Engineering Science Guidance Notes

Deflection of Beams and Cantilevers ES4

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Introduction

These Guidance Notes introduce you to the theory for a set of experiments in an engineering science topic. You fit different parts of your kit to a Work Panel to do an experiment. Figure 1 shows a typical experiment.

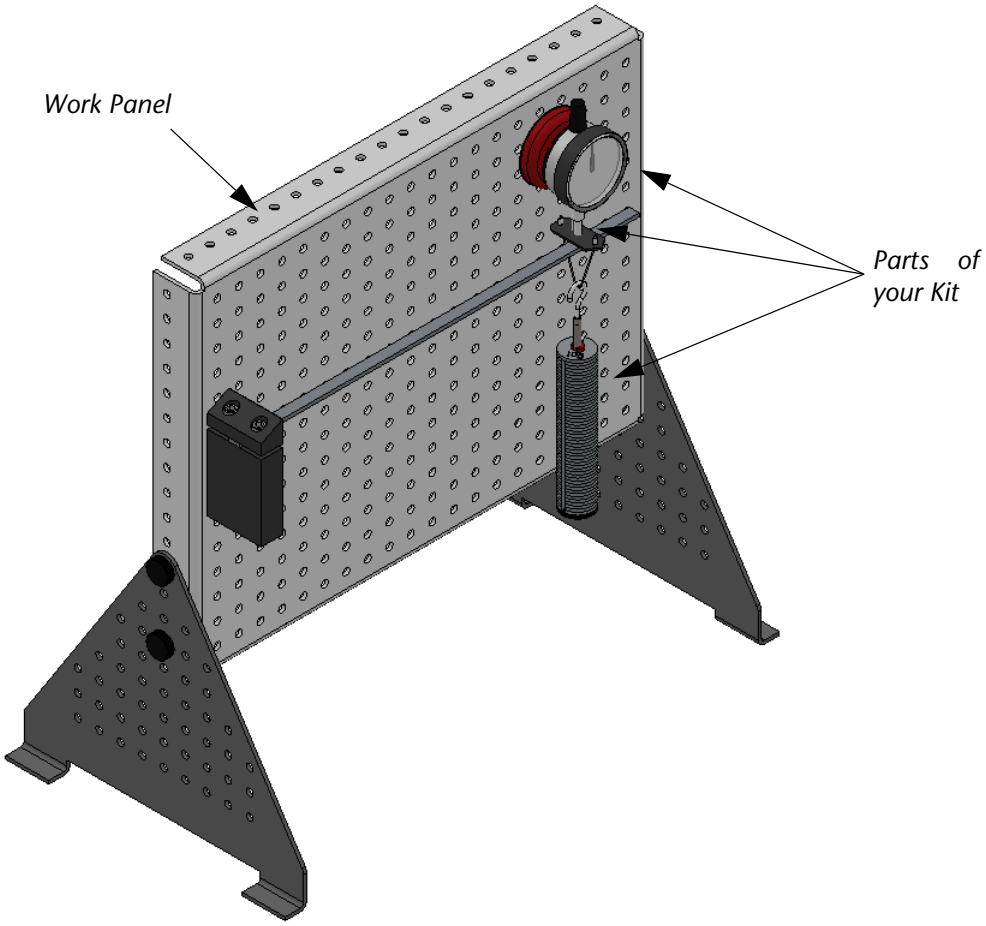


Figure 1 A Typical Experiment

Each kit can do one or more experiments and each experiment has Worksheets that tell you how to do the experiment. You must use the Worksheets with the Guidance Notes as they:

- Introduce the parts in the kit, and list the experiments that it can do.
- Give you important information you need to do the experiments or complete your Worksheet.

Your tutor may decide to ask you to do all the experiments or just a few. You must be sure what you need to do. To help you, Table 1 shows a list of the experiments for your kit.

The ES4 Kit - What is it and what can it do?

This kit helps you understand the factors that affect deflection (bending) in beams under load, and their importance to scientists and engineers.

The kit comes in a plastic box with a lid and contains all the parts you need to do the experiments shown in Table 1. Refer to the Parts List in your kit to see what parts are included.

Your tutor may decide to ask you to do all the experiments or just a few. You must be sure what you need to do.

List of Experiments

Experiment	Does my teacher need me to do this experiment?	Have I got the Worksheet?
1. Beam Load		
2. Beam Material		
3. Beam Dimensions		
4. Beam Supports		
5. Beam Length		

Table 1 List of Experiments

General Notes

The Work Panel

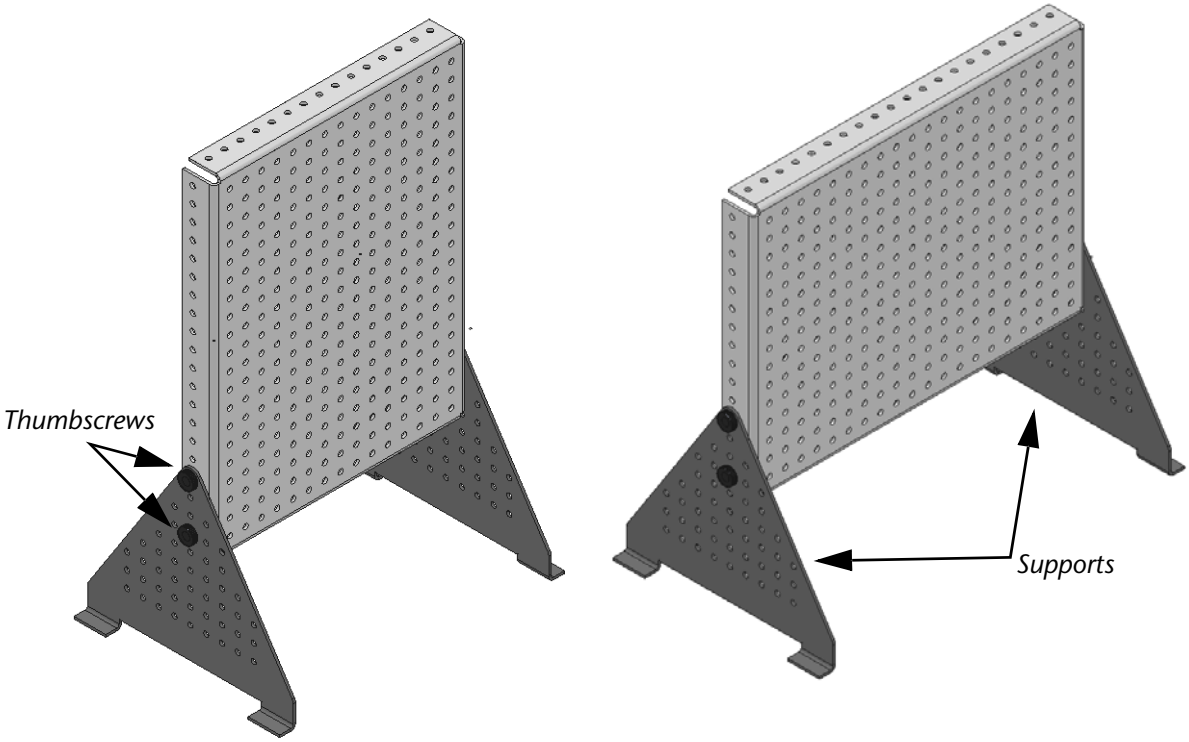


Figure 2 The Work Panel Mounted in Two Typical ways (Portrait and Landscape)

The Work Panel mounts on its Supports in different ways as needed by each experiment. The Worksheets show you which way TecQuipment recommends you to fit the Supports but you may find an alternative that fits better on your desk. To change how the Supports hold the Work Panel, ask your Teacher or a classmate to help you hold the Work Panel while you change the Supports around. However you mount the Work Panel, you must always use two Thumbscrews and Thumbnuts to hold each Support to the Work Panel.

Weights, Masses, Weight Hangers and Forces

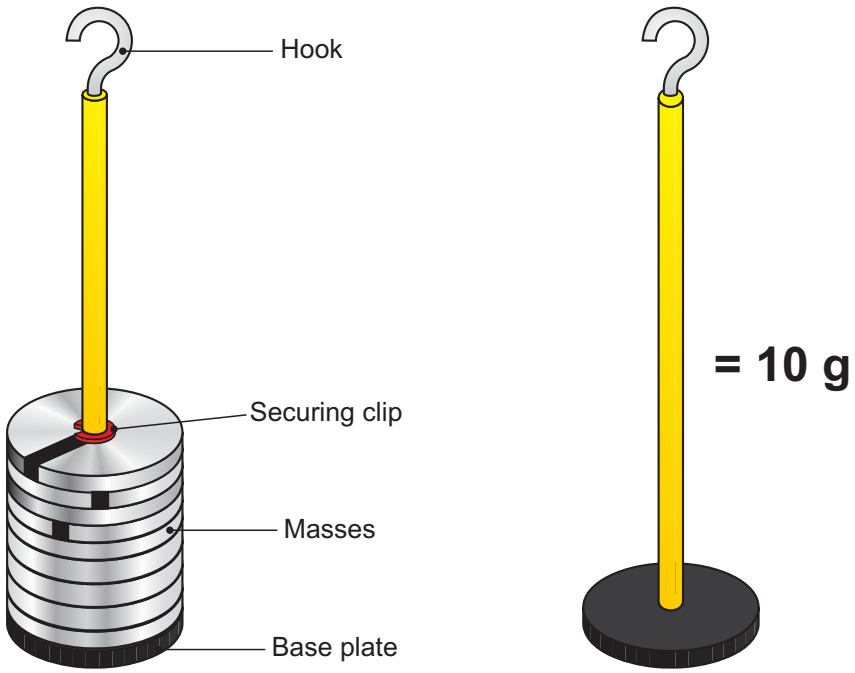


Figure 3 A Weight Hanger

The masses supplied with the Engineering Science Kits have markings in grams (or grammes). For your calculations, you use the unit of force (Newton - N) caused by the pull of gravity (downward) on the masses.

1 kg = 9.81 N or 100 g = 0.98 N

Note: Each weight hanger itself weighs 10 g, so you only need to add 9 x 10 g masses to get 100 g.

Weights	Force (N)
1 x 10 g = 10 g	0.098
10 x 10 g = 100 g	0.98
20 x 10 g = 200 g	1.96
30 x 10 g = 300 g	2.94
40 x 10 g = 400 g	3.92
50 x 10 g = 500 g	4.90

Using the Magnetic Base Safely

Carefully remove the metal 'keep' from the back of the Dial Indicator (see Figure 4). Store the keep safely with the other parts of the kit.

Carefully slide the Dial Indicator onto the Work Panel.

When you have finished your experiment, leave the Dial Indicator on the Work Panel or refit the keep back on the magnetic base.

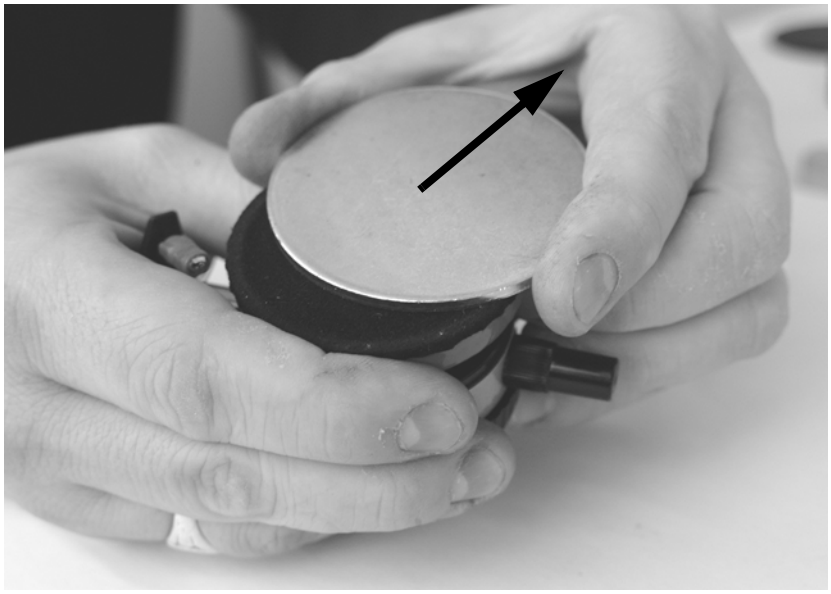


Figure 4 Carefully Remove the 'Keep'

CAUTION 

*Always **slide** the dial indicator onto the Work Panel, do not allow it to impact against the panel, as you may damage the Dial Indicator and the paintwork.*

The Dial Indicator contains a powerful magnet. Keep any sensitive mechanical watches or instruments away from the dial indicator when you handle it.

Always replace the 'keep' when you have finished using the Dial Indicator.

Setting up and Zeroing the Dial Indicator

Referring to Figure 5:

1. Slide the beam through the wire stirrup and fit it to the Fixings Blocks.
2. Slide the Dial Indicator Assembly down slowly until its tip just touches the top of the beam.
3. Turn the outer bezel of the Dial Indicator so that 0 is at the top.
4. Slide the Dial Indicator body further down with its tip resting lightly on the top of the beam until the smaller dial shows 0 and the large dial shows approximately 0. Do not push so far down that the tip pushes down (deflects) the beam.
5. Rotate the outer bezel of the dial indicator slightly if necessary so that the large pointer is at 0. If you have to rotate the bezel more than 30 degrees in either direction, readjust the height of the Dial Indicator slightly.
6. The smaller inner dial shows deflection in millimetres. The larger outer dial shows deflection in 0.01 mm divisions.

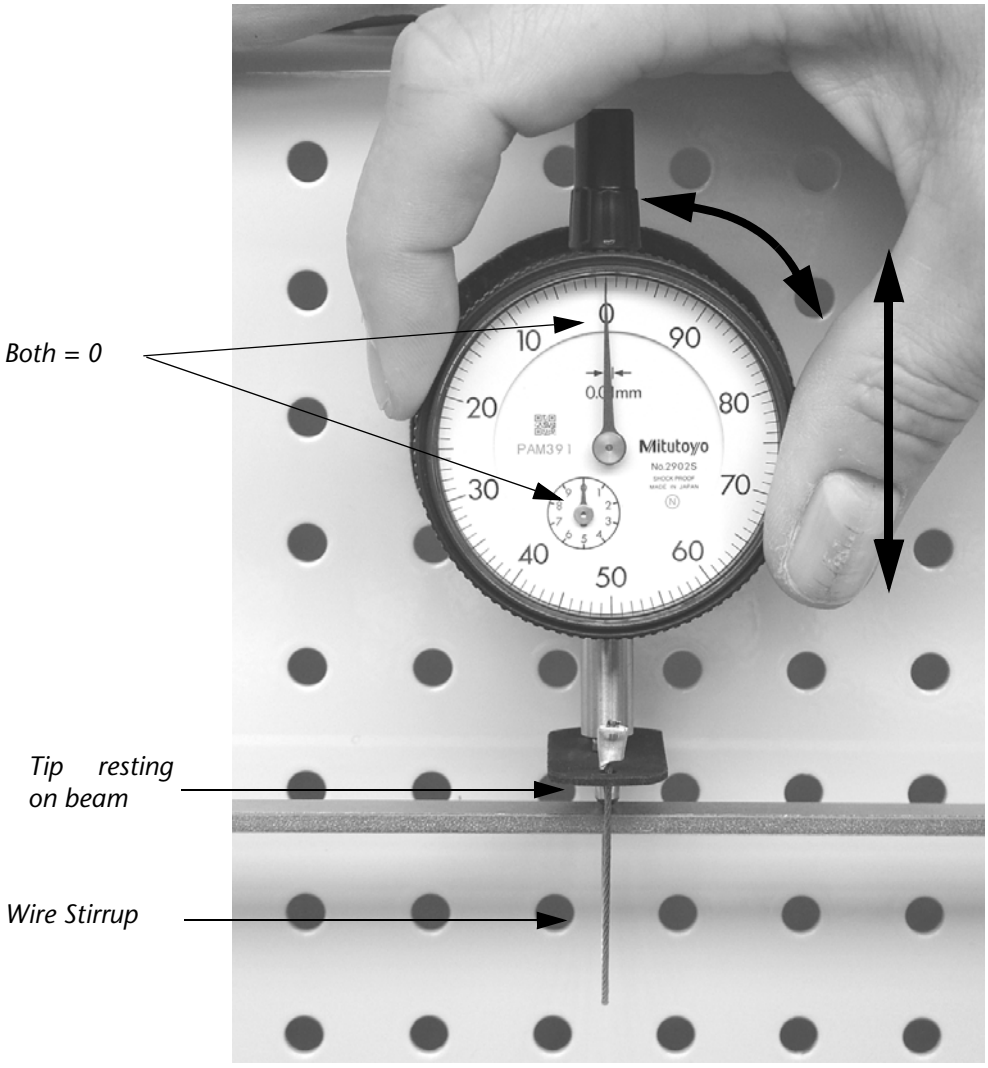


Figure 5 Setting up and Zeroing the Dial Indicator

Using the Fixing Blocks

The fixing blocks work as simple knife-edged supports (see Figure 6) or as clamps for fixed beams (see Figures 7 and 8). Use the hexagon tool (supplied) to undo and tighten the upper parts of the fixing blocks. Make sure the beam passes fully into the block for correct clamping and that clamp holds the beam evenly, so that the gaps each side are even (x = y) and the beam is in the middle (A = B) (see Figure 8).

CAUTION  *Do not overtighten the clamps, you may dent the beam and damage the clamps.*

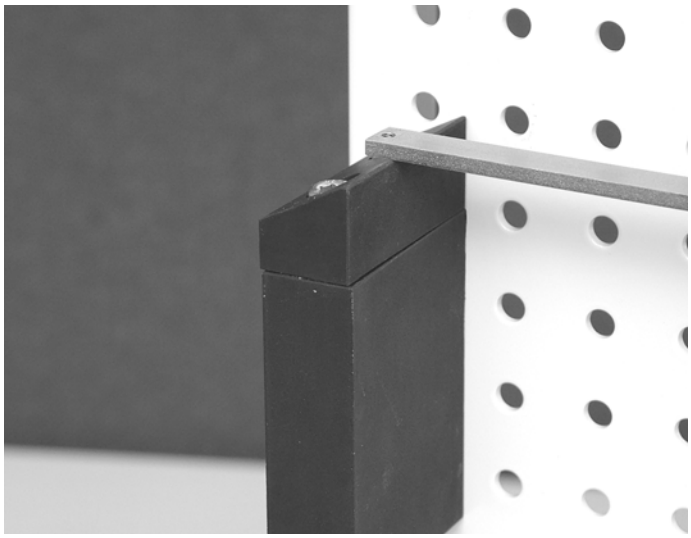


Figure 6 Fixing Blocks as Simple Supports

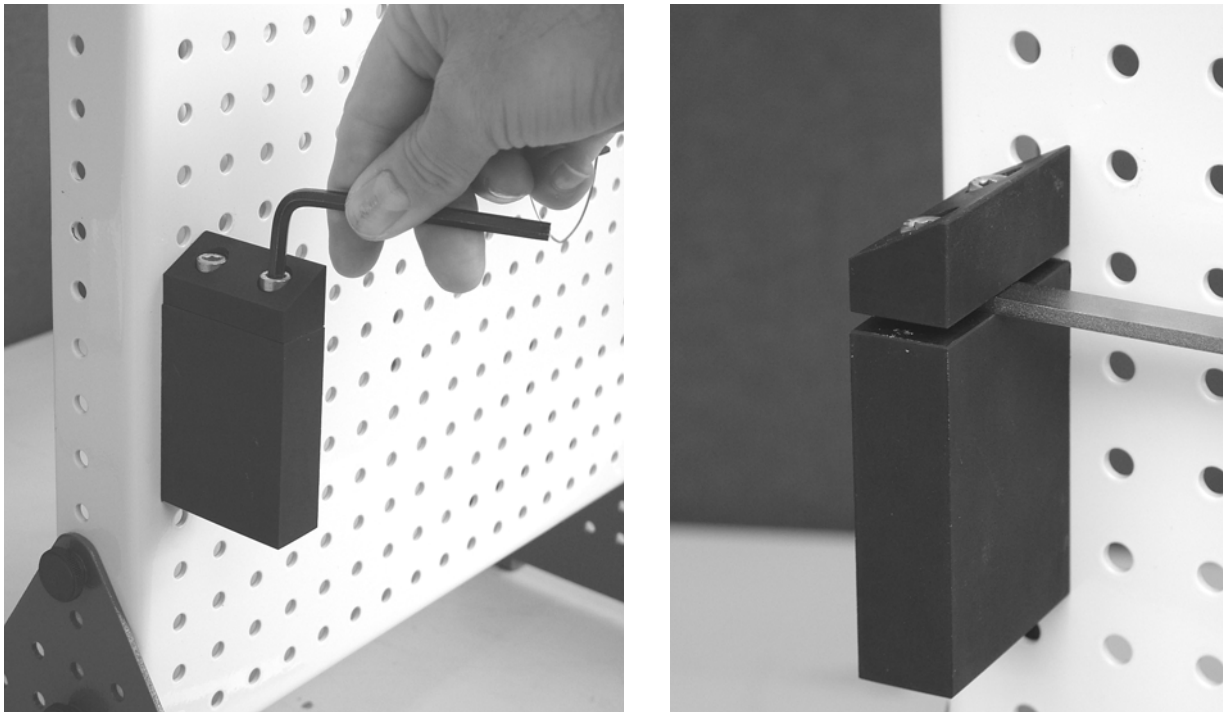


Figure 7 Fixing Blocks as a Clamp

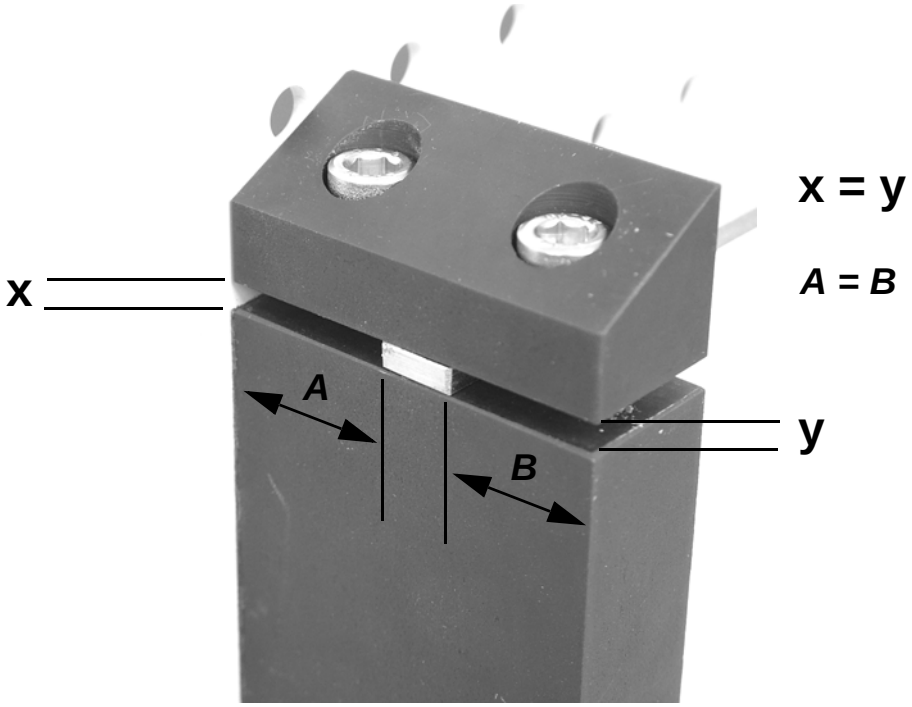


Figure 8 Fixing Blocks as Clamps for Fixed Beams

Accurate Results

For best results:

- Do not rush your experiment.
- Double-check each reading.
- Repeat the experiment if you are not sure of your results.
- Unless the instructions say otherwise, gently tap the Work Panel before taking a reading. Sometimes, mechanical parts 'stick' against each other (caused by friction). This is often called 'stiction' or static friction. Tapping the Work Panel helps to reduce this.

Do not expect your results to be exactly as shown in the theory. Theory always shows 'perfect' or 'ideal' results, based on perfect scientific conditions. Your 'actual' results will be slightly different to theory, based on the accuracy of the equipment and how carefully you do your experiment.

You may learn more about your experiments by making and finding mistakes than getting things right first time!

Theory

Introduction

A beam is a type of structure. The purpose of any structure is to support loads. This is true irrespective of how simple or complicated the structure is. Beams can be a structure in their own right, as well as being a part of more complicated structures. A cantilever is simply a particular application of a beam as you will see.

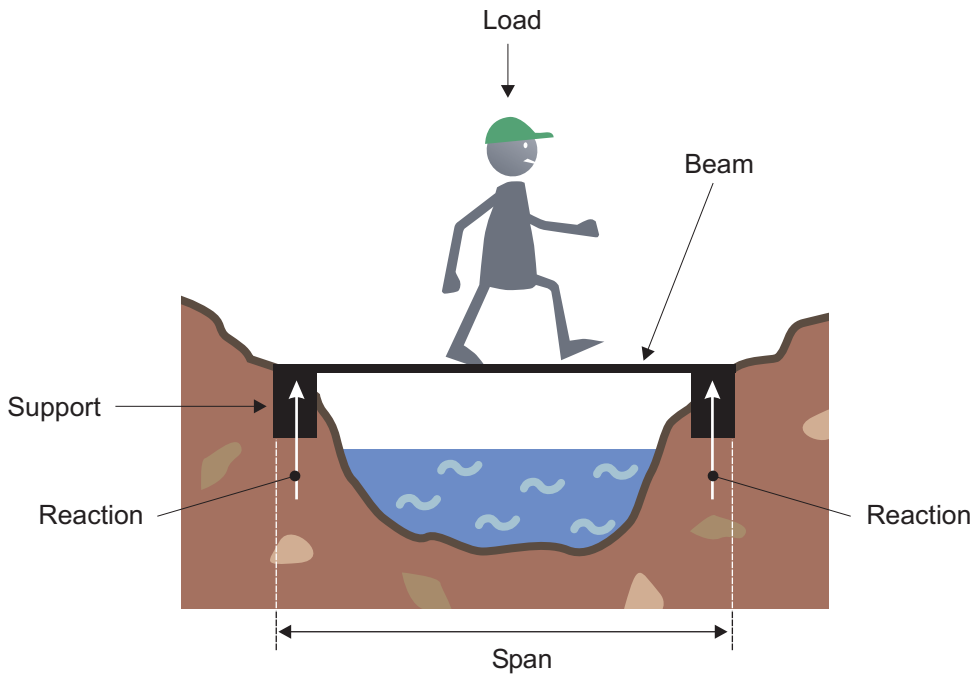


Figure 9 A Simple Beam Structure

It is obvious that a structure has to be strong enough to support the loads that are placed upon it, it should not break, however the structure also has to be stable to be functional. For instance we could build a bridge from Jelly (jello), it might well be able to support our weight - dependant on the strength of the jelly, but if the bridge were to wobble or deflect too much then its function as a structure would be questionable. In other words, as well as strength, structures need to have only reasonable and practical deflections.

It is therefore important to consider how much a structure will deflect under the loads placed upon it. In experiments we will look at simple beam structures and look at the factors that define how much they deflect.

Notation

Symbol	Meaning	Units
δ	Deflection	m
b	Breadth (or width)	m
d	Depth	m
K	A constant for the beam in its support conditions	-
E	Young's Modulus	$N.m^{-2}$ or Pa
I	Second moment of area	m^4
L	Length (span)	m
m	Masses	g
W	Load	N

Table 2 Notation

Factors Affecting Deflection

There are five main factors that affect how much a beam will deflect:

1. The load, the type, where it is placed, and how big it is.
2. The material the beam is made from.
3. The beam cross sectional dimensions - how wide and deep the beam is.
4. The type of support and its fixing conditions, that is how the support interacts with the beam.
5. The span or length of the beam.

These five factors combine together to form the general beam equation:

$$\delta = \frac{WL^3}{KEI}$$

The general beam equation as stated only works for certain simple types and arrangements of beams and loads, which will be explained along with each one of the terms in the equation. The kit helps you to understand these terms.

The Load

There are various types of loads that can be applied to beams. These include point loads (loads concentrated at a point on the beam), uniformly distributed loads (loads that are spread out) and moments (turning forces). To keep the analysis simple, the kit uses only point loads and measures deflection of the beam at the point of loading.

The Beam Material

The material property as used in the beam equation is its stiffness, different materials have different stiffness values.

The stiffness of a material is given by its Young's Modulus named after an English Physicist Thomas Young and is given the symbol "E". It is the ratio of stress to strain, or in simpler terms how much something stretches for a certain amount of force that you put on it. For most common engineering metals this ratio is constant until too much load is applied and the material deforms permanently. These materials are said to obey Hooke's Law. All the materials in the kit obey Hooke's law, and the experiments are designed make sure that you don't apply loads that will permanently bend the beams.

The Young's Modulus for a ***particular metal alloy*** is fairly constant or lies within a relatively small band of tolerance - even though the strengths of the alloys might be dramatically different. Table 3 shows textbook values that we have assumed for the materials that the beams in the ES4 kit are made from.

Material	Young's Modulus E (Pa)
Aluminium Alloy	69×10^9 (69 GPa)
Brass	105×10^9 (105 GPa)
Mild Steel	200×10^9 (200 GPa)

Table 3 Textbook Values of Young's Modulus

The Beam Dimensions

Imagine you have two beams which are identical other than their thickness. It is fairly intuitive that if you try to bend each one in turn the thicker one would bend less. ie the thicker one would be more rigid.

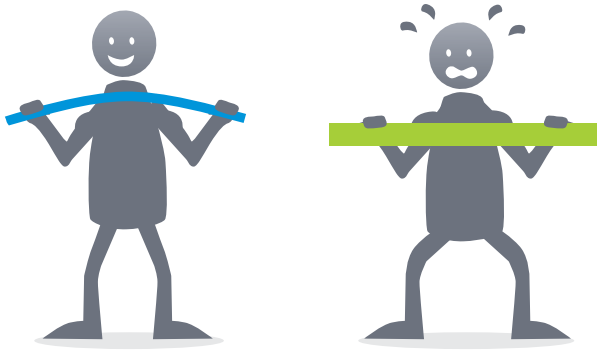


Figure 10 Beam Bending

However what is less intuitive is that this extra rigidity is not proportional to the thickness, it is in fact the cube of the thickness, therefore a small change in the thickness has a large effect on the rigidity.

This dimensional property of a beam is called the second moment of area 'I'.

For a rectangular section beam, Equation 1 gives the second moment of area.

$$I = \frac{bd^3}{12} \qquad (1)$$

Engineers use the terms 'breadth' and 'depth' instead of width and thickness to avoid confusion. To avoid mistakes as to what is breadth and depth, you must make sure that you know the direction of the load, for example, the same beam could have two I values determined by where you apply the force.

The easiest way to remember this is that the depth is the dimension of the beam in the same direction as the load.

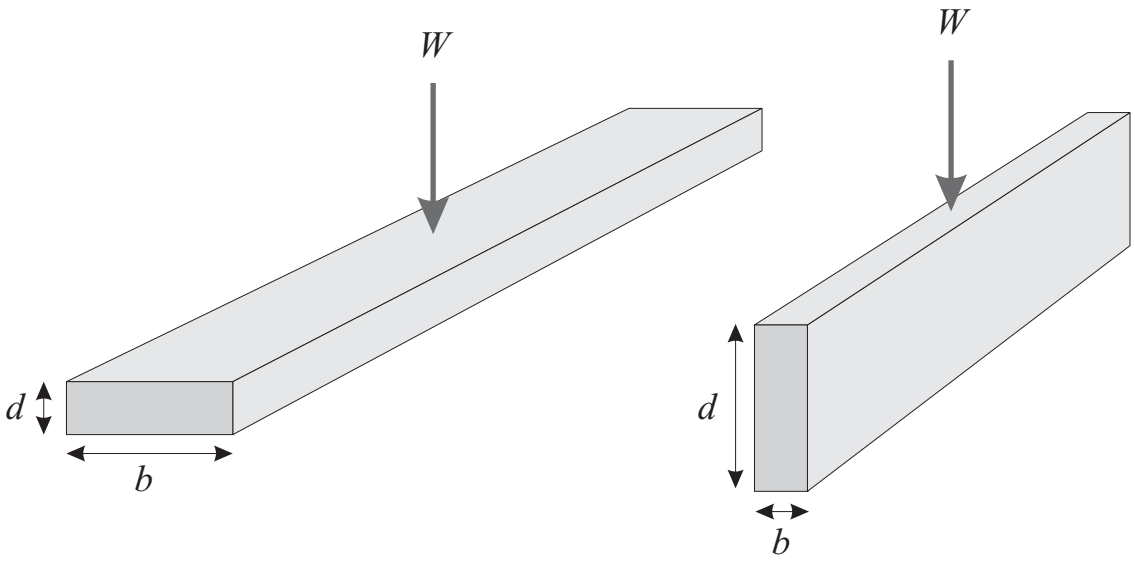


Figure 11 Second Moment of Area

Worked Examples:

Depth d = 3.1 mm (0.0031 m).

Breadth b = 9.5 mm (0.0095 m).

Second Moment of area $I = (0.0095 \times 0.0031^3)/12 = 2.36 \times 10^{-11}$

Depth d = 9.5 mm (0.0095 m).

Breadth b = 3.1 mm (0.0031 m).

Second Moment of area $I = (0.0031 \times 0.0095^3)/12 = 2.21 \times 10^{-10}$

Beam Supports and Fixing Conditions

Beam or cantilevers have to be in some way held or supported to be useful structures. In the example in Figure 9, the banks of the river form the supports.

In real life structures these supports can be very complicated, so at a basic level we used 'idealised' supports. These are supports with known attributes that make them relatively easy to analyse.

We will deal with three types of supports and fixing conditions, using drawings that you will see later in these notes. Combinations of these supports lead to the various overall fixing conditions for the beams, affecting the value of "K" in the general beam equation.

1. Pinned Supports

A pinned support allows the beam to rotate at the point of contact, but stops the beam falling down, or being pushed along.

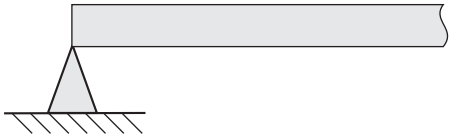


Figure 12 A Pinned Support

2. Pinned and Roller Supports

A pinned and roller support allows the beam to rotate at the point of contact, but stops the beam falling down. It does not stop it from being pushed along.

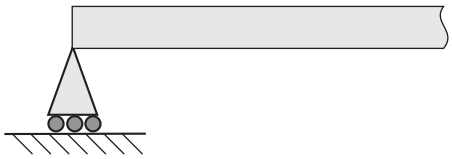


Figure 13 A Pinned and Roller Support

3. Encastré Fixing or 'Fixed' Supports

The encastré fixing stops the beam rotating, falling down or being pushed along.

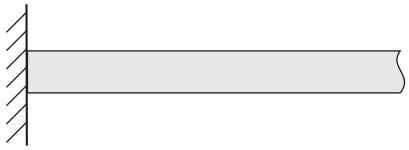


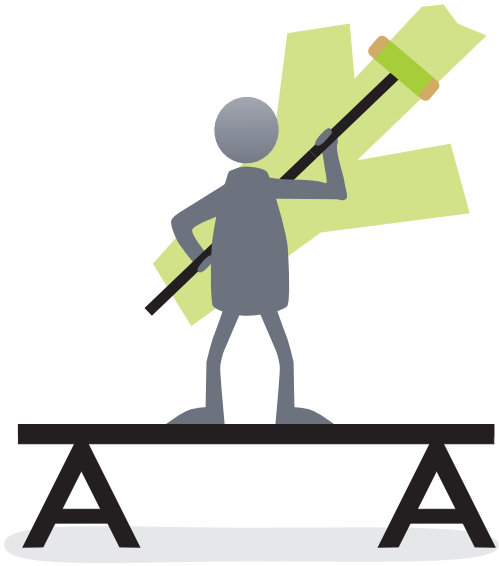
Figure 14 Encastre Fixing

Simply Supported Beams

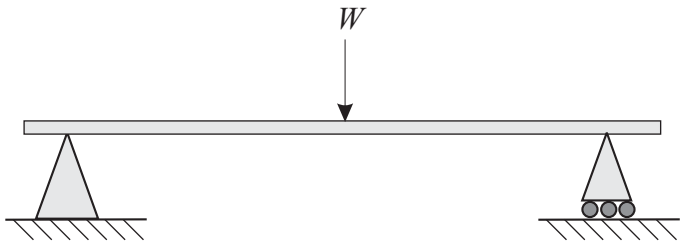
Both ends are free to rotate, but the beam does not fall off sideways. $K = 48$.

The knife-edge supports in the kit work as both Pinned and Pinned and Roller supports and so work as 'simple supports'.

Typical Example



Idealised Diagram



Setup using the kit

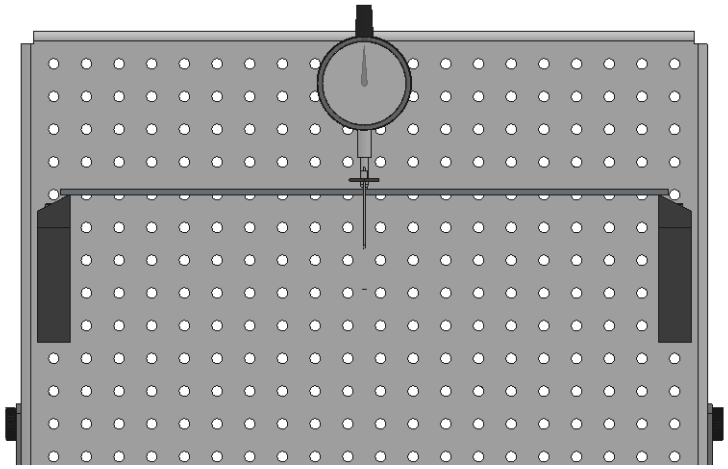
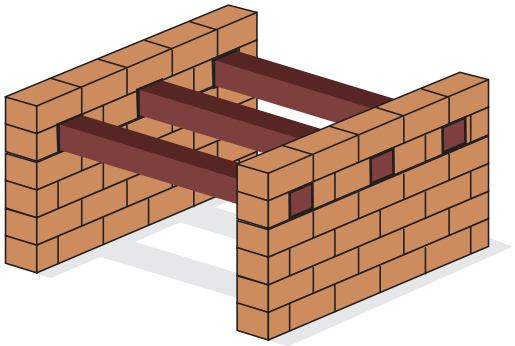


Figure 15 Simply Supported Beam

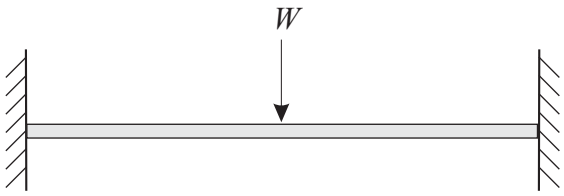
Fixed Beams

Both ends are rigidly fixed. The ends of the beam cannot move. K = **192.**

Typical Example



Idealised Diagram



Setup using the kit

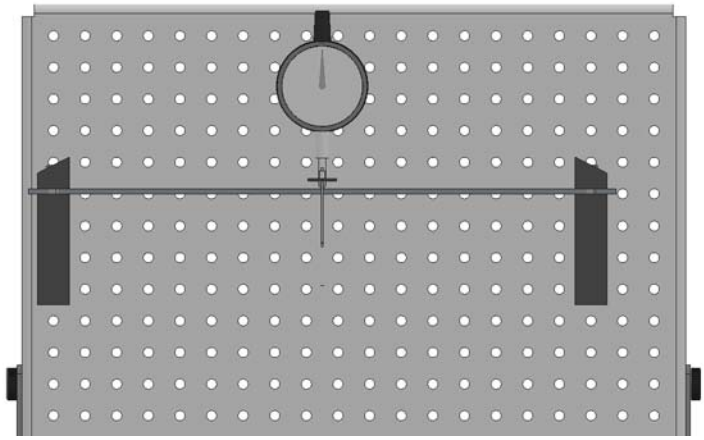
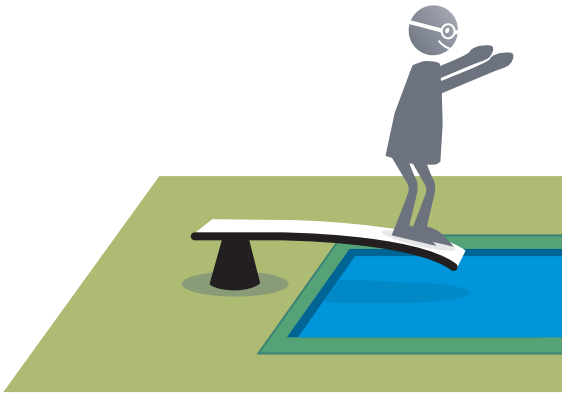


Figure 16 Fixed Beams

Cantilevers

One end is held rigidly, the other end is free to move. $K = 3$.

Typical Example



Idealised Diagram



Setup using the kit

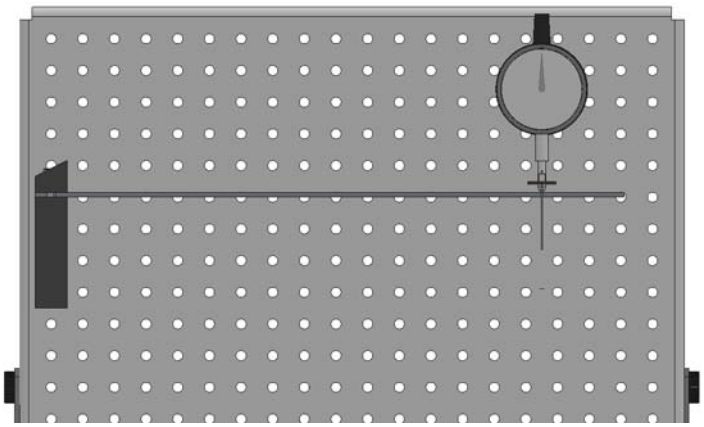
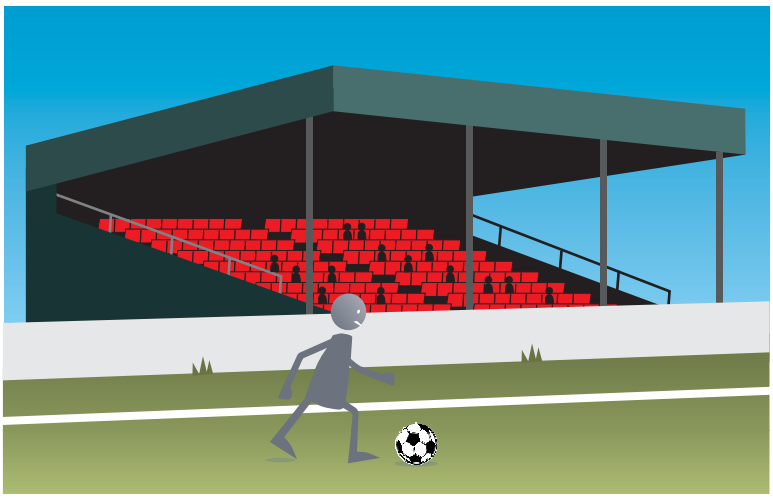


Figure 17 Cantilever

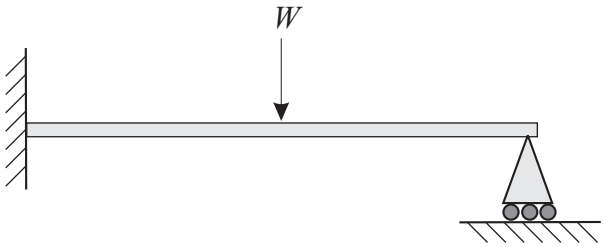
Propped Cantilevers

One end is held rigidly, the other is stopped from moving vertically. K = **110**.

Typical Example



Idealised Diagram



Setup using the kit

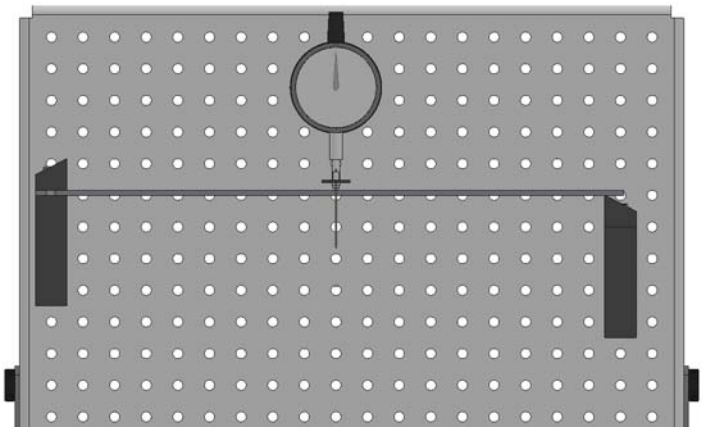


Figure 18 Propped Cantilever

Beam Length (Span) L

The final factor that affects the deflection of the beam is it's length or span 'L', which is simply the simply the distance between its supports, or the case of a cantilever, the distance from the encastre fixing and the point of loading (since the rest of the cantilever isn't actually doing anything from a structural point of view).

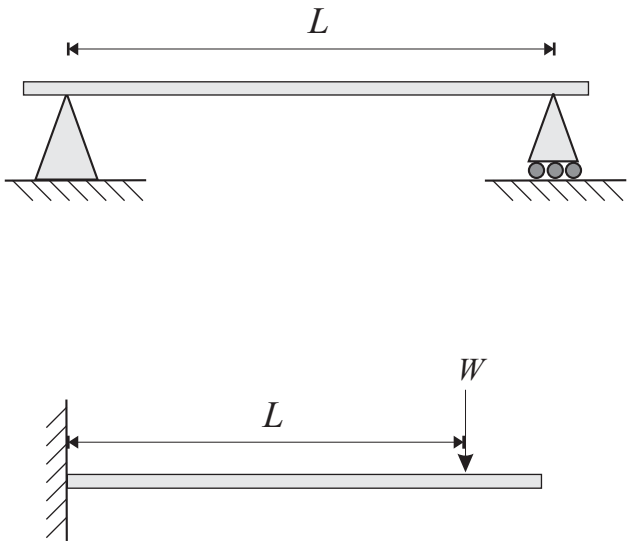
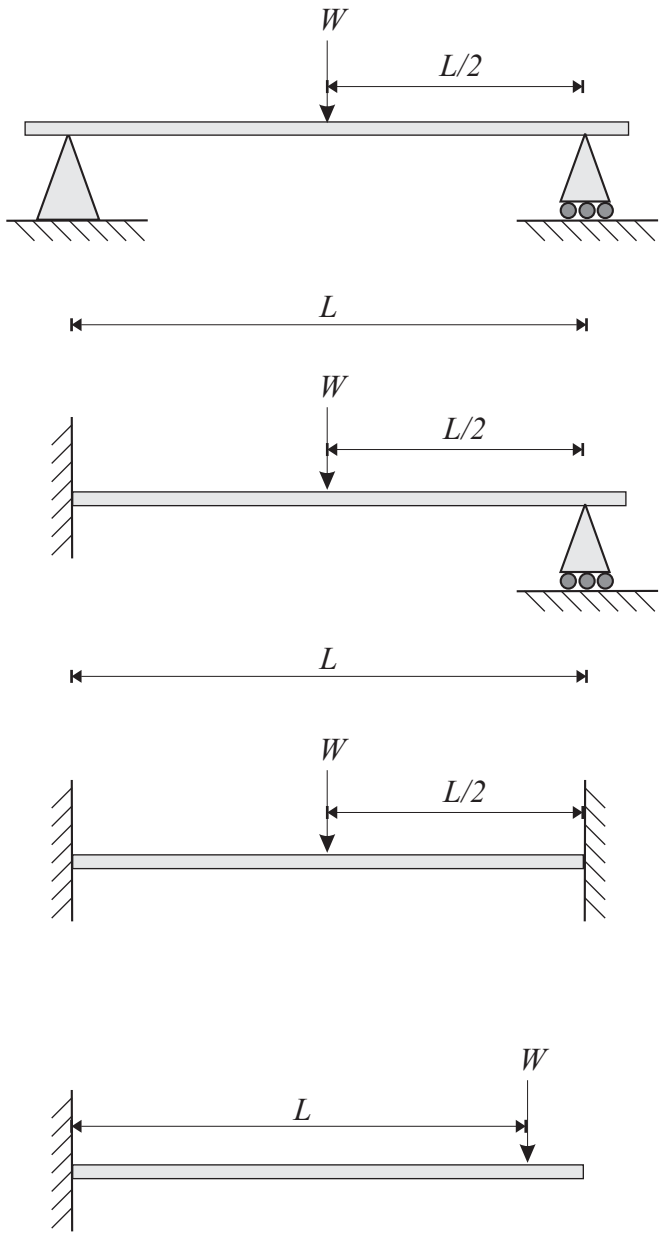


Figure 19 Length (Span) of Beam

Summary of Beam Types and Bending Equations



$$\delta = \frac{WL^3}{48EI}$$

$$\delta = \frac{WL^3}{110EI}$$

$$\delta = \frac{WL^3}{192EI}$$

$$\delta = \frac{WL^3}{3EI}$$

Figure 20 Summary of Beam Types

Relationships - Deflection and Load

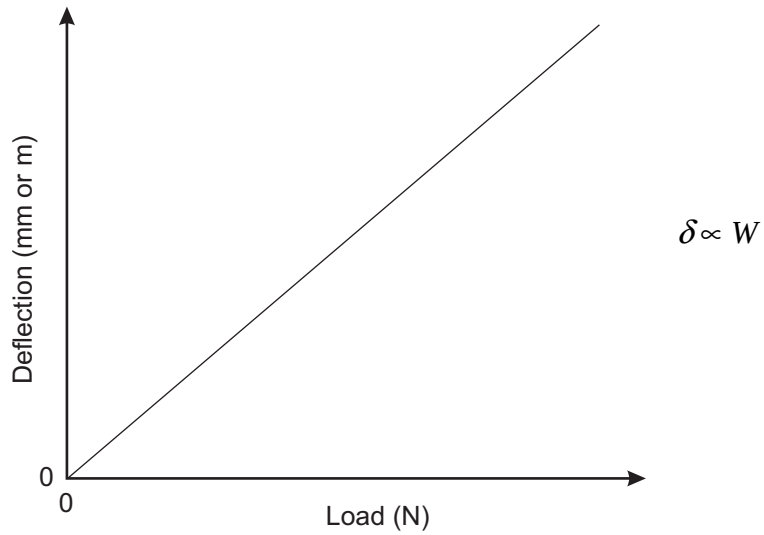


Figure 21 Deflection of a Beam

As Figure 21 shows, for the four beam support methods used in this kit, any charts of deflection against load should give linear results (a straight line). The line should pass through the origin (0, 0) of the chart. This shows that when everything else remains constant in the beam deflection equation, ***deflection is directly proportional to load.***

Relationships - Deflection and Young's Modulus

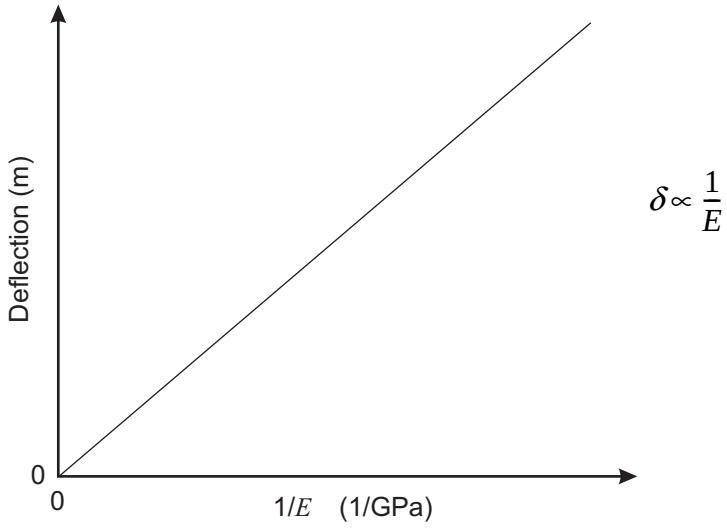


Figure 22 Deflection and Young's Modulus

As Figure 22 shows, for the beam support methods used in this kit, charts of deflection against the inverse of the Young's Modulus value should give linear results (a straight line). The line should pass through the origin (0, 0) of the chart. This shows that when everything else remains constant in the beam deflection equation, ***deflection is directly proportional to the inverse of Young's Modulus.***

Relationships - Deflection and I Value

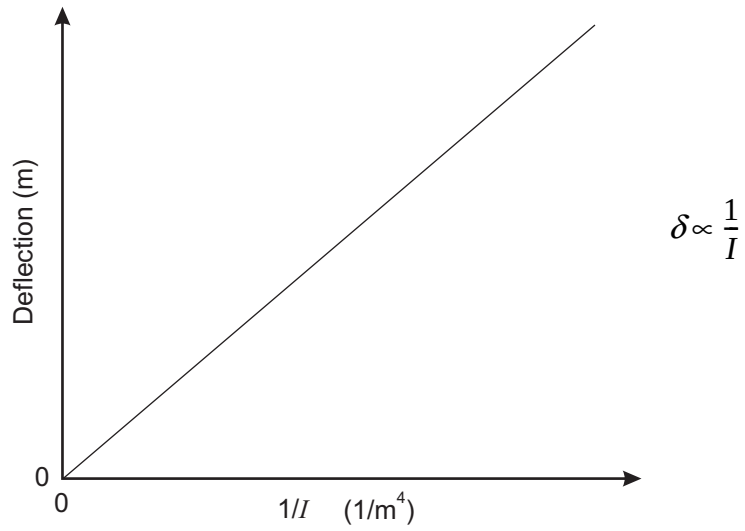


Figure 23 Deflection and I Value

As Figure 23 shows, for the beam support methods used in this kit, charts of deflection against the inverse of the I value should give linear results (a straight line). The line should pass through the origin (0, 0) of the chart. This shows that when everything else remains constant in the beam deflection equation, ***deflection is directly proportional to the inverse of the I value.***

Relationships - Deflection and Length

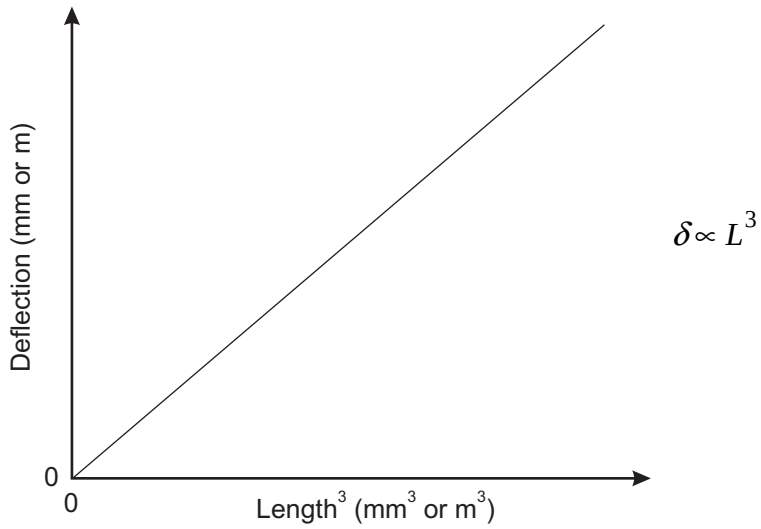


Figure 24 Deflection and Length of a Beam

As Figure 24 shows, for the four beam support methods used in this kit, any charts of deflection against the cube of length should give linear results (a straight line). The line should pass through the origin (0, 0) of the chart. This shows that when everything else remains constant in the beam deflection equation, ***deflection is directly proportional to length cubed.***