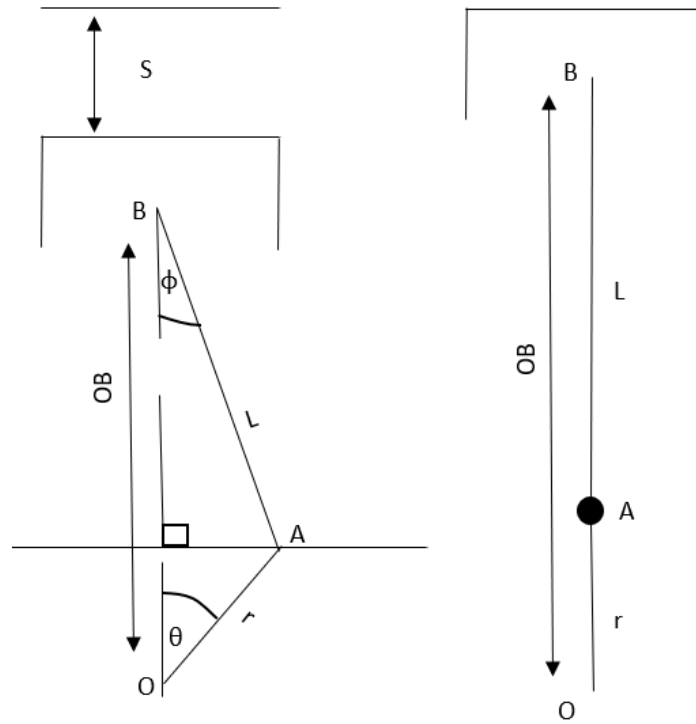


Figure 1 Trigonometric depiction of piston movement. [5]

Where;

S = Displacement [m]

r = Crank radius [m]

L = Stroke [m]

θ = Crank angle [$^{\circ}$]

ϕ = Rod angle [$^{\circ}$]

OB_{new} = The new distance between OB due to movement of the conrod.

To get the equation for piston travel, using trigonometry and functions of, start with;

$$S = r + L - OB_{new}$$

$$OB_{new} = OC + CB$$

From ΔOAC

$$OC = r \cdot \cos(\theta)$$

From ΔABC

$$CB = L \cdot \cos(\phi)$$

So, OB is;

$$OB = r \cdot \cos(\theta) + L \cdot \cos(\phi)$$

From ΔOAC

$$AC = r \cdot \sin(\theta)$$

From ΔABC

$$AC = L \cdot \sin(\phi)$$

Shown as;

$$r \cdot \sin(\theta) = L \cdot \sin(\phi)$$

Or;

$$\sin(\phi) = \frac{r}{L} \cdot \sin(\theta)$$

Using Pythagoras' Theorem;

$$\cos^2(\phi) + \sin^2(\phi) = 1$$

$$\cos^2(\phi) = 1 - \sin^2(\phi)$$

$$\cos(\phi) = (1 - \sin^2(\phi))^{\frac{1}{2}}$$

$$\cos(\phi) = (1 - \frac{r^2}{L^2} \cdot \sin^2(\theta))^{\frac{1}{2}}$$

Inputted into the equation;

$$S = r + L - r \cdot \cos(\theta) - L \cdot \cos(\phi)$$

Results in;

$$S = r + L - r \cdot \cos(\theta) - L(1 - \frac{r^2}{L^2} \cdot \sin^2(\theta))^{\frac{1}{2}}$$

Piston Velocity.

The equation for piston velocity with respect to time is;

$$v = \frac{ds}{d\theta} \cdot \omega$$

Piston instantaneous velocity is shown as;

$$v_{ins} = r\omega \left[\sin(\theta) + \frac{r}{L} \frac{\sin 2(\theta)}{2} \right] \quad [4]$$

Piston acceleration.

Acceleration with respect to angle is shown as;

$$\frac{d^2s}{d\theta^2} = r \cdot \cos(\theta) + \frac{r}{L} \cdot (r \cdot \cos 2(\theta))$$

The piston acceleration, with respect to time, is the second derivative of piston velocity. So, can be written as;

$$a = \frac{d^2s}{d\theta^2} \cdot \omega^2$$

Or;

$$\frac{d^2s}{dt} = r\omega^2 \cdot \left[\cos(\theta) + \frac{r}{L} \cdot \cos 2(\theta) \right]$$

Where;

r = Crank radius [m]

ω = Angular Velocity [rads/s]

L = Connecting Rod length [m]

θ = Crank angle [°]

Inertia Loads.

Regarding the inertia loads in of the engine internals, there were two types to be considered in this study. One was the force on the system due to inertia and secondly was the torque that is created from this force and how these would impact the results.

“Inertia force is obtained by multiplying the piston acceleration by the reciprocating mass and acts only in the line of the cylinders.” [4]

The inertia force equation is identified as;

$$F_{inertia} = M_{rec} \cdot r\omega^2 \cdot \left[\cos(\theta) + \frac{r}{l} \cdot \cos(2\theta) - \frac{1}{4} \cdot \frac{r^3}{l^3} \cdot \cos(4\theta) \right] \quad [4]$$

The inertia torque that occurs from the motion of the piston is represented as;

$$T_{inertia} = M_{rec} \cdot r^2\omega^2 \left[\frac{r}{4L} \cdot \sin(\theta) - \frac{1}{2} \cdot \sin 2(\theta) - \frac{3r}{4L} \cdot \sin 3(\theta) \right] \quad [4]$$

These equations are based on Newtons 2nd Law of motion;

$$F = M \cdot a$$

F = Force [N]

M = Mass [Kg]

a = Acceleration [m/s²]

And in the form of angular;

$$T = I\alpha$$

T = Torque [N/m]

I = Moment of inertia [Kg/m²]

α = Angular acceleration [rads/s²]

Highest Net forces on connecting rod.

The net forces acting on the connecting rod are shown by the following equations and it will be used to show both the highest compressive and tensile forces.

The net force acting down in the vertical plane is equal to;

$$F_{Net} = F_{Gas} - F_{inertia}$$

Using figure 1, to calculate rod angle Φ we can show that;

$$AC = L \cdot \sin(\Phi)$$

$$AC = r \cdot \sin(\theta)$$

So, rearranging for Φ which is the rod angle, we get;

$$\sin(\Phi) = \frac{r}{L} \cdot \sin(\theta)$$

$$\Phi = \sin^{-1} \left[\frac{r}{L} \cdot \sin(\theta) \right]$$

Then for the rod force, because the equation is;

$$F_{net} = F_{rod} \cdot \cos(\Phi)$$

$$F_{rod} = \frac{F_{gas} - F_{inertia}}{\cos(\Phi)}$$

So, the net compressive force is simply the product of the maximum force the connecting rod incurs.

Stresses.

To formulate the stresses that the rod will see, is simply shown in the form of;

$$\sigma = \frac{F_N}{A}$$

[For this study, $A = 139.58\text{mm}^2$]

Where F_N is the normal force that is perpendicular to the rod, which is either the compressive or the tensile forces acting on the rod, in newtons. And A is the area in

which it acts over, identified as the smallest cross-sectional area of the con-rod in millimetres squared.

Safety factors.

To return a safety factor based on the forces seen by the rod, with respect to the materials ultimate tensile strength (UTS) is written as;

$$S_f = \frac{UTS}{\sigma}$$

$$[UTS = 750N/mm^2]$$

The UTS should be in the unit of N/mm^2 because the stress force was also calculated in this unit format. Hence the reason, when divided, the units cancel out leaving a simple ratio named as ‘the safety factor’.

The safety factor is a way for larger manufacturers to identify lifetime of the parts and reduce the amount of warranties they must cover. Therefore, “failure of the connecting rod is not common since the big automobile companies try to keep very high factor of safety of 2 or 3 above” [6].

Compression and Stretch of Rod.

There is an important factor to consider during the manufacture process of the connecting rod regarding how much it may extend under high tensile loads because of possible interference with the valves. Therefore, using the following equation, it's simple to get an understanding of this unwanted potential travel could cause damage based on the tolerance between the piston crown and the valves.

$$x = \frac{F_S \cdot L}{E \cdot A}$$

Where;

x = Change in distance [mm].

F_S = Stress force [N].

L = Connecting rod length [mm].

E = Youngs modulus (Elastic modulus) [MPa].

A = Cross sectional area of the connecting rod [mm^2].

The equation comes from the relationship between stress and strain;

$$\sigma = \frac{F_N}{A}$$

$$\varepsilon = \frac{x}{L}$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon} = \frac{\frac{F_S}{A}}{\frac{x}{L}} = \frac{F_S \cdot L}{x \cdot A}$$

Make x the subject;

$$E = \frac{F_S \cdot L}{x \cdot A}$$

$$x = \frac{F_S \cdot L}{E \cdot A}$$

Side forces.

Because although the piston moves in a reciprocating motion, the connecting rod will travel in a rotary motion. And due to the components being based on a pin-to-pin jointed system, the force can only act down the rod. Consequently, there will be a resultant force acting tangentially from the angle of the connecting rod against the side of cylinder. This can be shown as;

$$\frac{\sin(\phi)}{\cos(\phi)} = \frac{F_{side}}{F_{net}} = \tan(\phi)$$

$$F_{side} = F_{net} \cdot \tan(\phi)$$

Radial force acting on the rings.

The force acting on the ring from the gas pressure can be modelled by using;

$$F = P \cdot A$$

With P being the pressure that is created from the gas inside the cylinder and A is there area in which it acts over. That area can be shown as the height of the piston ring [1.2mm] multiplied by the circumference of the ring.

[This is assuming that the full pressure from the gas has an effect along with the fact that acts equally over the piston ring area. Along with ignoring the tension force.]

Side and Radial frictional forces.

The frictional losses within an internal combustion engine is just one of the reasons why their efficiency struggles to get out the 20 – 35% range.

One of the biggest impacts on the friction is that caused by the piston rings and although the modelling is very complicated, we can make a simple model using the friction equation of;

$$F_{Friction} = \mu \cdot F_N$$

μ = Coefficient of friction.

[Boundary coefficient of friction (μ) is 0.1. [7]]

F_N = Normal force.

So, to result a simplified model and produce a rough idea of the frictional forces incurred from the reaction side force and pressure acting on the piston rings. We can then incorporate the force per degree from side force and then the radial force, multiplied by μ . Because the frictional force is always a constant loss through the cycle, we need to incorporate it as an absolute integer per degree and remove any negatives.

Work done.

Work done across the engine is simple when we have the force per degree and distance travelled by the piston.

The equation is written as;

$$W = F_{avg} \cdot \Delta s$$

$$W = \left(\frac{F_1 + F_2}{2} \right) \cdot (S_2 - S_1)$$

W = Work done [J/°].

F_{avg} = Average force at each degree [N].

Δs = Distance travelled per degree [m].

The total work done on the system by the gas and frictional forces and can summed per degree for a total per cycle;

$$W_{Gas} - W_{Friction} = W_{Total}$$

Torque.

The torque produced from the engine is the sum of the gas and inertia torque.

Starting with the instantaneous results of these;

$$T_{total_{ins}} = T_{Gas_{ins}} + T_{inertia_{ins}}$$

With the total torque per degree;

$$T_{Total} = \frac{\Sigma T_{total_{ins}}}{n_{degrees}} \cdot N$$

[For this study; $n = 720^\circ$, $N = 4$]

$n_{degrees}$ = Total number of degrees per cycle.

N = Number of cylinders.

Mean effective pressures.

There is three types of mean effective pressures that were calculated in this study; Indicated mean effective pressure [IMEP], Brake mean effective pressure [BMEP] and then Frictional mean effective pressure [FMEP].

The IMEP is a representative of the mean effective pressure over a cycle, that is resultant of the gas force and doesn't take into consideration the efficiency of the engine.

$$IMEP = \frac{\text{Total } WD_{Gas} \text{ (across engine)}}{\text{Engine displacement}}$$

Or per cylinder;

$$IMEP = \frac{\text{Total } WD_{Gas} \text{ (one cylinder)}}{\left[\frac{\text{Engine Displacement}}{\text{Number of cylinders}} \right]}$$

The BMEP is considering the frictional losses within the engine, it can be shown as;

$$BMEP = \frac{\text{Total } WD_{Gas} - \text{Total } WD_{Friction} \text{ (across engine)}}{\text{Engine Displacement}}$$

$$BMEP = IMEP - FMEP$$

Or per cylinder;

$$BMEP = \frac{\text{Total } WD_{Gas} - \text{Total } WD_{Friction} \text{ (Per cylinder)}}{\left[\frac{\text{Engine Displacement}}{\text{Number of cylinders}} \right]}$$

Then with IMEP & BMEP calculated, FMEP is;

$$FMEP = IMEP - BMEP$$

Mechanical efficiency can be written as;

$$n_m = 1 - \frac{FMEP}{IMEP}$$

4. Analysis.

Gas Force.

Firstly, recording the peak cylinder pressures and showing them as a maximum force the piston endures. This is important because it shows the maximum force the system components will ever see in a vertical direction, when in the OEM configuration. This is important for the manufacture process and specification of what will be required to meet Ford's targets expected lifetime of the component and reduce warranty returns.

	1000	1500	1700	2000	2500	3000	3500	4000	4500	5000	5500	5700	6000
Peak Gas Force (kN)	22.53	28.36	32.40	35.53	37.58	39.11	40.47	41.57	41.32	42.76	45.58	46.69	46.86
Peak Gas Force (kN) B	24.63	31.01	35.42	38.84	41.08	42.76	44.24	45.45	45.18	46.75	49.83	51.04	51.23
Peak Gas Force (kN) S	21.83	27.49	31.39	34.43	36.41	37.90	39.22	40.29	40.05	41.44	44.17	45.25	45.41

Table 2 Peak gas pressures across RPM for Standard and the modified engines.

As can be seen in the *Table 2*, the peak forces for each engine happen at the highest RPM. Because the most part of the compression and combustion strokes can be treated as an isochoric process, where pressure is directly proportional to temperature.

Therefore $\frac{P}{T} = \text{Constant}$, so at higher RPM there is less time for heat transfer, temperature remains higher, so pressures must increase for the relationship to remain as a constant.

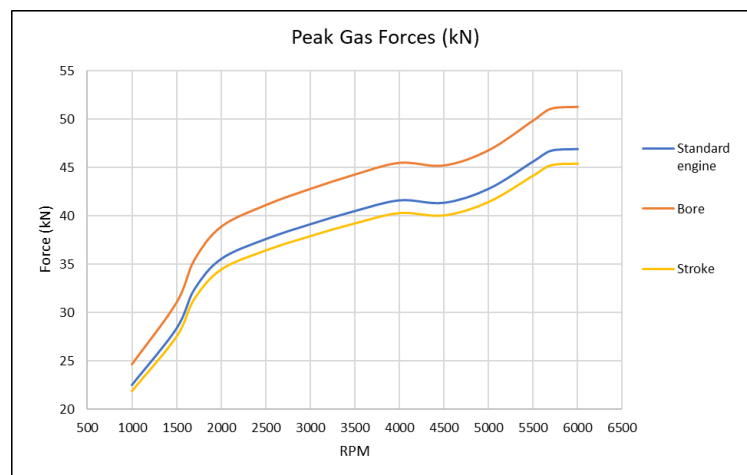


Figure 2 Peak gas force per RPM (Standard, modified bore and modified stroke)

Looking at the graph, the larger bore engine has higher peak forces because of the equation, larger area with the same pressure results in a larger force. And of course, the smaller the bore gets the less force, hence the pattern shown in the graph.

Inertia forces.

The inertia forces are a resultant of Newtons second law, in terms of $F = M \cdot a$, the object in motion will have a resultant inertia force when a change in direction occurs, that is opposite to said direction and this is why the piston will try to continue moving in the previous direction before the change. This being the primary force, will increase with engine speed, as seen in figure 3 and will cause serious unwanted oscillations if not counteracted.

These primary forces naturally balance themselves on an in-line four-cylinder engine because of the turning moment couples of cylinders 1 & 4 equally oppose the couple from 2 & 3. Counterbalances on the crankshaft itself are used more to reduce the bending moment caused.

But because of the nature of the reciprocating components having a shorter dwell period at the point of TDC in comparison to BDC (relationship between piston movement and crank motion), the inertia forces are much greater at top dead centre compared to bottom dead centre. [8]

Within figure 4 it can be seen at BDC there is a deviation in the wave that doesn't follow the pattern, this is pointing out the factor of dwell period difference and the impact of second order forces.

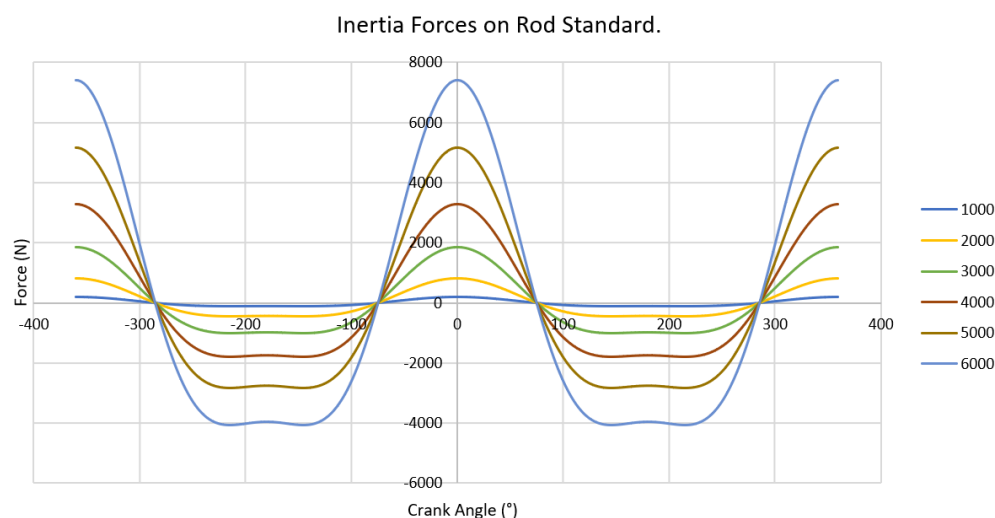


Figure 3 Standard engine inertia forces.

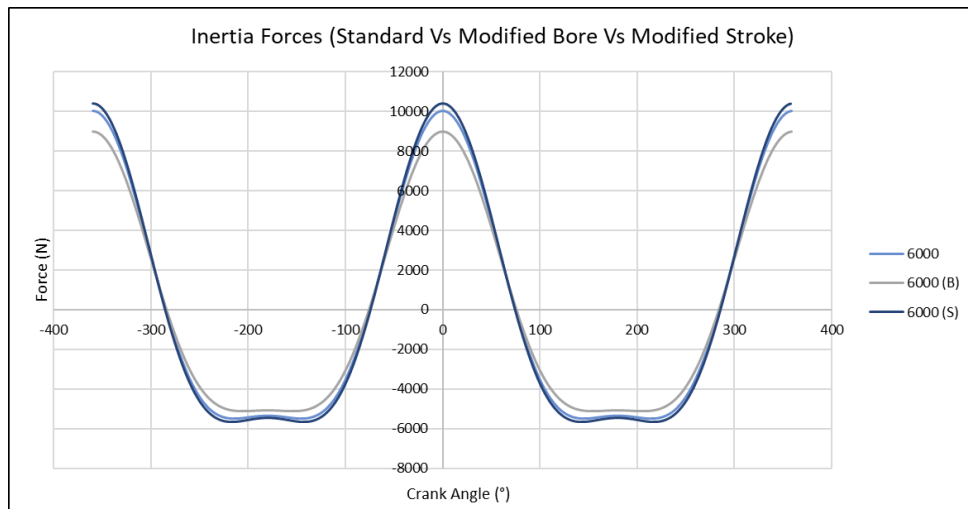


Figure 4 Inertia forces acting upon the rod for three RPMs on each engine.

Demonstrated in figure 5, the second order forces are happening at twice engine speed and because the upward peak of these are acting in the same direction as the primary [TDC], they work together and there is no disruption. But during BDC the second order are opposing the primary and are positive, therefore because they're added together, the force is reduced, and this causes the unbalanced vibration. The Fourth order terms are very minor (see figure 5) and can be neglected. The effect on the modified engines are either side of the standard, in terms of magnitude; the larger bore, shorter stroke has lower forces and vice versa for the smaller bore, longer stroke as seen in figure 4.

This is because the equation is resultant of crank radius and conrod, if the rod remains the same length, the ratio increases if crank radius is larger, increasing the force. And obviously it's the opposite for a shorted stroke.



Figure 5 Different order Inertia forces.

Highest Net compressive force.

The net compressive force is simply the highest force acting in a downwards motion and will act directly through the rod, because of its pin jointed nature. It's a product of the inertia force subtracted from the gas force. The gas force far outweighs the inertia force during the compression and combustion strokes of the engine, but during the exhaust and induction can be overcome by inertia forces, as will be discussed later.

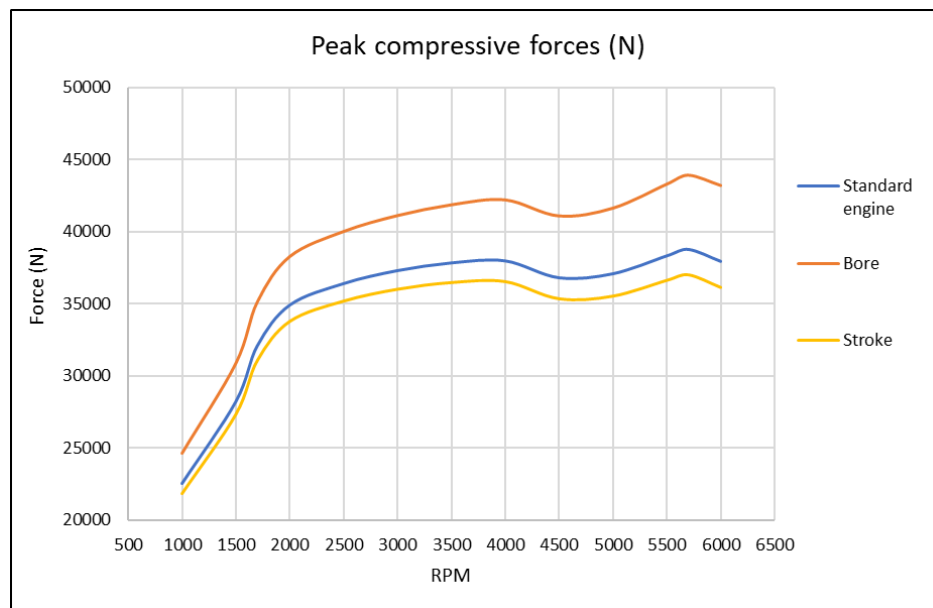


Figure 6 Peak Compressive Forces on rod for each engine.

The pattern shown is clearly showing that as the pressures inside the cylinder increase, so do the peak forces occurring down the rod. It naturally follows the peak gas force seen and as expected the larger bore experiences the most force (larger acting area). The drop in the curve at 4000RPM is suggestive of when the boost pressure from the turbocharger begins to fade, but as discussed in peak gas force, due to the relationship of pressure and temperature, the cylinder pressure will continue to increase. And as can be seen, it peaks at 5700RPM and this corresponds to data posted from ford, that the maximum power is at this point. The point after this for (6000RPM) decreases rather linearly, which is also the point that torque produced from the gas force is decreasing at its most aggressive rate, as seen in figure 7.

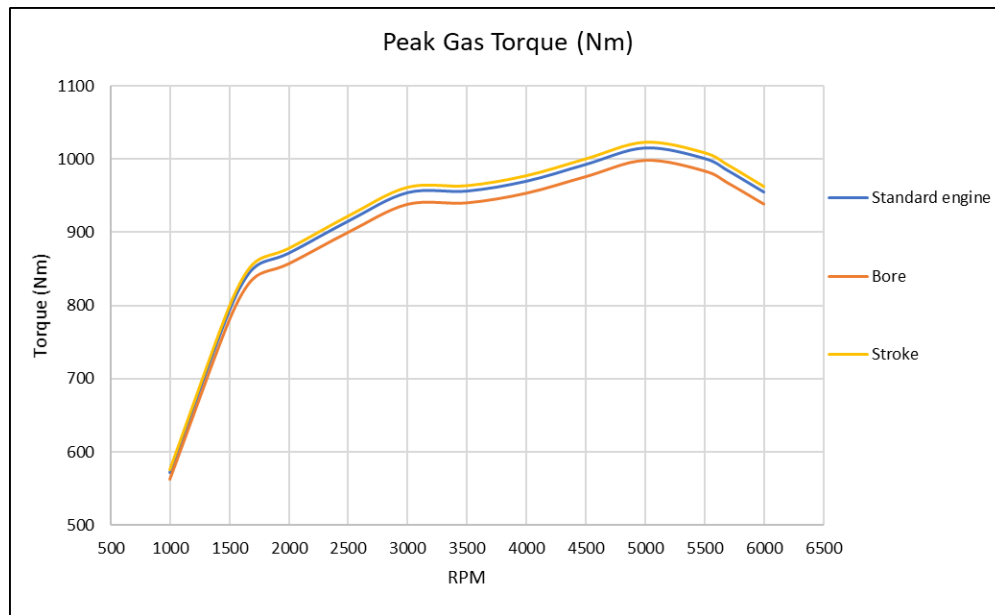


Figure 7 Peak gas torque figures for each engine.

Given the relationship between power and torque, $P = T\omega$, although the angular speed is increasing, the torque brings down the power produced. Along with this, the inertia forces are also increasing in an exponential fashion [see figure 8] and they are acting in the opposing direction of the compressive force, reducing the effect.

Highest Net tensile force.

The tensile force acting through the rod is an unwanted, but naturally occurring force. It acts against the change in direction when at TDC, stretching the connecting rod and resisting the change. It's based on newtons second law, $F = ma$, and although the mass of the reciprocating assembly concerned; "the piston, piston rings, piston pin, and the equivalent mass of the upper end of the connecting rod" [9] remains constant. The acceleration seen will increase with the engine speed, obviously increasing force and this is clearly depicted in figure 8.

Because of this natural correlation, the only way to reduce the inertia force is reduced the mass being reciprocated.

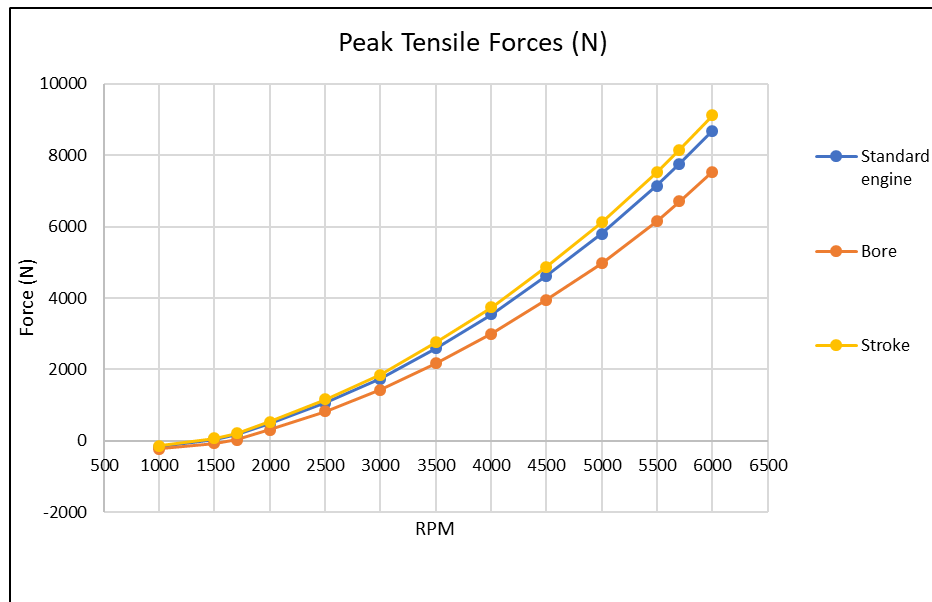


Figure 8 Peak Tensile Forces.

Looking at figure 8, the engine with a longer stroke is experiencing the highest tensile force as expected. This is because it is being accelerated at a greater rate than the standard engine or the engine with a modified bore. This is due to the crank radius being larger and given the acceleration equation;

$$\frac{d^2s}{dt^2} = r\omega^2 \cdot \left[\cos(\theta) + \frac{r}{L} \cdot \cos 2(\theta) \right]$$

The larger the r input, the larger the acceleration for a given angle θ , therefore explaining the decrease for the standard engine and again with the larger bore engine with a smaller stroke.

Maximum compressive stress.

“The stress applied to a material is the force per unit area applied to the material. The maximum stress a material can stand before it breaks is called the **breaking stress** or **ultimate tensile stress**.” [10]

The compressive stress on the rod is trying to squash the material together and using a simplified version on the ‘Rankine Gordon buckling equation’;

$$\text{Buckling Force} = \frac{\pi^2 \cdot E \cdot I}{L^2}$$

Where I is;

$$I = \frac{B \cdot D^2}{12}$$

B = Breadth [mm]

D = Depth [mm]

E = Young's modulus [MPa]

L = Connecting rod length [mm]

The force required to buckle the rod when theta is zero, is 680Kn of force.

This isn't achievable via gas force, as seen in figure 6, but despite this the connecting rod will experience compressive stress and it will, in turn, cause a change in length of the rod as discussed later.

The maximum compressive rod stresses are shown in figure 9 and as expected, because they're a product of gas force, the larger bored engine experiences the higher compressive forces.

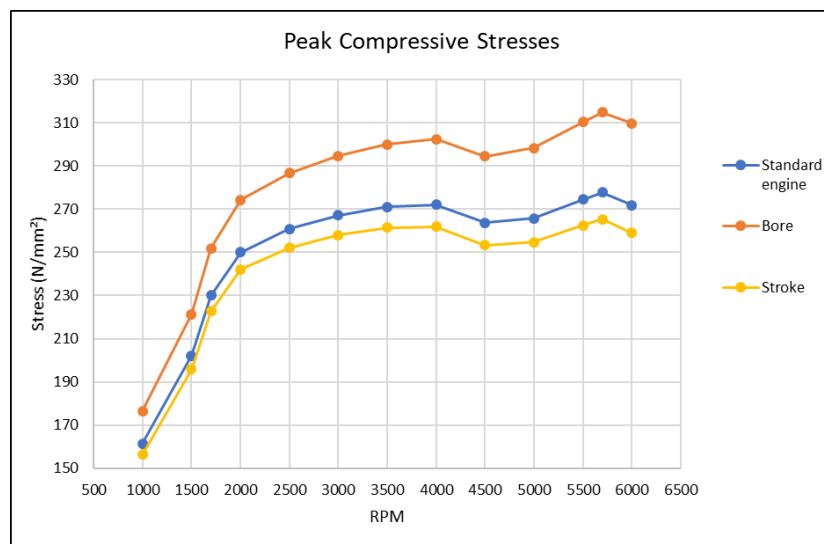


Figure 9 Peak compressive stresses.

Maximum tensile stress.

The tensile stress that occurs on the rod is a product of the inertia forces acting on the rod. The peak placement of these forces, with respect to crank angle, are at and around the point TDC during the end of the exhaust stroke due to the lack of force from gas pressure opposing it. These stresses increases in an exponential fashion regarding the engine speed, because of the inertia force relationship as discussed and this can be seen in figure 10.

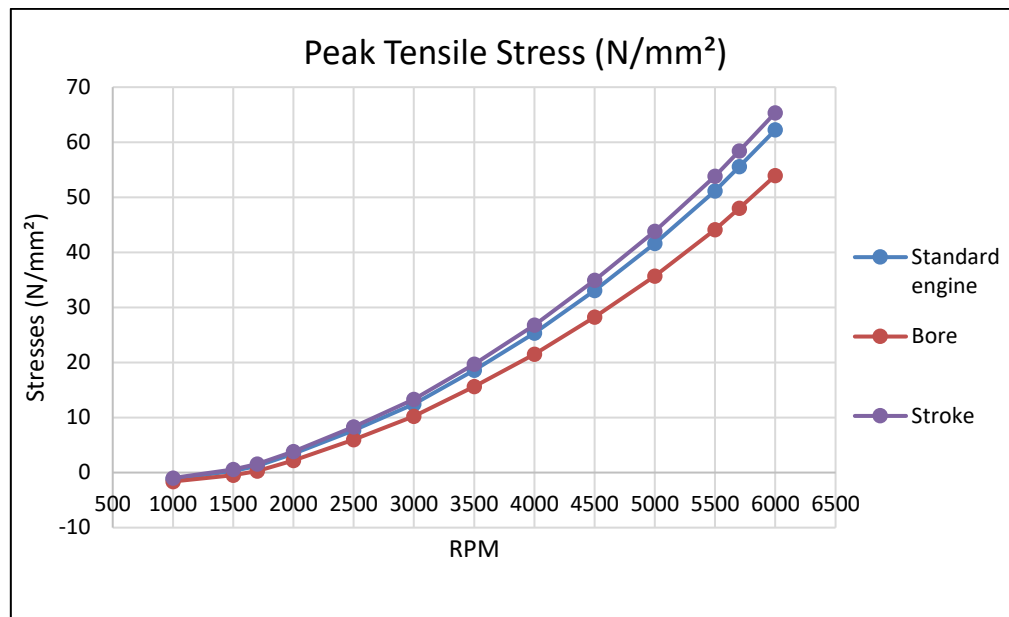


Figure 10 Peak tensile stresses.

At the lower engine speeds, the tensile force seen on the rod isn't as such, the inertia force that creates the stress has yet to have overcome the gas force that remains in the cylinder during the exhaust stroke, demonstrating compressive force and not tensile. This will have been a positive force, with the rest negative simply showing the direction of the force in comparison to the direction of travel. The results were flipped for ease of representation and therefore at 1000RPM the results is a negative value.

Static safety factors.

The static safety factor is simply a representation of the UTS of the material (750MPa) divided by the amount of stress force seen. With compressive stresses being much larger than the tensile, obviously the safety factor is going to be lower. For the major OEM's, they're confident in their designs when the compressive safety factor is around the 2 – 3 mark. As seen in figure 11, this is what ford had easily achieved and as for the

tensile safety factor, they're well above the mark because their inertia forces are nothing in comparison to combustion gas pressure forces.

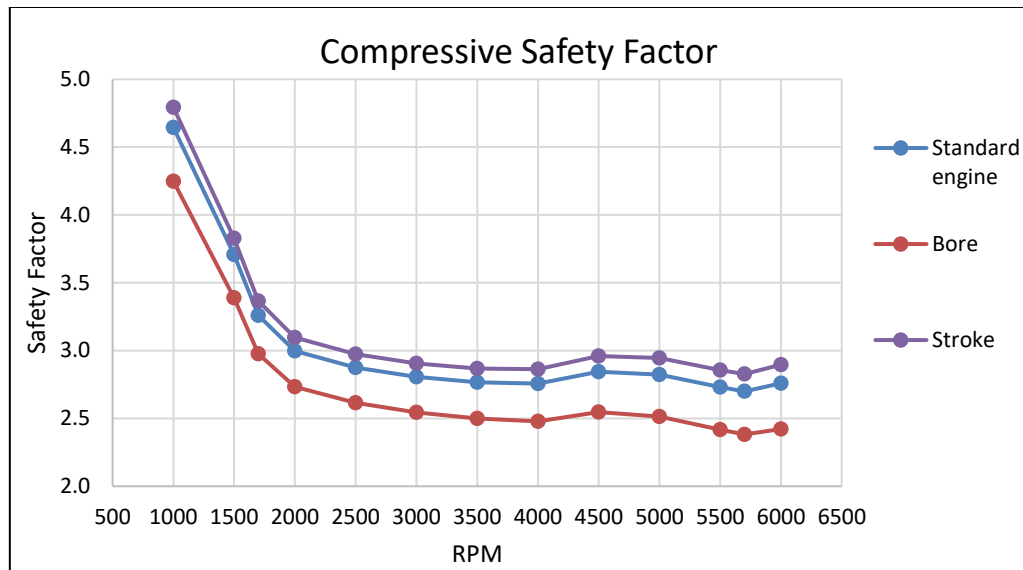


Figure 11 Compressive safety factor.

The relationship for between safety and stress is clear because figure 11 is representing the same trend shown in figure 9, simply in the opposite direction. The results seen show that Ford had specified the connecting rod to have a minimum safety factor of around 2.4.

As for tensile, the safety factor far exceeds the 2 – 3 limit as seen in figure 12, so there is no concern regarding breakage from stretch due to tensile force.

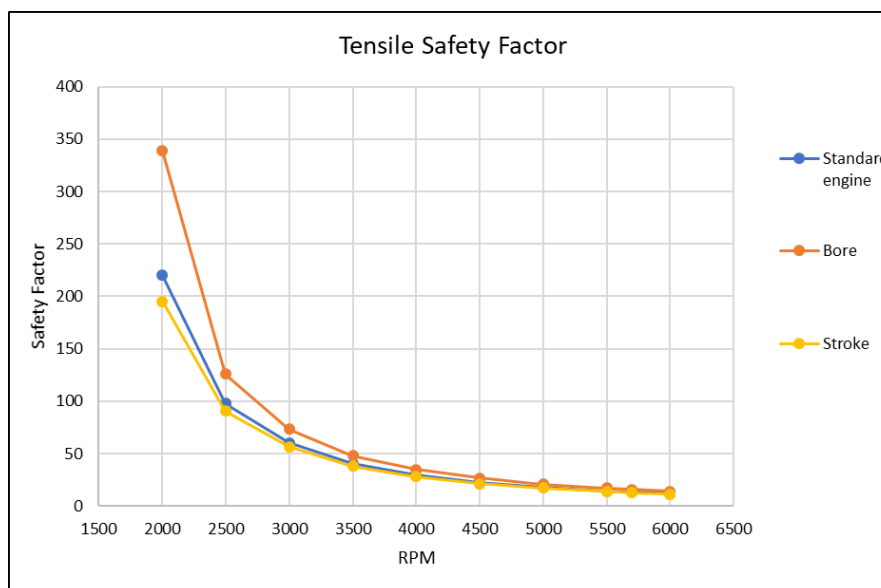


Figure 12 Tensile safety factor.

Maximum compression and stretch.

Due to the forces experienced by the connecting rod and the elastic properties of said material [Youngs modulus], there is a change in distance for the original shape. This change in length is important when characterising the internal clearances on the engine regarding potential collision/interference with the valve train during the representative strokes of the engine and therefore the stretch is important. With compression there isn't much to be concerned for an interference aspect, more in terms of a fatigue study and expected lifetime of the part. The change in length is very minimal as seen in figures 13 & 14.

The usual clearance between the valve and head of the piston is said to be around 2 – 2.5mm, and as seen on figure 14, the stretch of the rod isn't going to interfere in anyway.

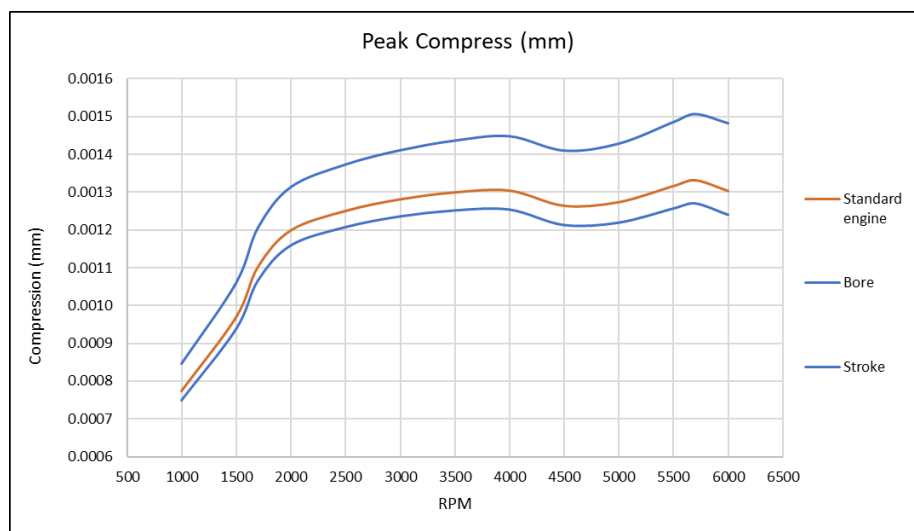


Figure 14 Peak compression (mm)

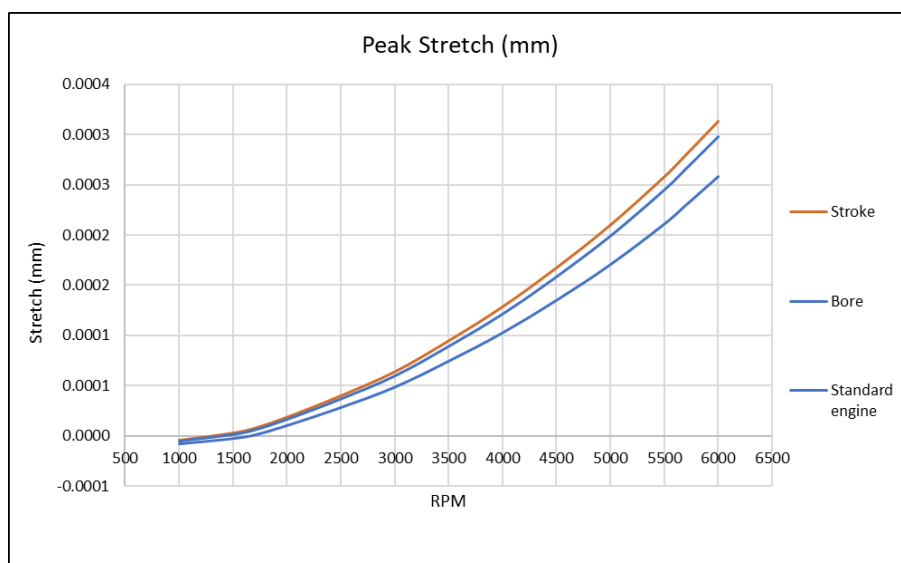


Figure 13 Peak stretch (mm).

Titanium [Ti-6Al-4V].

The study of the impacting forces acting on the connecting rod was also conducted considering the material used as titanium. The material properties were set as a UTS of 950MPa, an elastic modulus of 113.8GPa and a density of 4430Kg/m³ [11].

The inertia force acting on the rod were re-calculated using simple density relationship equation to achieve the new reciprocating mass value.

$$\rho = \frac{M}{V}$$

ρ = Density [Kg/m³]

M = Mass [Kg]

V = Volume [m³]

Using for the values of steel to return the volume of material used in the rod, returned a value of (4.88e⁻⁵ m³).

$$M = \rho \cdot V$$

Using the new mass of the connecting rod in the equations, it gave a more accurate representation on the forces now acting on the rod and the new parameters regarding the engine internals. As seen in the following figures;

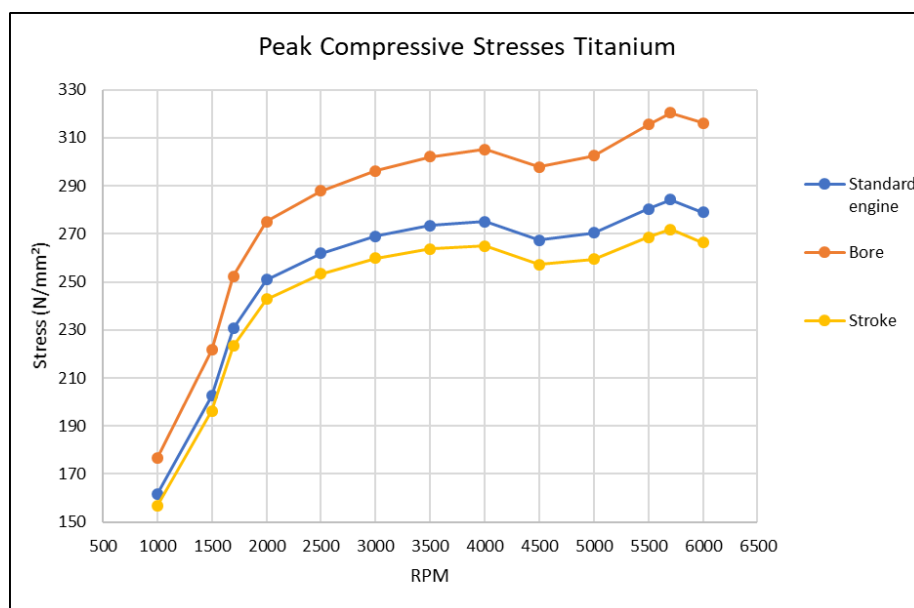


Figure 15 Peak compressive stresses for titanium rod.

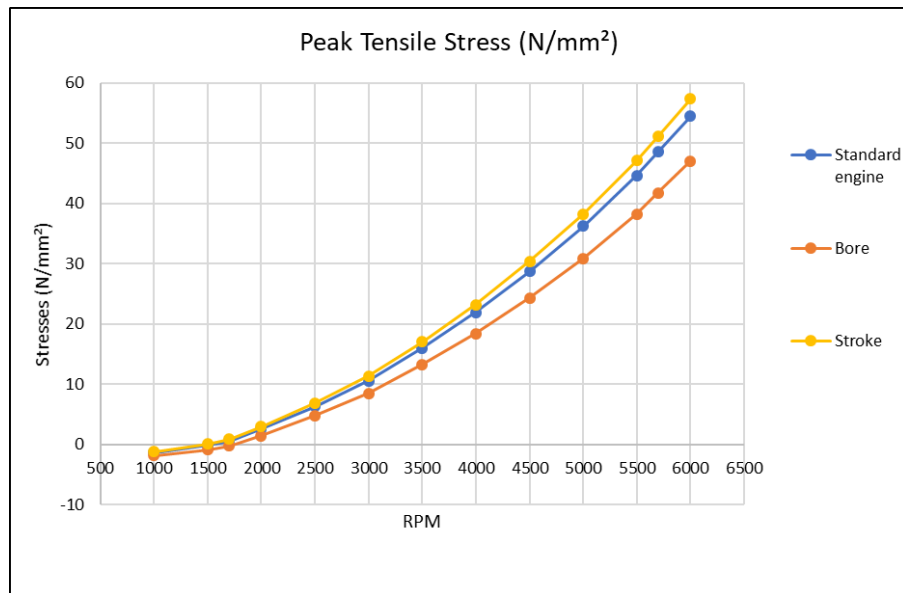


Figure 16 Peak tensile forces on titanium rod.

As can be seen when comparing figure 9 to that of figure 15, the compressive stresses on the rod are higher regarding the titanium rod, because there is less mass in consideration for the inertia forces being reciprocated, so the net force from the gas will increase. And as will be obvious, the engine types are representative of the peak gas force graph, the larger bore engine will experience the most compressive force.

But the engine with a longer stroke will see the highest tensile force, because of the relationship described earlier in tensile forces.

In terms of the safety factors, titanium has a much higher UTS [950MPa], therefore the obvious outcome is the safety factors across the RPM range is going to be higher, as seen in the following figures;

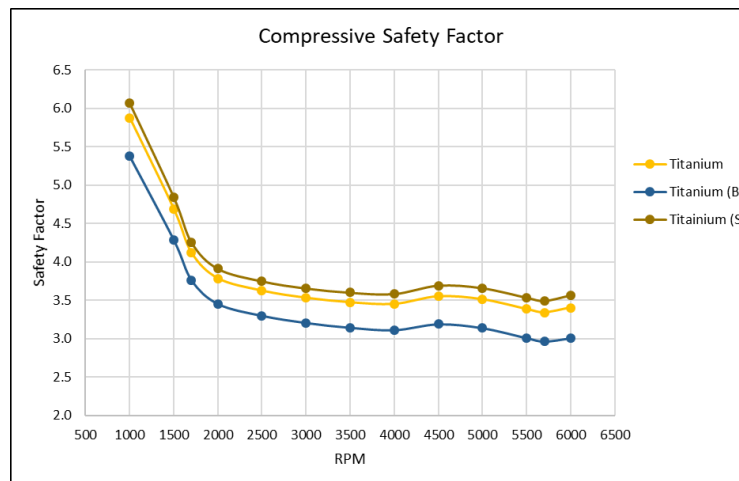


Figure 17 Compressive safety factors of Titanium.

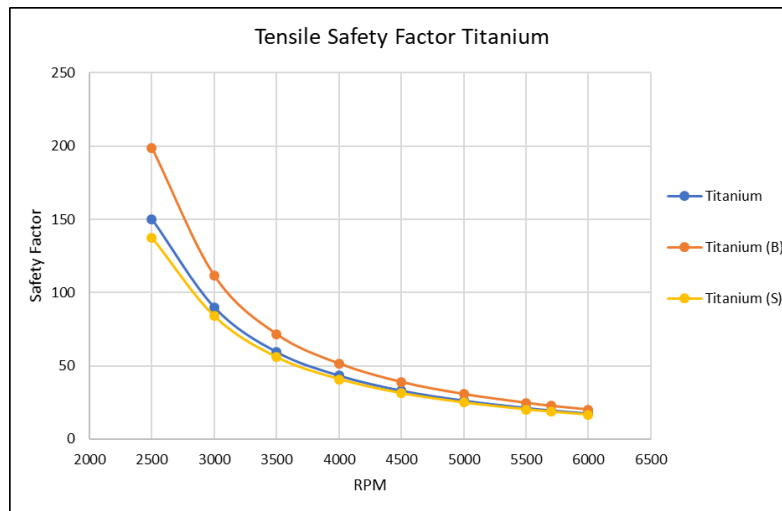


Figure 18 Tensile safety Factors of Titanium.

Figures 17 & 18 clearly showing the forces relationship again, as the engine with the larger bore has a lower compressive safety factor, it has the highest tensile safety factor because it experiences the least tensile forces that act negatively against the gas force. And then this relationship follows through, regarding the stroked engine having the highest compressive, but lowest tensile safety factor.

For the compression and stretch movement of the titanium rod, the elastic modulus is lower than that of the standard steel rod, therefore it has less resistance to deformation, and this can be seen in the figures below.

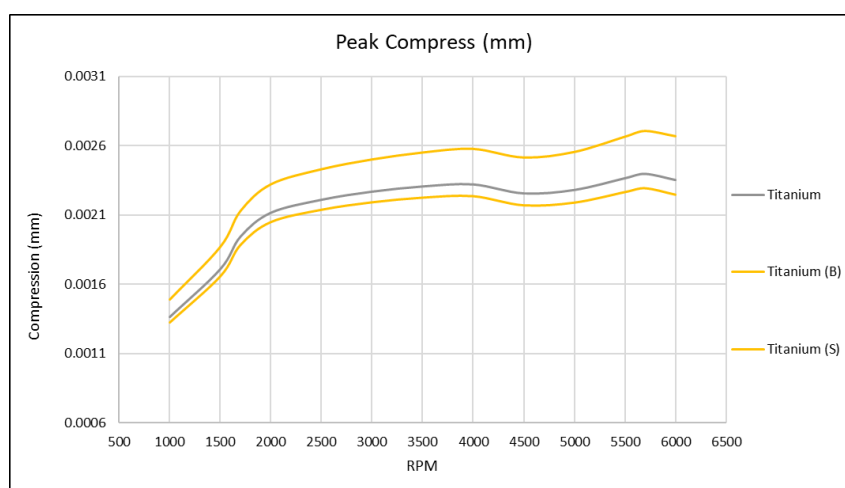


Figure 19 Compression movement of the titanium rod for each engine.

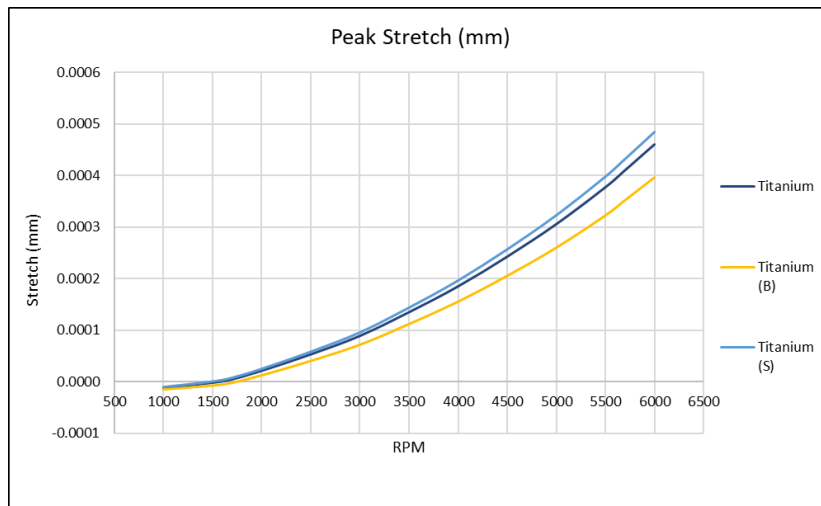


Figure 20 Stretch of the titanium rod for each engine.

The figures are clearly showing that the titanium rod is both compressing and stretching more than the of the standard steel. Again, the correlation between the stroke length effect is shown, longer stroke the higher the maximum stretch will be, therefore is will compress less and vice versa for the larger bored engine (shorter stroke).

If the clearance between the valves can be assumed as 2mm, then there is no reason no to use the titanium rod in the engine, because with the shorter stroke engine there is a gain in terms of positive effect gas force.

But of course, titanium is expensive to use, especially when considering a large-scale manufacture of production automotive vehicles.

5. Conclusion.

To conclude the findings of this report, characterising the internals of an engine is a difficult process and there are many factors to consider; cost, purpose, lifetime, warrantability, reliability along with many more. But from a kinematics prospective, the ideal would be an 'oversquare' engine because of the shorter stroke and this then bringing about less negative inertia forces if the mass is also kept low, with an added effect of higher peak gas forces. But this isn't the case, because the results of this in practice is higher side forces due to the larger rod angles seen, resulting in more frictional losses, whilst running the risk of reducing the expected lifetime of the components internally. Therefore, there is always a balance needed between efficiency and cost, when on a mass production level.

6. References

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7. Appendix 1. (Extra Graphs)

Graphical representations of the following;

Piston acceleration of the standard engine configuration.

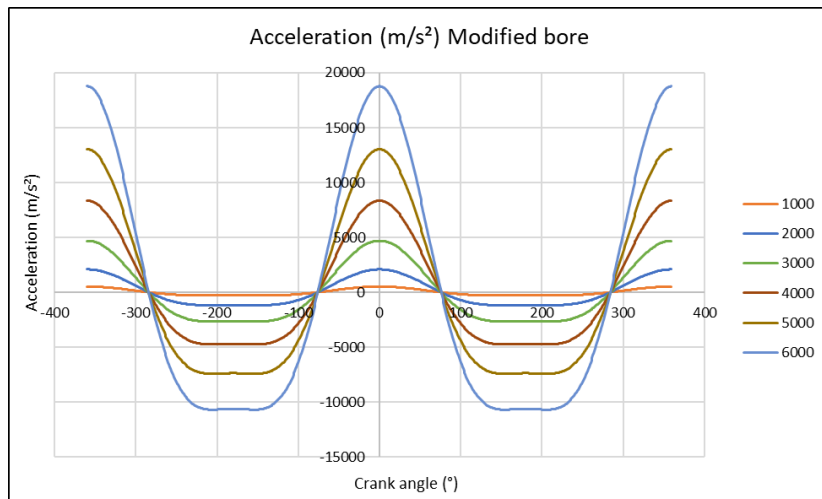


Figure 21 Standard engine piston acceleration.

Gas torque across most of the RPM range for the standard engine.

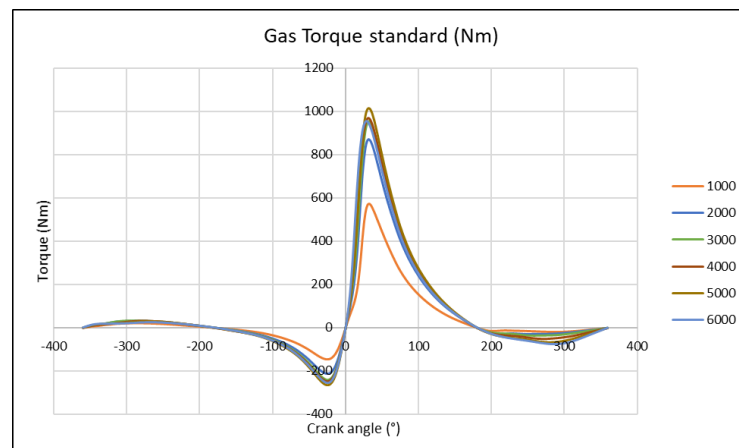


Figure 22 Gas torque from the standard engine.

Inertial torque across the RPM range for the standard engine.

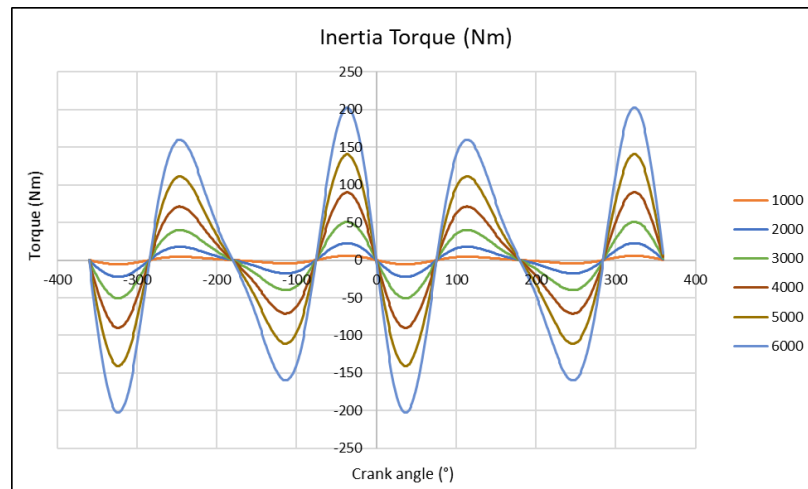


Figure 23 Inertial torque from the standard engine.

Total torque across the RPM range for the standard engine.

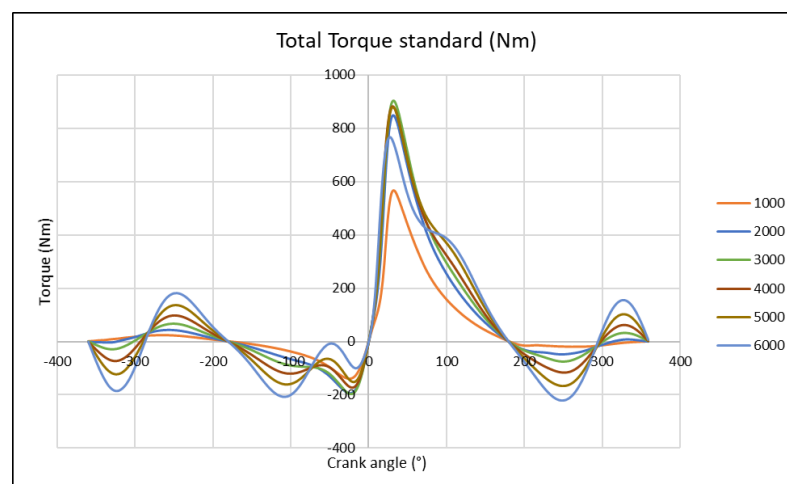


Figure 24 Total torque from the standard engine from one complete cycle.

Net force acting on the rod across the RPM range for the standard engine.

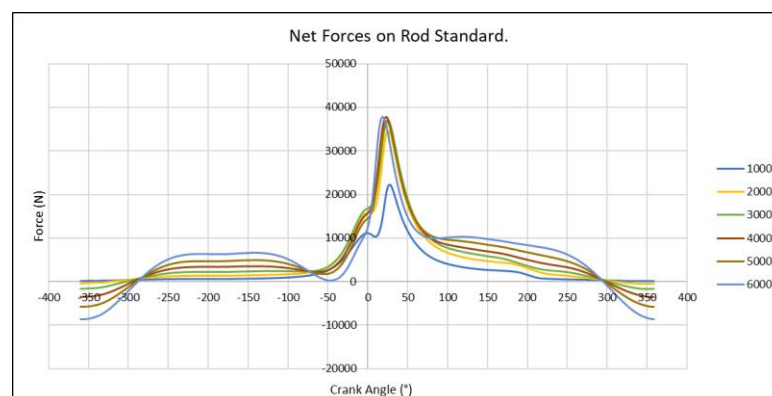


Figure 25 Net forces on standard engine.

Peak side forces from each engine.



Figure 26 Peak side forces on each engine across the RPM range.

Peak side force friction from each engine.

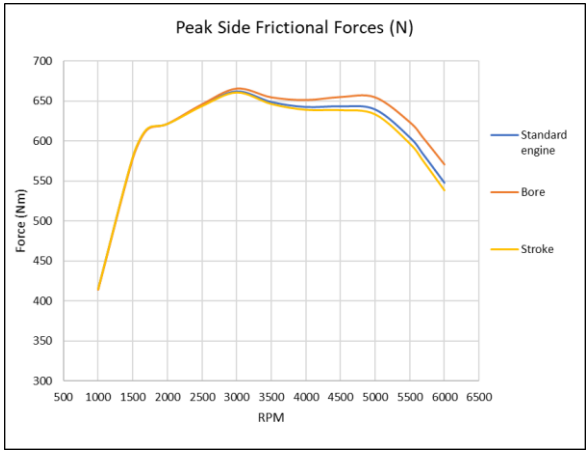


Figure 27 Peak frictional side forces on each engine.

Peak radial forces from each engine.

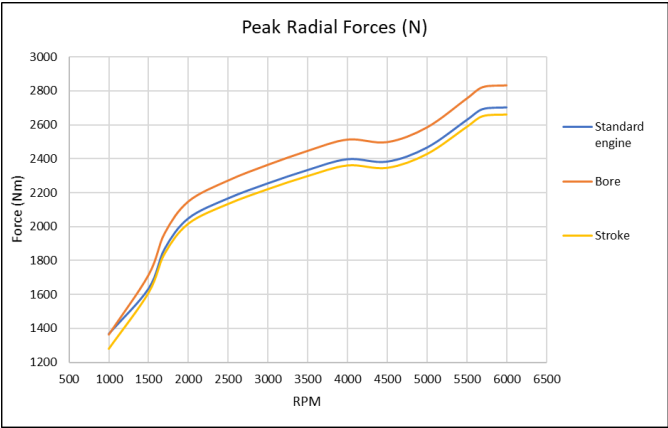


Figure 28 Peak radial forces for each engine.

Peak radial force friction from each engine.

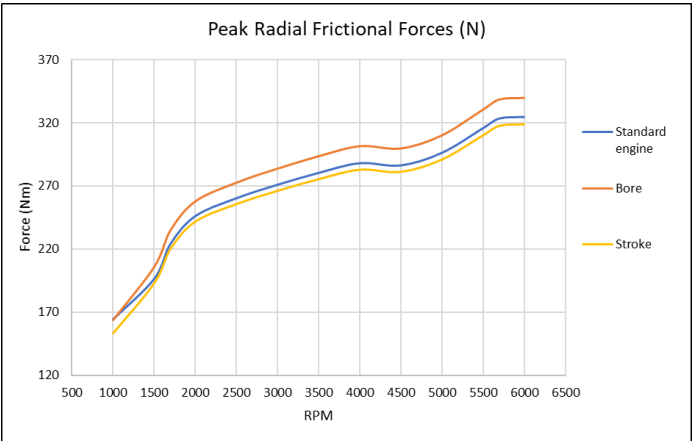


Figure 29 Peak radial frictional forces for each engine.

MEP's across the RPM range from the standard engine.

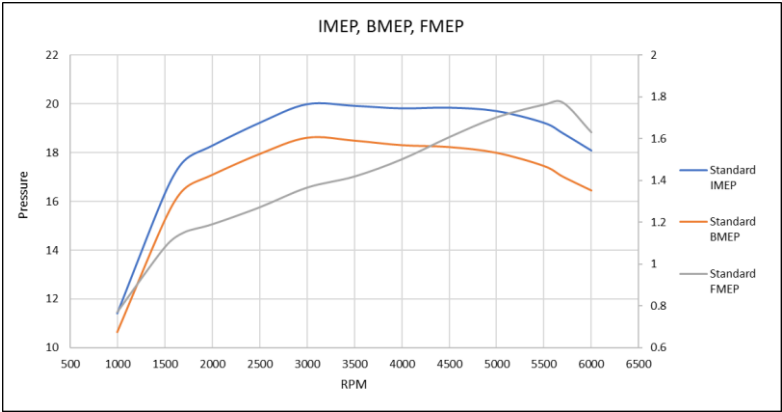


Figure 30 Indicated, Brake and Frictional mean effective pressures on standard engine across the RPM range.

Mechanical efficiency of the standard engine across the RPM range.

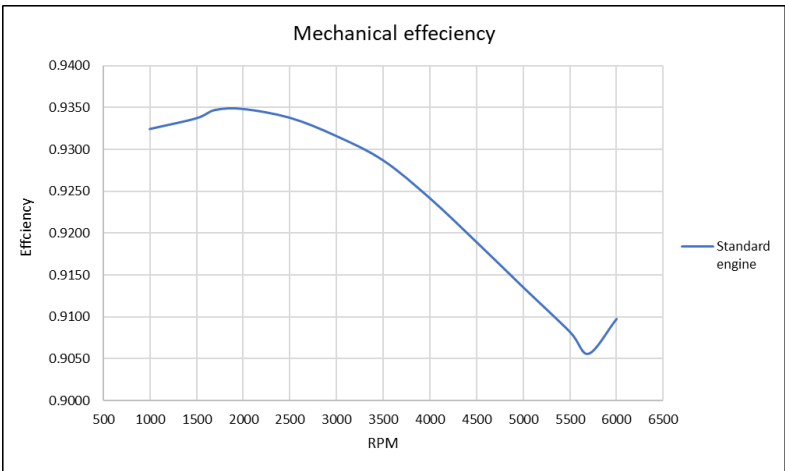


Figure 31 Mechanical efficiency of the standard engine.