

Phase Test 1 Formula Sheet

12 December 2018 16:43

Two-dimensional stress and strain equations:

$$\begin{aligned}\varepsilon_1 &= \frac{1}{E}(\sigma_1 - \nu\sigma_2) & \sigma_1 &= \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \\ \varepsilon_2 &= \frac{1}{E}(\sigma_2 - \nu\sigma_1) & \sigma_2 &= \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)\end{aligned}$$

Three-dimensional strain equations:

$$\begin{aligned}\varepsilon_x &= \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \\ \varepsilon_y &= \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \\ \varepsilon_z &= \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]\end{aligned}$$

Volumetric Strain:

$$\begin{aligned}\varepsilon_v &= \frac{\delta V}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z \\ \varepsilon_v &= \frac{\sigma_x + \sigma_y + \sigma_z}{E}(1 - 2\nu)\end{aligned}$$

Relationships between elastic constants:

$$E = \frac{\sigma}{\varepsilon} \quad K = \frac{\sigma}{\varepsilon_v} = \frac{E}{3(1-2\nu)} \quad G = \frac{E}{2(1+\nu)} \quad K = \frac{2G(1+\nu)}{3(1-2\nu)}$$

Stress and strains in thin walled pressure vessels:

Cylinders		Spheres
$\sigma_c = \frac{pD}{2t\eta_c}$	$\sigma_L = \frac{pD}{4t\eta_c}$	$\sigma = \frac{pD}{4t\eta}$
$\varepsilon_c = \varepsilon_D = \frac{pD}{4tE}(2-\nu)$	$\varepsilon_L = \frac{pD}{4tE}(1-2\nu)$	
$\varepsilon_v = \frac{pD}{4tE}(5-4\nu)$		$\varepsilon_v = \frac{3pD}{4tE}(1-\nu)$

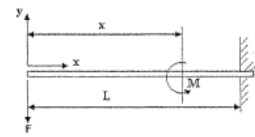
Stresses in thick walled cylinders: (Lame's Equations)

PhaseTest IBND auto

$$\sigma_R = a - \frac{b}{r^2} \quad \sigma_C = a + \frac{b}{r^2}$$

Where a and b are found by applying boundary conditions

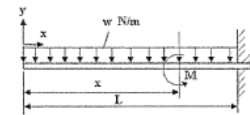
Cantilever Beams:



Max Slope

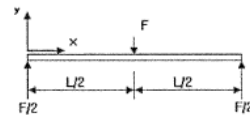
Max Deflection

$$\frac{FL^2}{2EI} \quad \frac{FL^3}{3EI}$$

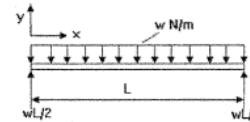


$$\frac{wL^3}{6EI} \quad \frac{wL^4}{8EI}$$

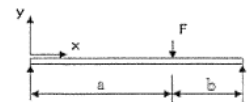
Simply Supported Beams:



$$\frac{FL^2}{16EI} \quad \frac{FL^3}{48EI}$$



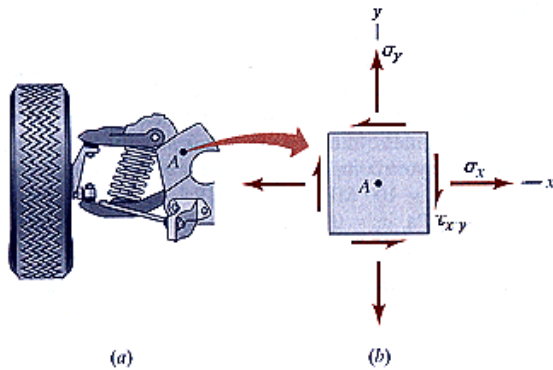
$$\frac{wL^3}{24EI} \quad \frac{5wL^4}{384EI}$$



$$\frac{Fb}{6EI}(2L^2 - 3Lb + b^2) \quad \frac{Fa^2b^2}{3EI}$$

Question 1

04 December 2018 14:56



- a) A pressed steel section of a suspension component has stresses of 180 MPa in the x direction and -123 MPa in the y direction. Given that the material has a Young's Modulus of 205 GPa and a Poisson's ratio of 0.25, calculate the resulting strains in both directions and state whether they are tensile or compressive.
- b) At a different section of the component, the strain is measured using a strain gauge and is found to be $+1052 \times 10^{-6}$ in the x direction and -380×10^{-6} in the y direction. Calculate the stresses at this point within the component.
- c) If the yield strength of the material is 250 MPa, calculate the minimum safety factor of the component, based on the stresses above.

a) $\sigma_x = 180 \times 10^6 \text{ N/m}^2$
 $\sigma_y = -123 \times 10^6 \text{ N/m}^2$
 $E = 205 \times 10^9 \text{ N/m}^2$
 $\nu = 0.25$

(10 marks)

$$\epsilon_x = \frac{1}{E} (\sigma_x - \nu \sigma_y) = \frac{1}{205 \times 10^9} (180 \times 10^6 - (0.25 \times -123 \times 10^6)) = 1028 \mu\epsilon$$

$$\epsilon_y = \frac{1}{E} (\sigma_y - \nu \sigma_x) = \frac{1}{205 \times 10^9} (-123 \times 10^6 - (0.25 \times 180 \times 10^6)) = -819.51 \mu\epsilon$$

b) $\epsilon_x = 1052 \times 10^{-6}$
 $\epsilon_y = -380 \times 10^{-6}$
 $E = 205 \times 10^9 \text{ N/m}^2$
 $\nu = 0.25$

$$\sigma_x = \frac{E}{1-\nu^2} (\epsilon_x + \nu \epsilon_y) = \frac{205 \times 10^9}{1-0.25^2} (1052 \times 10^{-6} + (0.25 \times -380 \times 10^{-6})) = 209.2 \text{ MPa}$$

$$\sigma_y = \frac{E}{1-\nu^2} (\epsilon_y + \nu \epsilon_x) = \frac{205 \times 10^9}{1-0.25^2} (-380 \times 10^{-6} + (0.25 \times 1052 \times 10^{-6})) = -27.6 \text{ MPa}$$

c) $FOS = \frac{\sigma_{allow}}{\sigma_{max}} = \frac{250 \times 10^6}{209.2 \times 10^6} = 1.2$

Question 2

04 December 2018 15:00

A sphere made from an Isotropic material, of diameter 150mm is subjected to an external fluid pressure of 25 MPa.

- If the modulus of elasticity of the ~~cube~~^{sphere} material is 163.2 GPa and the Poisson's ratio is 0.3, calculate the bulk modulus.
- If the sphere had a volume of $1.77 \times 10^{-3} \text{ m}^3$ before the pressure was applied, calculate the change in volume caused by the external pressure.
- Explain what is meant by the term 'isotropic'

$$\Delta = \delta$$

$$E = 163.2 \times 10^9 \text{ N/m}^2$$

(10 marks)

$$\nu = 0.3$$

$$\sigma = -P$$

$$a) K = \frac{E}{3(1-2\nu)} = \frac{163.2 \times 10^9}{3(1-(2 \times 0.3))} = 1.36 \times 10^{11}$$

$$b) V = 1.77 \times 10^{-3} \text{ m}^3$$

$$E_v = \frac{\Delta V}{V}$$

$$\Delta V = E_v V$$

$$K = \frac{\sigma}{E_v} = \frac{-P}{E_v}$$

$$E_v = \frac{-P}{K} = \frac{-25 \times 10^6}{1.36 \times 10^{11}} = -183.8 \times 10^{-6}$$

$$\Delta V = E_v V = -183.8 \times 10^{-6} \times 1.77 \times 10^{-3} = -0.33 \times 10^{-6} \text{ m}^3$$

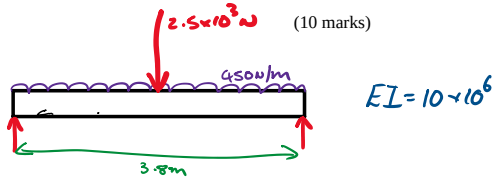
c) The material properties are the same in all directions.

Question 3

04 December 2018 15:01

A 3.8 m long simply supported beam carries a uniformly distributed load of 450 N/m along its entire length. A point load of 2.5 kN also acts at the midpoint of the beam span. The beam has a flexural stiffness of 10 MPa.

- Sketch the beam and the load case
- Calculate the corresponding deflection at the midpoint of the beam
- Calculate how much the point load may be increased before the deflection exceeds 0.5 mm



ii)

Diagram showing the superposition of the point load and UDL effects on the beam deflection.

$$y = \frac{FL^3}{48EI} + \frac{5wL^4}{384EI}$$

$$y = \frac{-2.5 \times 10^3 \times 3.8^3}{48 \times 10 \times 10^6} + \frac{5 \times 450 \times 3.8^4}{384 \times 10 \times 10^6}$$

$$y = -0.286 \times 10^{-3} - 0.122 \times 10^{-3}$$

$$y = -0.41 \times 10^{-3} \text{ m}$$

c)

$$y = 0.5 \times 10^{-3} \text{ m}$$

$$y = y_{PL} + y_{UDL}$$

$$-0.5 \times 10^{-3} = y_{PL} - 0.122 \times 10^{-3}$$

$$y_{PL} = -0.5 \times 10^{-3} + 0.122 \times 10^{-3} = -3.78 \times 10^{-4}$$

$$y_{PL} = -3.78 \times 10^{-4} = \frac{F'L^3}{48EI}$$

$$F' = \frac{y_{PL} 48EI}{L^3}$$

$$F' = \frac{-3.78 \times 10^{-4} \times 48 \times 10 \times 10^6}{3.8^3} = -3.3 \times 10^3 \text{ N}$$

$$\Delta F = 3.3 \times 10^3 - 2.5 \times 10^3 = 0.8 \times 10^3 \text{ N}$$

Question 4

04 December 2018 15:00



A Nitrogen Oxide storage tank for a drag vehicle may be modelled as a **thin walled** pressure vessel. If it has a diameter of 295mm, is 558mm in length and is made from 2mm thick sheet steel, calculate the maximum pressure allowable if the stress in the material must not exceed 190MPa. Take the longitudinal and circumferential joint efficiencies are 75% and 96% respectively,

(10 marks)

$$d = 0.295 \text{ m}$$

$$l = 0.558 \text{ m}$$

$$t = 2 \times 10^{-3} \text{ m}$$

$$\sigma_{\max} = 190 \times 10^6 \text{ N/m}^2$$

$$\eta_L = 0.75$$

$$\eta_c = 0.96$$

$$P_{\max} = ?$$

Circumferential

$$\sigma_c = \frac{pd}{2t\eta_L} \quad \checkmark$$

$$\sigma_c 2t\eta_L = pd$$

$$\frac{\sigma_c 2t\eta_L}{d} = p \quad \checkmark$$

$$P = \frac{190 \times 10^6 \times 2 \times 2 \times 10^{-3} \times 0.75}{0.295} \quad \checkmark$$

$$P = 19.3 \text{ Bar} \quad \checkmark$$

longitudinal

$$\sigma_L = \frac{pd}{4t\eta_c} \quad \checkmark$$

$$\sigma_L 4t\eta_c = pd$$

$$\frac{\sigma_L 4t\eta_c}{d} = p \quad \checkmark$$

$$P = \frac{190 \times 10^6 \times 4 \times 2 \times 10^{-3} \times 0.96}{0.295} \quad \checkmark$$

$$P = 49.5 \text{ Bar} \quad \checkmark$$

$$P_{\max} = 19.3 \text{ Bar} \quad \checkmark$$

Question 5

04 December 2018 15:00



Assuming that a clutch slave cylinder such as the one pictured may be modelled as a perfect cylinder with an external diameter of 38mm and a wall thickness of 3.2mm, determine the circumferential 'hoop' stress at the outer surface when the fluid pressure reaches 100 Bar.

(10 marks)

Thick walled

$$\sigma_r = a - \frac{b}{r^2}$$

$$\sigma_c = a + \frac{b}{r^2}$$

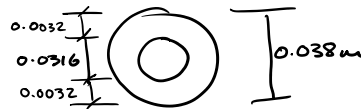
$$d = 38 \times 10^{-3} \text{ m}$$

$$r_o = 0.019 \text{ m}$$

$$t = 3.2 \times 10^{-3} \text{ m}$$

$$r_i = 0.0158 \text{ m}$$

$$P = 100 \times 10^5 \text{ N/m}^2$$



Boundary Conditions

@ outer surface

$$r = 0.019 \text{ m} \quad \sigma_r = 0$$

$$\sigma_r = a - \frac{b}{r^2}$$

$$0 = a - \frac{b}{0.019^2} \quad (\times 0.019^2)$$

$$0 = a(0.019^2) - b \quad (1)$$

$$(1) - (2)$$

@ inner surface

$$r = 0.0158 \text{ m} \quad \sigma_r = -P = -100 \times 10^5 \text{ N/m}^2$$

$$\sigma_r = a - \frac{b}{r^2}$$

$$-100 \times 10^5 = a - \frac{b}{0.0158^2} \quad (\times 0.0158^2)$$

$$-2496.4 = a(0.0158^2) - b \quad (2)$$

$$0 - -2496.4 = a(0.019^2) - a(0.0158^2) - b - -b$$

$$+2496.4 = a(0.019^2) - a(0.0158^2)$$

$$2496.4 = a(0.019^2 - 0.0158^2)$$

$$a = \frac{2496.4}{(0.019^2 - 0.0158^2)} = 22.42 \times 10^6$$

$$b = a(0.019^2) = 8092.7$$

$$\sigma_c = a + \frac{b}{r^2} = 22.42 \times 10^6 + \frac{8092.7}{(0.019^2)} = 44.84 \text{ MPa}$$

Question 6

04 December 2018 15:01

- a) A thin material has stresses of -12 MPa in the x direction and 28 MPa in the y direction. Given the elastic constants $E = 205 \text{ GPa}$ and $\nu = 0.27$, calculate the strains in both directions.
- b) If the strain in the y direction is increased by $+100\mu\epsilon$ without changing the strain in the x direction, calculate the new stresses in the material
- (10 marks)

Question 7

04 December 2018

15:01



A hydrogen storage tank for a vehicle may be modelled as a thin walled pressure vessel. If it has a diameter of 280mm, is 650mm in length and is made from sheet steel, calculate the minimum thickness if it is to withstand an internal pressure of 15 Bar with the maximum stress within the material not exceeding 150MPa. Take the longitudinal and circumferential joint efficiencies are 75% and 96% respectively,

(10 marks)

Question 8

04 December 2018

15:01



Assuming that the brake master cylinder shown above may be modelled as a perfect cylinder with an external diameter of 48mm and the internal diameter is 40mm, determine the circumferential 'hoop' stress at the outer surface when the fluid pressure reaches 150 Bar.

(10 marks)

Question 9

04 December 2018

15:02

A thin walled cylinder acts as an air receiver. It has a diameter of 0.25m, is 0.5m in length and is made from sheet steel with a thickness of 1.2mm. The longitudinal and circumferential joint efficiencies are 60% and 80% respectively, and the maximum allowable stress is 110MPa. What is the corresponding maximum allowable air pressure?

(10 marks)

Question 10

04 December 2018

15:02

A hydraulic cylinder has an external diameter of 300 mm and an internal diameter of 220 mm. If it is pressurised internally to 170 Bar gauge, determine the radial and circumferential stresses at the inner and outer surfaces.

(10 marks)

Question 11

04 December 2018 15:02

A 1.4 m long cantilever beam carries a uniformly distributed load (u.d.l) of 1100N/m along its entire length. A point load of 4 kN also acts at the free end of the beam. The beam has a flexural stiffness of 2.2 MPa.

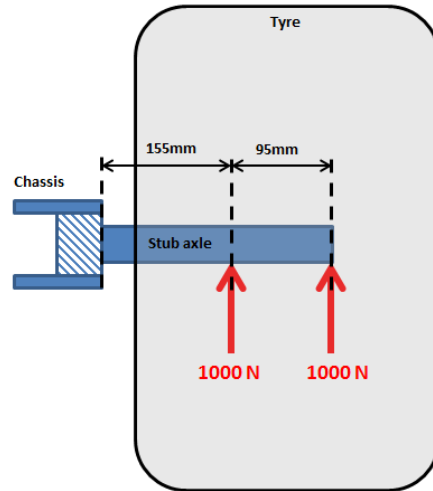
Calculate

- i) The corresponding maximum deflection at the end of the beam
- ii) How much the u.d.l may be increased before the deflection exceeds 2.5 mm

(10 marks)

Question 12

04 December 2018 15:03



The photo and diagram above shows a kart stub axle which may be modelled as a cantilever beam with a two point loads as shown. By applying Macaulay's method derive equations for the slope and deflection at any point along the axle. Calculate the deflection at the free end of the axle if the flexural stiffness is 100kPa.

(10 marks)