



EXAMINATION SOLUTIONS

(to accompany the examination paper sent to the external examiner
but NOT to be seen by students)

Programme:	BEng Motorcycle Engineering	Level:	5
Module:	Motorcycle Dynamics		
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Q	Question	Marks
1a	Assumptions : Rider mass = 75 Kg Ride Frequency = 2.1 Hz $\therefore$ Total Mass = <u>222 Kg</u> Front sprung mass = $(0.62 \times 222) - 7.5 = 130.14 \text{ Kg}$ Rear Sprung Mass = $(0.38 \times 222) - 12.5 = 71.86 \text{ Kg}$ with a ride frequency of 2.1 Hz where $f = \frac{1}{2\pi} \sqrt{k/m}$ Vertical spring rates : $K_s' = (2\pi f)^2 \times M = 174.1 \times 130.14 = 22.657 \text{ N/mm}$ $K_r' = (174.1 \times 71.86) = 12.510 \text{ N/mm}$ Front Fork spring stiffness: $K_s = (\frac{1}{2}) \times 22.657 \times \cos^2(24.8)$ $K_f = \underline{9.34 \text{ N/mm}}$	



Rear shock stiffness

$$K_r = 12.510 \times (\text{Link Ratio})^2$$

$$K_r = \underline{60.5 \text{ N/mm}}$$

(10)

- 6 for damping, assuming a damping ratio of 0.5
Calculate the critical damping

For rebound:

$$C_{crit} = 2\sqrt{KM}$$

$$= 2 \times \sqrt{9340 \times 130.14 \times 2}$$

$$= 3118.3 \text{ Ns/m}$$

For compression

$$C_{crit} = 2\sqrt{KM}$$

$$= 2 \times \sqrt{9340 \times 7.5 \times 2}$$

$$= 748.6 \text{ Ns/m}$$

Since Damping Ratio = $\frac{c}{C_{crit}}$

$$\begin{aligned} \text{For Compression damping} &= \frac{748.6}{2} \\ &= 374.3 \text{ Ns/m} \end{aligned}$$

$$\begin{aligned} \text{For Rebound damping} &= 0.5 \times 3118.3 \\ &= 1559.15 \text{ Ns/m} \end{aligned}$$

(5)



C Discussion to include:

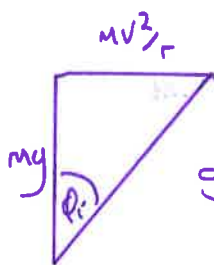
- Suspension at full compression at high lean angles, so limited functionality
- Flex of frame and swingarm are important
- Flex of fork leg and yokes also important
- Torsional and lateral stiffness requirements are both different.

- Lateral frame stiffness $\sim 2 \text{ kN/mm}$
- Torsional frame stiffness $\sim 5 \text{ kNm/}^\circ$
- Vertical frame stiffness $\sim 8 \text{ kN/mm}$

(5)

Total

20



So $\tan \phi = \frac{mv^2}{mgy}$

Hence

$$Q_i = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

4

$$Q = Q_i + \Delta Q$$

① $\sin Q_1 = \frac{x}{t}$

$$\therefore x = L \sin \theta;$$

(2) $\sin \Delta Q = \frac{x}{h-t}$

$$\therefore x = (h-t) \sin \Delta \theta$$

Equate (1) and (2)

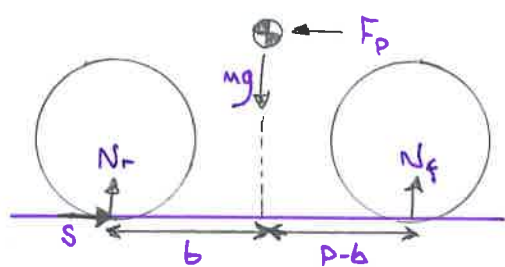
$$t \sin \phi = (h-t) \sin \Delta \phi$$

$$\frac{\sin \Delta \theta}{t} = \frac{\sin \theta}{(h-t)}$$

6



c



(1) $F_p = S$

(2) $mg = N_f + N_r$

Take moments about CoFG

$N_f(p-b) + Sh = N_r b$ Substitute (2) For N_r

$N_f p - \cancel{N_f b} + Sh = mg - \cancel{N_f b}$

$N_f p + Sh = mg b$

$N_f = mg \frac{b}{p} - S \frac{h}{p}$

Now do the same for N_r

$(mg - N_r)(p-b) + Sh = N_r b$

$mg p - N_r p - mg b + \cancel{N_r b} + Sh = \cancel{N_r b}$

$N_r p = mg p - mg b + Sh$

So $N_r = mg \left(\frac{p-b}{p} \right) + S \frac{h}{p}$

(7)

d.

At a lean angle of 55°

$\frac{mv^2}{r} = mg \times \tan 55 = \underline{1.428 mg}$

Resolve for wheel loads

$x = \sqrt{(1.428)^2 + (1)^2} = 1.743$

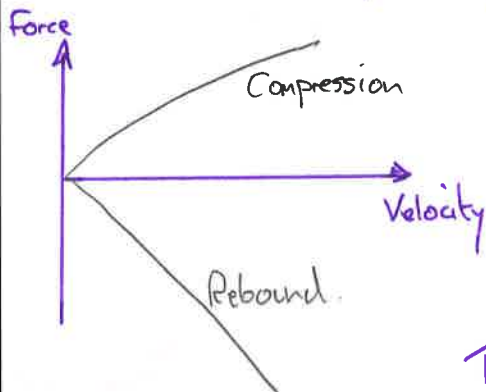
(3)

$\therefore$ at 55° Suspension load increased $1.743 mg$. T

20



- 3a The differential of the suspension position gives the Suspension velocity, which is needed in respect of damping force since $F = CV$. The basic damper dyno chart is of the form:



It shows the force at different velocities.

From the gradient, we get the damping constant.

The various lines correspond to the different settings 'clicks' available to the rider. The velocities from bike data can be interrogated and used with the damper charts to aid set-up.

(5)

b Using $m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0 \rightarrow$ we get

$$75r^2 + 900r + 13500 = 0 \quad (e^{rt} \text{ subs.})$$

divide by 75 $r^2 + 12r + 180 = 0$

$$r = \frac{-12 \pm \sqrt{12^2 - (4 \times 180)}}{2}$$

$$= \frac{-6 \pm \sqrt{144 - 720}}{2}$$

$$= -6 \pm i(24/2)$$

$$= -6 \pm 12i$$

so

$$x = e^{-6t} [A \cos(12t) + B \sin(12t)]$$

(5)



C For the initial conditions $x=0$, $\dot{x}=1.4 \text{ ms}^{-1}$ at $t=0$

First substitute $x=0$ at $t=0$

$$\therefore 0 = A + 0 \quad \text{so } A = 0$$

We can therefore neglect the Cos term in the equation

$$\therefore x = e^{-6t} [B \sin(12t)]$$

Now at $t=0$ $\dot{x}=1.4$

$$\dot{x} = -6e^{-6t} [B \sin(12t)] + e^{-6t} [\cancel{\sin(12t)} + 12B \cos(12t)]$$

$$\text{so } \dot{x} = 12B \cos(12t)$$

$$\dot{x} = 12B$$

$$1.4 = 12B$$

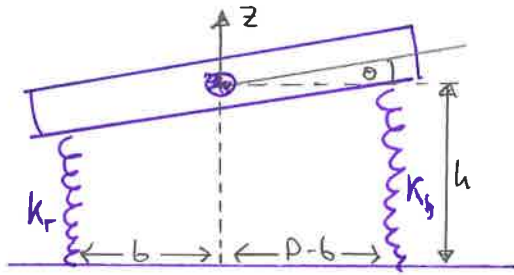
$$B = \frac{7}{60}$$

So we have

$$x = e^{-6t} \left[\frac{7}{60} \sin(12t) \right]$$



3d



The situation is coupled.
 $\therefore$ Bounce and Pitch
are linked.

$$M\ddot{z} + (k_f + k_r)z + k_f(P-b)\theta - k_rb\theta = 0$$

$$I\ddot{\theta} + k_f(P-b)z - k_rbz + k_f(P-b)^2\theta + k_rb^2\theta = 0$$

We can add damping if needed simplistically

$$\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} c_f + c_r & c_f(P-b) - c_rb \\ c_f(P-b) - c_rb & c_f(P-b)^2 + c_rb^2 \end{bmatrix} \begin{bmatrix} \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} k_f + k_r & k_f(P-b) - k_rb \\ k_f(P-b) - k_rb & k_f(P-b)^2 + k_rb^2 \end{bmatrix} \begin{bmatrix} z \\ \theta \end{bmatrix} = 0$$

(5)

Total

20



4a For this low speed condition, the method of Steer is given by the 'bicycle model' of dynamic steer
Description of model needed.

(4)

b at high speed the steering model is 'gyroscopic'
Description of gyroscopic steer model needed.

(4)

c Need lean angle $\phi_i = \tan^{-1}\left(\frac{v^2}{rg}\right) = \tan^{-1}\left(\frac{55 \times 5.18}{25g}\right)$

$$\phi_i = 43.6^\circ$$

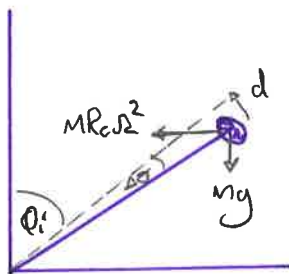
Gyroscopic Moment $G_m = I\omega\Omega \cos \phi_i$

$$G_m = (2 \times 0.44) \times \left(\frac{15.3}{0.3}\right) \times \left(\frac{15.3}{25}\right) \times \cos 43.6$$

$$G_m = 19.9 \text{ Nm}$$

(4)

d



$$\text{Torque} = F \times d = d \sqrt{(MR\omega^2)^2 + (mg)^2}$$

$$\text{Gyro torque} = I\omega\Omega \cos \phi_i$$

$$\text{so } d = \frac{I\omega\Omega \cos \phi_i}{\sqrt{(MR\omega^2)^2 + (mg)^2}}$$

$$d = \frac{19.9}{\sqrt{(285 \times 25 \times (\frac{15.3}{25})^2 + (mg)^2)}} = \frac{19.9}{3865.02} = 5.15 \text{ mm}$$

(5)



4d

The equivalent change in roll angle to achieve equilibrium would be

$$\Delta\theta = \sin^{-1}\left(\frac{d}{h}\right)$$

$$\Delta\theta = \sin^{-1}\left(\frac{5.15 \times 10^{-3}}{0.65}\right)$$

$$\Delta\theta = \sin^{-1}(8.48 \times 10^{-3})$$

$$\Delta\theta = \underline{0.49^\circ}$$

③

Total 20