

1909757

by Scott Jones

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Assignment Tasks

Task 1

The component values that I have chosen are:

$$R = 15\Omega$$

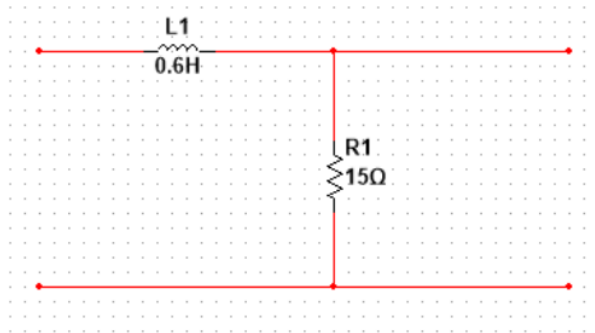
$$L = 0.6H$$

To find a value for the time constant in an RL circuit:

$$\tau = \frac{L}{R} = \frac{0.6}{15} = 0.04 \text{ Seconds}$$

Task 1.2

Below is the Laplace Model and the transfer function:



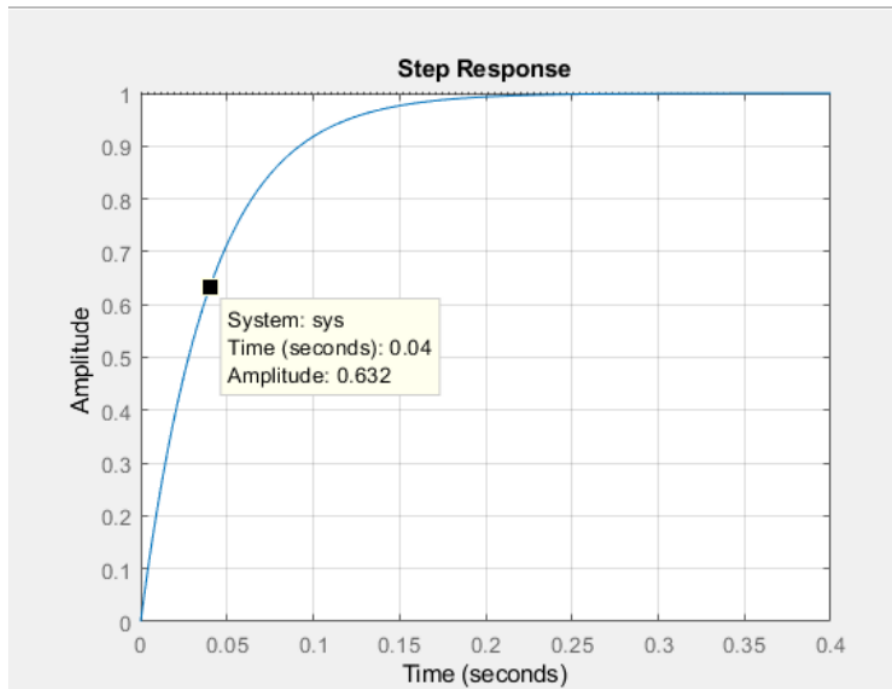
$$H(s) = \frac{1}{\tau s + 1} = \frac{1}{0.04s + 1} = 0.9615384$$

Incorrect



Task 1.3

The Matlab Model is shown below



```
>> num = [1];  
>> den = [0.04 1];  
>> step(num,den);  
grid
```

Task 1.4

Calculated Value	Calculated Results	Simulated Results
Time Constant (s)	0.04	0.04

Task 1.5

The circuit performed as expected and the results calculated for the time constant were identical to those simulated. Simulated values were taken from the point where the amplitude was 0.632, this is because the time constant reached at 63.2%.

Task 2

The component values that I have chosen are:

$$R = 22\text{k}\Omega$$

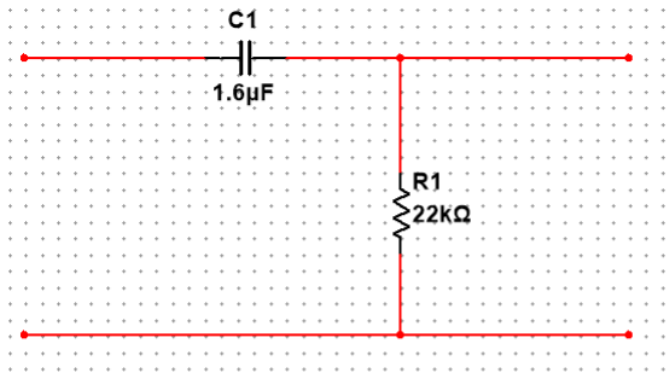
$$C = 1.6\mu\text{F}$$

To find the value for the time constant in an RC circuit:

$$\tau = RC = (22 \times 10^3) \times (1.6 \times 10^{-6}) = 0.0352 \text{ seconds}$$

Task 2.2

Below is the Laplace model and transfer function:

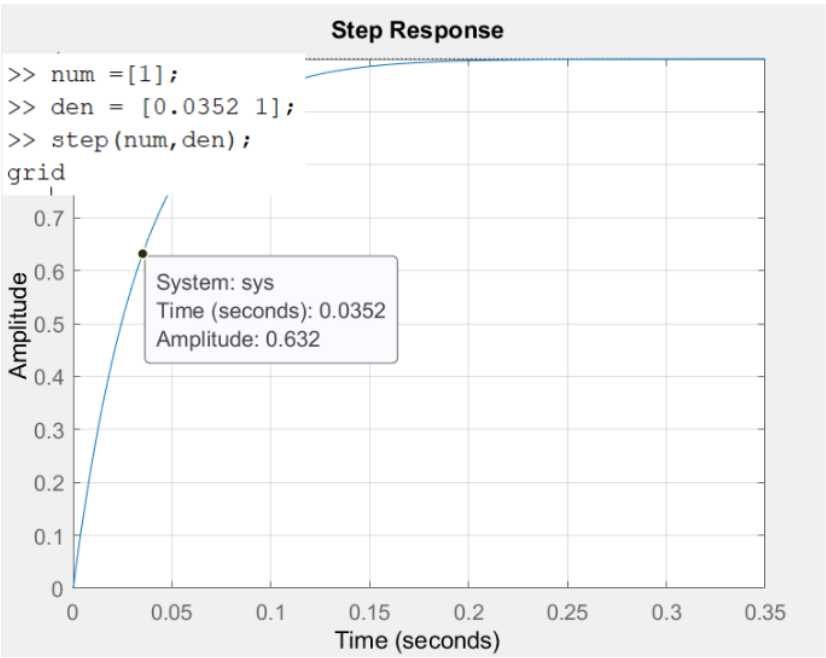


$$H(s) = \frac{1}{\tau s + 1} = \frac{1}{0.0352s + 1} = 0.9659969088$$

Incorrect

Task 2.3

The Matlab model is shown below



Task 2.4

Calculated Value	Calculated Results	Simulated Results
Time Constant (s)	0.0352	0.0352

Task 2.5

The circuit performed as expected and the results calculated for the time constant were identical to those simulated. Simulated values were taken from the point where the amplitude was 0.632, this is because the time constant reached at 63.2%.



Task 5

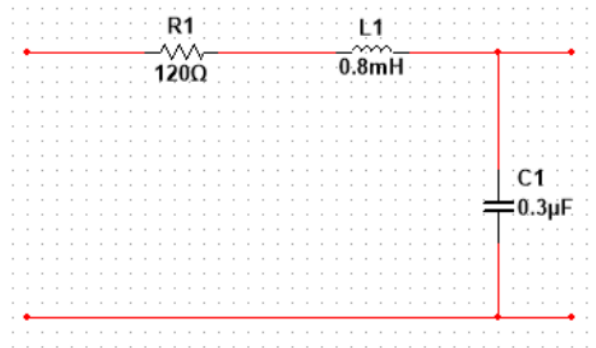
Component Values

$$R = 120 \, \Omega$$

$$L = 0.8 \, \text{H}$$

$$C = 30 \, \mu\text{F}$$

5.2



Transfer Function:

$$H(s) = \frac{k\omega n^2}{s^2 + 2\zeta\omega n s + \omega n^2}$$



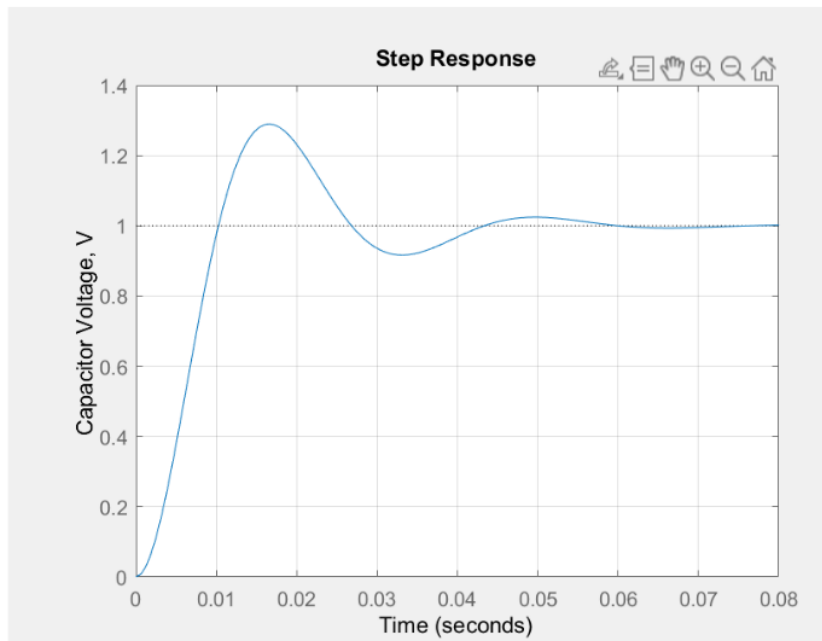
5.3/ 5.4/ 5.6

$$K = 1$$

$$\omega n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.8)(30 \times 10^{-6})}} = 204 \, \text{rad/s}$$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{120}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 0.367 \, (\text{Underdamped})$$

$$H(s) = \frac{41616}{s^2 + 149.736s + 41616}$$

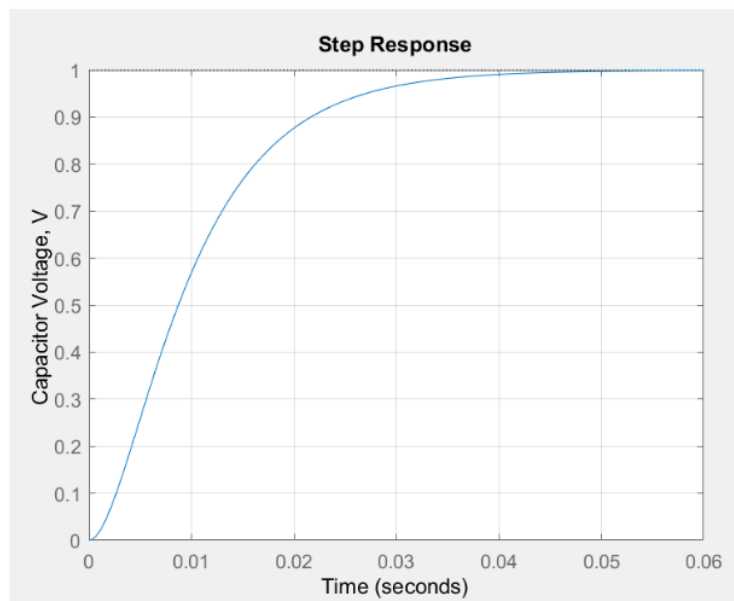


```
>> clear all
>> %RLC Response to Step Input
>> %R = 120, L = 0.8, C = 30u
>> %Input = 1V Step
>> num = [41616];
>> den = [1 149.736 41616];
>> step(num,den);
>> grid
>> xlabel('Time');
>> ylabel('Capacitor Voltage, V');
```

$$R = 360$$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{360}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 1.102 \text{ (Overdamped)}$$

$$H(s) = \frac{41616}{s^2 + 449.616s + 41616}$$

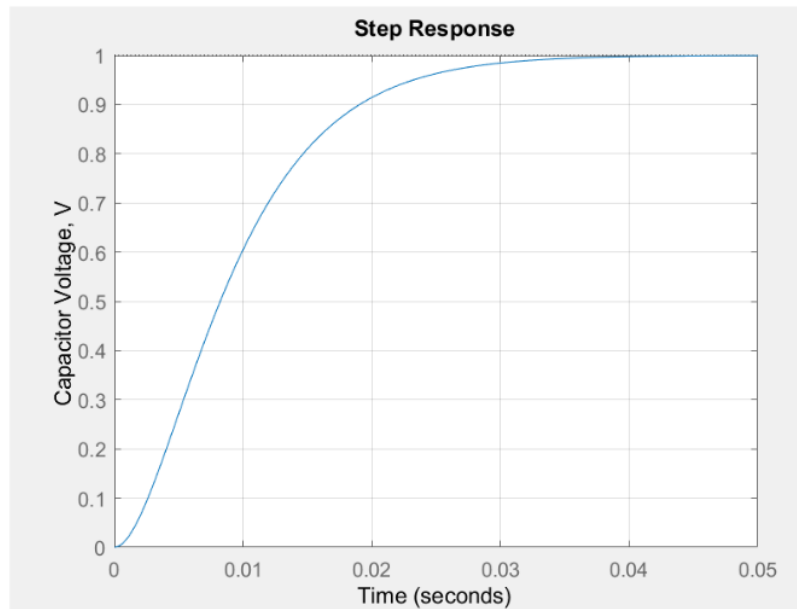


```
>> %RLC Response to Step Input
>> %R = 360, L = 0.8, C = 30u
>> %Input = 1V Step
>> num = [41616];
>> den = [1 449.616 41616];
>> step(num,den);
>> grid
>> xlabel('Time');
>> ylabel('Capacitor Voltage, V');
```

$$R = 327$$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{327}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 1 \text{ (Critically Damped)}$$

$$H(s) = \frac{41616}{s^2 + 408s + 41616}$$

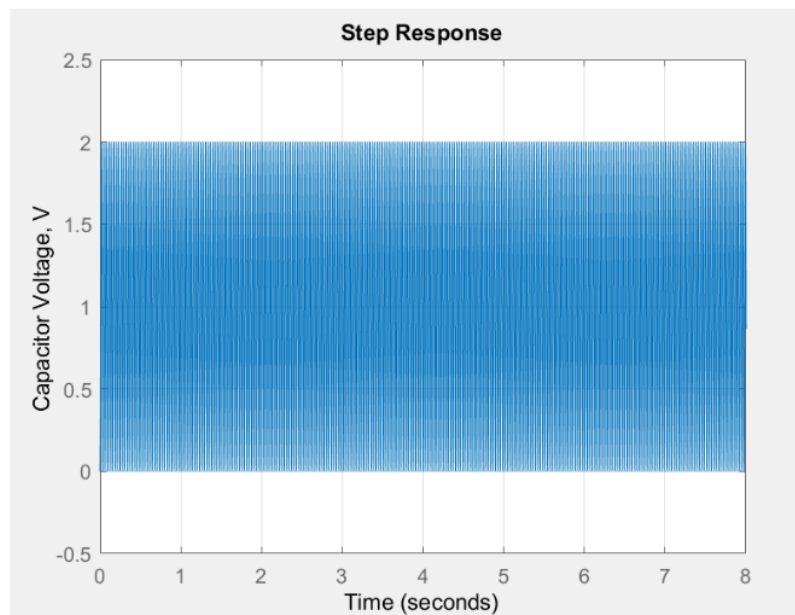


```
>> %RLC Response to Step Input
>> %R = 327, L = 0.8, C = 30u
>> %Input = 1V Step
>> num = [41616];
>> den = [1 408 41616];
>> step(num,den);
grid
>> xlabel('Time');
>> ylabel('Capacitor Voltage, V');
```

$$R = 0$$

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{0}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 0 \text{ (No Damping)}$$

$$H(s) = \frac{41616}{s^2 + 0 + 41616}$$



```
>> %RLC Response to Step Input
>> %R = 0, L = 0.8, C = 30u
>> %Input = 1V Step
>> num = [41616];
>> den = [1 0 41616];
>> step(num,den);
grid
>> xlabel('Time');
>> ylabel('Capacitor Voltage, V');
```

5.5

Below is a table comparing the values calculated for Sigma against the values chosen for resistance for each of the four types of damping.

Type of Damping	Resistance Value	Sigma (ζ) Value
Underdamped	120	0.367
Overdamped	360	1.102
Critically Damped	327	1
No Damping	0	0

5.7

From analysing the results above, it is clear to see that the resistance value chosen has an impact on the value that is calculated for Sigma. By using a higher resistance value, the value for Sigma will also increase, this means that higher resistances produce overdamping. In order to decrease the amount of Damping on a circuit you would need to decrease the Resistance value which in turn would reduce the Sigma value. Results calculated for Sigma are affected by calculation errors such as over rounding, so care must be taken when carrying out the calculations.

Task 6

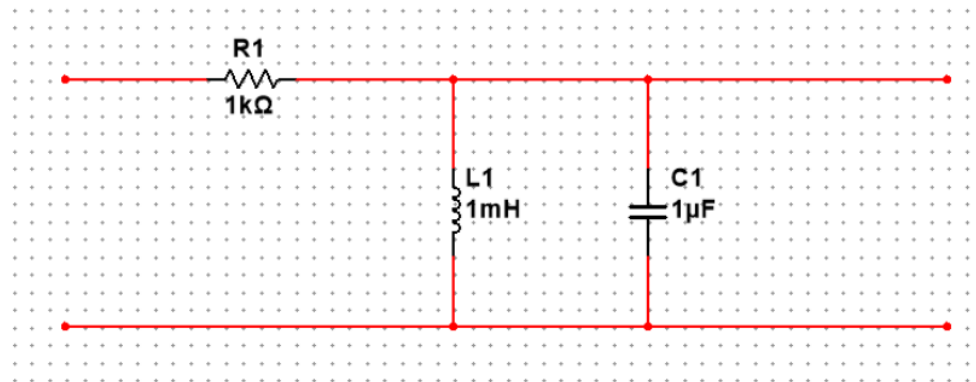
Component Values

$$R = 100 \, \Omega$$

$$L = 1.2 \, \text{H}$$

$$C = 60 \, \mu\text{F}$$

6.2



Transfer Function:

$$H(s) = \frac{s \times \frac{1}{RC}}{s^2 + \frac{1}{RC}s + 1/LC}$$

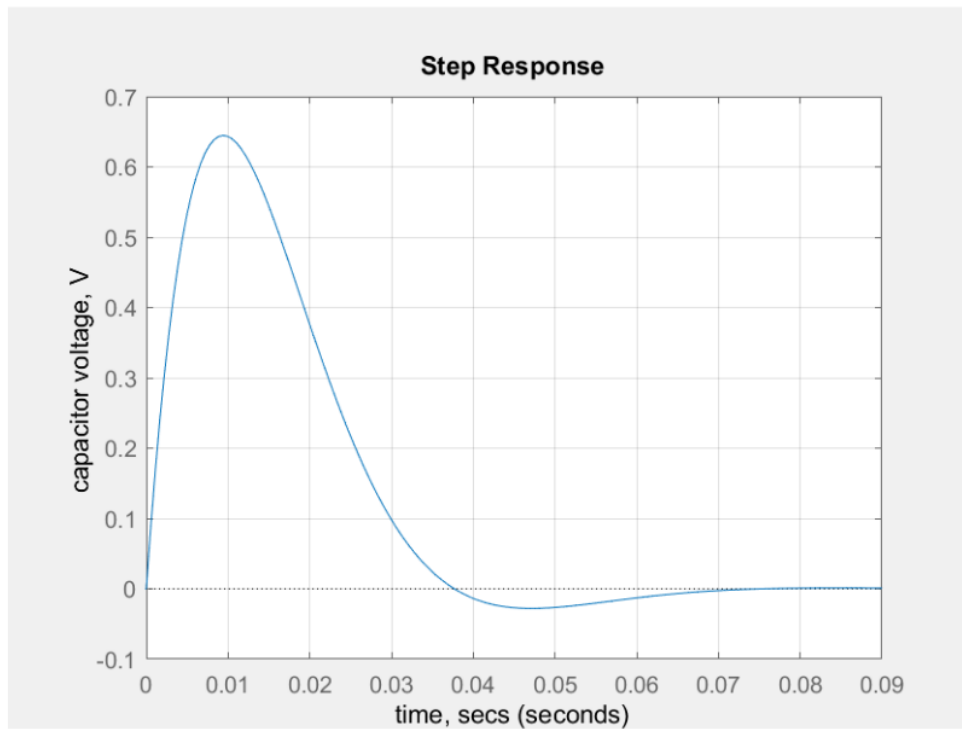
Incorrect

6.3

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.2)(60 \times 10^{-6})}} = 118 \, \text{rad/s}$$

$$\zeta = \frac{1}{(2)(R)} \sqrt{\frac{L}{C}} = \frac{1}{(2)(100)} \sqrt{\frac{1.2}{60 \times 10^{-6}}} = 0.707 \, (\text{Underdamped})$$

$$H(s) = \frac{166.6s + 0}{s^2 + 166.6s + 13888.8}$$

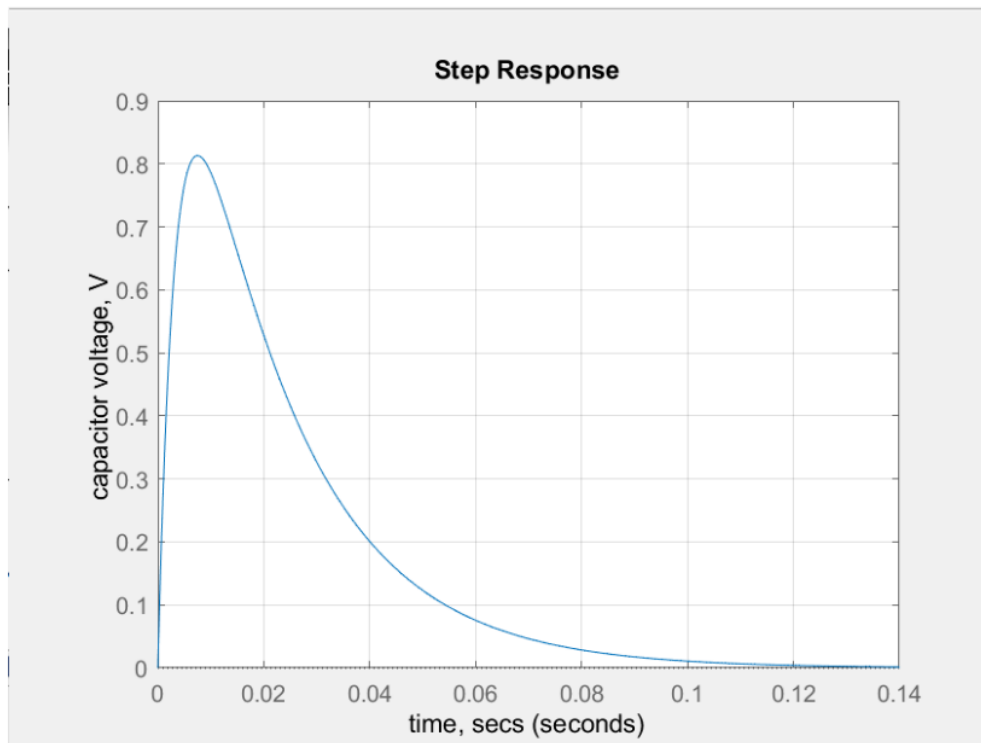


```
>> %Parallel RLC Response to Step Input
>> %R = 100, L = 1.2, C = 60u
>> %Input = 1V Step
>> num = [166.6 0];
>> den = [1 166.6 13888.8];
>> step(num,den);
>> grid
>> xlabel('time, secs');
>> ylabel('capacitor voltage, V');
```

$$R = 50$$

$$\zeta = \frac{1}{(2)(R)} \sqrt{\frac{L}{C}} = \frac{1}{(2)(50)} \sqrt{\frac{1.2}{60 \times 10^{-6}}} = 1.4 \text{ (Overdamped)}$$

$$H(s) = \frac{333.3s + 0}{s^2 + 333.3s + 13888.8}$$

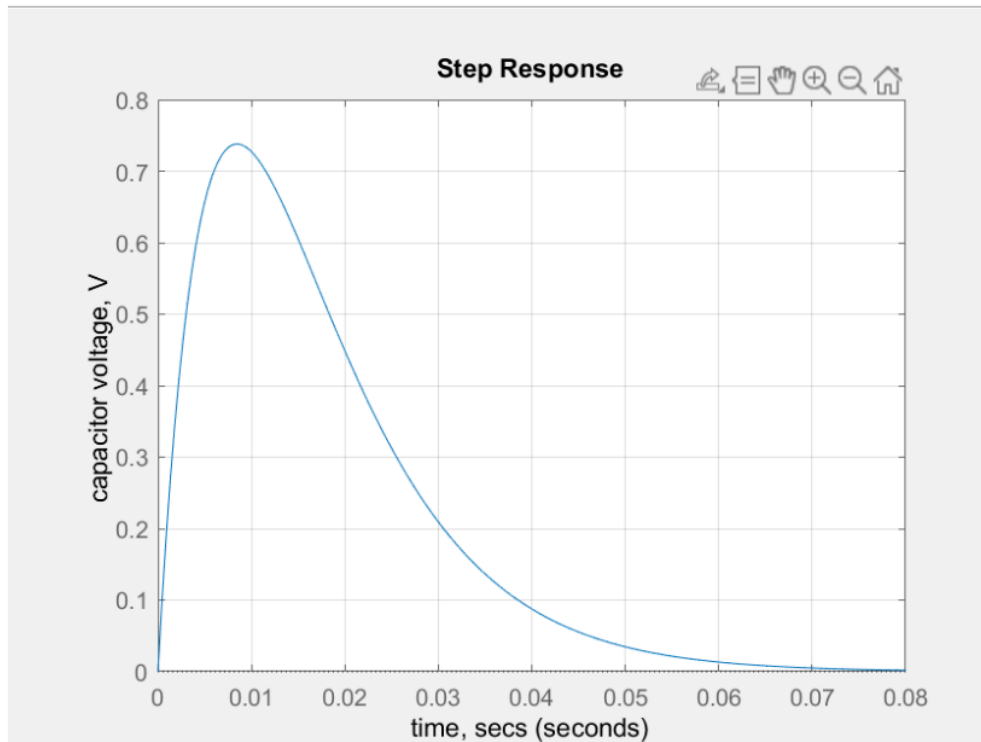


```
>> %Parallel RLC Response to Step Input
>> %R = 50, L = 1.2, C = 60u
>> %Input = 1V Step
>> num = [333.3 0];
>> den = [1 333.3 13888.8];
>> step(num,den);
>> grid
>> xlabel('time, secs');
>> ylabel('capacitor voltage, V');
```

$$R = 70$$

$$\zeta = \frac{1}{(2)(R)} \sqrt{\frac{L}{C}} = \frac{1}{(2)(70)} \sqrt{\frac{1.2}{60 \times 10^{-6}}} = 1 \text{ (Critically damped)}$$

$$H(s) = \frac{238s + 0}{s^2 + 238s + 13888.8}$$

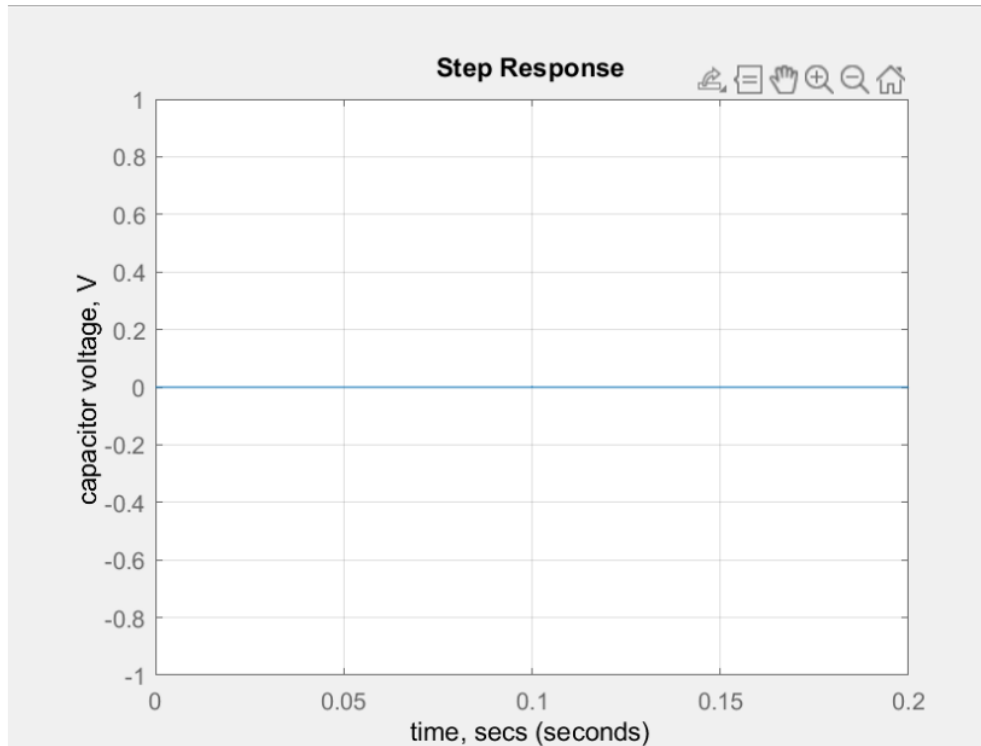


```
>> %Parallel RLC Response to Step Input
>> %R = 70, L = 1.2, C = 60u
>> %Input = 1V Step
>> num = [238 0];
>> den = [1 238 13888.8];
>> step(num,den);
>> grid
>> xlabel('time, secs');
>> ylabel('capacitor voltage, V');
```

R = 0

$$\zeta = \frac{1}{(2)(R)} \sqrt{\frac{L}{C}} = \frac{1}{(2)(0)} \sqrt{\frac{1.2}{60 \times 10^{-6}}} = 0 \text{ (No damping)}$$

$$H(s) = \frac{0s + 0}{s^2 + 0s + 13888.8}$$



```
>> %Parallel RLC Response to Step Input
>> %R = 0, L = 1.2, C = 60u
>> %Input = 1V Step
>> num = [0 0];
>> den = [1 0 13888.8];
>> step(num,den);
>> grid
>> xlabel('time, secs');
>> ylabel('capacitor voltage, V');
```

Conclusion

Higher resistance values produce circuits with higher levels of damping.



Task 7

Component Values

$$R = 120 \, \Omega$$

$$L = 0.8 \, \text{H}$$

$$C = 30 \, \mu\text{F}$$

Transfer function:

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{41666.6}{s^2 + 150 + 41666.6}$$

Natural Frequency:

$$\omega_n = \frac{1}{\sqrt{LC}} \, \text{rad/s}$$

Damping Factor:

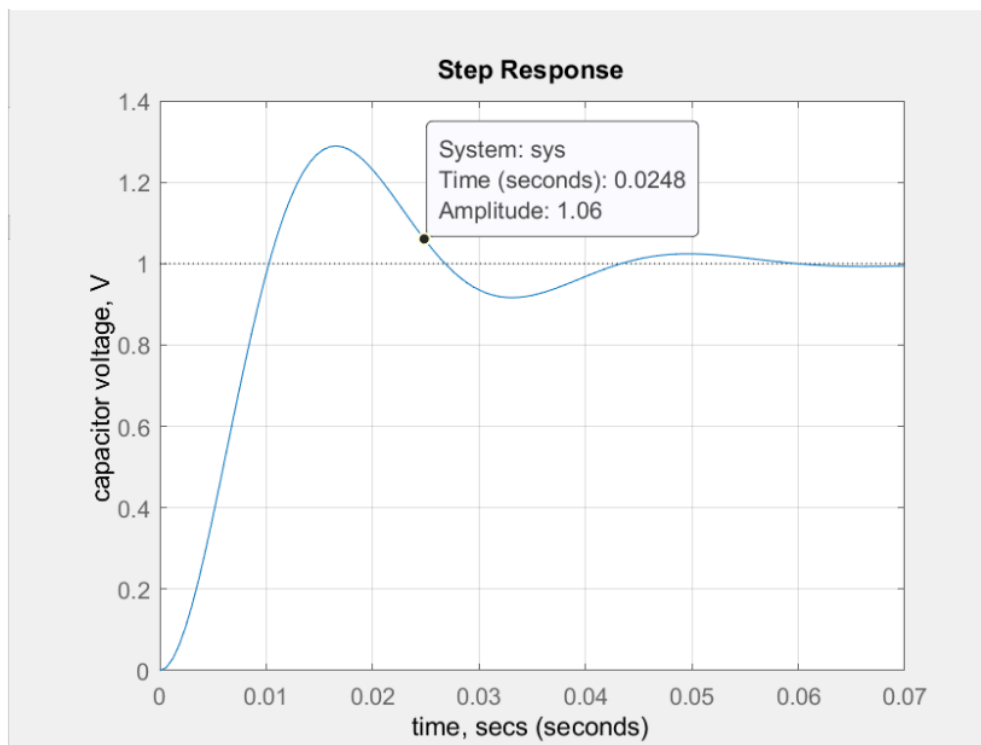
$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{120}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 0.367 \, \text{Underdamped}$$

$$\omega_m = \sqrt{\frac{1}{LC}} - \frac{1}{2} \left(\frac{R}{L}\right)^2 = \sqrt{\frac{1}{0.8 \times 30 \times 10^{-6}}} - \frac{1}{2} \left(\frac{120}{0.8}\right)^2 = -11045.88 \, \text{rad/s}$$



Quality Factor:

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{120} \sqrt{\frac{0.8}{30 \times 10^{-6}}} = 1.36$$



```
>> clear all
>> %RLC Response to Step Input
>> %R = 120, L = 0.8, C = 30u
>> %Input = 1V Step
>> num = [41666.6];
>> den = [1 150 41666.6];
>> step(num,den);
>> grid
```

7.4

R=200

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{200}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 0.612 \text{ Underdamped}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{41666.6}{s^2 + 250 + 41666.6}$$

R = 320

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{320}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 0.98 \text{ Underdamped}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{41666.6}{s^2 + 400 + 41666.6}$$

R = 340

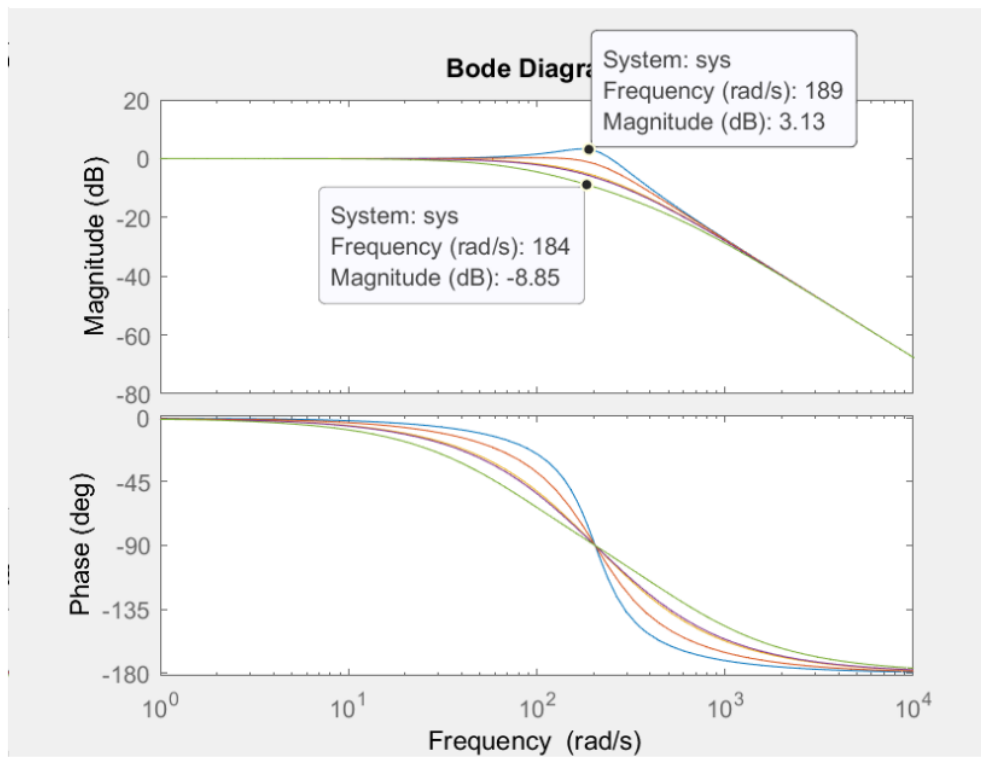
$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{340}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 1 \text{ Critically damped}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{41666.6}{s^2 + 425 + 41666.6}$$

R = 500

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}} = \frac{500}{2} \sqrt{\frac{30 \times 10^{-6}}{0.8}} = 1.53 \text{ Overdamped}$$

$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{41666.6}{s^2 + 625 + 41666.6}$$



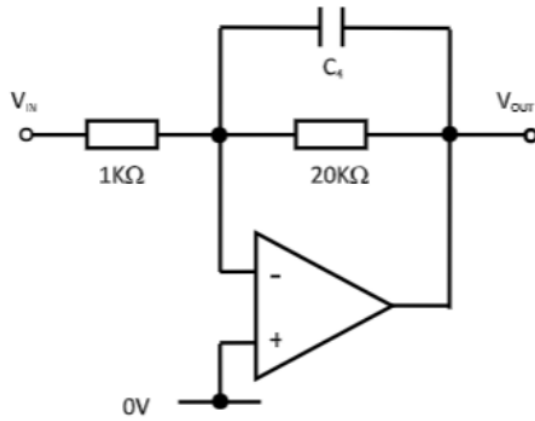
```
>> %Underdamped R = 120, L = 0.8, C = 30u
>> num = [41666.6];
>> den = [1 150 41666.6];
>> bode(num,den);
>> hold on
>> %Underdamped R = 200
>> den = [1 250 41666.6];
>> bode(num,den);
>> %Underdamped R = 320
>> den = [1 400 41666.6];
>> bode(num,den);
>> %Critically damped
>> den = [1 425 41666.6];
>> bode(num,den);
>> %Overdamped R = 500
>> den = [1 625 41666.6];
>> bode(num,den);
```

7.5

When the resistance of the circuit is increased more damping takes place. The higher the damping effect the lower the magnitude (dB) and less phase shift (Degrees) takes place. As shown in the graph above, the underdamped circuit with resistance value 120Ω shows a spike in magnitude at the drop off point whereas the overdamped circuit with a resistance value of 500Ω drops off smoothly.

Task 9

Determine a suitable value for C4 for the electronic circuit below.



$$C4 = 490\text{nF}$$

$$\text{Gain} = K = \frac{20}{1} = 20$$

$$\omega_c = \frac{1}{(490 \times 10^{-9}) \times (20000)} = 102.04 \text{ rad/s}$$

$$H(s) = -20 \left\{ \frac{102.04}{s + 102.04} \right\}$$

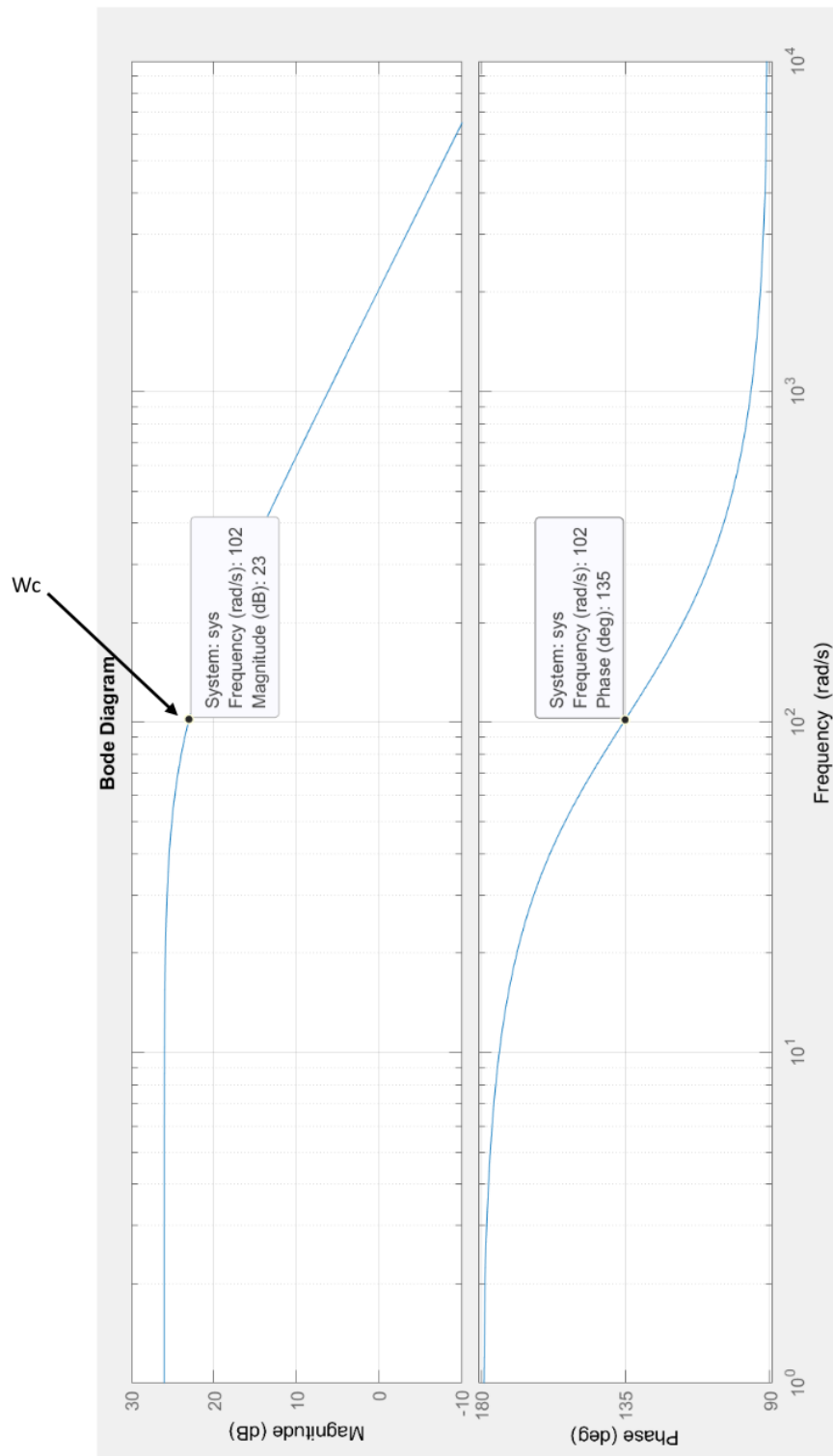
When using matlab

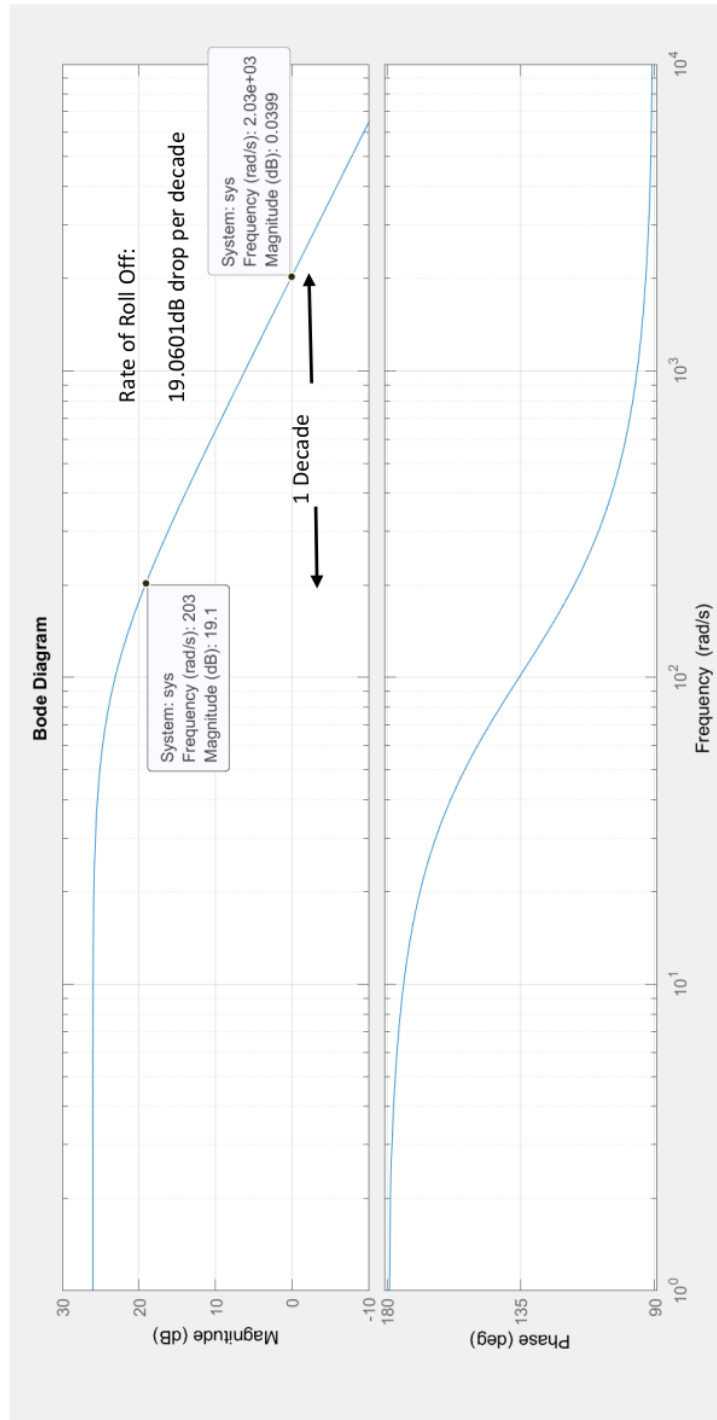
In dBs 20 is:

$$20 \log_{10}(20) = 26.02$$

Cut off occurs at $26.02 - 3 = 23.02\text{dB}$

```
>> clear all
>> num = [-2040.4];
>> den = [1 102.02];
>> bode(num,den);
>> grid
>> |
```

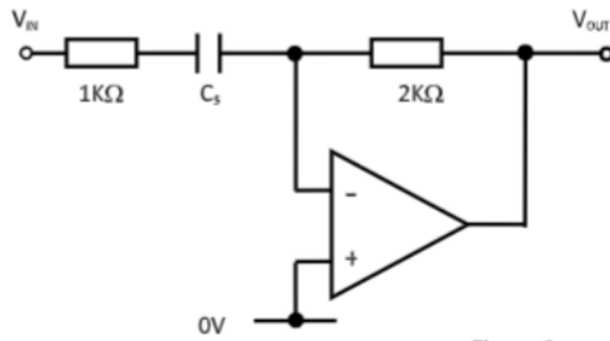




11

Task 10

Select a suitable value for C5



$$C5 = 250\text{nF}$$

$$\text{Gain} = K = \frac{2000}{1000} = 2$$

$$\omega_c = \frac{1}{(250 \times 10^{-9})(1000)} = 4000 \text{ rad/s}$$

$$H(s) = -2 \left\{ \frac{s}{s + 4000} \right\}$$

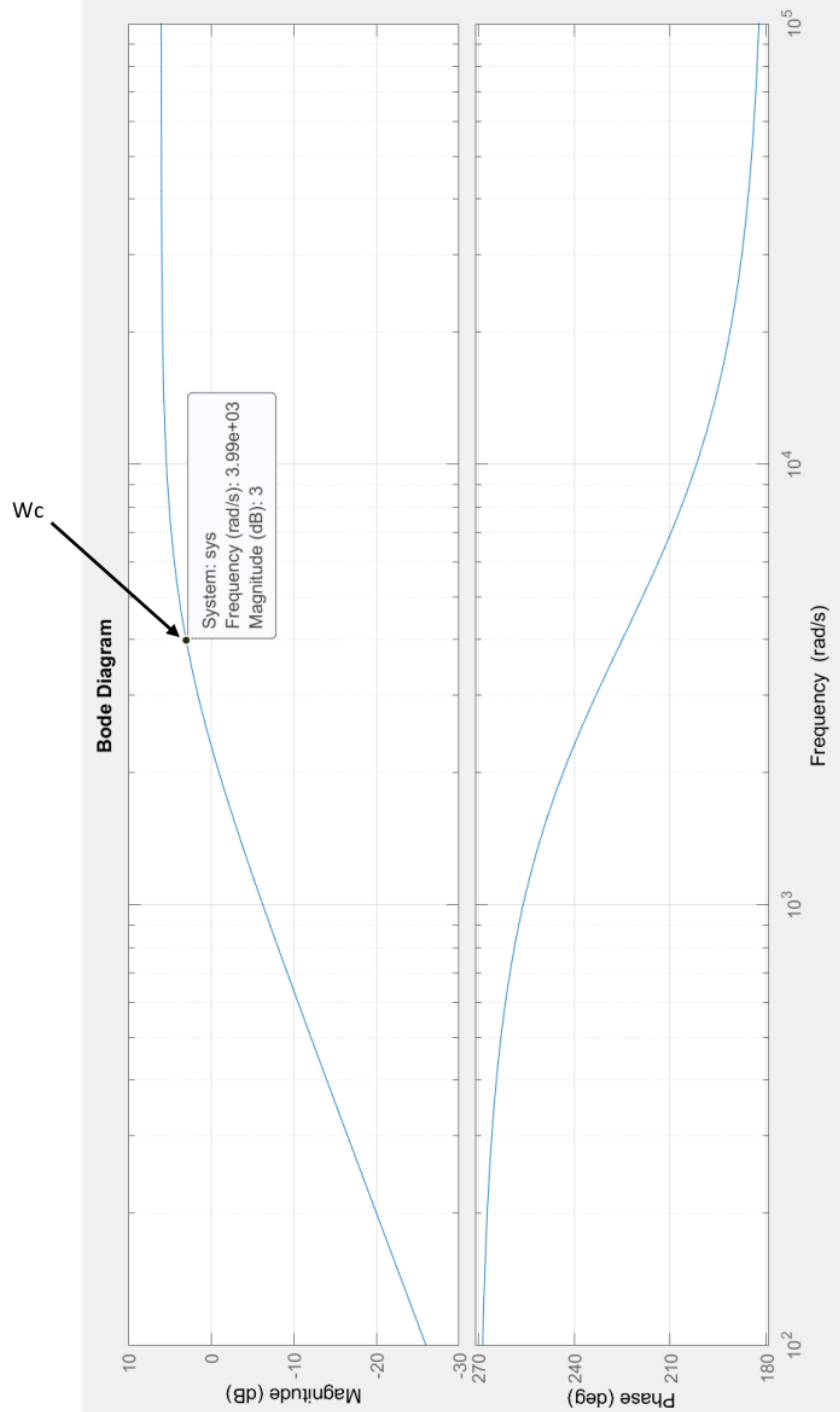
When using Matlab:

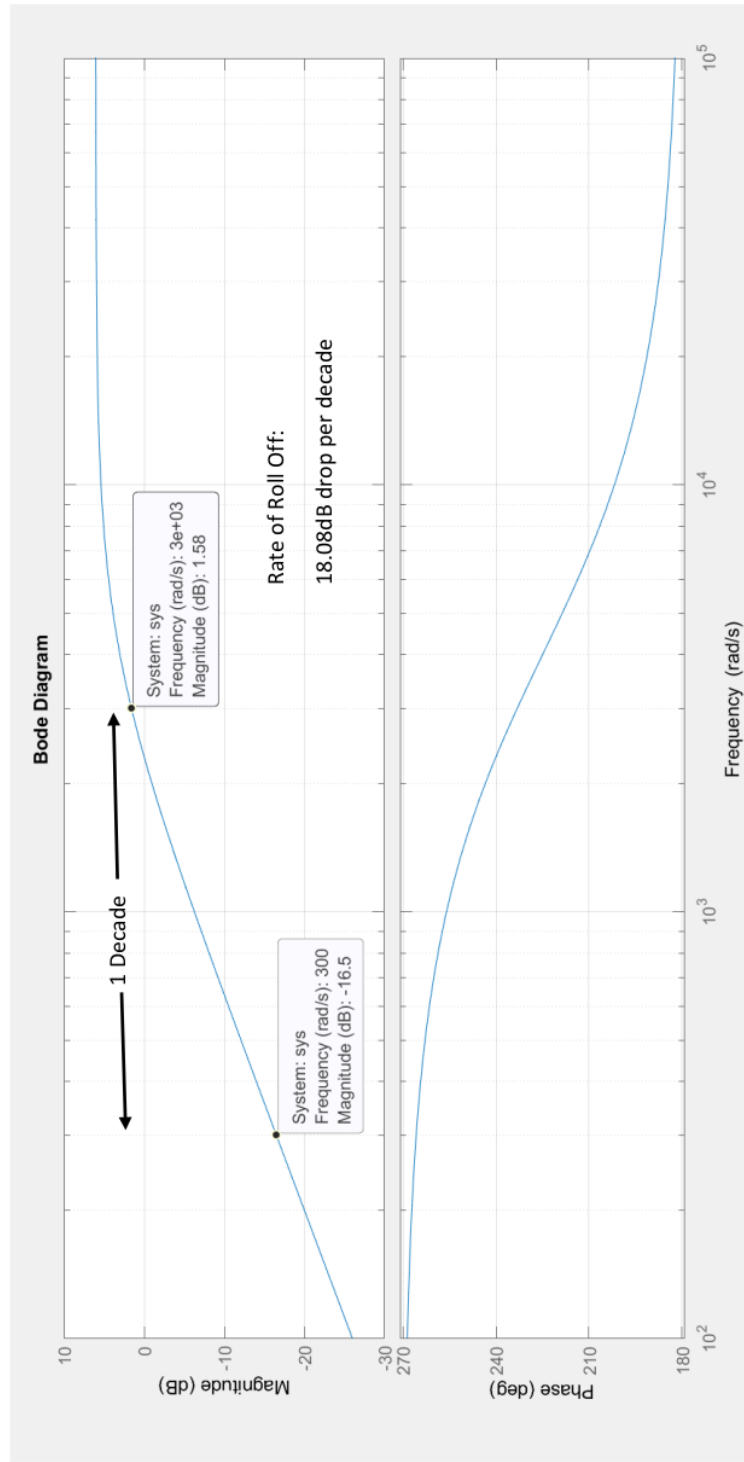
In dB 2 is:

$$20\log(2) = 6\text{dB}$$

So Cut-Off occurs at Magnitude = 3dB

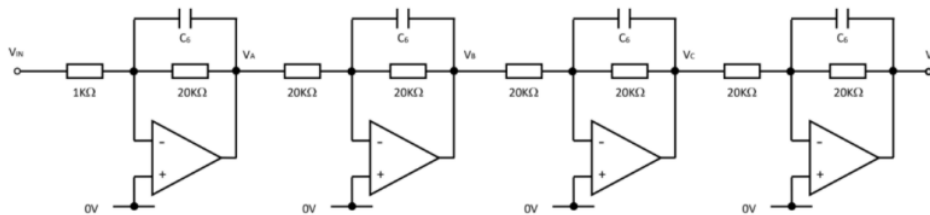
```
>> clear all
>> num = [-2 0];
>> den = [1 4000];
>> bode(num,den);
>> grid
```





Task 11

Select a suitable capacitor value for Ca-d



$$C = 400\text{nF}$$

$$\text{Gain} = K = \frac{R2}{R1} = \frac{20k}{1k} = 20$$

$$\text{Cut - Off Frequency} = W_c = \frac{1}{R_2 C} = \frac{1}{20000 \times 400 \times 10^{-9}} = 125 \text{ rad/s}$$

$$H(s) = \frac{K W_c^2}{s^2 + 2W_c s + W_c^2}$$

$$H(s) = \frac{20 \times 125^2}{s^2 + (2)(125)s + 125^2}$$

$$H(s) = \frac{312500}{s^2 + 250s + 15625}$$

For an Nth order filter the cut-off frequency will become:

$$W_{cn} = \sqrt[n]{\sqrt{2} - 1} \times W_c$$

The value for 'n' will vary depending on the order of the filter:

$$W_{c2} = \sqrt[2]{\sqrt{2} - 1} = 0.6436$$

$$W_{c2} = \sqrt[2]{\sqrt{2} - 1} \times 125 = 80.45 \text{ rad/s}$$

$$W_{c3} = \sqrt[3]{\sqrt{2} - 1} = 0.5098$$

$$W_{c3} = \sqrt[3]{\sqrt{2} - 1} \times 125 = 63.73 \text{ rad/s}$$

$$W_{c4} = \sqrt[4]{\sqrt{2} - 1} = 0.435$$

$$W_{c4} = \sqrt[4]{\sqrt{2} - 1} \times 125 = 54.37 \text{ rad/s}$$

To find the scaling factor:

$$W_{c2} = \frac{1}{\sqrt[2]{\sqrt{2} - 1}} = 1.554$$

$$W_{c3} = \frac{1}{\sqrt[3]{\sqrt{2} - 1}} = 1.962$$

$$W_{c4} = \frac{1}{\sqrt[4]{\sqrt{2} - 1}} = 2.299$$

Therefore, to find the scaled cut off frequencies the following calculations must be made:

$$W_{c2} = \frac{125}{0.6436} = 194.22 \text{ rad/s}$$

$$W_{c3} = \frac{125}{0.5098} = 245.19 \text{ rad/s}$$

$$W_{c4} = \frac{125}{0.435} = 287.36 \text{ rad/s}$$

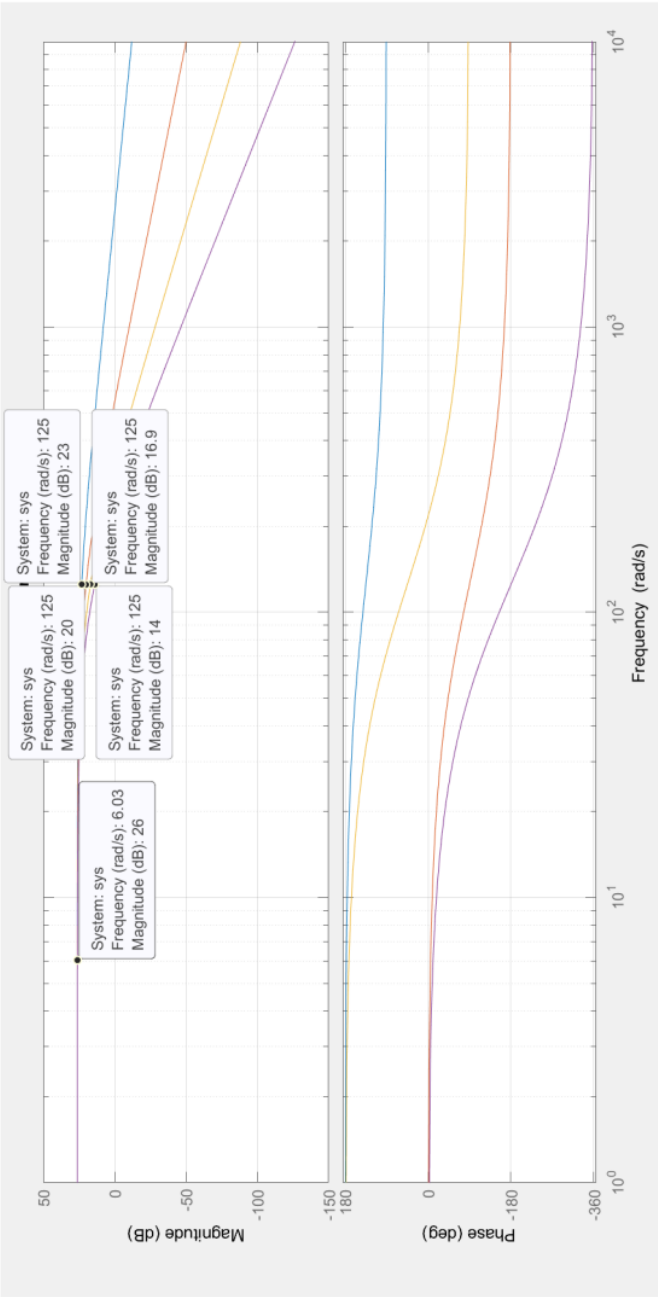
The transfer functions for each of the filter orders are also shown below:

$$n = 2 = H(s) = \frac{K W_c^2}{S^2 + 2W_c S + W_c^2}$$

$$n = 3 = H(s) = \frac{-K W_c^3}{S^3 + 3W_c S^2 + 3W_c^2 S + W_c^3}$$

$$n = 4 = H(s) = \frac{K W_c^4}{S^4 + 4W_c S^3 + 6W_c^2 S^2 + 4W_c^3 S + W_c^4}$$

The image below shows the cut-off frequency of the four stages before scaling was introduced to the system.



```

>> %Cascaded 4th Order Low Pass Filter
>> %First Stage
>> wc = 125;
>> k = 20 %Gain of Stage 1

k =

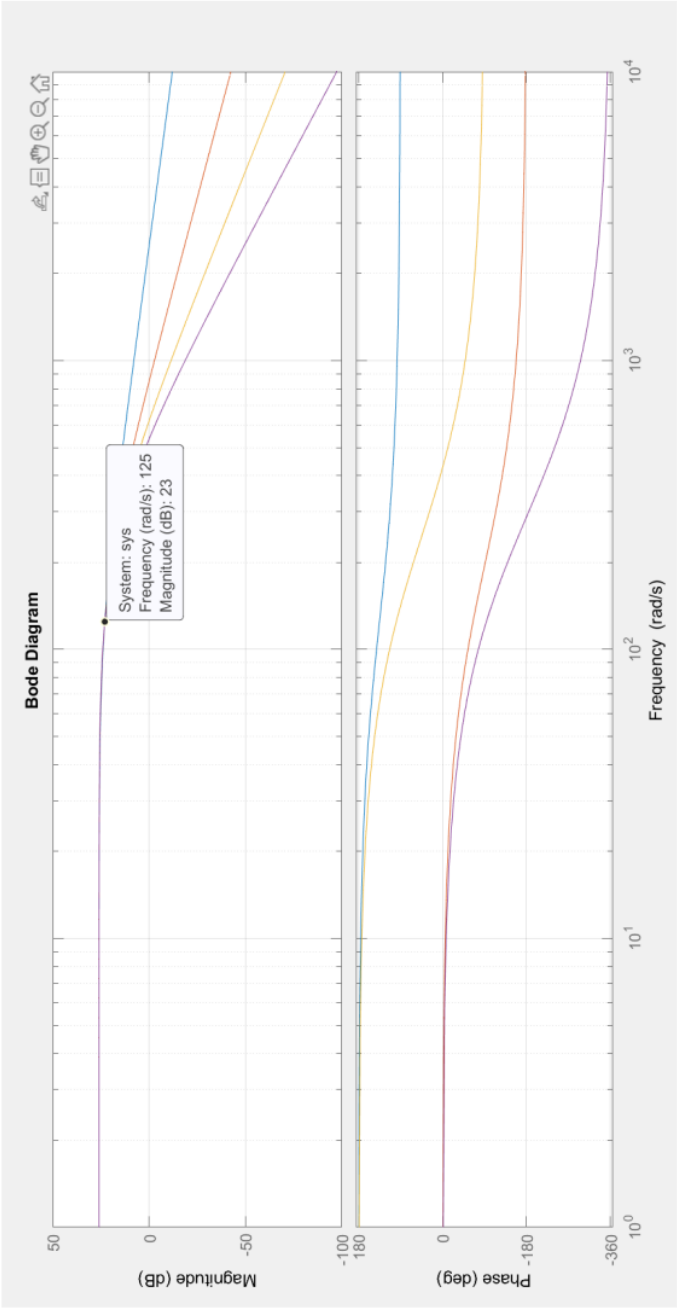
    20

>> num1 = [-k*wc];
>> den1 = [1 wc];
>> bode(num1,den1);
>> grid on
>> hold on
>>
>> %Second Stage
>> num2 = [k*wc^2];
>> den2 = [1 2*wc wc^2];
>> bode(num2,den2);

>> %Third Stage
>> num3 = [-k*wc^3];
>> den3 = [1 3*wc 3*wc^2 wc^3];
>> bode(num3,den3);
>>
>> %Fourth Stage
>> num4 = [k*wc^4];
>> den4 = [1 4*wc 6*wc^2 4*wc^3 wc^4];
>> bode(num4,den4);

```

The image below shows the cut-off frequency of the four stages after scaling the values



```

>> %Cascaded 4th Order Low Pass Filter Modified Design
>> %First Stage
>> wc1 = 125

wc1 =

    125

>> wc = wc1;
>> k = 20

k =

    20

>> num1 = [-k*wc];
>> den1 = [1 wc];
>> bode(num1,den1);
>> grid on
>> hold on

>> %Second Stage
>> %To get the same cut off as wc, use scaling factor of 1.554
>> wc = wc1*1.554;
>> num2 = [k*wc^2];
>> den2 = [1 2*wc wc^2];
>> bode(num2,den2);
>>

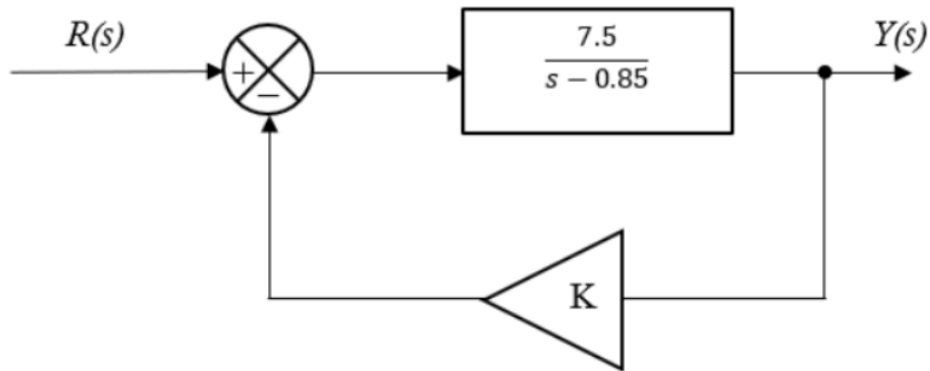
>> %Third Stage
>> %To get the same cut off as wc, use scaling factor of 1.962
>> wc = wc1*1.962;
>> num3 = [-k*wc^3];
>> den3 = [1 3*wc 3*wc^2 wc^3];
>> bode(num3,den3);
>>

>> %Fourth Stage
>> %To get the same cut off as wc, use scaling factor of 2.299
>> wc = wc1*2.299;
>> num4 = [k*wc^4];
>> den4 = [1 4*wc 6*wc^2 4*wc^3 wc^4];
>> bode(num4,den4);

```

Task 12

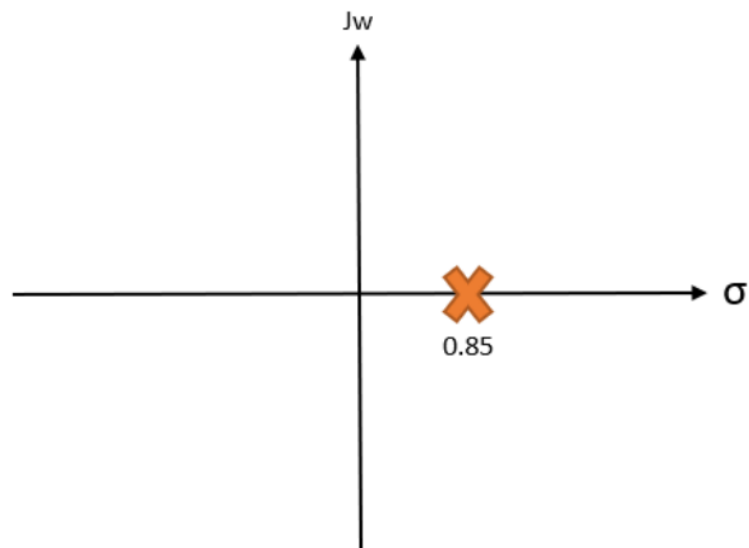
The Circuit below will be analysed in this task.



The transfer function for the above circuit is shown below:

$$H(s) = \frac{7.5}{s - 0.85}$$

From the transfer function it can be said that the pole occurs at $p = 0.85$, the following argand diagram can then be produced.



The Following equations can be produced from the block diagram above:

Equation 1:

$$E(s) = R(s) - (H(s))(Y(s))$$

Equation 2:

$$Y(s) = (G(s))(E(s))$$

From Equation 2:

$$E(s) = \frac{Y(s)}{G(s)}$$

Substitute back into Equation 1:

$$\frac{Y(s)}{G(s)} = R(s) - [H(s)][Y(s)]$$

Multiply through by G(s):

$$Y(s) = [G(s)][R(s)] - [(H(s))][Y(s)]$$

$$Y(s) + [G(s)][H(s)][Y(s)] = [G(s)][R(s)]$$

$$Y(s) [1 + G(s)H(s)] = G(s)R(s)$$

$$Y(s) = \frac{G(s)R(s)}{[1 + G(s)H(s)]}$$

$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$G(s) = \frac{7.5}{s - 0.85}$$

$$H(s) = K$$

To find the value for the stability of the system the following calculations need to be carried out:

$$Y(s) = \frac{\frac{7.5}{s - 0.85}}{1 + \frac{7.5}{s - 0.85} \times K}$$

Multiply the top and bottom of the equation by $(s - 0.85)$ to get:

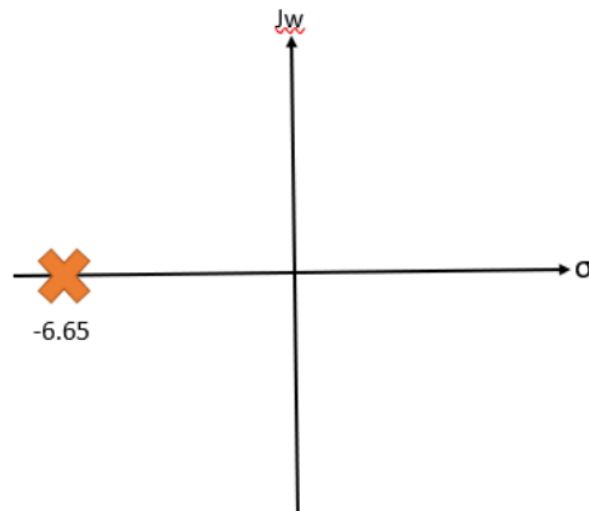
$$Y(s) = \frac{7.5}{s + 7.5K - 0.85}$$

Since K is given as 1:

$$Y(s) = \frac{7.5}{s + 7.5(1) - 0.85}$$

$$Y(s) = \frac{7.5}{s + 6.65}$$

From the calculations above the following argand diagram can now be created. This Argand Diagram shows a control system that is stable, this is because the pole is on the left-hand side of the Argand diagram.



To calculate the minimum value for the gain (k) the following calculations will need to be carried out:

$$s + (7.5K - 0.85) = 0$$

$$S = -(7.5K - 0.85)$$

$$\frac{0.85}{7.5} = 0.1133$$

You have worked this out correctly, well

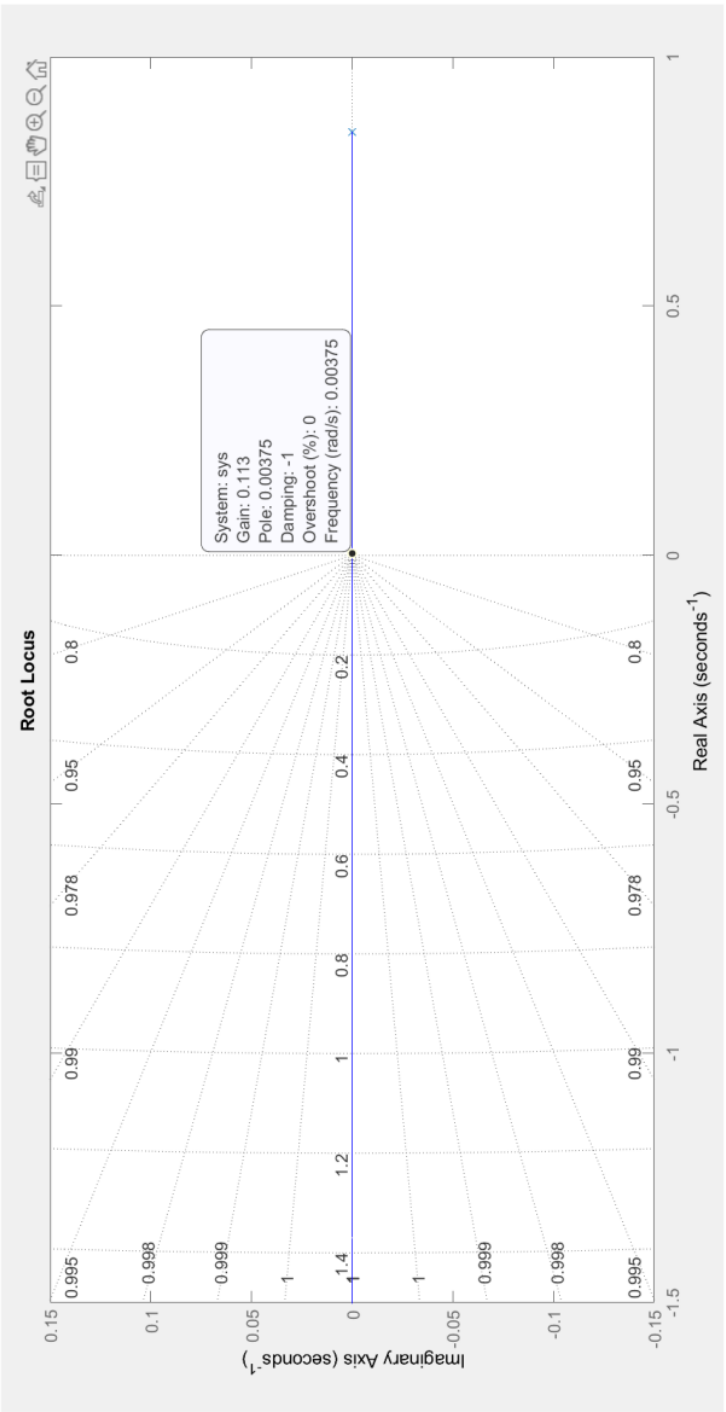
The minimum calculated value for the gain of the system is 0.1133, this will need to be proved on matlab by producing a root locus plot.

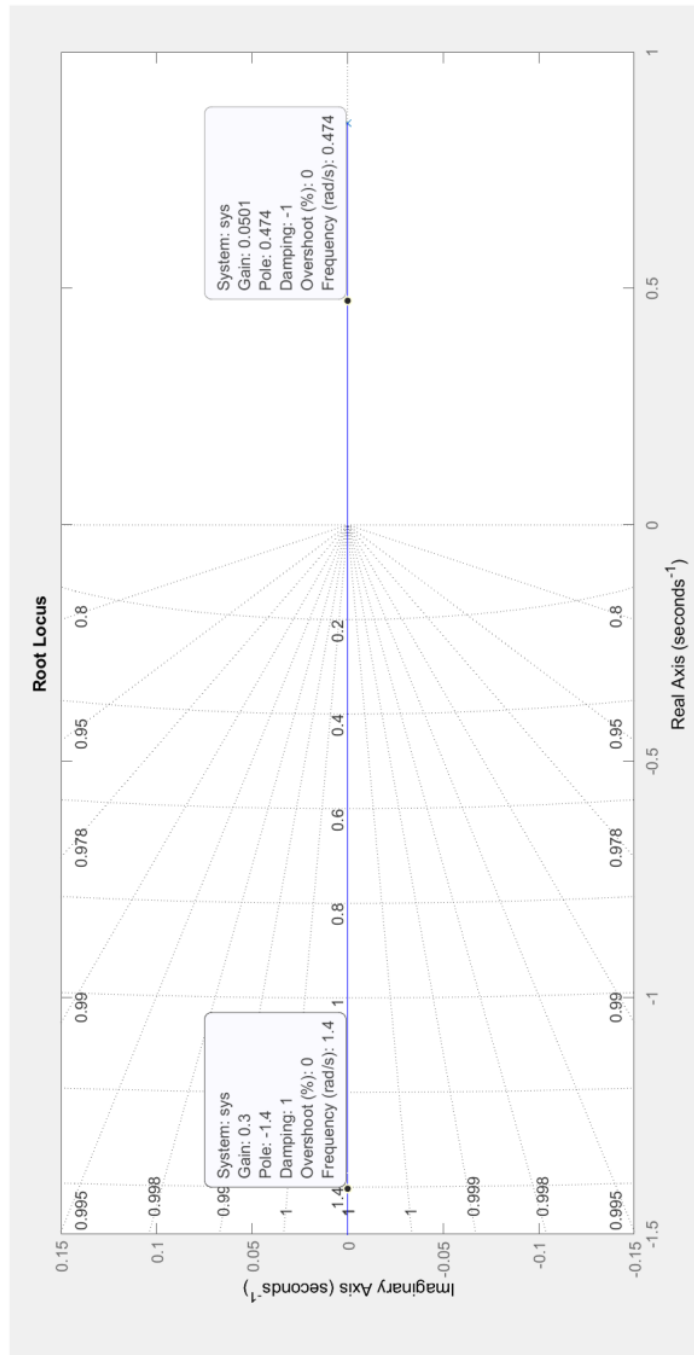
Below the root locus plot for the gain there is another root locus plot, this time the plot is used to show the difference in the pole reading when the gain value is increased to 0.3 and also when it is decreased to 0.05.

From looking at this root locus plot it can be said that increasing the gain to 0.3 stabilises the system as the pole value becomes negative, thus moving to the left-hand side of the argand diagram. Decreasing the gain will increase the pole value to 0.474, moving the pole further to the right of the argand diagram and therefore decreasing the stability of the system.

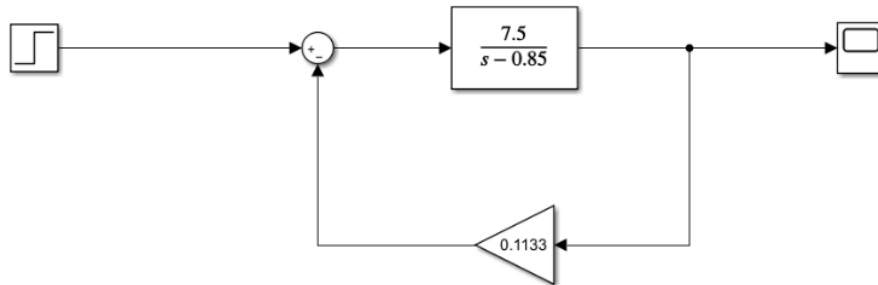
The Matlab code used to produce the Root Locus Plot is shown below:

```
>> num = [7.5];  
>> den = [1 -0.85];  
>> rlocus(num,den);  
>> grid
```

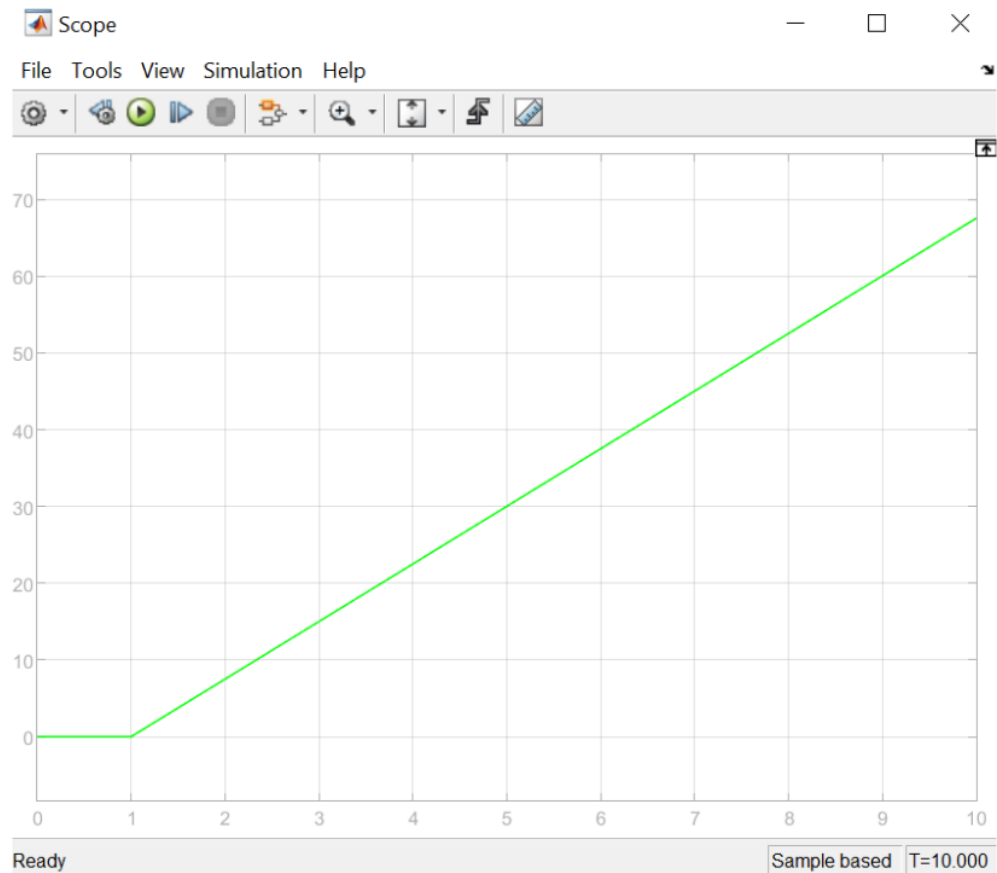




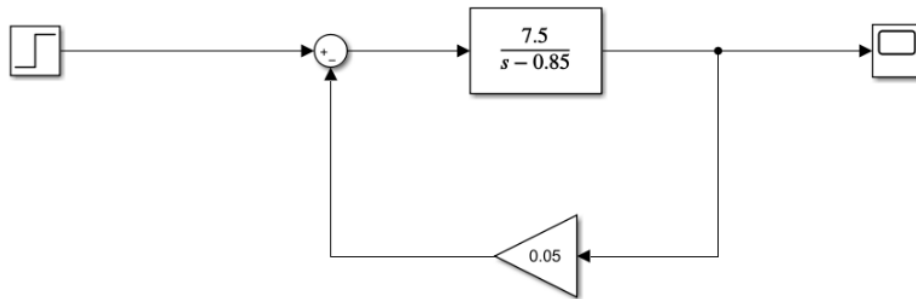
The control system being investigated was created on Simulink and a gain of 0.1133 was put into the system.



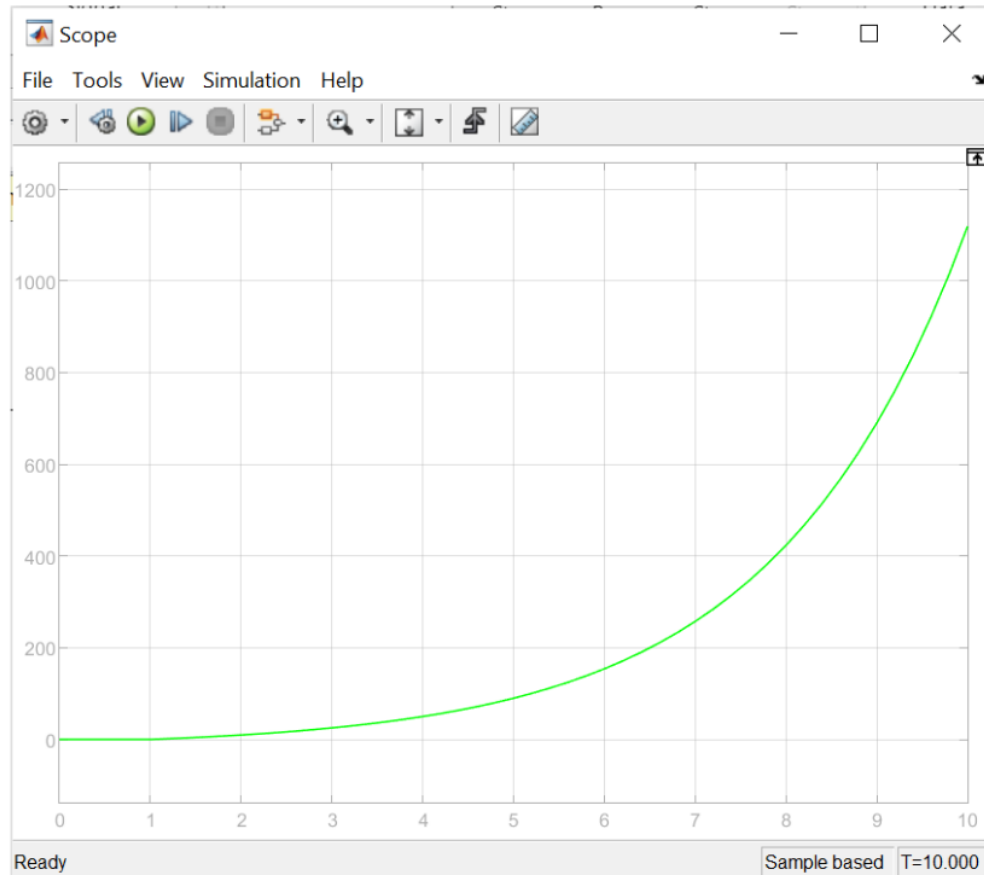
The circuit response with a gain on 0.1133 is shown below.



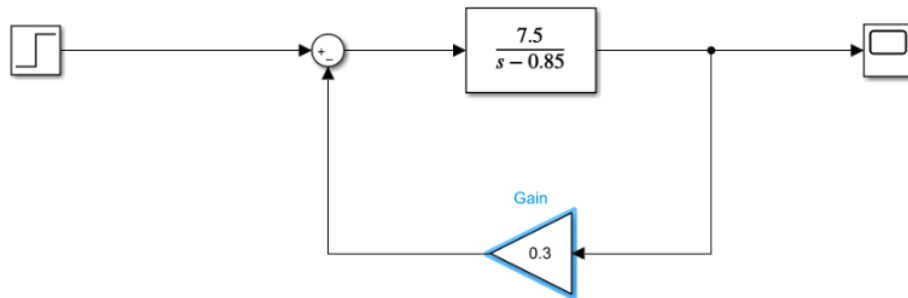
The Simulink circuit was then edited to include a gain of 0.05.



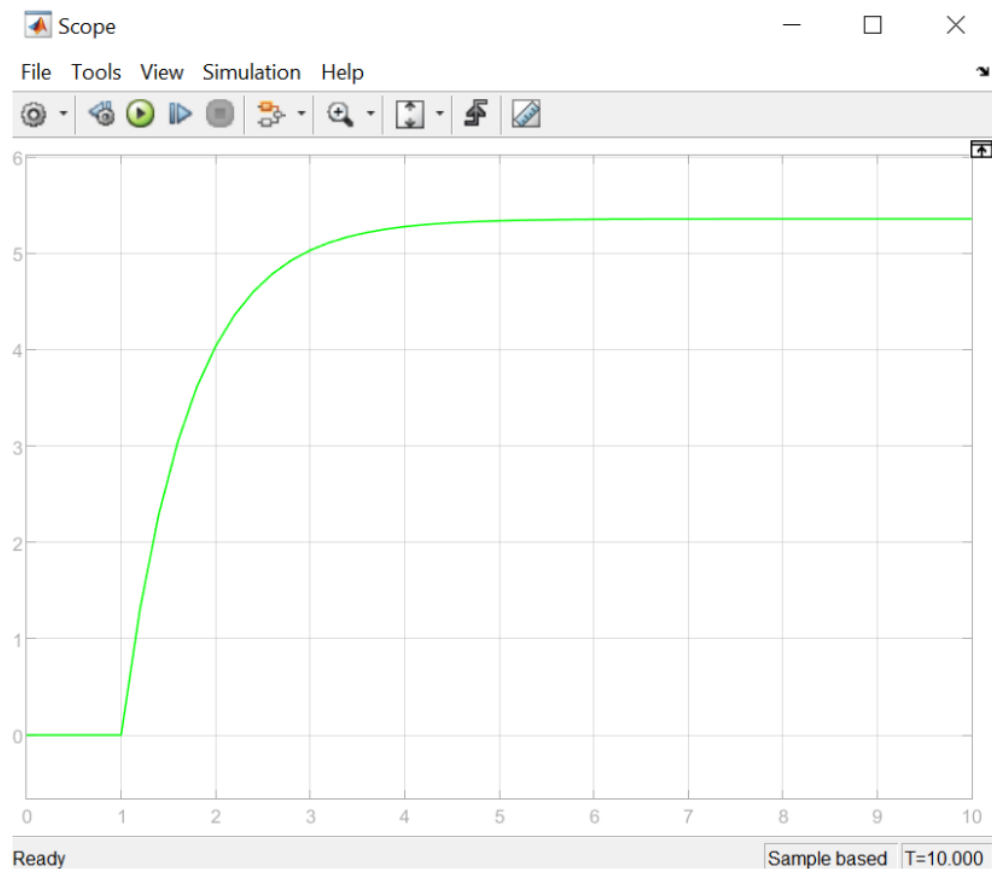
The circuit response for the system with a gain of 0.05 is shown below. This is unstable.



The Simulink circuit was then edited again, this time the gain was changed to 0.3. The gain is highlighted on the image below to show which value needs to be altered.



The circuit response for a gain of 0.3 is shown below. This is stable.



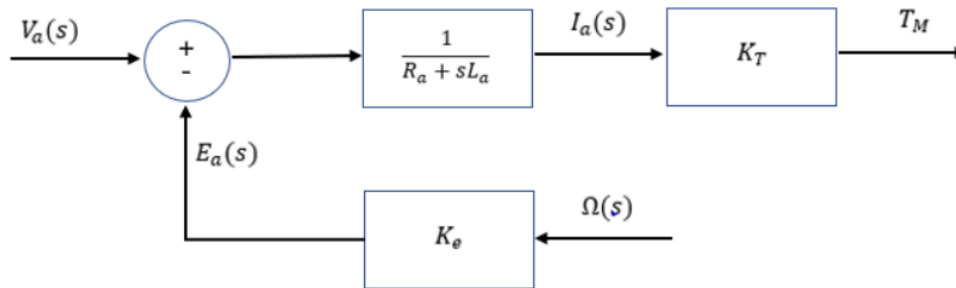
Task 13

A DC permanent magnet motor has the following parameters:

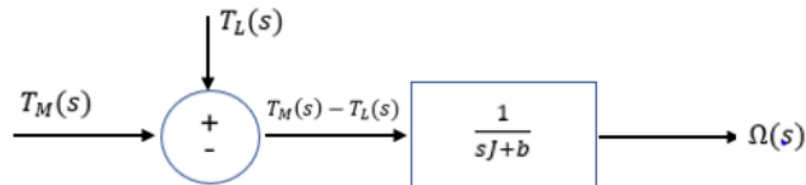
- (Ra) Armature resistance = 1.5 Ohms
- (La) Armature inductance = 0.165H
- (J) Rotor inertia = 0.00042 Kg m²
- (b) Friction coefficient = 4.6 x 10⁻⁶ Nm/rad/s
- (Ke) Motor voltage constant = 0.063 V/rad/s
- (Kt) Motor torque constant = 0.062 Nm/A
- Maximum armature voltage = 24V

When producing a mathematical model for a motor there are two parts required, these are the electrical and mechanical models. These two models will combine together to produce a complete model.

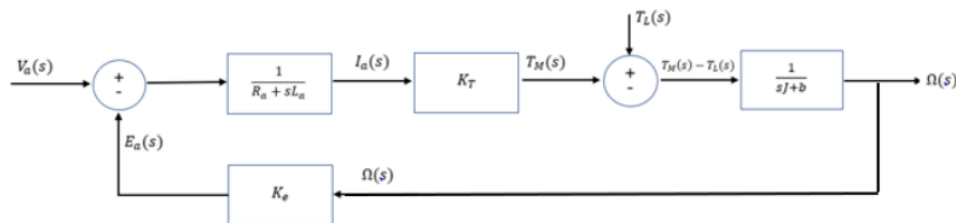
Electrical Model



Mechanical Model



Complete Block Diagram



The transfer function required is:

$$\frac{Kt/LaJ}{s^2 + S\left(\frac{b}{J} + \frac{Ra}{La}\right) + \frac{Rab}{LaJ} + \frac{KtKe}{LaJ}}$$

This transfer function can be simplified to:

$$\frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Time Constant:

$$\tau = \frac{L}{R} = \frac{0.165}{1.5} = 0.11s$$

The Natural Frequency is:

$$\omega_n^2 = \frac{Ra b + Kt Ke}{LaJ}$$

(From the transfer function)

$$\omega_n^2 = \frac{(1.5)(4.6 \times 10^{-6}) + (0.062)(0.063)}{(0.165)(0.00042)}$$

$$\omega_n^2 = 56.463$$

$$\omega_n = \sqrt{56.463}$$

$$\omega_n = 7.51 \text{ rad/s}$$

To find the Damping Factor:

$$\Sigma = \frac{b}{J} + \frac{R_a}{L_a} = \frac{4.6 \times 10^{-6}}{0.00042} + \frac{1.5}{2 \times 7.51}$$

(From the Transfer Function)

$$\Sigma = 0.606$$

To find the Stall Torque:

$$Tm(Stall) = \left(\frac{(Kt)(Va)}{Ra} \right) = \left(\frac{(0.062)(15)}{1.5} \right) = 0.62 \text{ Nm}$$

To find the theoretical frictionless no-load speed:

$$\Omega NL = \frac{Va}{Ke} = \frac{15}{0.063} = 238.09 \text{ rad/s}$$

You have worked this out correctly, well

Using the transfer function $\frac{K\omega_n^2}{s^2 + 2\Sigma\omega_n s + \omega_n^2}$ where:

- A = 1
- B = (2)(Σ)(ω_n) = (2)(0.606)(7.51) = 9.102
- C = ω_n^2 = 56.463

Solve the quadratic equation below to find the poles:

$$\frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

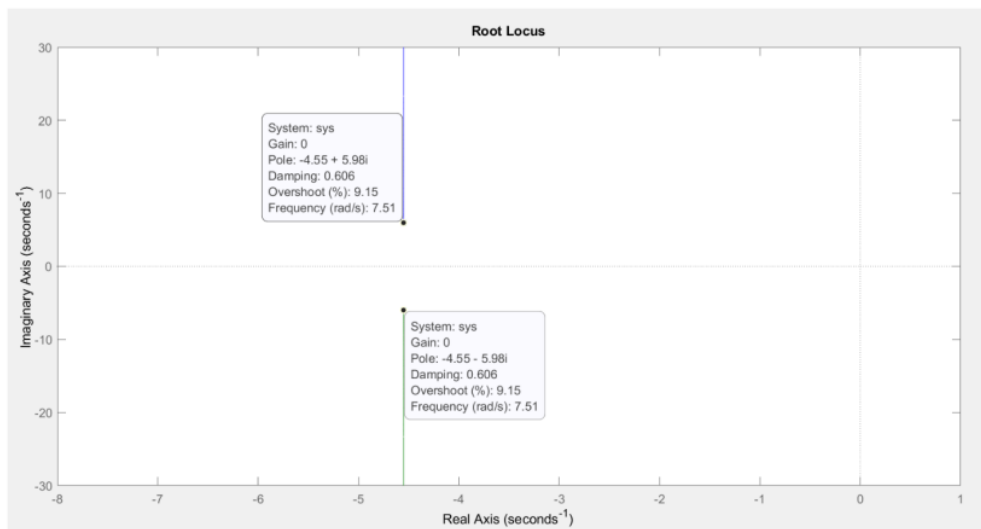
$$\frac{-(9.102) \mp \sqrt{(9.102)^2 - 4(1)(56.463)}}{2(1)}$$

$$\frac{-9.102 \mp \sqrt{-143.005596}}{2}$$

$$\frac{-9.102 \mp j11.9759}{2} = -4.551 \mp j5.9795$$

Now that the values for the poles have been calculated the argand diagram can be plotted on matlab. The matlab code and argand diagram are shown below.

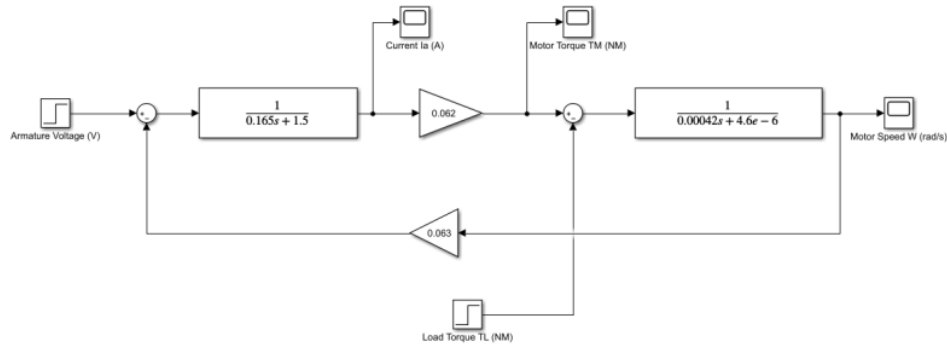
```
>> num = [849.661];  
>> den = [1 9.102 56.463];  
>> rlocus(num,den);
```



From the argand diagram produced on matlab you can see that the values shown for the poles $(-4.55 \pm j5.98)$ match when the gain is 0. It is also possible to see that the value for damping is 0.606 and the frequency shown is 7.51, these values match those calculated above.

You have worked this out correctly, well

The following system has been created using the values provided, it has been produced on Simulink and a simulation has been carried out over 20 seconds with an input of 15Vdc and a disturbance of 0.



Using the Simulink model above speed and torque responses were produced. A plot for the armature current was also recorded.

The following calculation was carried out to find a value for the motor speed under load, this will be required for comparison with the motor speed without a load.

$$\Omega_L = \frac{V_a - \frac{R_a T_L}{K_T}}{\frac{R_a b}{K_T} + K_e} = \frac{15 - \frac{(1.5)(0.03)}{0.062}}{\frac{(1.5)(4.6 \times 10^{-6})}{0.062} + 0.063} = 226.188 \text{ rad/s}$$

The value for motor load speed is 226.188 rad/s, this is less than the value for the no load motor speed. This difference is to be expected as the induced load will mean that the motor will need to work harder to maintain the same speed as when it had no load.

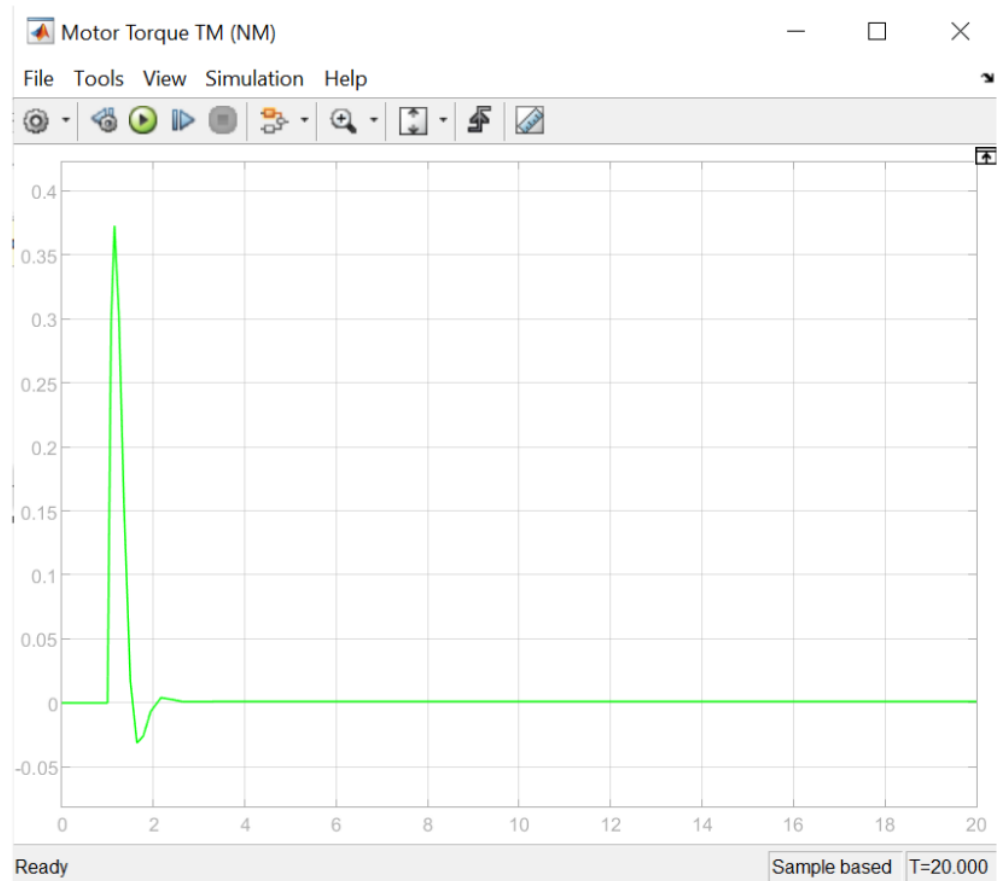
A value for the armature current under load can be found by carrying out the following calculation.

$$I_a = \frac{T_m}{K_T} = \frac{0.03104}{0.062} = 0.5006 \text{ A}$$

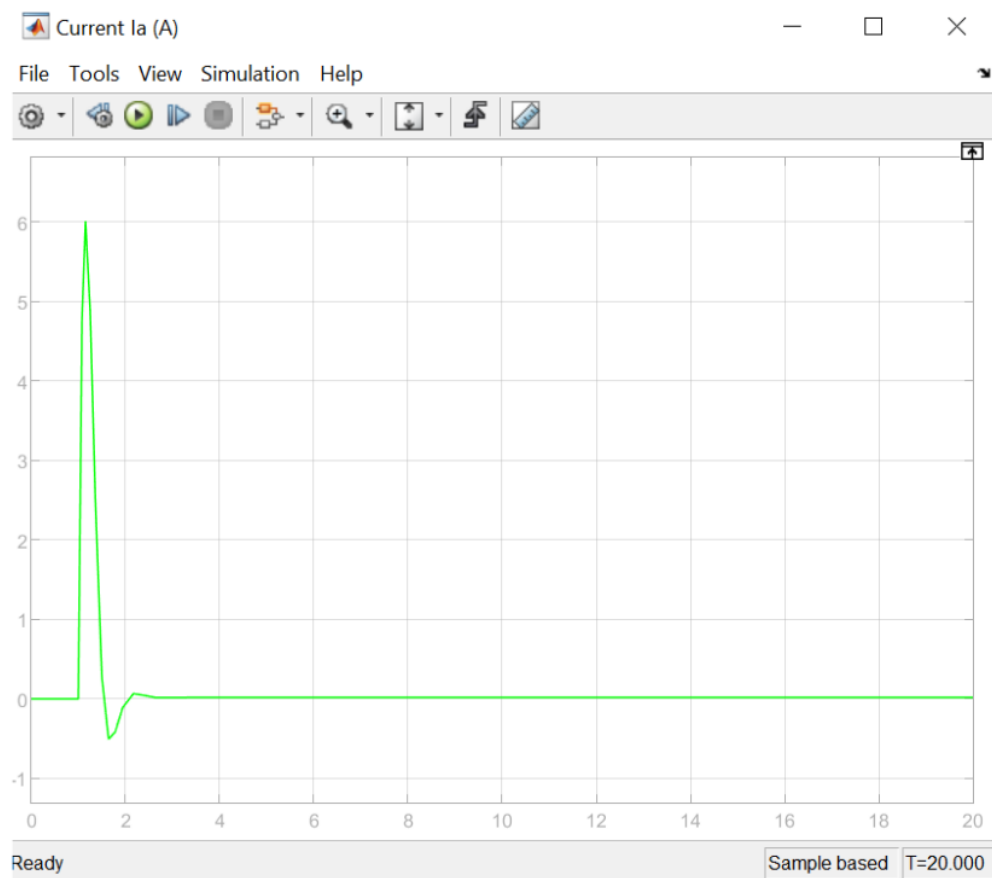
The simulation of the circuit shown above was carried out on Simulink, the plots produced are shown below. Motor speed, Motor torque and Motor current were measured over a 20 second timeframe with a step input of 15VDC and a disturbance of 0NM. When the motor starts up the speed increases rapidly and then settles to a steady value after around 2.5 seconds.



As shown below the motor torque spikes to overcome the initial inertia, once the motor is turning and the inertia has been overcome the torque value decreases before eventually tending to 0. This is because the motor is up to speed and there is no load present.



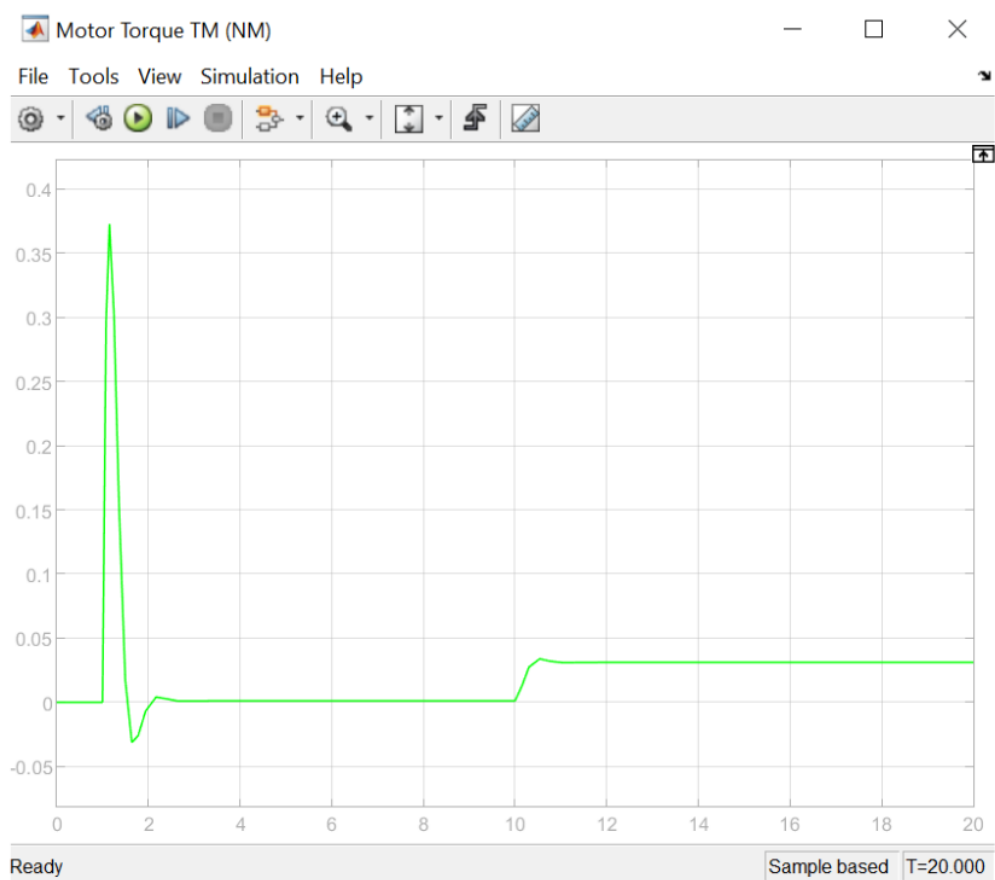
Like the motor torque, the armature current shows as initial spike as the motor tries to overcome the inertia, this value will also decrease once the motor is up to speed as there is no additional load on the motor.



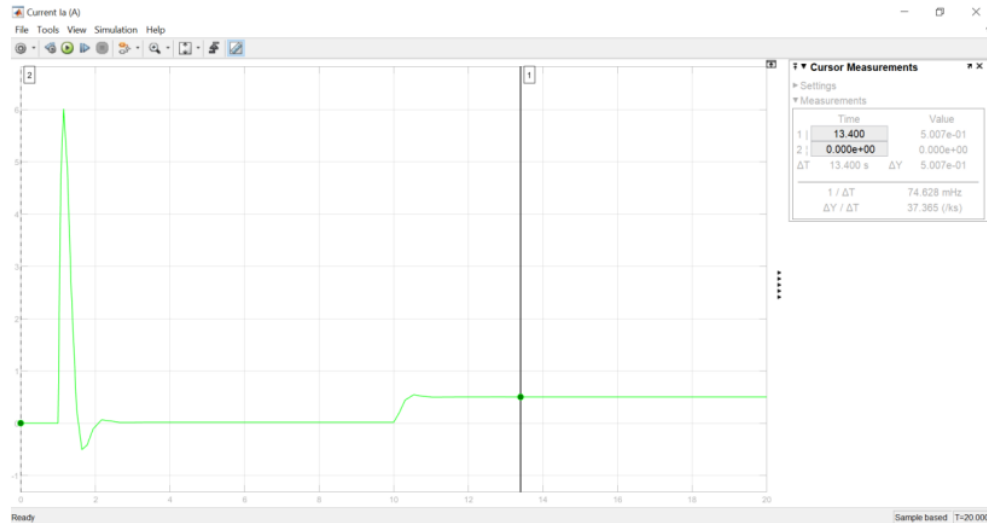
The disturbance was then set to 0.03NM after 10 seconds, the results are shown below. The motor speed decreases once a load is applied, this is due to the increased friction and forces that the motor is trying to work against.



Once a load of 0.03NM is applied to the motor the motor torque value increases to try and cope with the additional load. This is shown on the graph below.



Due to the relationship between current and torque the graph for current shown below looks the same as the graph for torque shown above. Once a load is applied at 10 seconds the current increases to cope with the additional forces acting against the motor.



	Calculated	Simulated
Damping Factor - $\sum$	0.606	0.606
Natural Frequency - ω_n	7.51rad/s	7.51rad/s
Load Speed - Ω_L	226.188rad/s	226.2rad/s
No-Load Speed - Ω_{NL}	238.09rad/s	237.7rad/s
Armature Current - I_a	0.5006A	0.5007A

Conclusion

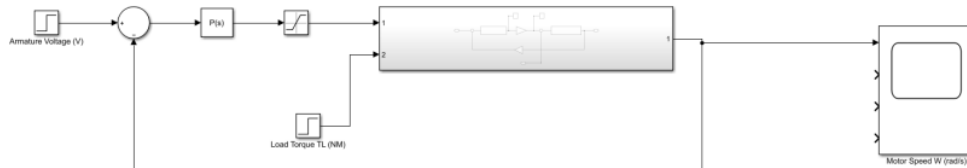
From the results table shown above which compares the calculated and simulated values it can be said that the calculations were carried out correctly as the simulated values match almost perfectly. The slight variation between some results would have most likely been caused by over rounding during the calculations.

As you can see from the results above when a load of 0.03 is applied to the system after 10 seconds the motor speed drops as expected. The motor torque and armature current both increase as the motor has to work harder to overcome the additional resistance created by introducing the disturbance.

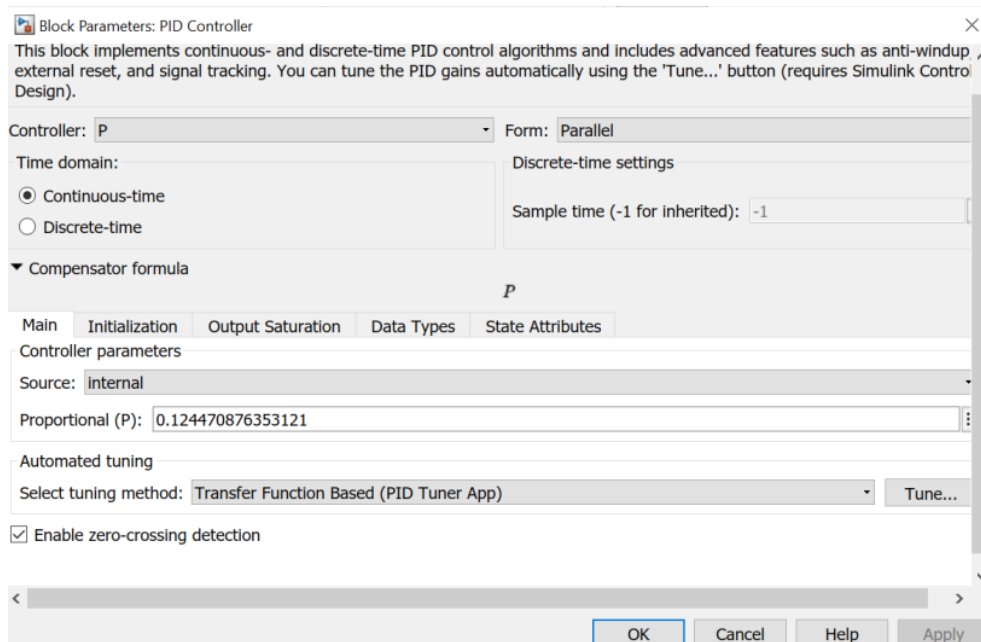
Task 14

The motor model has been simplified and converted into a subsystem. This subsystem has then been placed into a system with a feedback loop, step input, sum, PID controller and also a Saturation controller. The saturation controller has been set up so that it only allows signals between -24v and 24v to pass through, this will prevent an unexpected spike which may cause damage to the system.

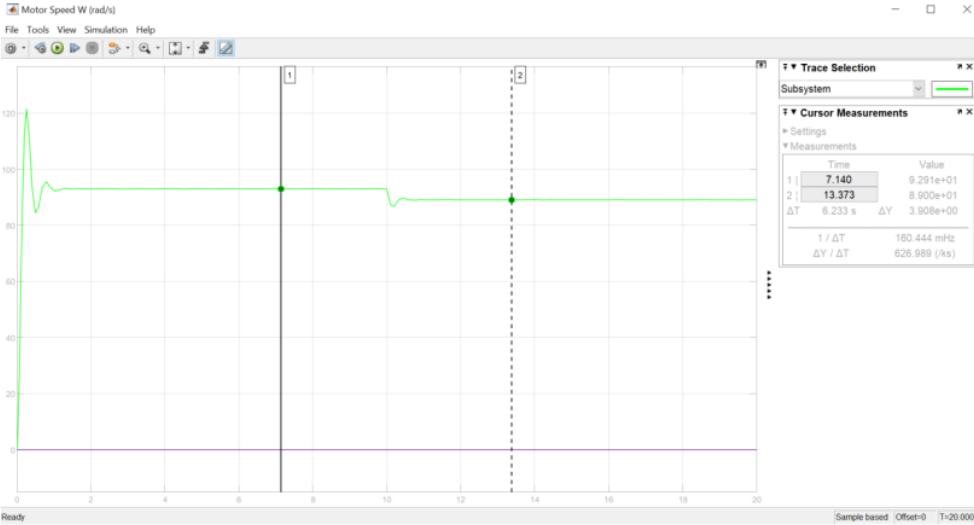
This system is shown below:



As shown in the image the PID controller is set up for P control and the system was autotuned to find the value for K_p that the system thinks is most appropriate. As shown below the system has calculated a K_p value of 0.124470876353121.



This value for K_p will produce the following response for the motor speed. The response is not good enough as the steady state value after settling is recorded as 92.9 rad/s and after the load was introduced it fell to 89 rad/s. A rise time of roughly 0.5 seconds can be observed.



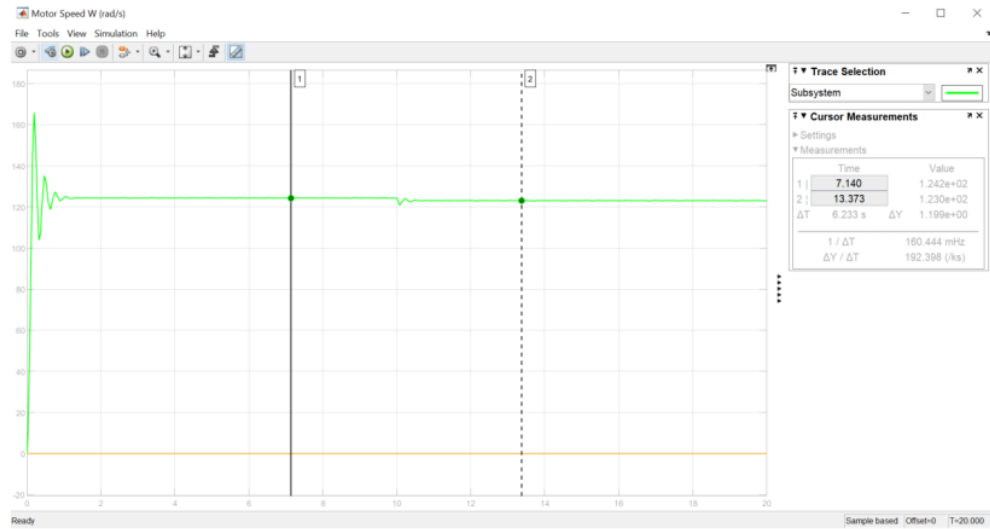
Shown in the table below is the effect that increasing the value of K_p has on the system.

	Rise Time	Overshoot	Settling Time	Steady-State Error
K_p	Decrease	Increase	Small Change	Decrease

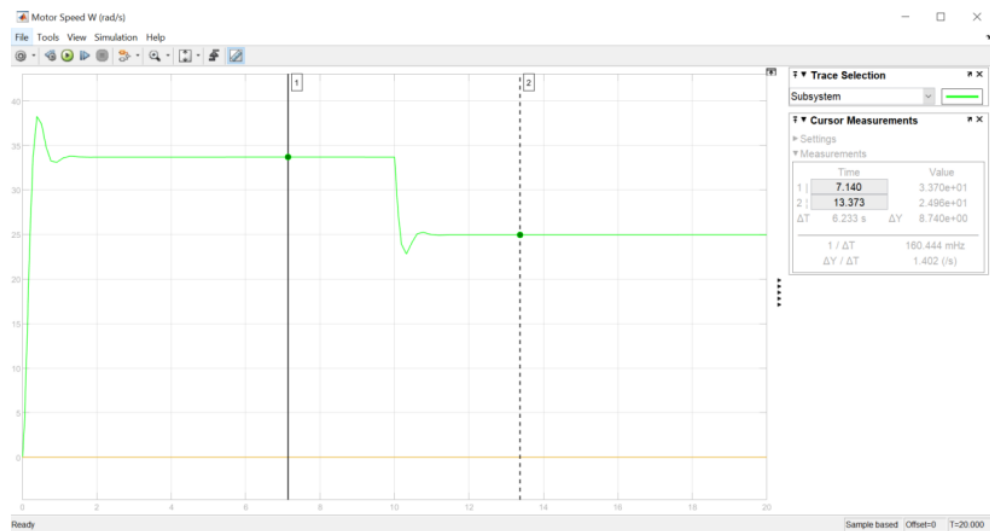
From this table it can be said that increasing the value of K_p will decrease the rise time so a response will be achieved faster and it will also increase the steady-state error, this means that the motor speed should be closer to the specified 140 rad/s.

From looking at the graphed results above it is clear to see that the value of K_p will need to be increased to allow the steady-state error to come anywhere near where it should be.

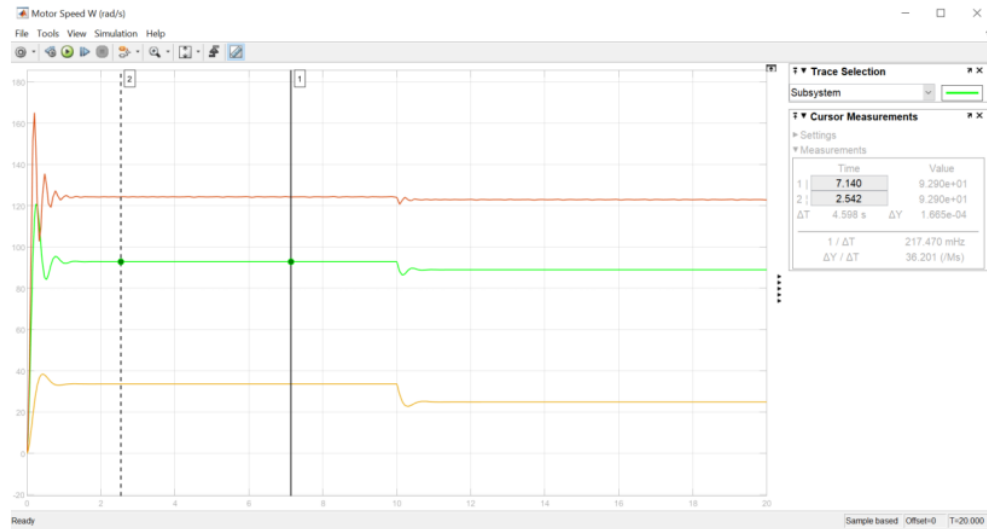
The graph below is showing the results when the value for Kp is increased to 0.5.



Shown below is what would happen if the value for Kp had been decreased from the autotuned value. It has been decreased to 0.02.



Shown below are all three graphs on one page so that the differences can be examined side by side.



The results have also been tabulated below:

Kp	%Overshoot	Steady-State Error	Settling Time (s)
0.02 (Yellow)	-72.5%	106.3	1.3s
0.124470876353121(Green)	-13.86%	47.1	1.5s
0.5 (Red)	+16.93%	15.7	1.6s

You have worked this out correctly, well

Conclusion

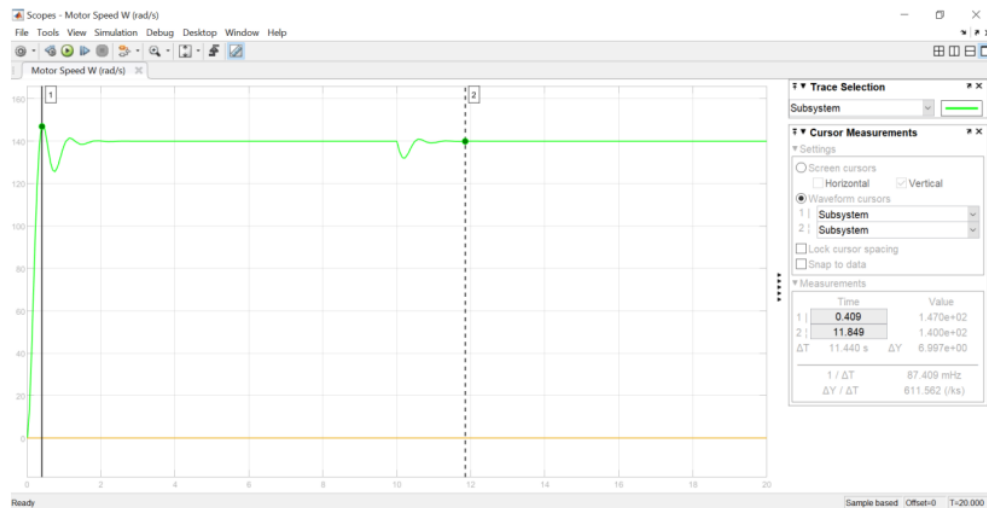
As expected, the rise time decreased when the value of Kp was increased, however it did not reach the required steady state value. The table above shows that the settling time increased as the value of Kp was increased, this was to be expected.

Task 15

For this task the controller was changed from a P controller to a PI controller. The system was autotuned and the following values were produced:

- $K_P = 0.0494486625230351$
- $K_I = 0.304329211373465$

Entering these values for K_p and K_i will produce the following response for the motor speed. The response is much better than when the P controller was used alone as the steady state value after settling is recorded as 140 rad/s and after the load was introduced it fell as expected however it took the system 1.8 seconds to recover to 140 rad/s. A rise time of roughly 0.4 seconds can be observed.

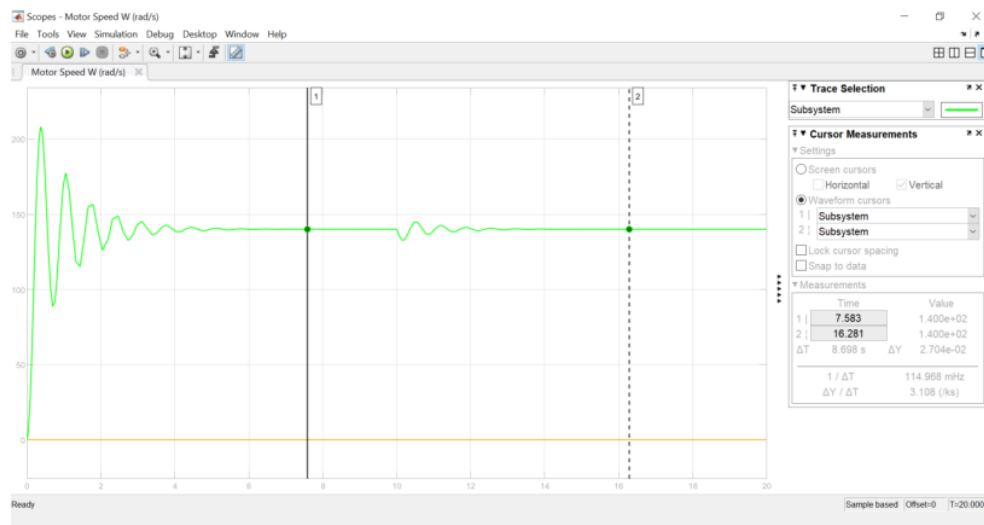


Shown in the table below is the effect that increasing the value of K_p and K_i has on the system.

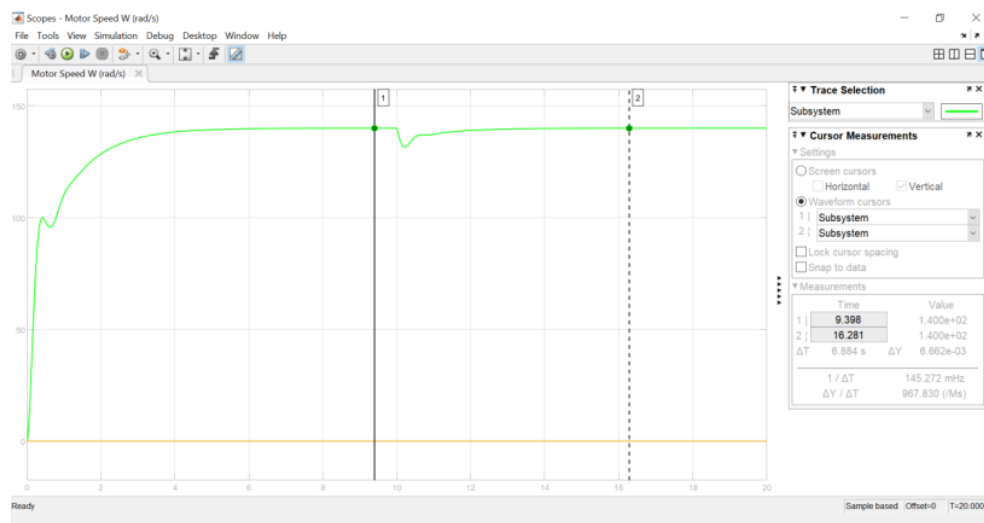
	Rise Time	Overshoot	Settling Time	Steady-State Error
K_p	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate

It can be said that increasing the value of K_i will reduce the time taken to rise and it will eliminate any steady state error, this means that once settled the graph should read 140 rad/s. Increasing K_i will cause the system to overshoot further and also will increase the time the system takes to settle to its steady-state.

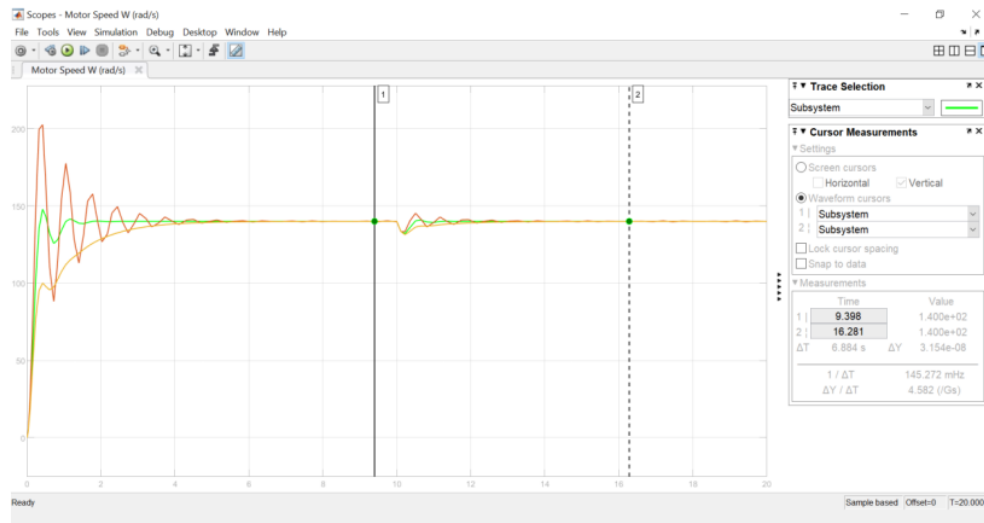
The graph below shows the effect of increasing the integral (Ki) to 0.7, as expected the settling time and overshoot have both increased dramatically.



On the graph shown below Ki was reduced to 0.1, the graph clearly shows that the time to rise is increased and the overshoot has been completely removed from the system. However there is a slight kink at about 0.5 seconds.



The three results have been plotted together so that they can be easily compared, this is shown below.



The results have also been tabulated below:

Ki	%Overshoot	Steady-State Error	Settling Time (s)
0.1 (Yellow)	N/A	0	7.66s
0.304329211373465 (Green)	+5.29%	0	2.42s
0.7 (Red)	+44.57%	0	8.14s

Conclusion

When K_i was introduced into the system the steady state error was completely removed and the graphs were able to rise to 140 rad/s each time. The overshoot (%) value was seen to rise when the K_i value was increased, this was expected and is shown as an effect of increasing K_i on the table at the beginning of the task. The Lowest value for K_i was found to be the best as the rise was smooth and consistent, there was no overshoot and the system did not need to settle once it reached 140 rad/s as it got to the set value and maintained it well.

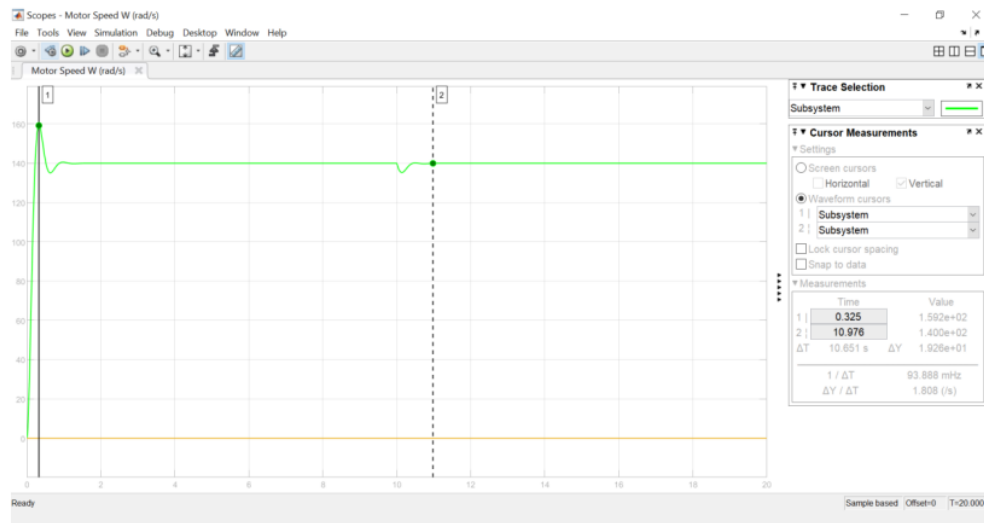
You have worked this out correctly, well

Task 16

For this task the controller was changed from a PI controller to a PID controller. The system was autotuned and the following values were produced:

- $K_P = 0.129029287980737$
- $K_I = 0.622411057171759$
- $K_D = 0.00661794482273913$

Entering these values for K_p , K_i and K_d will produce the following response for the motor speed. The response is similar to when the PI was used as the steady state value after settling is recorded as 140 rad/s and after the load was introduced it fell as expected however it only took the system 0.976 seconds to recover to 140 rad/s. A rise time of roughly 0.3 seconds can be observed.

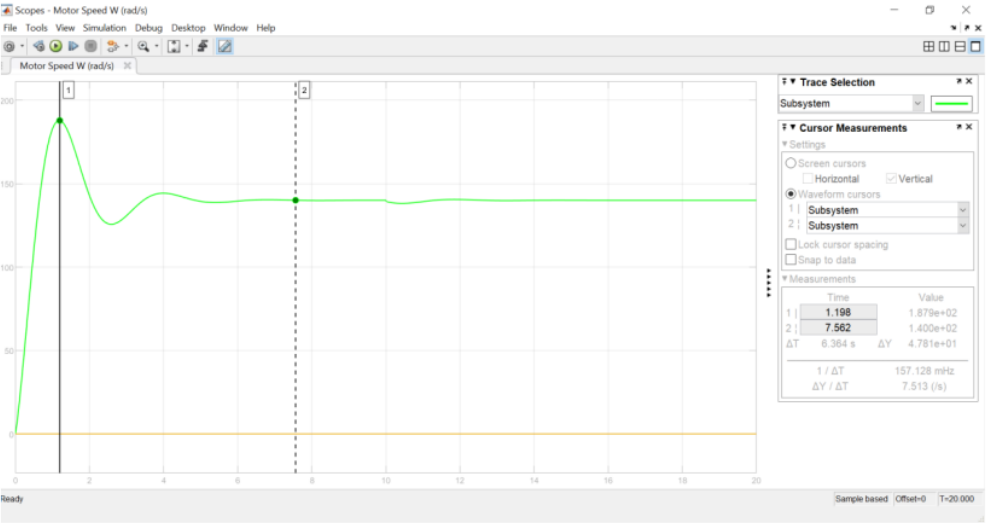


Shown in the table below is the effect that increasing the value of K_p , K_i and K_d has on the system.

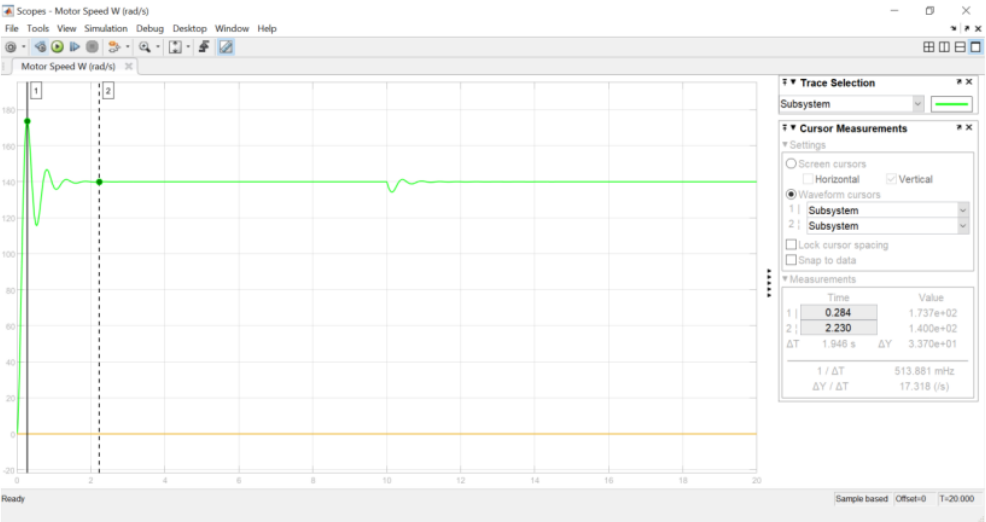
	Rise Time	Overshoot	Settling Time	Steady-State Error
K_p	Decrease	Increase	Small Change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small Decrease	Decrease	Decrease	None

It can be said that by implementing K_d into the system the rise time will be further decreased resulting in an overall faster response. Unlike the other stages the overshoot will also be decreased, this will provide the system with a smooth increase that will reach steady-state without overshooting. The time to settle is also decreased so the system will be quicker at providing an accurate condition.

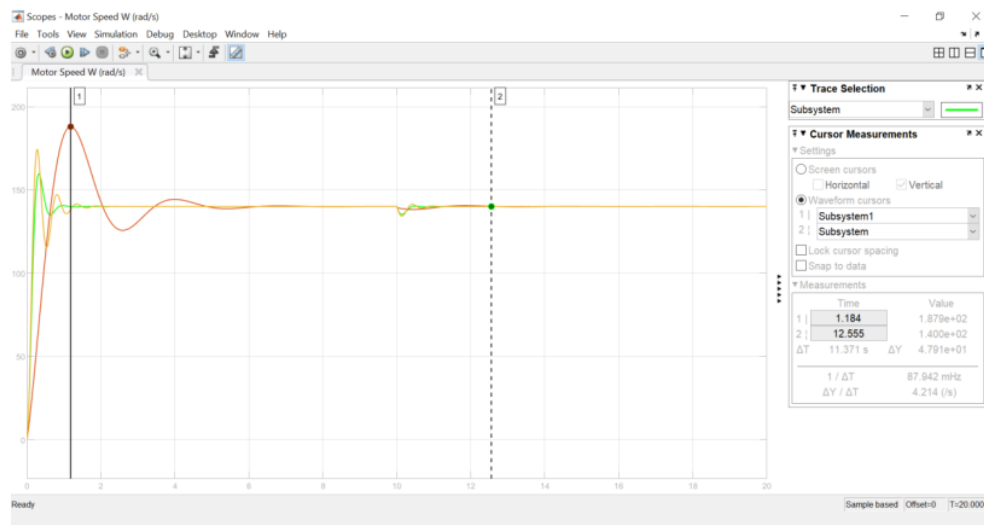
The Graph below shows the results when the value for K_d is increased to 0.1. This is the smoothest that the system has been able to handle the load being applied to it as it only allows the speed to reduce slightly before correcting itself.



The graph below shows the effects of reducing the derivative value to 0.001. The time to rise is reduced however when the load is added to the system the speed drops and takes about 1.5 seconds to recover.



The three results have been plotted together so that they can be easily compared, this is shown below.



Kd	%Overshoot	Steady-State Error	Settling Time (s)
0.001 (Yellow)	+23.14%	0	2s
0.304329211373465 (Green)	+14%	0	1.461s
0.7 (Red)	+34.21%	0	7.583s

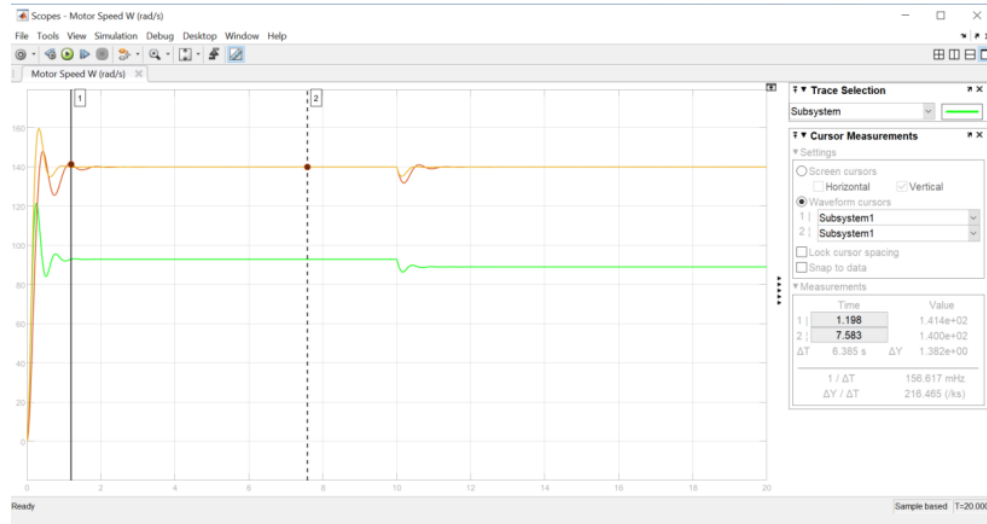
You have worked this out correctly, well

Conclusion

As expected, the introduction of K_d into the system had no effect on the systems ability to reach and maintain the correct steady state value of 140 rad/s. The Overshoot (%) was seen to decrease when K_d was increased however, if K_d was increased too much it had a negative impact on the overshoot value and it was seen to rise.

Task 17

The circuit below shows all 3 different control systems in use. The green line shows the effects that using only Proportional (K_p) control has on the system. The red line shows what happens when both Proportional and Integral (K_i) are used. The yellow line shows what happens when Proportional, Integral and Derivative (K_d) are used, for each type of control the system was autotuned so that the effects can be compared against each other.



Kd	%Overshoot	Steady-State Error	Settling Time (s)
P (Green)	-13.21%	47.11	1.994s
PI (Red)	+5.57%	0	2.44s
PID (Yellow)	+14%	0	1.45s

Task 18 – Summary of Conclusions

Task 1

The circuit performed as expected and the results calculated for the time constant were identical to those simulated. Simulated values were taken from the point where the amplitude was 0.632, this is because the time constant reached at 63.2%.  17

Task 2

The circuit performed as expected and the results calculated for the time constant were identical to those simulated. Simulated values were taken from the point where the amplitude was 0.632, this is because the time constant reached at 63.2%.  incorrect

Task 5

From analysing the results above, it is clear to see that the resistance value chosen has an impact on the value that is calculated for Sigma. By using a higher resistance value, the value for Sigma will also increase, this means that higher resistances produce overdamping. In order to decrease the amount of Damping on a circuit you would need to decrease the Resistance value which in turn would reduce the Sigma value. Results calculated for Sigma are affected by calculation errors such as over rounding, so care must be taken when carrying out the calculations.

Task 6

Higher resistance values produce circuits with higher levels of damping.  18

Task 7

When the resistance of the circuit is increased more damping takes place. The higher the damping effect the lower the magnitude (dB) and less phase shift (Degrees) takes place. As shown in the graph above, the underdamped circuit with resistance value 120Ω shows a spike in magnitude at the drop off point whereas the overdamped circuit with a resistance value of 500Ω drops off smoothly.  19

Task 14-17

It was also found that using PID provided the most control over a system as there was more that could be done to alter it. If there had been time the system could have been analysed further and the relevant values for K_p , K_i and K_d could have been found which would give the system the best response to the increased torque load, the system would then be able to react in a way that would keep the speed as close to the required 140 rad/s as possible when a load was applied.

1909757

GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

Instructor

54/100

PAGE 1

PAGE 2



Comment 1

use meaningful titles

PAGE 3



Incorrect

You have worked this out wrongly. It is not correct, check work more carefully.



Comment 2

missing s

PAGE 4



Comment 3

why?

PAGE 5



Incorrect

You have worked this out wrongly. It is not correct, check work more carefully.

PAGE 6



Comment 4

graph, table and analysis do not match!



Comment 5

task 3 and 4 missing

PAGE 7



Comment 6

missing circuit TF

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PAGE 10

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PAGE 12



Comment 7

Is this possible?

PAGE 13

QM

Incorrect

You have worked this out wrongly. It is not correct, check work more carefully.

PAGE 14

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Comment 8

missing table of results

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Comment 9

why is it negative?

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Comment 10

Too vague, does not show understanding.

LPF with resonant point, what about Q and effect on ω_m ?

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PAGE 25



Comment 11

missing table of results and conclusion

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Comment 12

see previous comment

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Comment 13

see previous 2 comments

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QM

You have worked this out correctly, well

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QM

You have worked this out correctly, well

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QM

You have worked this out correctly, well

PAGE 47

QM

You have worked this out correctly, well

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Comment 14

does your calculation account for effect of friction?

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Comment 15

incorrect

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You have worked this out correctly, well

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You have worked this out correctly, well

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You have worked this out correctly, well

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Comment 16

your recommendation?

PAGE 66



Comment 17

repetition and too vague



Incorrect

You have worked this out wrongly. It is not correct, check work more carefully.



Comment 18

Include the importance of the TF in your conclusion



Comment 19

Large gaps

GRADING FORM: NEW CSG-RUBRIC

SCOTT JONES

54

1. ENGAGEMENT WITH LITERATURE



NA

2. KNOWLEDGE AND UNDERSTANDING



Explanations for electrici and electronic circuits lack essential detail and include erros and gaps.
Explanation for control systems very good and detailed. 16/30

16

3. COGNITIVE AND INTELLECTUAL



Analysis of electrici and electronic circuits lack essential detail and include erros and large gaps.
Analysis for control systems very good and detailed. 16/30

16

4. PRACTICAL APPLICATION



Simulations for electric and electronic circuits are good with some gaps. Simulation for control systems very good and detailed. 16/30

16

5. TRANSFERABLE SKILLS



Communication is good, some repetition and vagueness exist. 6/10

6