

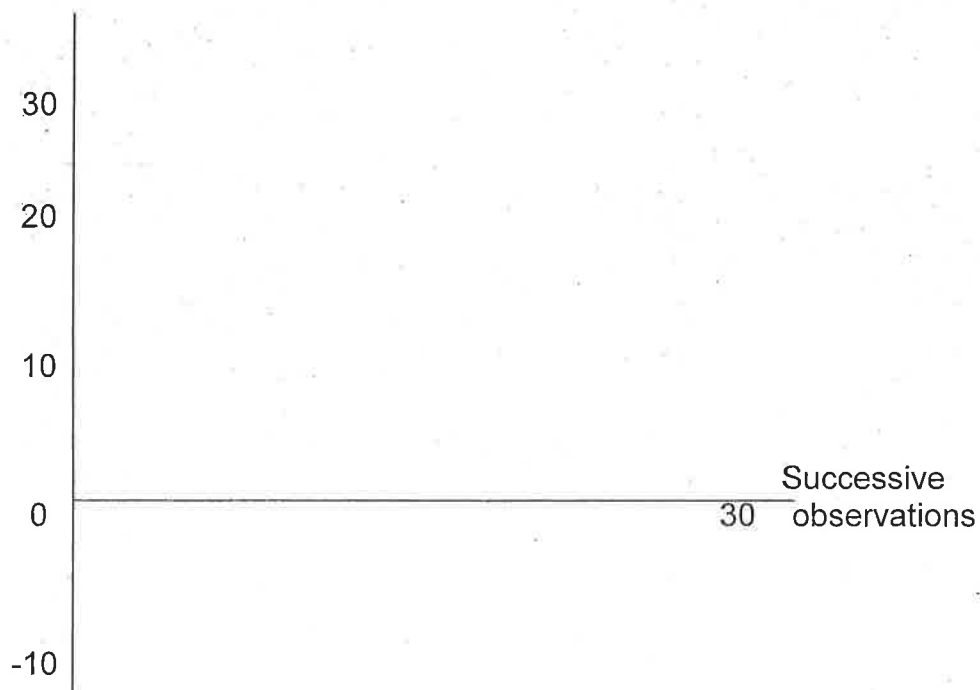
Cusum Charting

The Cumulative sum chart (cusum chart) has been designed to identify slight but sustained shifts in a process. It incorporates all past data by plotting cumulative sums of the deviations of sample values from a target or reference value. This target value T , which corresponds to the target (best) level of our quality characteristic, is subtracted from each observation. The cumulative sum of these deviations from target is the plot point.

Consider the following data, obtained from a process at intervals in order shown, where a target value of 10 has been set:

Observation	Value	Deviation from target ($T = 10$)	Cumulative sum of deviation
1	10	0	0
2	11	1	1
3	9	-1	0
4	10	0	0
5	12	2	2
6	9	-1	1
7	10	0	1
8	8	-2	-1
9	10	0	-1
10	11	1	0

Set up the graph sheet on the next page in the format of the figure below and plot the first ten points



Suppose the observations continue like this

Observation	Value	Deviation from target (T = 10)	Cumulative sum of deviation
11	11	1	1
12	10	0	1
13	12	2	3
14	11	1	4
15	10	0	4
16	11	1	5
17	13	3	8
18	12	2	10
19	9	-1	9
20	11	1	10

Complete the calculations and plot the points on the cusum chart.

Suppose the observations continue like this

Observation	Value	Deviation from target (T = 10)	Cumulative sum of deviation
21	11	1	11
22	12	2	13
23	13	3	16
24	13	3	19
25	12	2	21
26	10	0	21
27	12	2	23
28	13	3	26
29	12	2	28
30	12	2	30

Complete the calculations and plot these points on the cusum chart.

As can be seen the cusum method of plotting results in the average level being represented by the SLOPE of the chart. Observations 1 to 10 had an average value of 10 (equal to the target value). Observations 11 to 20 had an average value of 11 and observations 21 to 30 had an average value of 12.

Interpretation of cusum charts

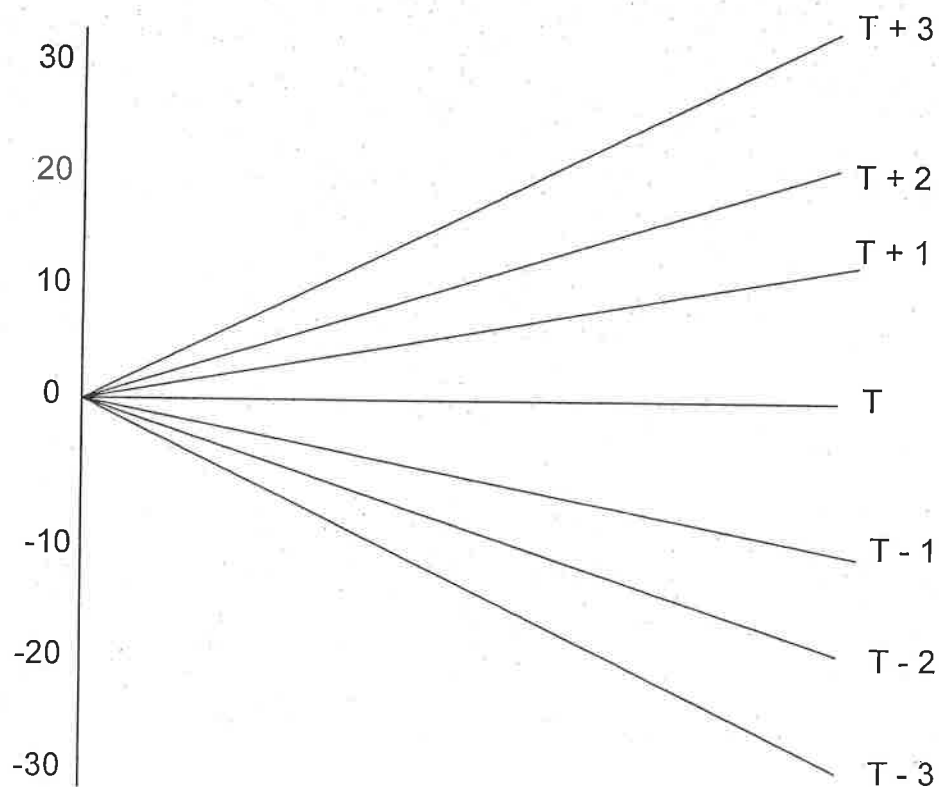
1. A horizontal line of points means that the process is holding an average value equal to the target value.
2. A straight line upwards means that the process has changed to a new steady average level, which is above the target value. The steeper the line, the greater the difference between the new average and the target

value. Conversely if the straight line is downwards, the new average is below the target value.

3. If the line is curved upwards, the new average is above the target, and still increasing. Similarly if it is curved downwards, the average is below the target and still reducing.
4. It should be noted that the position of a point on the chart represents the cumulative past history of the process, since observations were started. Thus there is no particular significance in whether the point is high up or low down on the paper. If points run over the top of the chart, it is common practice to restart them again near the bottom. All that matters is the direction in which the points are heading. In our example we must adjust the process so that they keep a horizontal course if we wish to control the process at the target value of 10.

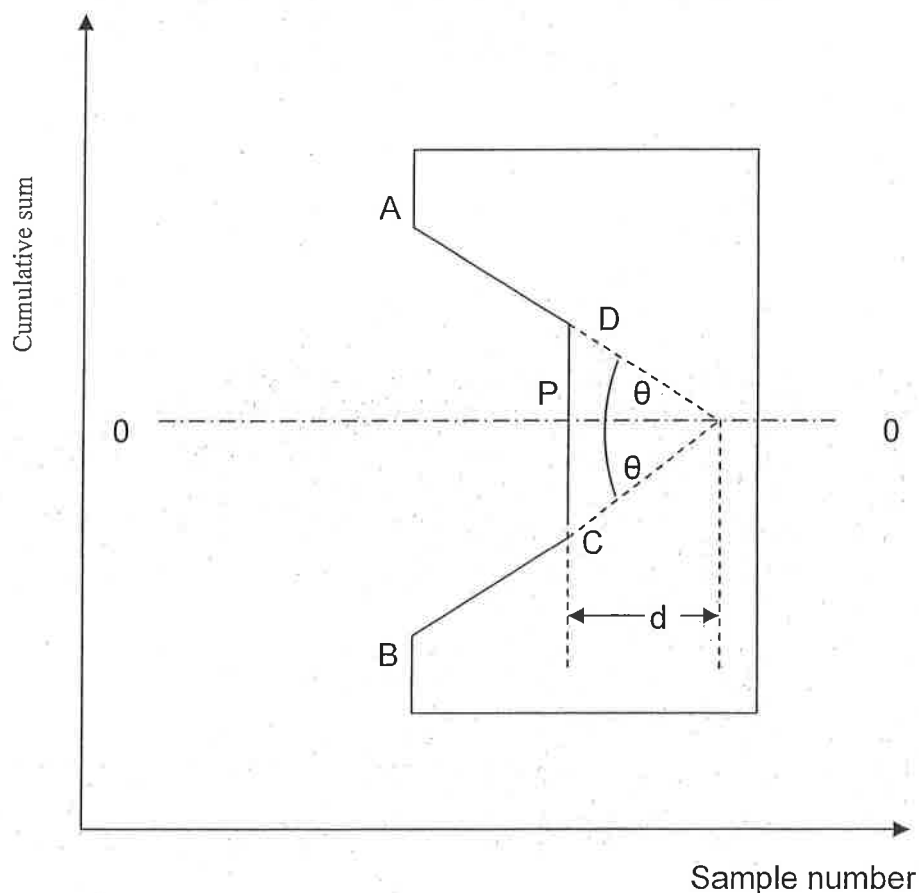
Calculation of averages

The cusum chart clearly shows up changes in average value. To assess the magnitude of these changes a slope guide or protractor can be put on the chart. A horizontal line ten samples long represents an average level on target (T). Other lines can then be constructed at angles corresponding to a suitable range of average levels. An example is shown below



Control limits

The control limits on the cusum chart take the form of a truncated V mask and consist of a location pointer P and two angled control limits. The distance d and the angle θ are the design parameters of the mask and are chosen from tables to fit a range of probability levels and run lengths. The mask is usually made of clear plastic so that points lying under it can still be seen. In use the mask is placed with its axis $0 - 0 - 0$ horizontal, and with P over the last point plotted. If the mask covers any of the plotted points, the process average is deemed to have changed. If it is too high, then the plotted points will rise too steeply, and some of them will be hidden by the *lower* side of the mask BC . Conversely, if the process average is too low, the plotted points will fall too steeply, so that some will be hidden by the *upper* limb AD .



Advantages and Disadvantages

The chief advantage of a cusum chart is that it is very sensitive to small changes in the process average in the region of the reference value. It will also show large changes in average, but conventional charts will do this just as well.

The chart is not very sensitive to changes in variability, but where (as is sometimes the case) each plotted point is an average of several observations, a cusum chart can be used in conjunction with an ordinary range chart. Another drawback of the cusum chart is that operators find them difficult to interpret. Hence they tend to be charts for quality engineers.

Question

Items of bar are cropped to length. The target size is 175mm. Twenty samples of five items were taken periodically from the process and the lengths measured, to the nearest 0.01mm. The mean and range of each sample was calculated and are given below.

Sample No	1	2	3	4	5	6	7	8	9	10
Mean	175.10	175.24	174.86	175.22	174.92	175.06	175.00	174.94	175.06	175.14
Range	0.9	0.9	0.8	0.8	0.4	0.9	0.7	1.10	0.6	0.5

Sample No	11	12	13	14	15	16	17	18	19	20
Mean	175.48	175.06	175.16	174.90	175.28	175.10	174.86	174.98	175.30	174.90
Range	0.6	0.9	0.9	0.4	0.8	0.9	1.0	0.6	0.6	0.7

- Produce a Cusum Chart for sample means and plot the data onto the chart. It is essential you establish a suitable scaling factor.
- In terms of modern manufacturing concepts, compare and contrast the use of Cusum Charts with the traditional charts for measured data. Your discussion should identify the advantages and disadvantages of both methods.

