



②

3

C
(S
S
F

1. Please give the information requested in the boxes
2. Enter your name in the space provided to the right, fold and seal down (using the label provided) to hide your name
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

1	Student Number	2415958
2	Title of Examination Paper	Mathematics Exam Paper 1 & 2
3	Module Code	ACDA4008
4	Date and Time of Examination	07/01/25 9:00 am
5	Examination Venue	IQ310
6	Seat Number	52
7	Number of Answer Books handed in	1

[illegible]

64% quite a good effort!

✓ AT Feb 25

Q1:

ai) $y = x + 3$
 $\frac{dy}{dx} = 1 + 0 = \underline{1}$

(2)

ii) $y = 4x^3 + 10x^{\frac{1}{2}}$
 $\frac{dy}{dx} = 4(3x^{3-1}) + 10(\frac{1}{2}x^{\frac{1}{2}-1}) = \underline{12x^2 + 5x^{-\frac{1}{2}}}$

(3)

iii) $y = \frac{1}{x^5} + x^3 \rightarrow y = x^{-5} + x^3$
 $\frac{dy}{dx} = -5x^{-5-1} + 3x^{3-1} = \underline{-5x^{-6} + 3x^2}$

(3)

iv) $y = \sin(8x)$
 $\frac{dy}{dx} = \cancel{8\sin(8x)} \underline{8\cos(8x)}$

(2)

v) $y = x \cdot \cos(x)$

✓

let $u = x$, $v = \cos(x)$
 $\frac{du}{dx} = 1$, $\frac{dv}{dx} = -\sin(x)$

$\frac{dy}{dx} = x \cdot (-\sin(x)) + \cos(x) \cdot 1 = \underline{-x\sin(x) + \cos(x)}$

(5)

vi) $y = (2x-3)^{10}$

let $u = 2x-3$, $y = (u)^{10}$
 $\frac{du}{dx} = 2-0$, $\frac{dy}{du} = 10u^9$

$\frac{dy}{dx} = 2 \cdot 10u^9 = \underline{20u^9 = 20(2x-3)^9}$

(3)

vii) $y = (3x + \tan(x))$

$\frac{dy}{dx} = \underline{3 + \sec^2(x)}$

(2)

✓

b) $y = \frac{1}{x} + \cos(2x) \rightarrow y = x^{-1} + \cos(2x)$

$$\frac{dy}{dx} = -1x^{-1-1} + -2\sin(2x) = -x^{-2} - 2\sin(2x)$$

$$\frac{d^2y}{dx^2} = +2x^{-2-1} - 2(2\cos(2x)) = +2x^{-3} - 4\cos(2x)$$

$$\frac{d^3y}{dx^3} = +2(-3x^{-3-1}) - 4(-2\sin(2x)) = \underline{\underline{-6x^{-4} + 8\sin(2x)}}$$

c) $y = \frac{(x-1)}{(4+x^2)}$

let $u = x-1$, $v = 4+x^2$

$$\frac{du}{dx} = 1-0$$

$$\frac{dv}{dx} = 0 + 2x$$

$$\frac{dy}{dx} = \frac{(4+x^2) \cdot 1 - (x-1) \cdot 2x}{(4+x^2)^2}$$

$$\frac{(4+x^2)(4+x^2) - (x-1) \cdot 2x}{(4+x^2)^2}$$

$$\frac{16 + 4x^2 + 4x^2 + x^4 - 2x^2 + 2x}{(4+x^2)^2}$$

$$\frac{x^4 + 8x^2 + 16}{(4+x^2)^2}$$

$$\frac{dy}{dx} = \frac{4+x^2 - 2x^2 - 2x}{(4+x^2)^2} = \underline{\underline{\frac{4-x^2-2x}{(4+x^2)^2}}}$$

when $x = 0$

$$\frac{dy}{dx} = \frac{4 - (0)^2 - 2(0)}{(4 + (0)^2)^2} = \frac{4}{16} = \underline{\underline{\frac{1}{4}}}$$

5

Q2:

ai) $\int (10x + x^3) dx$

$$= 10 \frac{x^{1+1}}{1+1} + \frac{x^{3+1}}{3+1} = 10 \frac{x^2}{2} + \frac{x^4}{4} = 5x^2 + \frac{x^4}{4} + C$$

ii) $\int (x - 3e^x) dx$

$$= \frac{x^{1+1}}{1+1} - 3e^x = \frac{x^2}{2} - 3e^x + C$$

iii) $\int (\frac{1}{x} + 4) dx$

$$= \ln(x) + 4x + C$$

iv) $\int 2x \cdot (3x + 4) dx = \int (6x^2 + 8x) dx$

$$= 6 \frac{x^{2+1}}{2+1} + 8 \frac{x^{1+1}}{1+1} = 6 \frac{x^3}{3} + 8 \frac{x^2}{2} = 2x^3 + 4x^2 + C$$

v) $\int_0^2 (3x^2 + 2x) dx$

$$= 3 \frac{x^{2+1}}{2+1} + 2 \frac{x^{1+1}}{1+1} = 3 \frac{x^3}{3} + 2 \frac{x^2}{2} = [x^3 + x^2]_0^2$$

$$(2^3 + 2^2) - (0^3 + 0^2) = 12$$

b) $\int_{x=0}^1 \frac{x}{\sqrt{x^2+1}} dx$

let $u = x^2 + 1$

when $x = 0$, $u = 1$

when $x = 1$, $u = 2$

$$\int_{u=1}^2 \frac{\sqrt{u-1}}{\sqrt{u}} du = \int_1^2 \frac{\sqrt{u-1}}{u^{\frac{1}{2}}} du = \int_1^2 (\sqrt{u-1}) \cdot u^{-\frac{1}{2}} du$$

$$\int_1^2 (u-1)^{\frac{1}{2}} \cdot u^{-\frac{1}{2}} du$$

$$= \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}$$

$$= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C$$

ci) $\int \underbrace{x \cdot \sin(2x)}_{\substack{dv \\ dx} \quad u} dx$

$$\frac{dv}{dx} = \sin(2x) \\ v = -\frac{\cos(2x)}{2}$$

$$u = x \\ \frac{du}{dx} = 1$$

No other way around ✓

$$\text{let } u = \sin(2x), \quad \frac{dv}{dx} = x \\ \frac{du}{dx} = 2\cos(2x), \quad v = \int x dx = \frac{x^{1+1}}{1+1} = \frac{x^2}{2}$$

$$\begin{aligned} \int x \cdot \sin(2x) dx &= \sin(2x) \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot 2\cos 2x \\ &= \frac{1}{2} x^2 \sin(2x) - \frac{1}{2} \left(\frac{x^{2+1}}{2+1} \right) \cdot 2 \left(\frac{\sin(2x)}{2} \right) \\ &= \underline{\underline{\frac{1}{2} x^2 \sin(2x) - \frac{x^3}{6} \cdot \sin(2x) + C}} \end{aligned}$$

ii) $\int_1^e \underbrace{x \cdot \ln(x)}_{\substack{dv \\ dx} \quad u} dx$

$$\text{let } u = \ln(x), \quad \frac{dv}{dx} = x \\ \frac{du}{dx} = x^{-1}, \quad v = \int x dx = \frac{x^2}{2}$$

$$\int_1^e x \cdot \ln(x) dx = \ln(x) \cdot \frac{x^2}{2} - \int_1^e \frac{x^2}{2} \cdot \frac{1}{x} dx \quad \text{needed}$$

$$\begin{aligned} &= \frac{1}{2} x^2 \ln(x) - \frac{1}{2} \left(\frac{x^{2+1}}{2+1} \right) \cdot \ln(x) \\ &= \frac{1}{2} x^2 \ln(x) - \frac{x^3}{6} \cdot \ln(x) = \left[\frac{1}{2} x^2 \ln(x) - \frac{1}{6} x^3 \ln(x) \right]_1^e \end{aligned}$$

$$\left(\frac{1}{2} (e)^2 \ln(e) - \frac{1}{6} (e)^3 \ln(e) \right) - \left(\frac{1}{2} (1)^2 \ln(1) - \frac{1}{6} (1)^3 \ln(1) \right)$$

$$\approx 0.35 - 0$$

$$= \underline{\underline{0.35}} \quad \text{to 2.d.p.}$$

Q3:

ai) $(3 + 2i)(1 - i)$

$$3 - 3i + 2i - 2i^2 = 3 - i - 2(-1) = 3 - i + 2 = 5 - i$$

$$= 3 - i - 2 = 1 - i$$

iii) $\frac{4}{i^5} = \frac{4}{i^2} \times \frac{1}{i^2} \times \frac{14}{i^2} = \frac{4}{-1} \times \frac{1}{-1} \times \frac{1}{i}$

$$= -4 \times -1 \times i^{-1}$$

$$= 4i^{-1} = 0 + 4i^{-1}$$

$$\frac{4 \cdot i}{i \cdot i} = \frac{4i}{-1} = 0 - 4i$$

d) $z = \sqrt{3} + i$

$$r = |z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2$$

$$\theta = \arg(z) = \tan^{-1} \frac{1}{\sqrt{3}} = 30^\circ$$

$$z = 2(\cos 30 + i \sin 30) = 2e^{i30}$$

aii) $\frac{2-i}{2+j}$

$$= \frac{2-i}{2+j} \times \frac{\sqrt{2^2+1^2}}{\sqrt{2^2+1^2}} = \frac{\sqrt{5}}{\sqrt{5}} e^{j(-26.57-26.57)}$$

$$\tan^{-1}\left(\frac{-1}{2}\right) = -26.57 \text{ to 2.d.p.}$$

$$\tan^{-1}\left(\frac{1}{2}\right) = 26.57 \text{ to 2.d.p.}$$

$$\frac{\sqrt{15}}{5} e^{j(-53.13)} \rightarrow \frac{\sqrt{15}}{5} (\cos(-53.13) + j \sin(-53.13))$$

$$\frac{3\sqrt{15}}{25} + -j 0.62$$

$$0.46 - 0.62j$$

Q4:

ai)

$$3^{2x} = 2^{x+1}$$

$$\begin{aligned} 2x \cdot \log 3 &= (x+1) \cdot \log 2 \\ 2x \log 3 - x \log 2 &= \log 2 \\ 2x \log 3 &= x \log 2 + \log 2 \end{aligned}$$

$$2x \log 3 - x \log 2 = \log 2$$

$$x = \frac{\log 2}{2 \log 3 - \log 2}$$

(2)

$$\text{ii)} \quad 2 \ln(x) = \ln(5x-6)$$

$$\text{bi)} \quad \log(x^6 y z) = 6 \log x + \log y + \log z$$

$$\text{ii)} \quad \ln\left(\frac{x^3 y}{z}\right) = 3 \ln x + \ln y - \ln z$$

(2)

(2)

This is quite a good effort