

Mechanical Principles

Semester 1 Revision Examples.

- 1) A material has a stress in the x -direction of -40 MPa and a stress of $+20 \text{ MPa}$ in the y direction. Given the modulus of elasticity is 205 GPa ~~and~~ ^{and} ~~calculate~~ $\nu = 0.25$, calculate the strain in each direction.

$$\epsilon_x = \frac{1}{E} [\sigma_x - \nu \sigma_y] = \frac{1}{205 \times 10^9} [-40 \times 10^6 - (0.25 \times 20 \times 10^6)]$$

$$\epsilon_x = \underline{\underline{-219.5 \mu\epsilon}}$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \nu \sigma_x] = \frac{1}{205 \times 10^9} [20 \times 10^6 - (0.25 \times -40 \times 10^6)]$$

$$\epsilon_y = \underline{\underline{146.3 \mu\epsilon}}$$

- 2) Calculate the stress in the x and y directions which would result if a material has a strain in the x direction of $48 \mu\epsilon$ and in the y direction of $-36 \mu\epsilon$, given the elastic constants $E = 200 \text{ GPa}$ and $\nu = 0.27$.

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{200 \times 10^9}{1-0.27^2} (48 \times 10^{-6} + (0.27 \times -36 \times 10^{-6})) = 8.26 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{200 \times 10^9}{1-0.27^2} (-36 \times 10^{-6} + (0.27 \times 48 \times 10^{-6})) = -4.97 \text{ MPa}$$

3) A solid piece of metal of volume 6000 mm^3 is compressed by a pressure of 60 MPa . Calculate the bulk modulus given $E = 205 \text{ GPa}$ and $\nu = 0.29$. Calculate the change in volume.

$$K = \frac{E}{3(1-2\nu)} = \frac{205 \times 10^9}{3(1-(2 \times 0.29))} = 162.7 \times 10^9 \text{ Pa.}$$

$$K = \frac{\sigma}{\epsilon_r} = \frac{-P}{\epsilon_r} \quad \epsilon_r = \frac{-P}{K} = \frac{-60 \times 10^6}{162.7 \times 10^9} = -0.369 \times 10^{-3}$$

$$\epsilon_r = \frac{\Delta V}{V} \quad \Delta V = \epsilon_r V = -0.369 \times 10^{-3} \times 6000$$

$$\Delta V = \underline{\underline{-2.21 \text{ mm}^3}}$$

4) A solid steel sphere with a diameter of 0.2 m is subjected to a uniform external pressure of 40 Bar . Determine the decrease in volume if $E = 200 \text{ GPa}$ and $\nu = 0.3$.

$$V = \frac{4}{3} \pi r^3$$

$$40 \text{ Bar} = 4 \times 10^6 \text{ MPa.}$$

$$K = \frac{E}{3(1-2\nu)} = \frac{200 \times 10^9}{3(1-0.6)} = 166.67 \times 10^9 \text{ Pa.}$$

$$\epsilon_r = \frac{-P}{K} = \frac{-4 \times 10^6}{166.67 \times 10^9} = -2.4 \times 10^{-5}$$

$$\Delta V = \epsilon_r V = -2.4 \times 10^{-5} \times \frac{4}{3} \pi (0.1)^3$$

$$\Delta V = -1.005 \times 10^{-7} \text{ m}^3$$

5) A cantilever is 3 m long and has a flexural stiffness of 55 MNm^2 . If a point load of 3 kN is applied at the end of the beam calculate the maximum slope and deflection. What would the point load need to be reduced to, to limit the maximum deflection to -0.25 mm ?

$$\text{Max Slope} = \frac{FL^2}{2EI} = \frac{-3000 \times (3)^2}{2 \times 55 \times 10^6} = \underline{\underline{-0.245 \times 10^{-3}}}$$

$$\text{Max deflection} = \frac{FL^3}{3EI} = \frac{-3000 (3)^3}{3 \times 55 \times 10^6} = \underline{\underline{-0.491 \times 10^{-3} \text{ m}}}$$

$$\text{Max load for deflection} = -0.25 \text{ mm}$$

$$F = \frac{3EI \Delta y}{L^3} = \frac{3 \times 55 \times 10^6 \times -0.25 \times 10^{-3}}{3^3}$$

$$\underline{\underline{F_{\text{max}} = -1527.8 \text{ N}}}$$

6) A simply supported beam is 2m long and has a flexural stiffness of 20 MNm^2 . The beam carries a u.d.l of 2000 N/m along its length and a point load of 5 kN at the centre of the beam. Calculate the maximum slope and deflection and state where these occur.

$$\text{Max deflection} = \frac{FL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$= \frac{-5000(2)^3}{48 \times 20 \times 10^6} + \frac{5 \times 2000 \times (2)^4}{384 \times 20 \times 10^6}$$

$$= -0.6417 \times 10^{-3} - 0.0208 \times 10^{-3}$$

$$= \underline{\underline{-0.063 \text{ mm}}}$$

At centre of beam.

$$\text{Max slope} = \frac{FL^2}{16EI} + \frac{wL^3}{24EI}$$

$$= \frac{-5000(2)^2}{16 \times 20 \times 10^6} + \frac{-2000 \times (2)^3}{24 \times 20 \times 10^6}$$

$$= -6.25 \times 10^{-5} - 3.33 \times 10^{-5}$$

$$= \underline{\underline{-95.83 \times 10^{-6}}}$$

At each end of the beam.

7) A cantilever is 3m long with a flexural stiffness of 20 MNm^2 . Calculate the deflection at the free end if it carries a udl of 600 N/m along its entire length and a point load of 2 kN at its free end.

$$\text{Max. deflection} = \frac{FL^3}{3EI} + \frac{wL^4}{8EI}$$

$$= \frac{-2000(\frac{3}{3})^3}{3 \times 20 \times 10^6} - \frac{600 \times 3^4}{8 \times 20 \times 10^6}$$

$$= -9 \times 10^{-4} - 3.04 \times 10^{-4}$$

$$= \underline{\underline{-1.2.04 \text{ mm}}}$$

- 8) A thin walled pressure vessel with closed ends, has an inside diameter of 100mm. It has a joint efficiency of ~~45%~~ 45% and ~~40%~~ 40% in the longitudinal and circumferential directions, respectively, and is made from 3mm thick steel plate. Calculate the circumferential and longitudinal stress in the cylinder if it has an internal pressure of 10 Bar.

$$\sigma_c = \frac{PD}{2t\eta_c} = \frac{1 \times 10^6 \times 0.1}{2 \times 0.003 \times 0.45} = \underline{\underline{37.04 \text{ MPa}}}$$

$$\sigma_L = \frac{PD}{4t\eta_c} = \frac{1 \times 10^6 \times 0.1}{4 \times 0.003 \times 0.4} = \underline{\underline{20.83 \text{ MPa}}}$$

- 9) A thin walled pressure vessel with closed ends is made from 2mm thick plate and has an internal diameter of 0.12 m. If it has a joint efficiency of 40% in the longitudinal and 60% in the circumferential directions, calculate the maximum internal pressure if the maximum stress is to be limited to 30 MPa.

$$\sigma_c = \frac{PD}{2t\eta_c}$$

$$\sigma_L = \frac{PD}{4t\eta_c}$$

$$P_{\max} = \frac{\sigma_c 2t\eta_c}{D}$$

$$P_{\max} = \frac{30 \times 10^6 \times 2 \times 0.002 \times 0.4}{0.12}$$

$$P_{\max} = 4 \times 10^5 \text{ N/m}^2 = \underline{\underline{4 \text{ bar}}}$$

$$\therefore P_{\max} = \underline{\underline{4 \text{ bar}}}$$

$$P = \frac{4\sigma_L t\eta_c}{D}$$

$$P_{\max} = \frac{4 \times 30 \times 10^6 \times 0.002 \times 0.6}{0.12}$$

$$P_{\max} = 1.2 \times 10^6 \text{ N/m}^2 = \underline{\underline{12 \text{ Bar}}}$$

10) A hydraulic cylinder has an internal diameter of 80mm and an external diameter of 120mm. If it is pressurised internally to 150 Bar gauge, calculate the radial and circumferential stresses at the internal and external surfaces.

@ Inner surface $r = 0.04$ $\sigma_r = -15 \times 10^6$ Pa. (compressive)

@ outer surface $r = 0.06$ $\sigma_r = 0$ Pa.

$$\sigma_r = a - \frac{b}{r^2} \quad \sigma_c = a + \frac{b}{r^2}$$

$$-15 \times 10^6 = a - \frac{b}{0.04^2}$$

$$0 = a - \frac{b}{0.06^2}$$

$$-2.4 \times 10^4 = 0.04^2 a - b$$

$$0 = 0.06^2 a - b$$

$$-2.4 \times 10^4 = -0.002 a$$

$$a = 12 \times 10^6$$

$$b = 4.32 \times 10^4$$

@ inner surface

$$\sigma_c = 12 \times 10^6 + \frac{4.32 \times 10^4}{0.04^2} = \underline{\underline{39 \text{ MPa}}}$$

@ outer surface

$$\sigma_c = 12 \times 10^6 + \frac{4.32 \times 10^4}{0.06^2} = \underline{\underline{24 \text{ MPa}}}$$

q1) A material has a stress in the x and y directions of 100 MPa and -90 MPa respectively. Calculate the strain in each direction if the Young's modulus of the material is 208 GPa and the poisson's ratio is 0.22

$$\epsilon_x = \frac{1}{E} [\sigma_x - \nu \sigma_y] = \frac{1}{208 \times 10^9} [100 \times 10^6 - (0.22 \times -90 \times 10^6)]$$

$$= 575.96 \times 10^{-6}$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \nu \sigma_x] = \frac{1}{208 \times 10^9} [-90 \times 10^6 - (0.22 \times 100 \times 10^6)]$$

$$= -538.46 \times 10^{-6}$$

2) b) If a material is found to have a strain of 1080×10^{-6} in the y direction and 894×10^{-6} in the x direction, calculate the stress in the material if $E = 205$ GPa and $\nu = 0.28$.

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y)$$

$$= \frac{205 \times 10^9}{1-0.28^2} (894 \times 10^{-6} + (0.28 \times 1080 \times 10^{-6})) = 266.13 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x)$$

$$= \frac{205 \times 10^9}{1-0.28^2} (1080 \times 10^{-6} + (0.28 \times 894 \times 10^{-6})) = 295.92 \text{ MPa}$$

Q3a). A solid piece of metal of volume 3000 mm^3 ~~is excited~~ is compressed by an external pressure of 10 MPa . Calculate the bulk modulus and change in volume if Young's modulus is 208 GPa and Poisson's ratio is 0.26 .

$$K = \frac{E}{3(1-2\nu)} = \frac{208 \times 10^9}{3(1-0.52)} = 144.4 \text{ GPa.}$$

$$K = \frac{\sigma}{\epsilon_v} \rightarrow \epsilon_v = \frac{\sigma}{K} = \frac{-P}{K} = \frac{-10 \times 10^6}{144.4 \times 10^9} = -0.069 \times 10^{-3}$$

$$\epsilon_v = \frac{\Delta V}{V} \quad \Delta V = \epsilon_v V = -0.069 \times 10^{-3} \times 3000 = -0.208 \text{ mm}^3 \quad \left(\begin{array}{l} 3000 \text{ mm}^3 \\ = 3 \times 10^{-6} \text{ m}^3 \end{array} \right)$$

Q5a) A simply supported beam ^{is 4m long.} carries a udl along its entire length of 2800 N/m . Calculate the maximum slope and deflection and state where they occur. What would the udl have to be reduced by to reduce the maximum deflection to 0.25 mm ? Flexural stiffness $= 25 \text{ MN/m}^2$.

$$\text{Max } \overset{\text{slope}}{\text{deflection}} = \frac{5\omega L^4}{384EI} = \frac{5 \times 2800 \times 4^4}{384 \times 25 \times 10^6} = -3.73 \times 10^{-4}$$

$= -0.373 \text{ mm}$,
at centre of span

$$\text{Max slope} = \frac{\omega L^3}{24EI} = \frac{-2800 \times 4^3}{24 \times 25 \times 10^6} = -298.7 \times 10^{-6}$$

at ends/supports

$$\omega = \frac{y \times 384EI}{5L^4} = \frac{-0.25 \times 10^{-3} \times 384 \times 25 \times 10^6}{5 \times 4^4} = -1875 \text{ N}$$

$$\text{Reduction} = 2800 - 1875 = 925$$

Q6. A $\sqrt{2\text{m}}$ long cantilever carries a udl of 750 N/m along its entire length, and a point load of 4 kN at its free end. If the beam has a flexural stiffness of 30 MN/m^2 calculate the maximum deflection and slope. What 'prop' force would be required at the free end of the beam to limit deflection to 0.3 mm ?

$$\text{Max deflection} = \frac{FL^3}{3EI} + \frac{\omega L^4}{8EI}$$

$$= \frac{-4 \times 10^3 \times 2^3}{3 \times 30 \times 10^6} + \frac{750 \times 2^4}{8 \times 30 \times 10^6}$$

$$= -3.56 \times 10^{-4} + 5 \times 10^{-5}$$

$$= -0.406 \text{ mm.}$$

$$\text{max slope} = \frac{FL^2}{2EI} + \frac{\omega L^3}{6EI}$$

$$= \frac{-4000 \times 2^2}{2 \times 30 \times 10^6} + \frac{750 \times 2^3}{6 \times 30 \times 10^6}$$

$$= -2.67 \times 10^{-4} + 3.33 \times 10^{-5}$$

$$= 300 \times 10^{-6}$$

$$\text{Vertical deflection required} = 0.406 - 0.3 = 0.106 \text{ mm}$$

$$F = \frac{3EI\delta}{L^3} = \frac{3 \times 30 \times 10^6 \times 0.106 \times 10^{-3}}{2^3} = 1.1925 \text{ kN}$$

$$\underline{\underline{1192.5 \text{ N}}}$$

Q7/ A thin walled pressure vessel with closed ends is made from 1.6 mm thick steel plate and has an internal diameter of 0.3 m. If it has joint efficiencies of 75% and 68% in the longitudinal and circumferential directions respectively, calculate the maximum internal pressure if the stress is to be limited to 90 MPa.

$$t = 1.6 \times 10^{-3} \text{ m}$$

$$\phi = 0.3 \text{ m}$$

$$\eta_L = 0.75$$

$$\eta_C = 0.68$$

$$\sigma_{\max} = 90 \times 10^6 \text{ N/m}^2$$

$$\sigma_C = \frac{PD}{2t\eta_L} \quad \sigma_L = \frac{PD}{4t\eta_C}$$

$$P = \frac{2t\eta_L\sigma_C}{D} \quad P = \frac{4t\eta_C\sigma_L}{D}$$

$$P = \frac{2 \times t \times \eta_L \times \sigma_C}{D}$$

$$= \frac{2 \times 1.6 \times 10^{-3} \times 0.75 \times 90 \times 10^6}{0.3}$$

$$P = 720 \text{ kPa}$$

$$= 0.72 \text{ Bar}$$

$$P_{\max} = 0.72 \text{ Bar}$$

$$P = \frac{4 \times t \times \eta_C \times \sigma_C}{D}$$

$$P = \frac{4 \times 1.6 \times 10^{-3} \times 0.68 \times 90 \times 10^6}{0.3}$$

$$P = 1305.6 \text{ kPa}$$

$$= 13.056 \text{ Bar}$$

A thick walled cylinder has an external diameter of 200 mm and an ^{inner} internal thickness of 20 mm. If it is pressurised to 190 Bar gauge Calculate the radial and circumferential Stresses at the internal + external Surfaces.

$$\sigma_r = a - \frac{b}{r^2}$$

$$r_o = 0.1 \text{ m}$$

$$r_i = \frac{(200 - (2 \times 20))}{2} = 0.08 \text{ m.}$$

$$P = 190 \times 10^5 \text{ N/m}^2$$

$$@ r_i = 0.08 \quad \sigma_r = -190 \times 10^5 \text{ Pa.}$$

$$-190 \times 10^5 = a - \frac{b}{(0.08)^2}$$

$$-1.216 \times 10^5 = (0.08)^2 a - b$$

$$\textcircled{1} \quad 1.216 \times 10^5 = b - (0.08)^2 a$$

$$@ r_o = 0.1 \quad \sigma_r = 0$$

$$0 = a - \frac{b}{(0.1)^2}$$

$$0 = (0.1)^2 a - b$$

$$\textcircled{2} \quad 0 = b - (0.1)^2 a$$

$\textcircled{1} - \textcircled{2}$

$$1.216 \times 10^5 = (0.08)^2 a + (0.1)^2 a$$

$$a = \frac{1.216 \times 10^5}{(0.1)^2 - (0.08)^2}$$

$$a = 3.37 \times 10^7$$

$$b = (0.1)^2 a$$

$$b = 3.37 \times 10^5$$

$$\sigma_c = a + \frac{b}{r^2}$$

$$@ r_i = 0.08$$

$$\sigma_c = 33.7 \times 10^6 + \frac{3.37 \times 10^5}{(0.08)^2} = 86.36 \text{ MPa}$$

$$@ r_o = 0.1$$

$$\sigma_c = 33.7 \times 10^6 + \frac{3.37 \times 10^5}{(0.1)^2} = \begin{matrix} 67.7 \text{ MPa} \\ \cancel{86.36 \text{ MPa}} \end{matrix}$$