



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

MSc - Motorsport Engineering

**Investigating Vehicle Dynamic Effects of Passive and
Active Limited Slip Differentials.**

Module Name	Research Methods and Professional Developments
Module Number	ACFA 7004
Student Name	Sean Doyle
Student Number	2305948
Date of Submission	02/03/2025

University of Wales Trinity Saint David

School of Engineering

2024-2025

Contents

Table of Figures	3
Table of Tables	4
Nomenclature.....	5
Introduction.....	6
1 Industrial Relevance	7
2 Literature Review	8
2.1 Tyre Dynamics Fundamentals.....	8
2.2 Differential Fundamentals.....	9
2.2.1 Open Differential – The Traction Problem	10
2.3 Limited Slip Differentials Fundamentals	11
2.4 Passive Limited Slip Differentials.....	12
2.4.1 Viscous Limited Slip Differential	12
2.4.2 Clutch Plate Limited Slip Differentials.....	13
2.4.3 Torque Sensing Limited Slip Differential.....	15
2.5 Active Limited Slip Differentials	16
2.5.1 Electronic Limited Slip Differential.....	16
2.5.2 Magnetorheological Fluid Limited Slip Differential.....	18
2.5.3 Torque Vectoring Differential.....	20
3 Proposed Methodology.....	22
3.1 Vehicle Ride Model	23
3.2 Vehicle Handling Model	25
3.3 Software Modelling.....	27
4 Results Collection and Analysis	28
5 Project Planning.....	29
6 Evaluation.....	30
7 Reference List.....	31
Appendix A.....	36
Appendix B.....	38
Appendix C.....	39

Table of Figures

Figure 1: Tyre Characteristics and Associated Axis (Pacejka & Besselink, 2012).	8
Figure 2: Comparing Open (left), and Locked (right) Differentials where (top) Absence of Drive Torque and (Bottom) with Driveline Torque (Blundell & Harty, 2015).	9
Figure 3: Open Differential (Top) Components (Crouse & Anglin, 1983), (Bottom) Traction Problem (AuburnGear, n.d.).	10
Figure 4: LSD Cornering Characteristics (Polyphony Digital Inc, 2025).	11
Figure 5: Viscous Couple LSD Diagram (Jin, et al., 2021).	12
Figure 6: ‘Salisbury’ Clutch Plate and Ramp (Top) Diagram and Components (Guiggiani, 2018), (Bottom) Phases of Locking Torque (Tremlett, et al., 2014).	13
Figure 7: (Left) ‘Trac-Loc’ Clutch Plate Type LSD Components (Guiggiani, 2018), (Right) Bevel Gear Separation Forces (Sharma & Purohit, 2004).	14
Figure 8: Cone Clutch LSD, (Left) Construction (Stone & Ball, 2004), (Right) Operation (Autodata Training, 2019).	15
Figure 9: Torque Sensing LSD (Shih & Bowerman, 2002).	15
Figure 10: Eaton 'IntelliTrac' Electronic LSD (Eaton, 2016).	17
Figure 11: Schematic Diagram of Active Electronic LSD (Rubin & Arogeti, 2015).	17
Figure 12: Schematic Diagram of MRF LSD (Russo, et al., 2016).	18
Figure 13: MRF LSD Current and Locking Torque Graph (Abd elhafiz & Metered, 2018).	19
Figure 14: (Left) Active Front and Centre Differentials, (Right) Torque Vectoring Example in Active Powertrain (Jalali, et al., 2013).	20
Figure 15: Mitsubishi TVD Torque Transfer to (Left) Left Wheel, (Right) Right Wheel (Cypress, 2007).	20
Figure 16: Mitsubishi TVD Final Drive Model (Ankinovich, et al., 2020).	21
Figure 17: 7 DOF Vehicle Ride Model (Setiawan, et al., 2009).	23
Figure 18: Handling Model (Top) Bicycle Model Simplest Form (Aripin, et al., 2014), (Bottom) 7 DOF Vehicle Handling Model (Aripin, et al., 2014).	25
Figure 19: 14-DOF Vehicle Ride and Handling Model (Setiawan, et al., 2009).	26
Figure 20: (Left) Standard Simulink Modelling, (Right) Vehicle Dynamics Blockset (MathWorks (2), n.d).	27

Figure 21: Vehicle Dynamics Modelling (Left) Simulink Multibody (Strässle, 2025), (Right) Vi-CarRealTime.....	27
Figure 22: Project Plan Gannt Chart.....	29
Figure 23: ICE Drivetrain Layouts (AutoPivot, 2023).	36
Figure 24: BEV Drivetrain Architecture. (a) ICE Based, (b) Fixed Gear no Clutch, (c) Integrated Fixed Gear, Motor and Differential, (d) Dual Motors with Fixed Gears, (e) Dual Motors, Geared in Wheel Drives, (f) Gearless Dual Motor In-Wheel Drive (Zhou, et al., 2023).....	37
Figure 25: Detroit Locker Differential (Strong, 2011).....	38
Figure 26: Yaw Stability Control System Architectural Flow Chart (Bhatt, et al., 2019).....	39
Figure 27: Yaw Stability Control System Types (Rajamani, 2011).	40
Figure 28: Multi-Input and Multi-Output Handling of MPC (Ulusoy, 2018).....	40

Table of Tables

Table 1: MRF LSD Components and Materials (Russo, et al., 2016).	18
---	----

Nomenclature

<i>4WD</i>	4 Wheel Drive
<i>AWD</i>	All Wheel Drive
<i>BEV</i>	Battery Electric Vehicle
<i>DOF</i>	Degrees-of-freedom
<i>FWD</i>	Front Wheel Drive
<i>eLSD</i>	Electronic Limited Slip Differential
<i>LSD</i>	Limited Slip Differential
<i>ICE</i>	Internal Combustion Engine
<i>MF</i>	Pacejka Magic Formula
<i>MPC</i>	Model Predictive Control
<i>MRF</i>	Magnetorheological Fluid
<i>PID</i>	Proportional-Integral-Derivative
<i>RWD</i>	Rear Wheel Drive
<i>SMD</i>	Spring-mass-damper
<i>TVD</i>	Torque Vectoring Differential
<i>YSC</i>	Yaw Stability Controller
<i>A</i>	Acceleration (m/s/s)
<i>a</i>	Distance from CoG to Front Axle (m)
<i>b</i>	Distance from CoG to Rear Axle (m)
<i>F</i>	Force (N)
<i>I</i>	Inertia (Kg.m ²)
<i>i_{FD}</i>	Final drive gear ratio
<i>K</i>	Spring rate (N/m)
<i>k</i>	Longitudinal slip
<i>M</i>	Mass (Kg)
<i>n_{VC}</i>	Number of plates in viscous coupling limited slip differential.
<i>r</i>	Yaw rate (rad/s)
<i>r₁</i>	Inner radius of outer discs (m).
<i>r₂</i>	Outer radius of inner discs (m).
<i>r_{mid}</i>	Pitch radius of gear under consideration at tooth mid-point (m)
<i>t</i>	Temperature (K).
<i>T</i>	Torque (Nm).
<i>u</i>	Ratio between differential housing and first (1), and second (2) auxiliary links.
<i>v</i>	Velocity (m/s)
<i>V_s</i>	Kinematic Viscosity of silicone fluid (Pa.s).
<i>Z</i>	Vertical displacement of mass (m)
<i>α</i>	Distance from CoG to Vehicle Tyres Right (m)
<i>β</i>	Vehicle sideslip angle (rad)
<i>ρ</i>	Distance from CoG to Vehicle Tyres Left (m)
<i>Ω</i>	Wheel angular velocity (rad/s)
<i>φ</i>	Roll Angle (rad)
<i>θ</i>	Pitch Angle (rad)
<i>δ</i>	Front wheel steer angle (rad).
<i>λ</i>	Plate separation distance (m).
<i>ρ_s</i>	Silicone oil density (Kg/m ³).
<i>ω</i>	Angular velocity (rad/s).
<i>ξ</i>	Pressure Angle (rad).

Introduction

This project “Investigating Vehicle Dynamic Effects of Passive and Active Limited Slip Differentials” stems from the constant development of new technology that has led to the inevitable evolution of the differential. Over the years, developments from the standard open differential have led to traction improved, motorsport approved LSDs. From passive to active devices, the number of differentials available today is staggering. Previously published pieces of literature have conducted studies evaluating the effectiveness of a single mainstream device. Unfortunately, a comparison between these different devices regarding operation and vehicle dynamic effects is often overlooked. Additionally, when modelling vehicle dynamics, unless the focus is the differential, these devices are often overlooked. Gadola et al. (2018) confirmed the author's initial assessment of existing literature in their report, stating, “The limited slip differential (LSD) has been in use for decades, however its impact on vehicle dynamics is somehow neglected in literature”. The author of this report and project found a lack of industry research knowledge where a comprehensive comparison of the main technologies and effects of these devices is compared. The sheer number of effects these devices have can vary due to many different parameters, such as drivetrain configuration, the LSD in operation, locking effects/parameters of this device, and much more. The independent variable of this project is the LSD, whilst dependent variables are: torque distribution, locking effect, yaw rate and other vehicle data results. Test vehicle control variables: suspension kinematics, vehicle mass and inertia, powertrain and general configuration, ensure only the LSD and drivetrain configurations impact results. By also ensuring control parameters are implemented in testing manoeuvres and road conditions, a fair test promises comparable results. The project aims are therefore:

- Provide motorsport and automotive industry with comprehensive comparison of different LSDs mechanical workings and vehicle dynamic effects.
- Fill in gaps within industry knowledge (motorsport and vehicle dynamics industry).
- Provide a guide to race teams/customers for selecting the appropriate LSD based on requirements and desired handling.
- Define overall handling implications and positive/negative effects of each LSD investigated.

Project objectives are:

- Investigate both passive and active LSDs.
- Build vehicle dynamics models in Simulink that access handling characteristics due to LSDs.
- Model a suitable YSC for active LSDs testing.
- Consider different drivetrain layouts (i.e. FWD, RWD and AWD).
- Use simulated and real-world data to compare validity and differentials.
- Conduct a comprehensive analysis comparing the different effects.

1 Industrial Relevance

A differential is present in most automotive and motorsport vehicles on the road/track today. For many race teams and some automotive customers, the installation of an LSD provides added benefits to the vehicles handling capabilities. Unfortunately, misconceptions, lack of understanding and published literature surround this field of vehicle technology, specifically in the aspects of torque distribution (Blundell & Harty, 2015). Comparisons between these different devices are the most neglected aspect when evaluating LSDs.

By completing this comprehensive comparison of mechanical operations and vehicle dynamic effects, an insight into the operation and predicted vehicle response characteristics can allow race teams and consumers to adjust parameters within their devices or purchase a more suitable differential for their vehicle or drivers racing style. In motorsport some restrictions such as racing regulations or financial limitations may limit the ability to change LSD device type. However, with this project, guidance can be provided to adjust allowable parameters within the current device to improve handling characteristics. Manufacturers of the different devices understand the different effects; however, publicly available published literature is very limited.

As this project is virtual simulation based, validation of results will rely on the results from others experiments, both virtual and real-world testing. The validity and credibility of these results will be heavily relied on and cited, therefore preventing any ethical issues. This project, to the authors knowledge has no environmental impact or associated risks.

2 Literature Review

The literature review examines the fundamental principles of various LSDs. Three devices from each LSD category (active or passive) were selected based on industry popularity, promising development, extensive research, relevance to industries, and suitability for this project. Drivetrain configurations are detailed in Appendix A.

2.1 Tyre Dynamics Fundamentals

Tyres significantly influence vehicle dynamics, prompting extensive research by Hans Pacejka. The 'Pacejka Magic Formula' (MF) models tyres, selecting specific variants of this formula based on known parameters and model requirements. Key tyre parameters are represented by shaded regions and peak point in figure 1. LSDs highly depend on tyre dynamics, making tyre slip crucial for modelling locking effect. Equations 1-4 calculate rolling radius (r_e), longitudinal slip (k), lateral slip angle (α), and an MF example (MathWorks (3), n.d.) where B, C, D and E are MF coefficients. The final tyre slip, 'spin' is detailed in "Tire and Vehicle Dynamics" (Pacejka & Besselink, 2012).

$$r_e = \frac{V_x}{\Omega_o} \quad (1)$$

$$k = -\frac{V_x - r_e \Omega}{V_x} = -\frac{\Omega_o - \Omega}{\Omega_o} \quad (2)$$

$$\tan \alpha = -\frac{V_y}{V_x} \quad (3)$$

$$F_x = F_z \times D \times \sin(C \times \tan^{-1}\{B_k - E[B_k - \tan^{-1}(B_k)]\}) \quad (4)$$

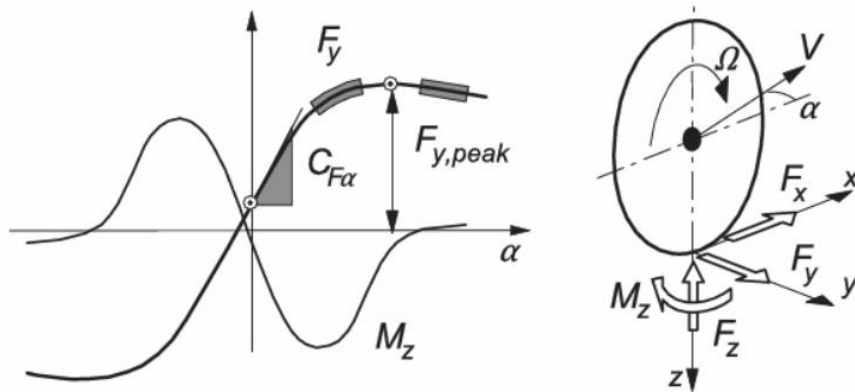


Figure 1: Tyre Characteristics and Associated Axis (Pacejka & Besselink, 2012).

2.2 Differential Fundamentals

A differential has three main purposes (Smith, 2013): providing power and torque to drive wheels, enable different wheel speeds on driven axle during cornering, and acts as the final drive gear within the drivetrain (see Appendix B for locked differential). Blundell and Harty (2015) provide insight into vehicle dynamic drive torque behaviours resulting from differentials: “How these speeds turn into torques is governed entirely by the wheel to which they are attached - it is a function of vertical load, friction coefficient and slip state. The latter is influenced by vehicle states and, of course, the differential locking conditions. These comments apply whichever differential is under consideration” (figure 2). Conducting simplistic vehicle dynamic studies with different LSDs reveals the locking characteristics of each device. This data can then be used in complex simulation models to analyse and compare resultant vehicle dynamic characteristics.

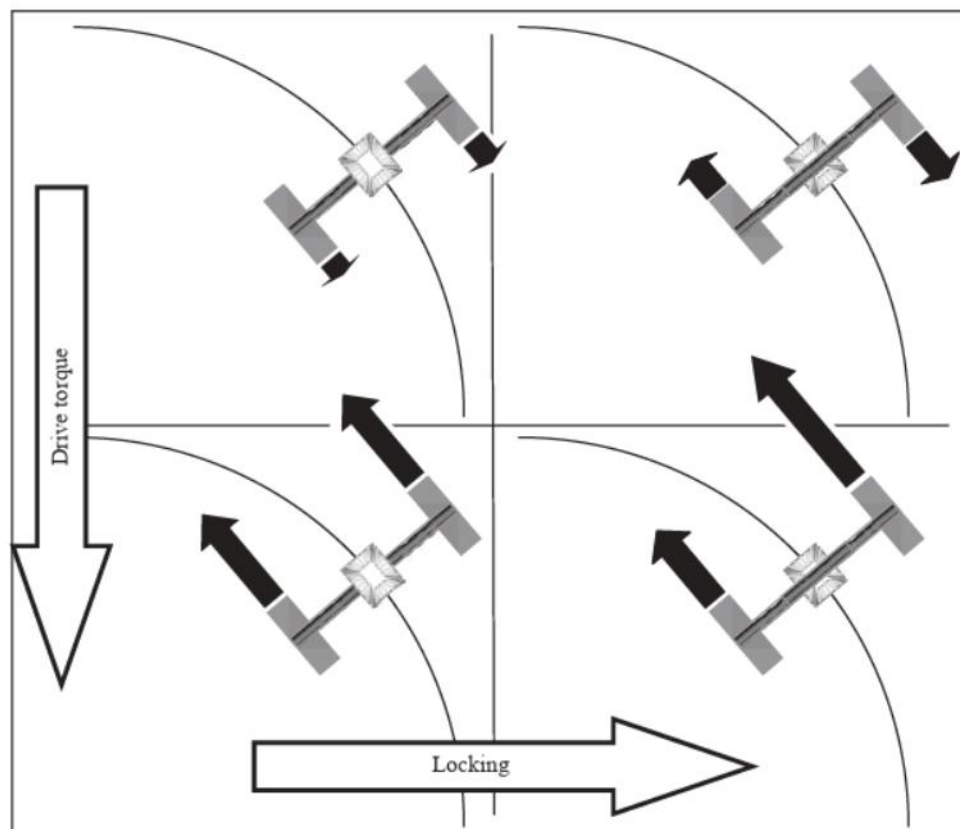


Figure 2: Comparing Open (left), and Locked (right) Differentials where (top) Absence of Drive Torque and (Bottom) with Driveline Torque (Blundell & Harty, 2015).

2.2.1 Open Differential – The Traction Problem

The open differential (figure 3) consists of the ring gear, pinion gears, drive pinion, pinion shaft, and differential case (Crouse & Anglin, 1983). Torque is always evenly split between the left and right axle, where the primary torque path ‘travels’ from ring gear to pinion shaft, pinion gear, side gear, and finally the wheels (Crolla, 2015). Equations 5-8 demonstrate the principal workings, where torque left equals torque right, and the division of the sum of left and right shaft angular velocities by two equals input speed. The associated traction problem with open differentials refers to the variable wheel speeds during a tyre slip event.

$$T_{Left} = T_{Right} \quad (5)$$

$$T_{in} = T_{out} \quad (6)$$

$$\therefore T_{in} = T_{Left} + T_{Right} \quad (7)$$

$$\omega_{In} = \frac{(\omega_{Left} + \omega_{Right})}{2} \quad (8)$$

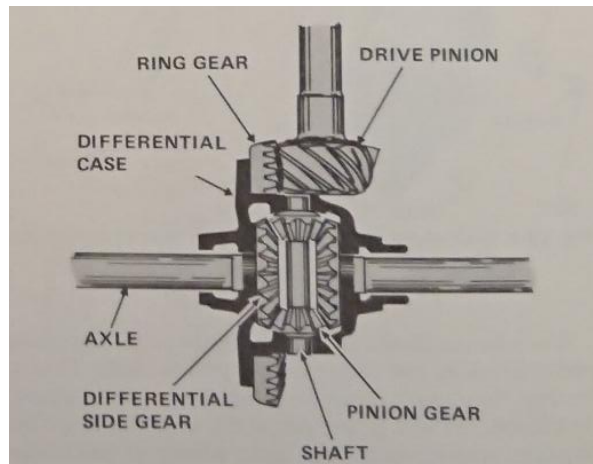


Figure 3: Open Differential (Top) Components (Crouse & Anglin, 1983), (Bottom) Traction Problem (AuburnGear, n.d.).

2.3 Limited Slip Differentials Fundamentals

Considering the LSD as a generic device, it enhances traction and performance (figure 4) over a standard open differential by limiting tyre slip and wheel angular velocity differences (VanGelder, 2022). There are two main categories of LSDs: active and passive. Passive differentials rely solely on mechanical forces to transfer torque between wheels, while active differentials are electronically controlled (see Appendix C for controller) (Forstinger, et al., 2015). An additional benefit of an LSD is the secondary torque path, which provides torque to the wheel with the most traction (Palazzolo, 2009). This secondary torque path is unique to each type of LSD due to their construction and dictates the vehicles dynamic response. The differences in these responses are the focus of this project.

The locking effect of an LSD refers to the amount of torque transferred within a differential. An open differential has a locking effect of 0%, while a fully locked differential can achieve 100% (Allen, 2009). These values vary for LSDs based on their mechanical workings, thus affecting vehicle handling characteristics. Formulas and values for locking effect will be detailed in the final report alongside the simulations.

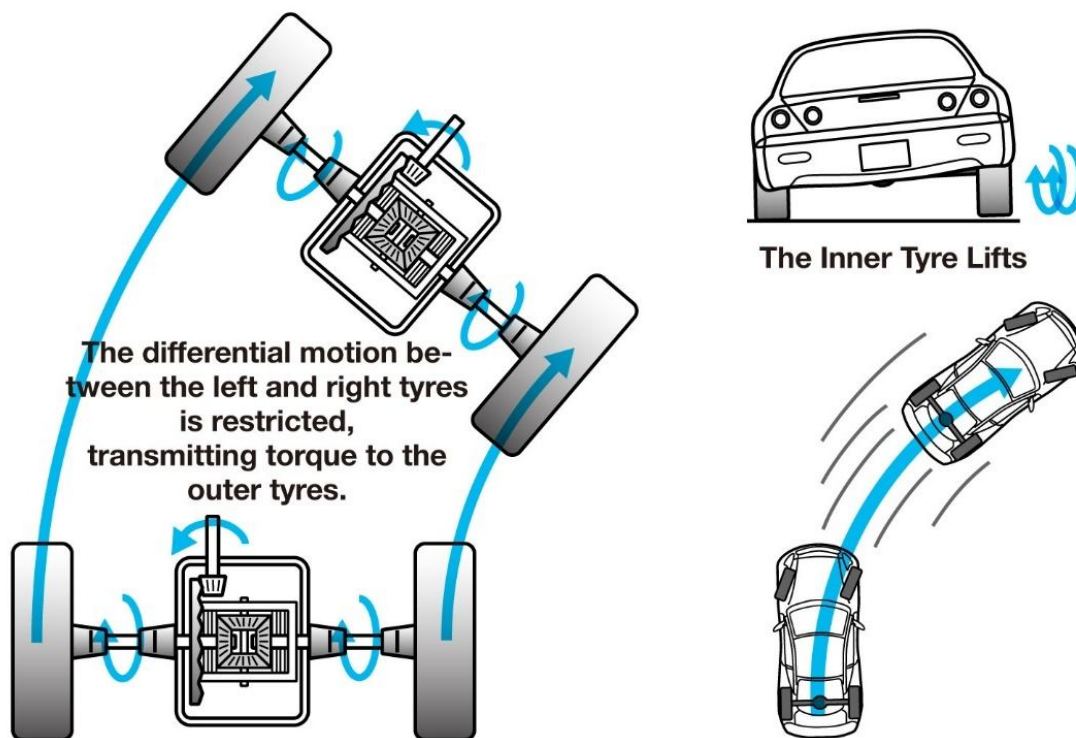


Figure 4: LSD Cornering Characteristics (Polyphony Digital Inc, 2025).

2.4 Passive Limited Slip Differentials

2.4.1 Viscous Limited Slip Differential

The speed sensing viscous LSD operates based on torque transmission due to fluid shearing, resulting in friction (Frömmig, 2023). A highly viscous fluid, usually silicon, is sealed within the viscous coupling unit attached to the end of a standard open differential (figure 5). The outer and inner discs are separated by a specified distance and form two separate groups, one connected to either the left or right axle, and the other to the casing. 'Speed sensing' refers to the requirement of different angular velocities of the two wheels to initiate the locking mechanism. However, the effectiveness of this type of LSD can be reduced by temperature increases caused by shearing friction, which decreases the fluids viscosity and therefore locking effect and torque, as shown in equations 9 and 10 (Jin, et al., 2021). Where: t_e current temperature, $V_{t=298}$ absolute viscosity at 298K, ' A_{VC} ' constant related to silicone oil viscosity, $\Delta\omega$ angular velocity difference of input and output shafts.

$$T_q = \frac{n_{VC}\pi\rho_s V_s \Delta\omega (r_2^4 - r_1^4)}{\lambda} \quad (9)$$

$$V = V_{t=298}^{(t_e/298)^{A_{VC}}} \quad (10)$$

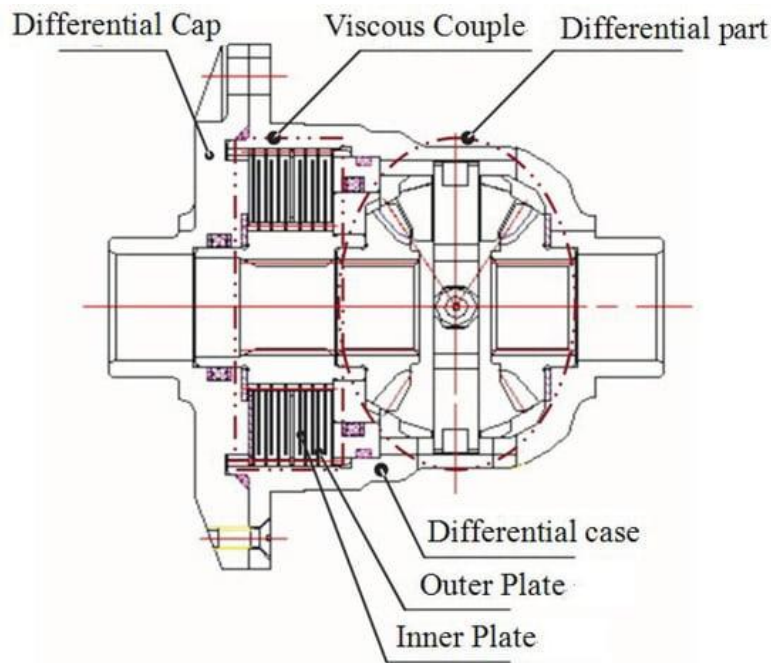


Figure 5: Viscous Couple LSD Diagram (Jin, et al., 2021).

2.4.2 Clutch Plate Limited Slip Differentials

The clutch plate LSD is torque sensitive and comes in three main variations: clutch plate and ramp ('Salisbury') (figure 6), clutch plate (figure 7) (Guiggiani, 2018), and cone clutch type (figure 8) (Shepard, 2013). The Salisbury and clutch plate LSDs are popular in motorsport, with adjustable parameters such as ramp angles (Salisbury only), friction faces and preload. Altering these values affects differential locking characteristics. In a Salisbury LSD, preload is controlled by a Belleville spring, typically applying positive preload to support torque transfer in low-torque situations, such as jacking events (Tremlett, et al., 2014). Increasing preload allows more torque transfer in these situations, as shown in figure 6 (Bottom) during the initial linear region. The Salisbury LSD locking effect is caused by acceleration or deceleration forces, depending on ramp block design (1-way, 1.5-way, or 2-way), which causes the shaft to separate the ramp blocks and compress against the wet clutch packs (Tremlett, et al., 2014).

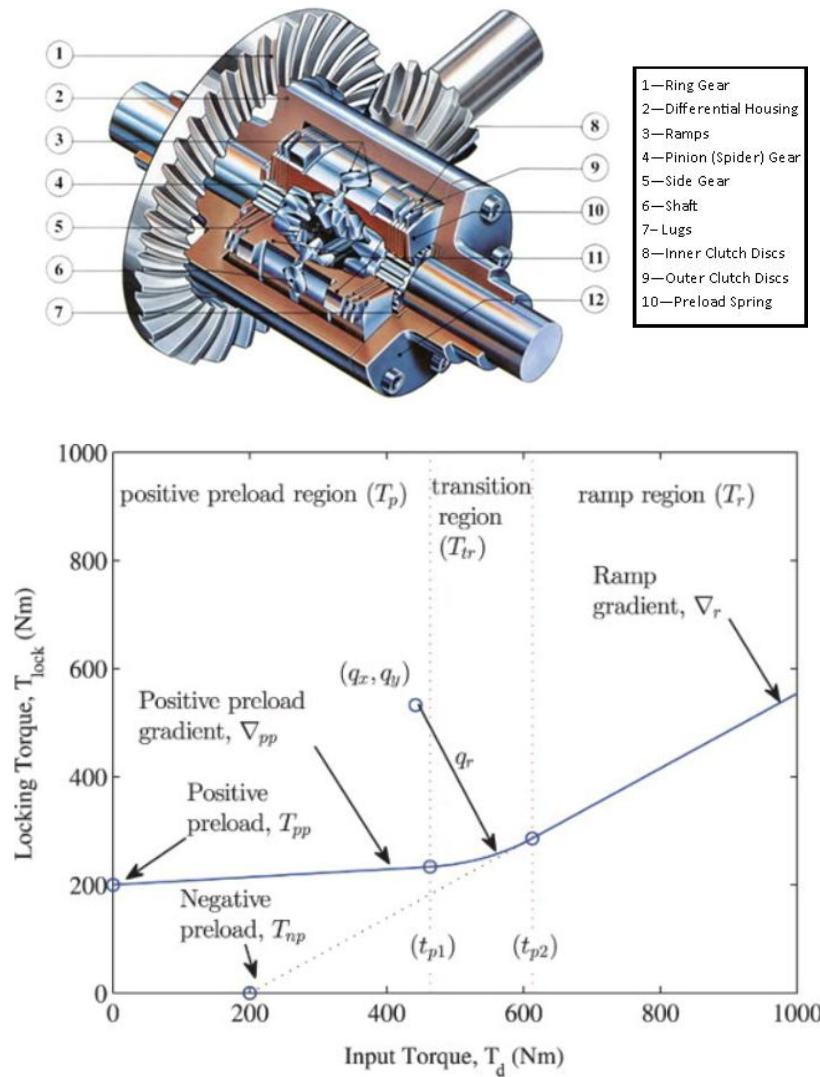


Figure 6: 'Salisbury' Clutch Plate and Ramp (Top) Diagram and Components (Guiggiani, 2018), (Bottom) Phases of Locking Torque (Tremlett, et al., 2014).

A simpler version of the Salisbury is a ‘clutch plate’ fitted with either four springs or s-shaped spring (figure 7 left). Instead of using a ramp block, this differential relies solely on the high axial thrust separation forces provided by the inherent design of the bevel gears (figure 7 right) to compress the clutch packs, causing locking effects. These separation forces can be approximated in equations 11-14, with an assumption error of the force acting along the mid-point of the tooth width, rather than “actually acting somewhere between the mid-point and larger end of tooth” (Sharma & Purohit, 2004). The calculation also excludes the force acting on the bevel gear from the preload springs. These springs are utilized with the same principles as the Salisbury LSD, where preload can be calculated using spring rate equation.

$$F_t = \frac{T}{r_{mid}} \quad (11)$$

$$F_s = F_t \tan \xi \quad (12)$$

$$F_a = F_s \sin \theta \quad (13)$$

$$F_r = F_s \cos \theta \quad (14)$$

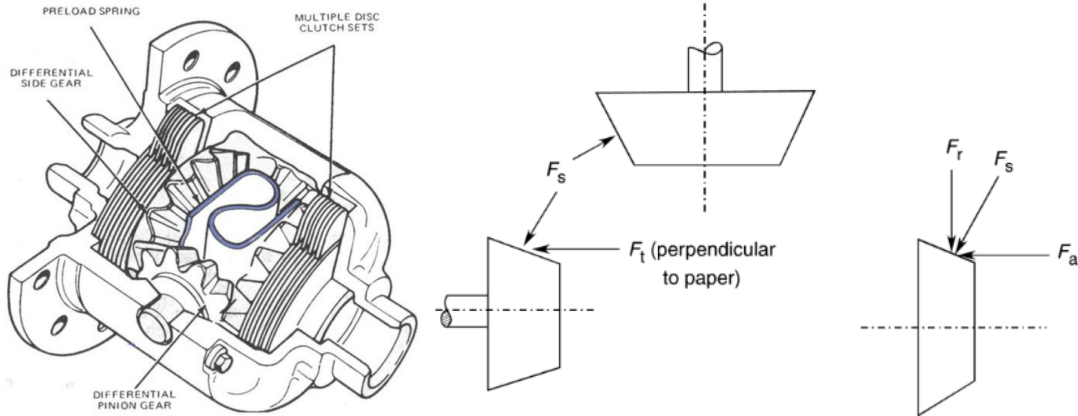


Figure 7: (Left) ‘Trac-Loc’ Clutch Plate Type LSD Components (Guiggiani, 2018), (Right) Bevel Gear Separation Forces (Sharma & Purohit, 2004).

A cone clutch LSD (figure 8) operates on bevel gear separation forces (similar to clutch plate), instead of a multi-disc clutch pack. This design depends on frictional forces generated when the cone is pressed into the concave region of the differential case, resisting wheel slippage events (Stone & Ball, 2004). This initiates the differential locking effect.

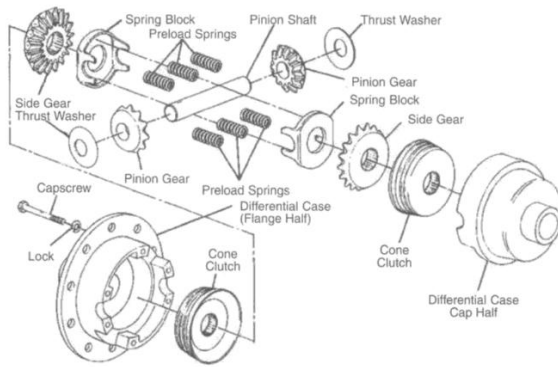


Figure 8: Cone Clutch LSD, (Left) Construction (Stone & Ball, 2004), (Right) Operation (Autodata Training, 2019).

2.4.3 Torque Sensing Limited Slip Differential

A mechanical helical gearing alternative is presented as a torque sensing LSD, contrasting with the clutch plate design. This LSD offers benefits such as reduced maintenance intervals and instantaneous torque biasing (VanGelder, 2022). Its components include worm wheels, worm gears, and spur gears. The fundamental principle is that a worm wheel cannot rotate a worm gear, causing instantaneous locking of the differential. Figure 9 shows a conventional torque sensing differential often used in RWD and AWD vehicles, with a torque biasing ratio typically between 2.5:1 and 4.5:1. The torque bias ratio (equation 15) is the ratio of torque transferable from the low grip wheel to the high grip wheel. The torque bias in this LSD results from frictional forces at various points, such as contact surfaces between the side gear and housing, between two side gears, and at the "6 gear meshes at the crossed-axis-helical side gear and element gear" (Shih & Bowerman, 2002).

$$TBR = \frac{T_{High}}{T_{Low}} \quad (15)$$

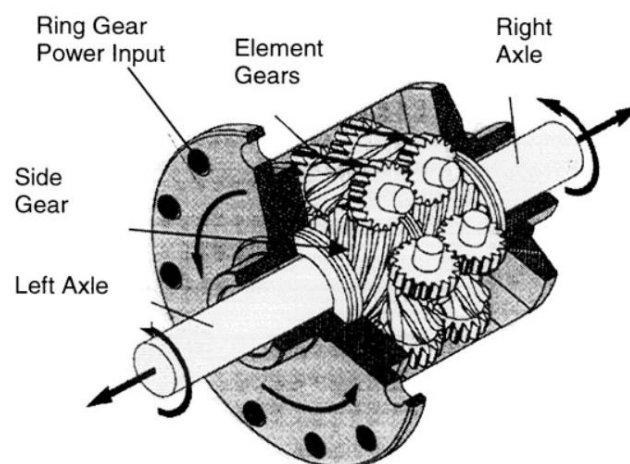


Figure 9: Torque Sensing LSD (Shih & Bowerman, 2002).

2.5 Active Limited Slip Differentials

2.5.1 Electronic Limited Slip Differential

The eLSD is a variation of the active differential. This project investigates a generic actuator with a single clutch pack design (figures 10 and 11). Main actuator types include electromechanical, electromagnetic, electric motor, and pneumatic (Rubin & Arogeti, 2015). These actuators are controlled by an electronic control module such as YSC. The signal from the control module causes the actuator to compress or decompress the clutch pack, determining the differentials locking effect and torque transferring capacity (Eaton, 2016). The following equations (16-20) are fully derived and available at Rubin and Arogeti (2015):

Considering figure 11 and the driveline components, the input torque to the differential ring gear is denoted as T_d with subscript g; transmission outputs in equation 16:

$$\bar{I}_{in}\dot{\omega}_g = T_g - n^{-1}T_d \quad (16)$$

Equation 17 is used to calculate the equivalent drive shaft and differential inertia:

$$\bar{I}_{in} = I + n^{-2}I_d \quad (17)$$

Equation 18 calculates transmission output speed as:

$$\omega_g = \frac{n}{2}(\omega_{2l} + \omega_{2r}) \quad (18)$$

Whilst individual wheel velocities are calculated from half shaft dynamics in equations 19 and 20:

$$I_l\dot{\omega}_{2l} = \frac{1}{2}T_d - \frac{1}{2}T_c - T_{2l} \quad (19)$$

$$I_r\dot{\omega}_{2r} = \frac{1}{2}T_d - \frac{1}{2}T_c - T_{2r} \quad (20)$$

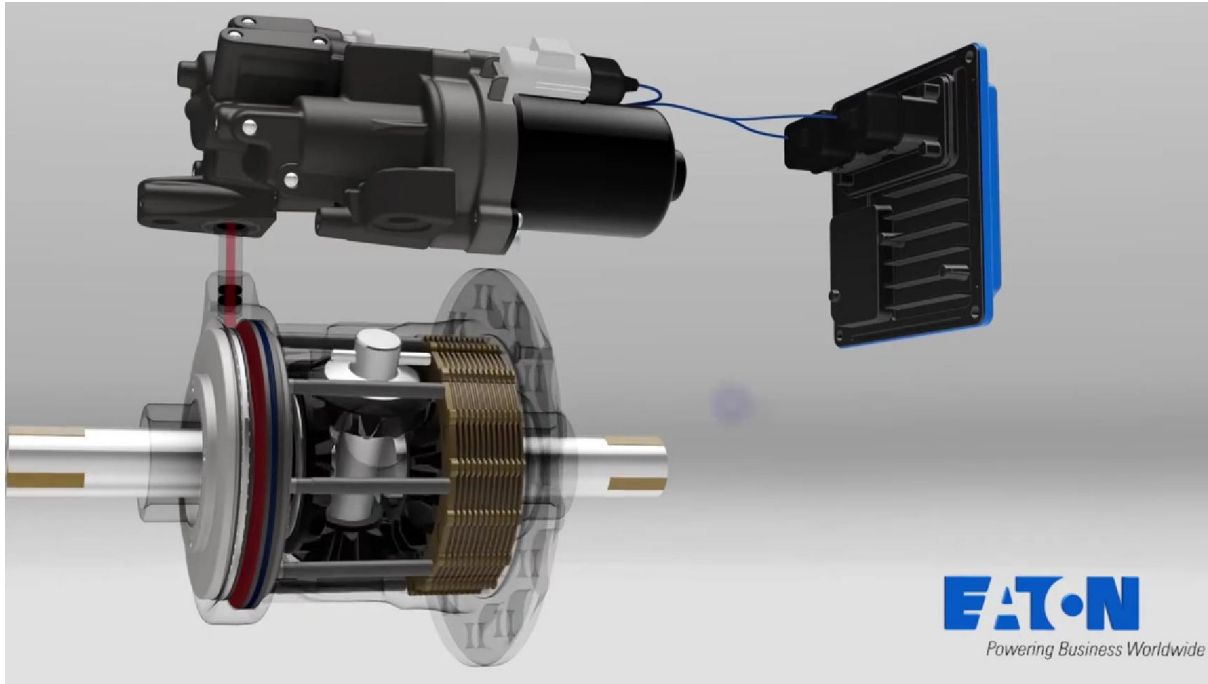


Figure 10: Eaton 'IntelliTrac' Electronic LSD (Eaton, 2016).

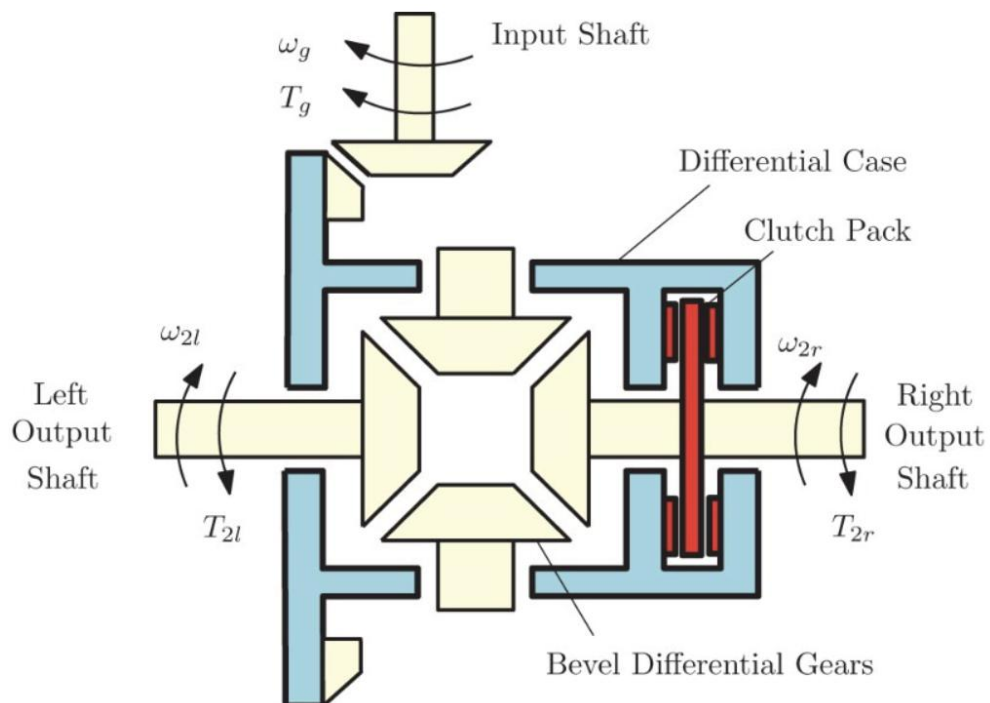


Figure 11: Schematic Diagram of Active Electronic LSD (Rubin & Arogeti, 2015).

2.5.2 Magnetorheological Fluid Limited Slip Differential

A relatively new device in semi-active differential technology is the MRF LSD. MRF fluid contains micron-sized ferrous particles. By applying a magnetic field, the shear stress and rate of the fluid are altered, changing locking torque (Russo, et al., 2016). Due to low viscosity of MRF, the LSD behaves close to open differentials when magnetic fields are absent. Table 1 lists the notations used in Figure 12, which demonstrates a proposed schematic diagram of this LSD.

Table 1: MRF LSD Components and Materials (Russo, et al., 2016).

Number	Description	Material
1	Side Gear A	Standard Steel
2	Side Gear B	Standard Steel
3	Planetary Gear	Standard Steel
4	Differential Case P	Low Carbon Steel
5	Differential Gear	Low Carbon Steel
6	Inner Disk	Low Carbon Steel
7	Outer Disk	Low Carbon Steel
8	Spacer Element	Austenitic Stainless Steel
9	Disk Shaft	Austenitic Stainless Steel
10	Disk Housing	Austenitic Stainless Steel
11	Coil Outer Housing	Low Carbon Steel
12	Coil Inner Housing	Austenitic Stainless Steel
13	Coil	Copper
14	Sealing	Steel
15	Sealing	Steel

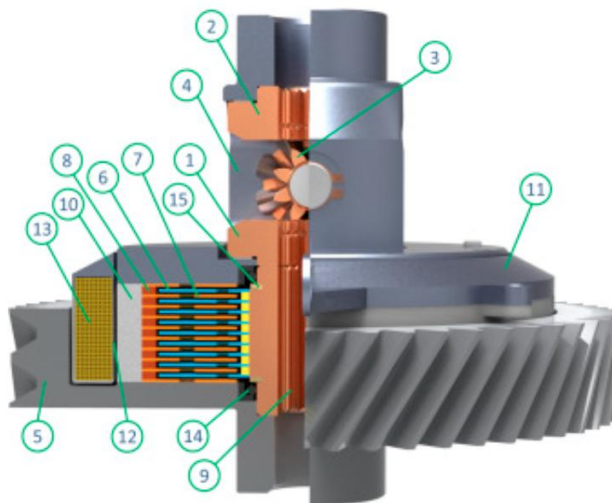


Figure 12: Schematic Diagram of MRF LSD (Russo, et al., 2016).

To control the LSD in slip events, an electromagnetic field requires a calculated supply voltage to the coil, dependent on the LSD controller output. In real-world applications, understanding supply voltage effect on clutch torque and differential locking torque is essential. Figure 13 from Abd elhafiz, & Metered, (2018) shows that higher current increases differential locking torque, with significant effects at lower currents (0-1 Amps). A relationship exists between clutch torque, coil voltage, and shaft relative rotary speed, expressed as a function in equation 21 (Abd elhafiz & Metered, 2018). The angular velocity of the differential case can be calculated as an average of the left and right axle angular velocities (Russo, et al., 2016).

$$M = \frac{n\omega}{g}AR^2 - 0.0264u^2 + 0.3538u + 0.105 \quad (21)$$

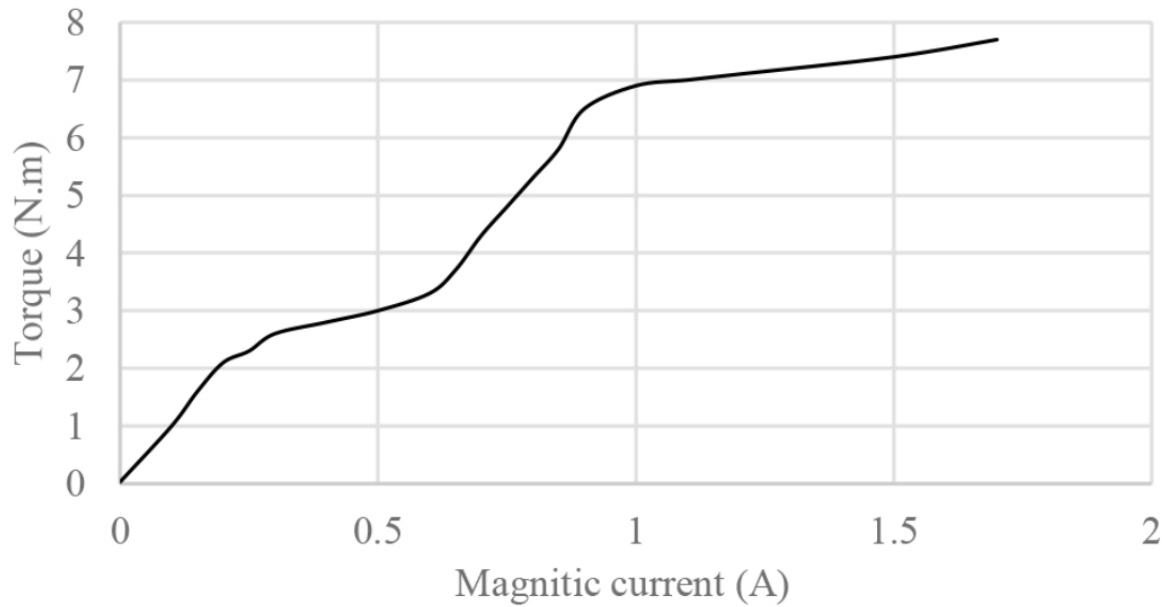


Figure 13: MRF LSD Current and Locking Torque Graph (Abd elhafiz & Metered, 2018).

2.5.3 Torque Vectoring Differential

The TVD is an active device that distributes engine output torque between two differential outputs, providing torque on demand (Jalali, et al., 2013). It can be used in either a centre or axle differential depending on drivetrain layout and vehicle requirements. This active differential relies on sensor signals analysed by a controller module using a specific algorithm. The controller calculates a desired output based on input data, suspected manoeuvre, and desired trajectory, then sends voltage signals to actuators on the differential (Trubachev, 2025). In a centre differential application, the device can apportion more/less torque to the front/rear axle based on driving dynamics. The same principle applies to a rear differential where torque is apportioned to left/right axles (effects shown figure 14). Figure 15 demonstrates torque paths through a Mitsubishi TVD. This project will consider the multi-clutch plate design due to literature surrounding this device.

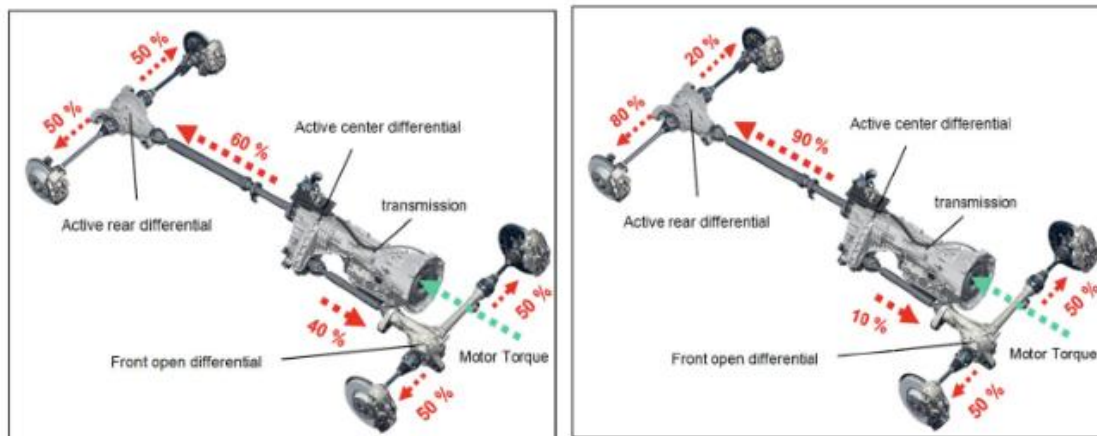


Figure 14: (Left) Active Front and Centre Differentials, (Right) Torque Vectoring Example in Active Powertrain (Jalali, et al., 2013).

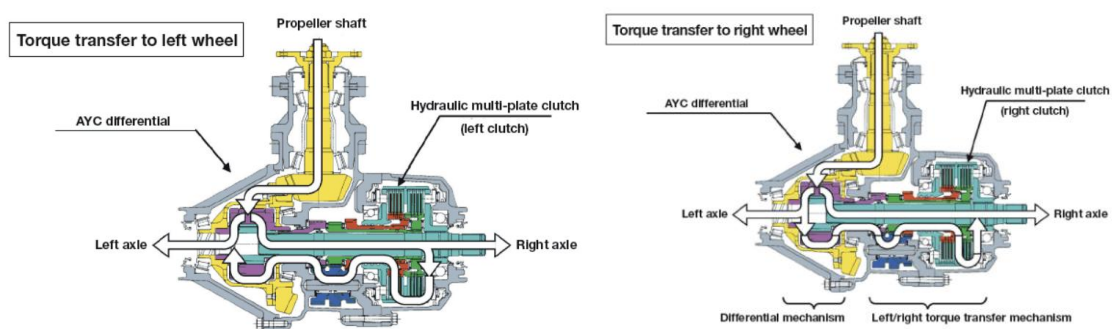


Figure 15: Mitsubishi TVD Torque Transfer to (Left) Left Wheel, (Right) Right Wheel (Cypress, 2007).

The mathematical model of the Mitsubishi TVD is based on diagrams in figures 15 and 16, and detailed in equations 22-29, retrieved from Ankinovich, et al., (2020).

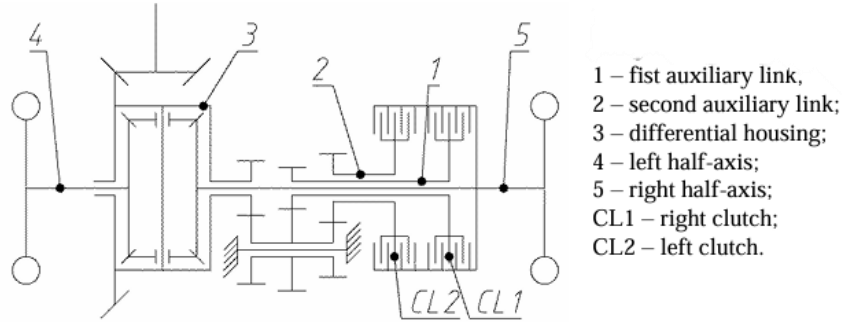


Figure 16: Mitsubishi TVD Final Drive Model (Ankinovich, et al., 2020).

Equations 22 and 23 describe the auxiliary links dynamics:

$$I_1 \dot{\omega}_1 = T_{link\ 1} - T_{CL1} \quad (22)$$

$$I_2 \dot{\omega}_2 = T_{link\ 2} - T_{CL2} \quad (23)$$

Equations 24 and 25 describe the auxiliary link torques:

$$I_w \dot{\omega}_{w\ right} = \frac{T_{diff}}{2} - T_{link\ 1} + T_{link\ 2} - T_{right}^{dr} \quad (24)$$

$$I_w \dot{\omega}_{left} = \frac{T_{diff}}{2} - T_{left}^{dr} \quad (25)$$

Equations 26-28 include final drive to describe the links interactions:

$$\dot{\omega}_{input} = i_{FD} \times \frac{\dot{\omega}_{w\ right} + \dot{\omega}_{left}}{2} \quad (26)$$

$$\dot{\omega}_{input} = i_{FD} \times \frac{\dot{\omega}_1}{u_1} \quad (27)$$

$$\dot{\omega}_{input} = i_{FD} \times \dot{\omega}_2 \times u_2 \quad (28)$$

Whilst equation 29 represents dynamics for the differential housing (reduced to input shaft):

$$I_{input} \dot{\omega}_{input} = T_{input} - \frac{T_{diff}}{i_{FD}} - \frac{T_{link\ 1}}{i_{FD} \times u_1} - \frac{T_{link\ 2} \times u_2}{i_{FD}} \quad (29)$$

3 Proposed Methodology

This project employs a theoretical simulation and modelling based methodology to compare differential effects on handling/vehicle dynamics. Although real-world testing would be ideal, financial and time constraints make it unsuitable. Therefore, some errors are anticipated in simulation results. A detailed review comparing these results to real-world studies is vital for validating accuracy. The simulation software used in this study includes MATLAB/Simulink and Vi-Grade, the following subsections detail the potential modelling methods implemented.

Suitable vehicle dynamics modelling techniques for this project include multibody systems (Blundell & Harty, 2015), SMD (vehicle ride modelling), and vehicle handling models. Combining some of these systems provides a more comprehensive vehicle representation suitable for MATLAB/Simulink and Vi-Grade/Vi-CarRealTime (Setiawan, et al., 2009). Each modelling method contains assumptions impacting result validity, an important factor when selecting modelling methods and analysing results. It is crucial to consider the following parameters and output results when modelling vehicle dynamic responses due to differentials (Zhao & Lin, 2024):

- Drive torques and forces.
- Tyre data (friction coefficient, lateral and longitudinal slip and forces, and tyre loads).
- Load transfer.
- Pitch, roll and yaw angles and angular velocities.
- Masses and inertia.
- Driveline system/configuration.
- Differential properties (efficiency, gear ratios unique LSD parameters).
- Controller implementation (active LSDs).
- Other factors and variables can also be implemented to improve accuracy.

3.1 Vehicle Ride Model

By representing a vehicle as springs, masses, and dampers (SMD system), vehicle ride quality, mass movements, pitch and roll angles, and angular velocities can be calculated to understand responses to lateral and longitudinal acceleration inputs (Guglielmino, et al., 2008). However, calculating these parameters requires a 7-DOF system rather than a simpler half-car (4-DOF), 2-DOF, or 1-DOF model. The 7-DOF ride model (figure 17) is essential for understanding vehicle ride characteristics and tyre loads when cornering. This model must be integrated into the project to provide tyre data for calculating LSD behaviours. Since the LSD depends on each driving wheels tyre slip, systems representing fewer than 7-DOF are unsuitable.

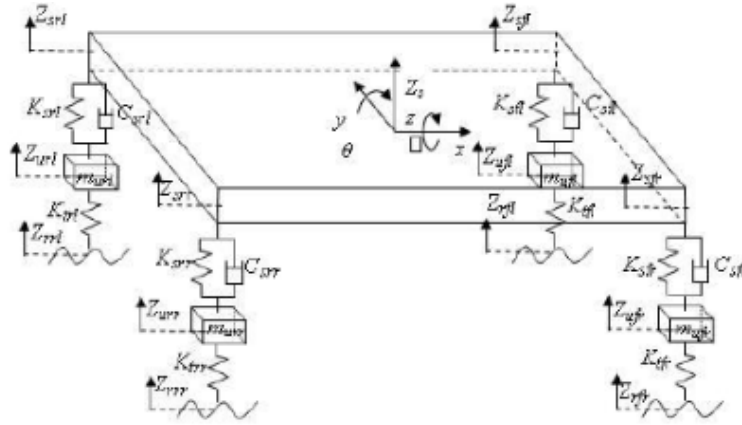


Figure 17: 7 DOF Vehicle Ride Model (Setiawan, et al., 2009).

The equations of motion for this model are derived from Newton's second law, Hooke's law for spring rates, and the damping coefficient formula. The combined result is the equations of motion for sprung and unsprung masses, shown in equations 30-36. Dampers are excluded from report for simplicity.

For unsprung mass (Tudor, 2024):

$$-M_{f1}\ddot{Z}_{f1} = K_{tf1}(Z_{f1} - y_{f1}) - K_{f1}(Z_s - Z_{f1} - a\theta + \varrho\phi) \quad (30)$$

$$-M_{f2}\ddot{Z}_{f2} = K_{tf2}(Z_{f2} - y_{f2}) - K_{f2}(Z_s - Z_{f2} - a\theta - \alpha\phi) \quad (31)$$

$$-M_{r1}\ddot{Z}_{r1} = K_{tr1}(Z_{r1} - y_{r1}) - K_{r1}(Z_s - Z_{r1} + b\theta + \varrho\phi) \quad (32)$$

$$-M_{r2}\ddot{Z}_{r2} = K_{tr2}(Z_{r2} - y_{r2}) - K_{r2}(Z_s - Z_{r2} + b\theta - \alpha\phi) \quad (33)$$

For sprung mass (Tudor, 2024):

$$-I_{xx}\ddot{\phi} = \beta K_{f1}(Z_s - Z_{f1} - a\theta + \varrho\varphi) + \beta K_{r1}(Z_s - Z_{r1} + b\theta + \varrho\varphi) - \alpha K_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) - \alpha K_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) \quad (34)$$

$$-I_{yy}\ddot{\theta} = bK_{r1}(Z_s - Z_{r1} + b\theta + \varrho\varphi) + bK_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) - aK_{f1}(Z_s - Z_{f1} - a\theta + \varrho\varphi) - aK_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) \quad (35)$$

$$-M_s\ddot{Z}_s = K_{f1}(Z_s - Z_{f1} - a\theta + \varrho\varphi) + K_{f2}(Z_s - Z_{f2} - a\theta - \alpha\varphi) + K_{r1}(Z_s - Z_{r1} + b\theta + \varrho\varphi) + K_{r2}(Z_s - Z_{r2} + b\theta - \alpha\varphi) \quad (36)$$

While this modelling method allows for a good initial interpretation of handling characteristics with simpler mathematical models, assumptions can significantly reduce data validity compared to a multibody system.

The following points detail some of these key assumptions (others are also made):

- Sprung mass is assumed to be a rigid body.
- Kinematics and geometry of suspension is excluded (Reimpell, et al., 2001).
- Linear modelling of suspension components (damper hysteresis) (Guglielmino, et al., 2008).
- Exclusion of external forces (aerodynamics, rolling resistance).
- Small angle theorem implementation (Kumar & Medhavi, 2022)
- Simplified tyre model as spring and damper system (Pacejka & Besselink, 2012).

These assumptions can have significant implications. By excluding suspension geometry, modelling suspension components linearly, and using a basic tyre model, the tyre-road interactions can differ greatly between simulation and real-world application. For example, if a wheel jacking event occurred, the system could not calculate the vehicle's loss in overall grip. The small angle theorem simplifies the mathematical model but may not accurately represent ride characteristics in extreme circumstances with larger angles (Kumar & Medhavi, 2022).

3.2 Vehicle Handling Model

A bicycle model (figure 18 top) is often integrated into YSC to calculate vehicle yaw rates for active systems control algorithms (Aripin, et al., 2014). The simplest form of the bicycle model has 2-DOF for lateral and yaw motions, making it suitable for conventional YSC. However, to accurately represent vehicle dynamics, a 7-DOF model that includes longitudinal, vertical, lateral motions and rotation of all four wheels (figure 18 bottom). Similar to the ride model, this approach includes assumptions, effecting result validity.

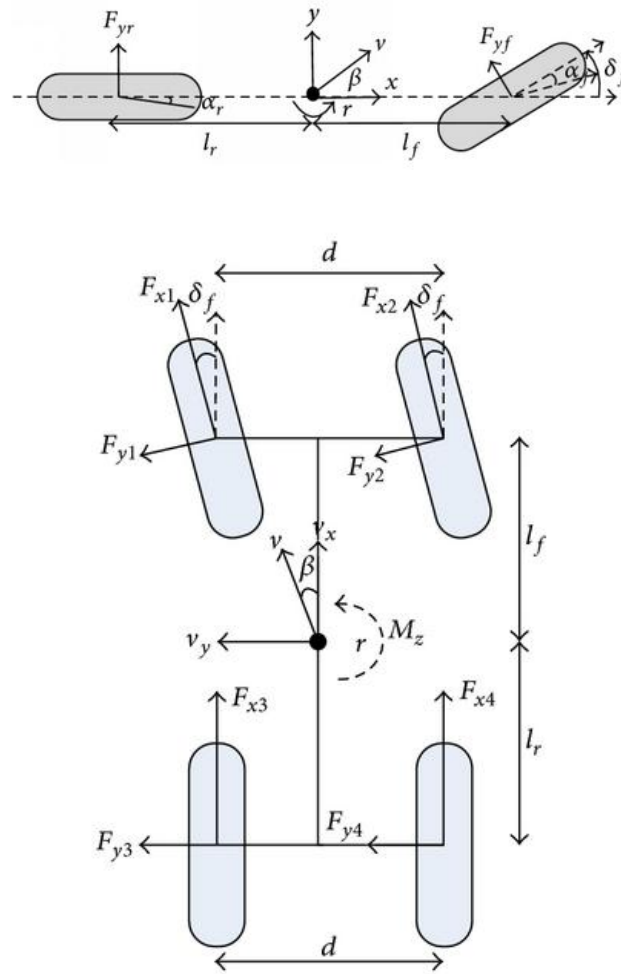


Figure 18: Handling Model (Top) Bicycle Model Simplest Form (Aripin, et al., 2014), (Bottom) 7 DOF Vehicle Handling Model (Aripin, et al., 2014).

Many assumptions in this model are similar to those in the ride model, such as: lack of suspension kinematics, aerodynamic forces and small angle theorem. However, this model includes parameters excluded from the ride model, such as wheel rotational velocities, lateral and longitudinal velocity, and yaw (Aripin, et al., 2014), calculations shown equations 37-41.

Longitudinal motion:

$$-MA_x = (F_{x1} + F_{x2}) \cos \delta_f + F_{x3} + F_{x4} - (F_{y1} + F_{y2}) \sin \delta_f \quad (37)$$

Lateral motion:

$$-MA_y = (F_{x1} + F_{x2}) \sin \delta_f + (F_{y1} + F_{y2}) \cos \delta_f + F_{y3} + F_{y4} \quad (38)$$

Yaw motion:

$$\begin{aligned}
 -I_Z \dot{r} = & l_f (F_{y1} \cos \delta_f + F_{y2} \cos \delta_f + F_{x1} \sin \delta_f + F_{x2} \sin \delta_f) \\
 & -l_r (F_{y3} + F_{y4}) + M_z
 \end{aligned} \quad (39)$$

Lateral acceleration:

$$A_y = v(r + \dot{\beta}) \quad (40)$$

Output sideslip:

$$\dot{\beta} = \frac{1}{mv} \begin{bmatrix} \cos \beta (\cos \delta_f (F_{x1} + F_{x2}) - \sin \delta_f (F_{y1} + F_{y2})) \\ -\sin \beta (\sin \delta_f (F_{x1} + F_{x2}) - \sin \delta_f (F_{y1} + F_{y2})) - r \end{bmatrix} \quad (41)$$

Combining ride and handling models produces a 14-DOF system (figure 19) that includes all previously mentioned parameters. Setiawan, et al. (2009) details the full equations of motion.

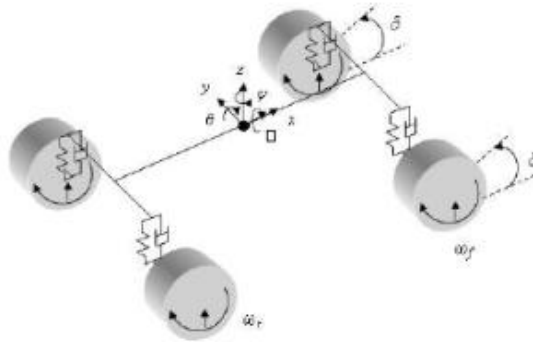


Figure 19: 14-DOF Vehicle Ride and Handling Model (Setiawan, et al., 2009).

3.3 Software Modelling

Figures 20 and 21 illustrate different methods for vehicle dynamics modelling within Simulink and VI-grade. Figure 20 (left) shows a typical Simulink subsystem for a full ride model, adaptable to a 14-DOF full car model. Figure 20 (right) demonstrates the MATLAB built-in vehicle dynamics block set, which includes blocks for differentials and various vehicle dynamic models. This add-ons suitability is uncertain due to the lack of required vehicle data and limited adjustability in differentials; therefore it will not be used in this project. Figure 21 (left) highlights MATLAB's multibody add-on for incorporating suspension kinematics (Blundell & Harty, 2015) and modelling differential gears, which can also be modelled mathematically using standard Simulink. The downside to multibody systems is processing power required for complex systems, this is where parametric systems are beneficial. Although not shown, Simscape Driveline can model various differentials for longitudinal dynamics only (MathWorks (3), n.d.), providing an initial understanding of differential locking effects and torques. This project will use standard Simulink modelling, Simscape Driveline, multibody systems, and VI-Grade (parametric based) methods (figure 21, right). Running co-simulations with Simulink and VI-CarRealTime will benefit the validation and modelling of a YSC (VI-Grade, 2020).

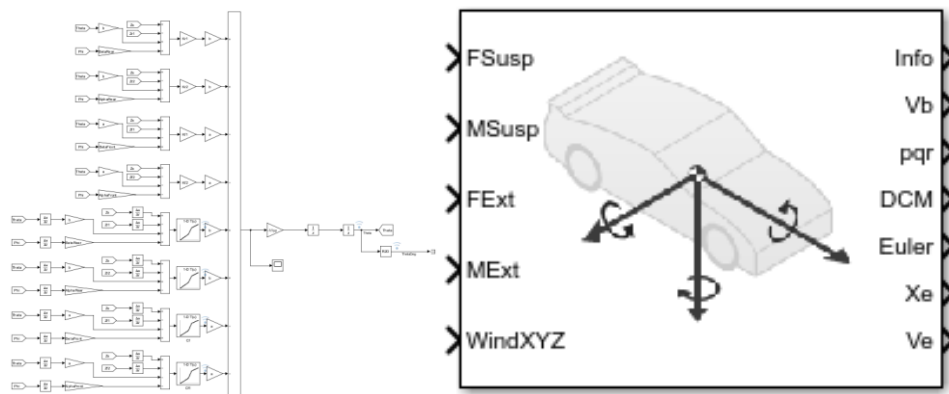


Figure 20: (Left) Standard Simulink Modelling, (Right) Vehicle Dynamics Blockset (MathWorks (2), n.d).

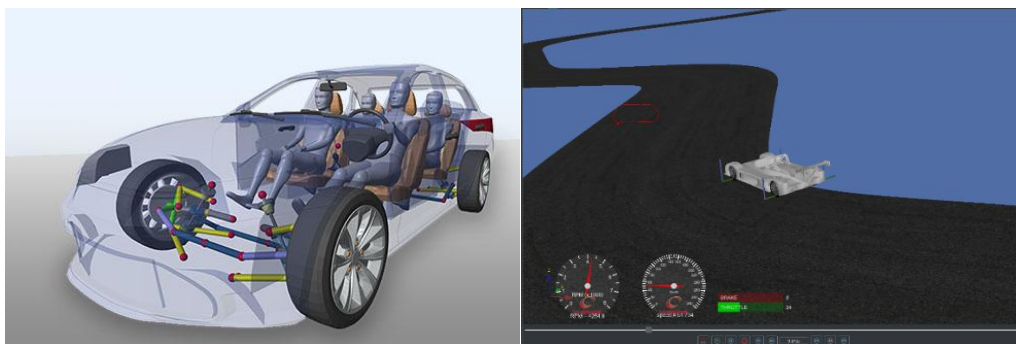


Figure 21: Vehicle Dynamics Modelling (Left) Simulink Multibody (Strässle, 2025), (Right) Vi-CarRealTime.

4 Results Collection and Analysis

The primary data type for this project is quantitative. Key parameters include yaw rate, tire slip and forces, torque distribution, wheel speeds, locking effect and torque, and vehicle velocity (OptimumG, 2010). Data from the simulation software will be imported into MATLAB for post-processing and graphical results generation. Graphs will assist in analysing vehicle handling and ride characteristics resulting from different LSD devices.

During post-processing, error estimates will be calculated/approximated, and data filtering and synchronization will be applied for comparable results. Performance metrics for evaluating the devices include locking effect and characteristics, angular velocities of drive wheels, vehicle yaw rates, torque distribution, and vehicle velocity during test manoeuvres.

Statistical analysis methods, such as mean (equation 42), absolute mean (equation 43), and standard deviation (equation 44) of test maneuver results (Rumsey, 2016), will be used to compare vehicle dynamic characteristics and data spread. Time series analysis, with data collected at specific intervals, will provide a better understanding of the differential implications on handling at critical time stamps and random timings.

$$\bar{x} = \frac{\sum x}{n} \quad (42)$$

$$|\bar{x}| = \frac{\sum |x|}{n} \quad (43)$$

$$\sigma = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n}} \quad (44)$$

Model validation is required throughout the project to ensure result accuracy and reliability. Validation will use published literature from real-world experimental data and simulated results, including comparison of result trend lines, data correlation values, and vehicle response behaviour. Potential problems may include errors due to model inaccuracies, incorrect parameters, and assumptions.

5 Project Planning

A project plan timeline is demonstrated in figure 22:

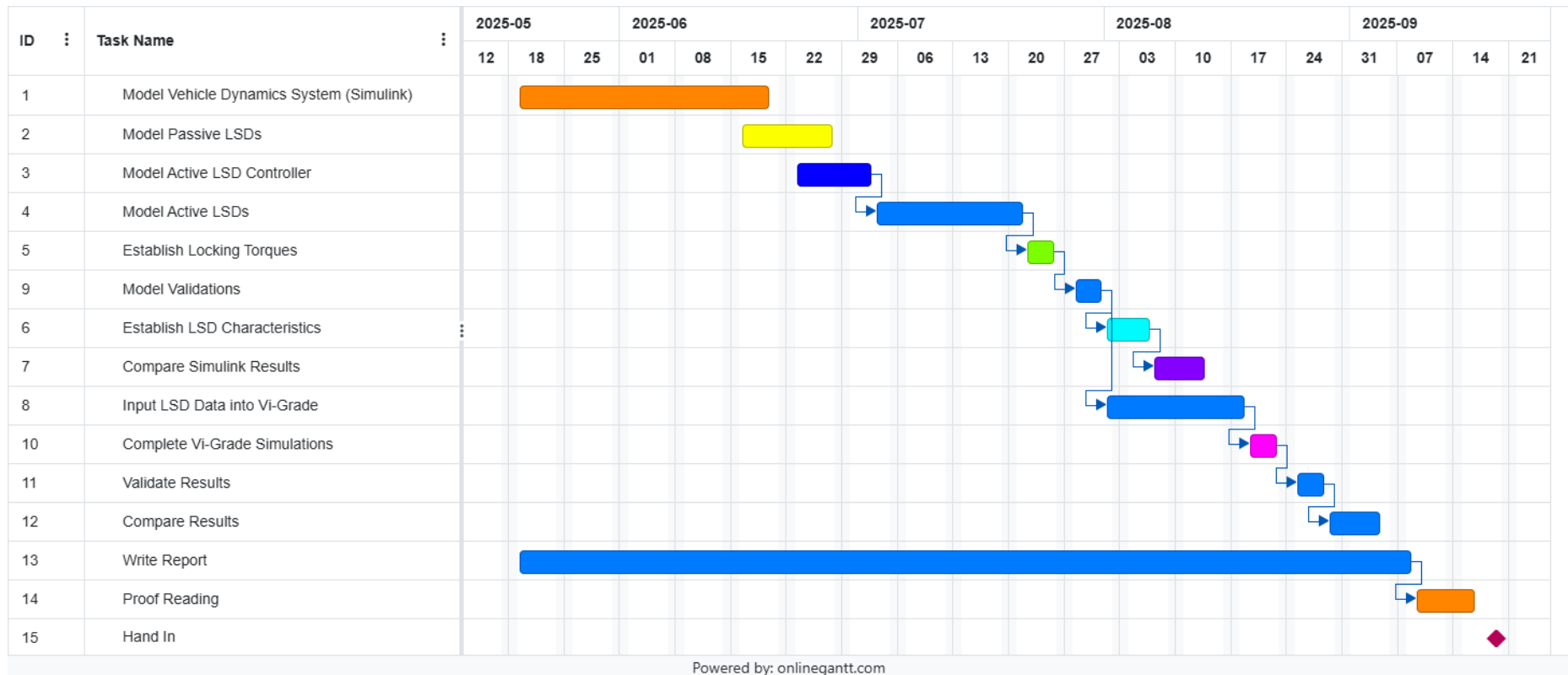


Figure 22: Project Plan Gantt Chart.

6 Evaluation

This initial literature review shows the numerous variants of both active and passive LSD differentials. Although there is existing literature detailing schematics and equations of specific LSD types, few compare these devices in simulated and real-world applications. The purpose of this review is to compile and establish current knowledge which includes: older, current, and new technologies, to provide insight into the operation and workings of a wide range of these devices. Future developments aim to model virtual testing systems to compare the characteristics of these devices, offering a comprehensive analysis for the motorsport and automotive industry. This will help consumers understand these device operations and vehicle dynamics effects, enabling them to select a suitable differential for their needs.

This project will use built-in MATLAB Simscape Driveline blocks to interpret initial longitudinal characteristics and locking effects of LSDs. A complex vehicle dynamic system using multibody mechanics and mathematical gear models, alongside a separate 'standard' Simulink 14-DOF system, will both be used to model cornering and handling implications. Using Vi-CarRealTime, performance tests/lap time simulations for each device can be completed, incorporating advanced driver-in-the-loop controllers to evaluate controllability and overall performance impacts.

The comprehensive project plan and methodologies study will help ensure the project stays on track for completion, no aspects are neglected and key aims and objectives are met. Analysis plans based on published literature results and appropriate statistical methods will enable a detailed and effective evaluation and comparison of differential operation, performance, and effects.

7 Reference List

Abd elhafiz, M. & Metered, H., 2018. *Performance Evaluation of Magnetorheological Limited Slip Differential for Automotive Applications*, Warrendale: SAE Technical Paper.

Allen, J., 2009. *Four-Wheeler's Bible*. United States: MBI Pub. Company.

Ankinovich, G., Verzhbitski, A. & Antonyan, A., 2020. Mathematical Modelling of Torque Vectoring Differentials. *IOP conference series. Materials Science and Engineering*, 820(1).

Ansys, 2024. *What is an EV Powertrain?*. [Online]
Available at: <https://www.ansys.com/en-gb/simulation-topics/what-is-an-ev-powertrain>
[Accessed 17 February 2025].

Aripin, M. et al., 2014. A Review of Active Yaw Control System for Vehicle Handling and Stability Enhancement. *International Journal of Vehicular Technology*.

AuburnGear, n.d.. *New Grip-N-Loc Limited Slip Differential: Built to Exceed your Expectations..*
[Online]
Available at: <https://auburngeardiffs.com/grip-n-loc/>
[Accessed 19 February 2025].

Auburn, n.d.. *Pro Series Differentials*. [Online]
Available at: <https://auburngeardiffs.com/pro-series-main/>
[Accessed 21 February 2025].

Autodata Training, 2019. *Youtube: Limited Slip Differentials Training Module Trailer*. [Online]
Available at: <https://www.youtube.com/watch?v=jp5KR-tf0Ic>
[Accessed 21 February 2025].

AutoPivot, 2023. *Youtube: Drive Configurations Explained AWD, 4WD, RWD, FWD which do you Prefer?*. [Online]
Available at: <https://www.youtube.com/watch?v=ILFYBH4ciqI>
[Accessed 17 February 2025].

Bhatt, G., Manoharan, K., Chauhan, P. & Bhattacharya, S., 2019. MEMS Sensors for Automotive Applications: A Review. In: *Sensors for Automotive and Aerospace Applications*. Germany: Springer Nature Singapore, pp. 223-239.

Blundell, M. & Harty, D., 2015. *The Multibody Systems Approach to Vehicle Dynamics*. 2nd ed. Coventry: Elsevier Science & Technology.

- Crolla, D., 2015. *Encyclopedia of Automotive Engineering*. United Kingdom: Wiley.
- Crolla, D. & Mashadi, B., 2011. *Vehicle Powertrain Systems*. Germany: Wiley.
- Crouse, W. & Anglin, D., 1983. *Automotive Manual Transmission and Power Trains*. 6th ed. United Kingdom: McGraw-Hill.
- Cypress, C., 2007. *S-AWC Super All Wheel Control - AYC (Active Yaw Control)*. [Online]
Available at: <https://media.mitsubishicars.com/en-US/releases/release-6db1181cb9e9c13c8ca591324b06fc60-?form=MG0AV3>
[Accessed 1 March 2025].
- Eaton, 2016. *Youtube: Eaton IntelliTrac electronic limited slip differential rear wheel drive features*. [Online]
Available at: <https://www.youtube.com/watch?v=GYUdFWzwgeM>
[Accessed 24 February 2025].
- Eaton, 2025. *What is a Locking Differential?*. [Online]
Available at: <https://www.eaton.com/gb/en-gb/products/differentials-traction-control/locking-differentials.html>
[Accessed 20 February 2025].
- Forstinger, M., Bauer, R. & Hofer, A., 2015. Modelling and Simulation of Passive Limited-slip Differentials. *IFAC-PapersOnLine*, 48(1), pp. 502-507.
- Frömmig, L., 2023. *Basic Course in Race Car Technology: Introduction to the Interaction of Tires, Chassis, Aerodynamics, Differential Locks and Frame*. Germany: Springer Fachmedien Wiesbaden.
- Guglielmino, E. et al., 2008. *Semi-active Suspension Control: Improved Vehicle Ride and Road Friendliness*. Netherlands: Springer London.
- Guiggiani, M., 2018. *The Science of Vehicle Dynamics Handling, Braking, and Ride of Road and Race Cars*. Germany: Springer International Publishing.
- Jalali, K., Uchida, T., Lambert, S. & McPhee, J., 2013. Development of an Advanced Torque Vectoring Control System for an Electric Vehicle with In-Wheel Motors using Soft Computing Techniques. *SAE International Journal of Alternative Powertrains*, 2(2), pp. 261-278.
- Jin, L. et al., 2021. Calculation and Verification of the Real-Time Working Characteristics of a Viscous Coupling. *Applied Science*, 11(3).

Kumar, S. & Medhavi, A., 2022. Modeling and Analysis of Active Full Vehicle Suspension Model Optimized Using the Advanced Fuzzy Logic Controller. *International Journal of Acoustics and Vibration*, 27(1), pp. 26-36.

MathWorks (2), n.d. *Help Centre: Vehicle Body 6DOF*. [Online]
Available at: <https://uk.mathworks.com/help/vdynblks/ref/vehiclebody6dof.html>
[Accessed 28 February 2025].

MathWorks (3), n.d.. *Matlab Help Centre: Tire-Road Interaction (Magic Formula)*. [Online]
Available at: <https://uk.mathworks.com/help/sdl/ref/tireroadinteractionmagicformula.html>
[Accessed 24 February 2025].

MathWorks, n.d.. *MATLAB Help Centre: Tire (Magic Formula)*. [Online]
Available at: <https://uk.mathworks.com/help/sdl/ref/tiremagicformula.html>
[Accessed 28 February 2025].

OptimumG, 2010. *Differential Behavior*, s.l.: OptimumG Vehicle Dynamics Solutions.

Pacejka, H. & Besselink, I., 2012. *Tire and Vehicle Dynamics*. 3rd ed. Amsterdam: Elsevier/BH.

Palazzolo, J., 2009. *High-Performance Differentials, Axles & Drivelines*. United Kingdom: CarTech.

Polyphony Digital Inc, 2025. *Beyond the ApexThe Gran Turismo Magazine*. [Online]
Available at: <https://www.gran-turismo.com/tr/gt7/apex/mechanics/07>
[Accessed 20 February 2025].

Radzevich, S., 2021. *Recent Advances in Gearing Scientific Theory and Applications*. Germany: Springer International Publishing.

Rajamani, R., 2011. *Vehicle Dynamics and Control*. United Kingdom: Springer.

Reimpell, J., Stoll, H. & Betzler, J., 2001. *The Automotive Chassis: Engineering Principles*. United Kingdom: Elsevier Science.

Rubin, D. & Arogeti, S., 2015. Vehicle Yaw Stability Control using Active Limited-Slip Differential via Model Predictive Control Methods. *International Journal of Vehicle Mechanics and Mobility*, 53(9), pp. 1315-1330.

Rumsey, D., 2016. *Statistics For Dummies*. Italy: Wiley.

Russo, R., Terzo, M. & Strano, S., 2016. Enhancement of vehicle dynamics via an innovative magnetorheological fluid limited slip differential. *Mechanical Systems and Signals Processing*, Volume 70-71, pp. 1193-1208.

Setiawan, J., Safarudin, M. & Singh, A., 2009. *Modeling, Simulation and Validation of 14 DOF Full Vehicle Model*, Semarang: Diponegoro University.

Sharma, C. & Purohit, K., 2004. *Design of Machine Elements*. India: PHI Learning.

Shepard, L., 2013. *Jeep, Dana and Chrysler Differentials: How to Rebuild the 8-1/4, 8-3/4, Dana 44 and 60 and AMC 20*. United States: CarTech Incorporated.

Shih, S. & Bowerman, W., 2002. *An Evaluation of Torque Bias and Efficiency of Torsen Differential*, s.l.: SAE Technical Paper.

Smith, J., 2013. *Smith's Fundamentals of Motorsport Engineering*. United Kingdom: Oxford Nelson Thornes Ltd.

Stone, R. & Ball, J., 2004. *Automotive Engineering Fundamentals*. United Kingdom: SAE International.

Strässle, T., 2025. *Simscape Vehicle Templates*. [Online]
Available at: <https://www.speedgoat.com/knowledge-center/reference-examples/simscape-vehicle-templates>
[Accessed 26 February 2025].

Strong, M., 2011. *American High-Performance Differentials*. Missouri: Racing Net Source.

Tremlett, A. et al., 2014. Quasi-Steady-State Linearisation of the Racing Vehicle Acceleration Envelope: a Limited Slip Differential Example. *Vehicle System Dynamics*, 52(11), p. 1416–1442.

Trubachev, E., 2025. *Theory and Practice of Gearing and Transmissions*. 172 ed. n.p: Springer Nature.

Tudor, T., 2024. *Derivation of 7-DOF Equations for Full Ride Model [Lecture Notes]*, s.l.: University of Wales Trinity Saint David in Advanced Motorsport Dynamics.

Ulusoy, M., 2018. *MathWorks: Why Use MPC? | Understanding Model Predictive Control, Part 1*. [Online]
Available at: <https://uk.mathworks.com/videos/understanding-model-predictive-control-part-1-why-use-mpc--1526484715269.html>
[Accessed 23 February 2025].

VanGelder, K., 2022. *Fundamentals of Automotive Technology*. United States: Jones & Bartlett Learning.

VI-Grade, 2020. *VI-CarRealTime 20.1 Documentation*. s.l.: VI-Grade GmbH.

Zhao, Y. & Lin, F., 2024. *Vehicle System Dynamics: Theoretical Modeling and Application*. Singapore: Springer Nature Singapore.

Zhou, W. et al., 2023. Cost, Range Anxiety and Future Electricity Supply: A Review of how Today's Technology Trends may Influence the Future Uptake of BEV's. *Renewable and Sustainable Energy Reviews*, 173(3).

Appendix A

The differential is an important part of the vehicle drivetrain. There are four types of drivetrain layouts available for a vehicle, these being: AWD, 4WD, FWD and RWD. The number of differentials present in a vehicle is dependent on the powertrain and drivetrain configuration (Ansys, 2024). Figures 23 and 24 demonstrate the differences between presence/absence of differentials based on these configurations. To summarise these principles, an internal combustion engine requires a differential regardless of drivetrain configuration. BEVs and hybrid vehicles, however, are dependent on the drivetrain system chosen, demonstrated in figure 24. When a singular motor is used to power the front or rear axle, a differential is required; if instead an individual motor is applied to each desired drive wheel, a differential is then no longer required (Zhou, et al., 2023). Due to the numerous different powertrains available and their drivetrain requirements, and this project focus on investigating vehicle dynamic effects of differentials. AWD, RWD, FWD and 4WD configurations will be focused for internal combustion engines (ICE).

Another important factor to consider when investigating drivetrain configurations is drivetrain losses. Each component within the drivetrain contributes to the loss of power delivered to the drive wheels from the powertrain due to friction and rotational inertia. The main contributing aspects to driveline losses are ICE's, gearboxes and differentials. The effects of drivetrain losses depend uniquely on each drivetrain component and drive cycle, however, rough estimates for manual transmission drivetrains are 87-91% and automatics as low as 80% (Crolla & Mashadi, 2011). For differentials, the key contributing factors are lubrication condition, gear: temperatures, geometry and type (Radzevich, 2021); this therefore also contributes to a differential's performance.

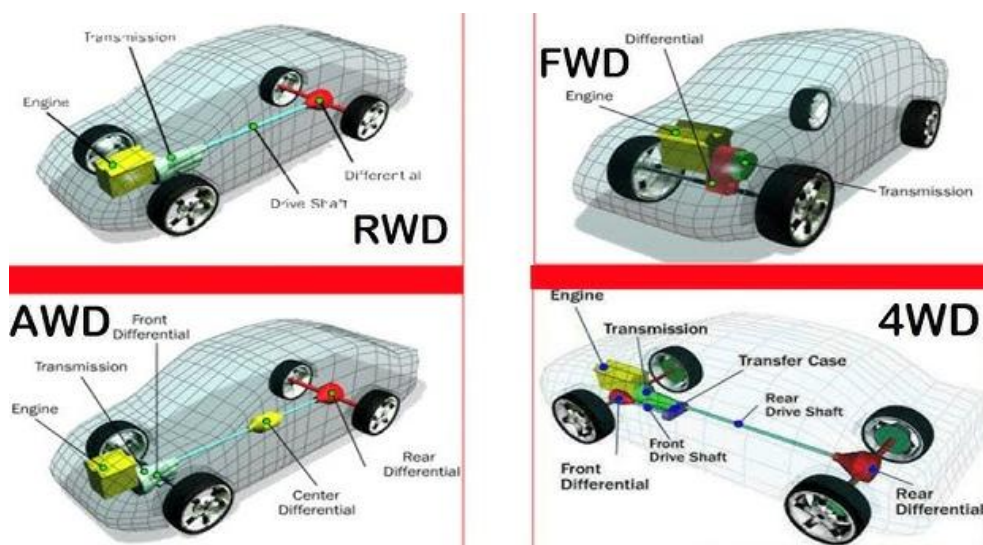


Figure 23: ICE Drivetrain Layouts (AutoPivot, 2023).

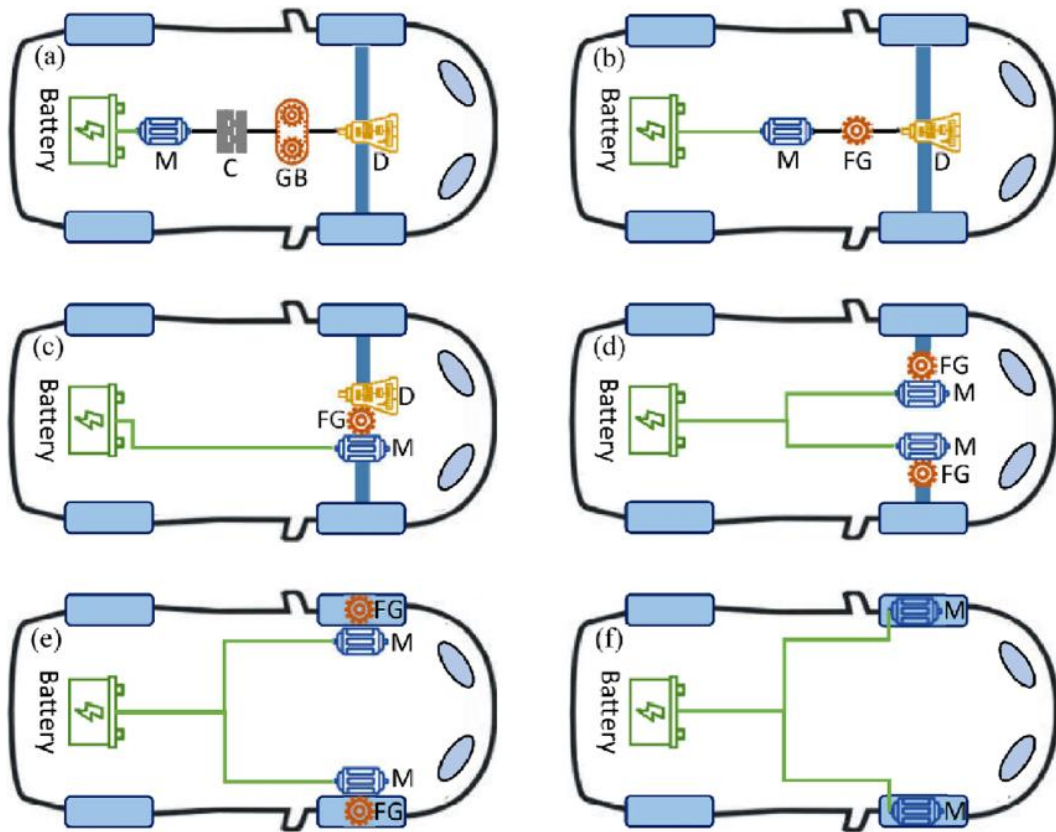


Figure 24: BEV Drivetrain Architecture. (a) ICE Based, (b) Fixed Gear no Clutch, (c) Integrated Fixed Gear, Motor and Differential, (d) Dual Motors with Fixed Gears, (e) Dual Motors, Geared in Wheel Drives, (f) Gearless Dual Motor In-Wheel Drive (Zhou, et al., 2023).

Appendix B

A locking differential can operate in many different methods depending on the control system (automatic or driver controlled), control module programming (conditions required to unlock or lock the differential), mechanical construction/design type (for example conventional locking differential or Detroit Locker, figure 25) and more (Eaton, 2025). A more conventional approach for the automatic locking differential is to act as an open differential until tyre slip is detected, usually only at low speeds. An alternative to this is the Detroit Locker which is considered locked normally and unlocking during cornering. When the differential is locked, no variation in wheel speeds is permitted on the drive axle. This can be advantageous for certain applications (such as off-roading) but has limited benefits in circuit racing or general automotive vehicles. However, when considering a permanently locked or spool axle, the main disadvantage of this system during circuit racing or automotive applications is the prevention of driven axle wheels from rotating at different angular velocities during cornering (Trubachev, 2025). The result of this differential can cause binding or skipping of a tyre during cornering which can lead to premature wear and tyre damage. The relationship between angular velocities is demonstrated in equation 45. The torque distribution in a locked differential varies based on the applied torque from the engine and tyre slip values; for instance, if applied torque results in a tyre force that is less than both tyres slipping conditions equation 46 is true. However, if one tyre's force is greater than the maximum tyre force without slipping, equation 47 is true, and torque is transferred based on the traction difference of the two tyres and available grip.

$$\omega_{Left} = \omega_{Right} \quad (45)$$

$$T_{Left} = T_{Right} \quad (46)$$

$$T_{Left} \neq T_{Right} \quad (47)$$



Figure 25: Detroit Locker Differential (Strong, 2011).

Appendix C

An active differential utilises a combination of sensors, electronic control units and actuators to control the active LSD. The sensors used in these systems are most often: accelerometers, wheel speed, throttle position sensor and steering angle sensors (Bhatt, et al., 2019). The data from these sensors is then processed within a control system calculating an output value for an actuator located at the differential (figure 26). The electronic control unit consists of a YSC which can include a model predictive control (MPC) system integrated to control vehicle response based on real world constraints of the vehicle, this approach is suitable for both academic and industry applications (Rubin & Arogeti, 2015). The type of YSC system applied (figure 27) depends on the differential in operation, the vehicle's application and the overall requirements. For example, a motorsport application would significantly benefit from a twin clutch torque transfer differential over a controlled LSD due to the capabilities of bi-directional torque transfer across an axle improving cornering characteristics whilst also maintaining vehicle acceleration (Rajamani, 2011).

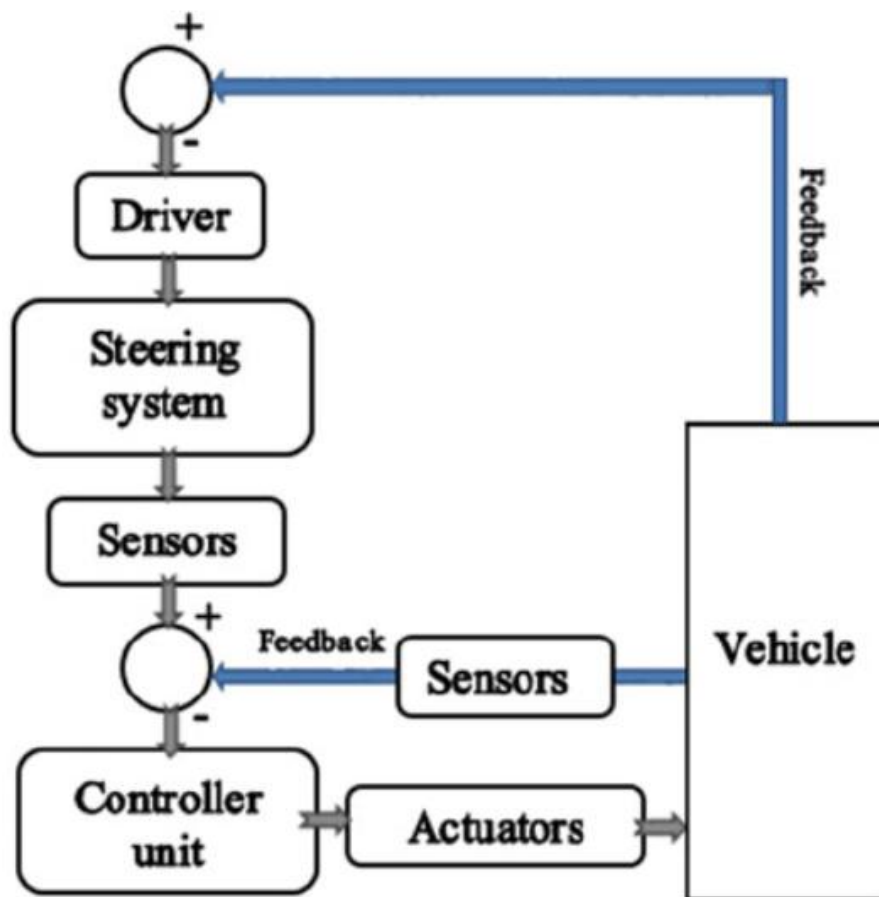


Figure 26: Yaw Stability Control System Architectural Flow Chart (Bhatt, et al., 2019).

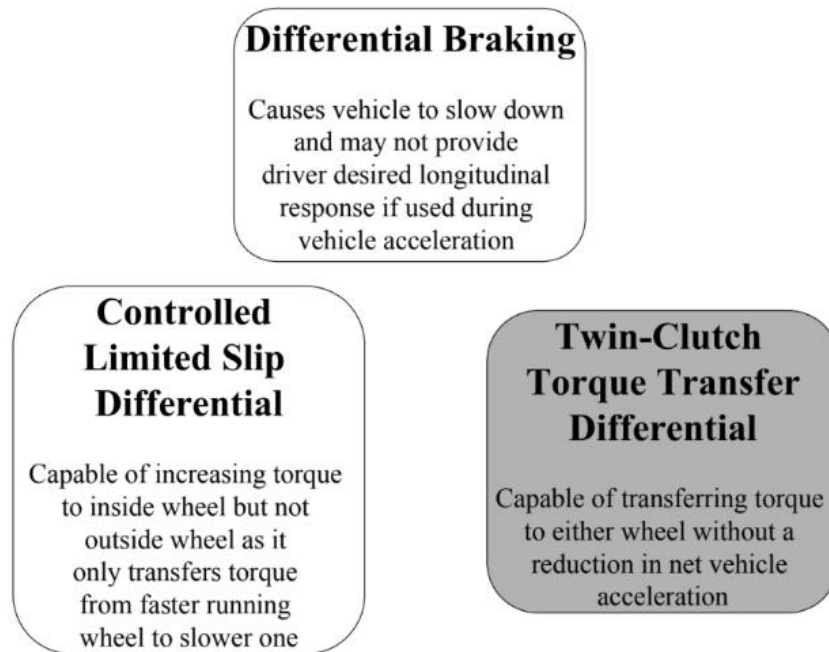


Figure 27: Yaw Stability Control System Types (Rajamani, 2011).

To construct and model the outputs from the control system, the sensor reference data is processed within the controller handling model. This output, along with the target signal, can be input into a proportional-integral-derivative (PID) controller. By also implementing an MPC system (figure 28), a more accurate controller (YSC) can be achieved because each individual parameter/signal reference data can influence other PID parameters within the same time iteration (Ulusoy, 2018). An MPC also benefits from the ability to add constraints to improve model realism, such as implementing a limitation on the vehicle's maximum or minimum achievable longitudinal acceleration rate.

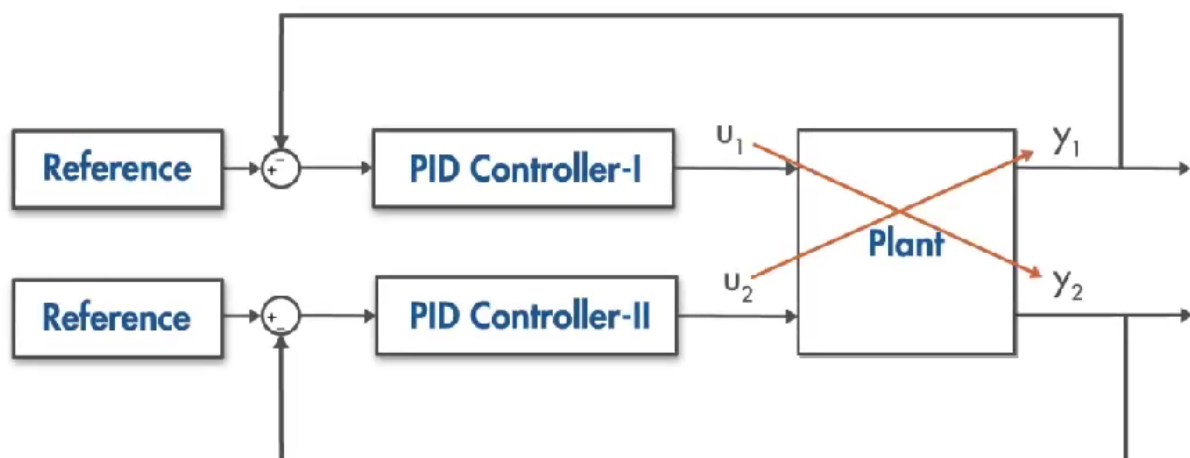


Figure 28: Multi-Input and Multi-Output Handling of MPC (Ulusoy, 2018).