

Analytical Methods Assignment 12.01.2025

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Question One:

i) $x = -\frac{41}{14} \text{ or } -2.93$

ii) $x = \frac{5 \log(2)}{\log(3)} \text{ or } 3.15$

iii) $x = \frac{-31}{7} \text{ or } 4.43$

iv) $x = \frac{345}{257} \text{ or } 1.34$

Question Two:

$$x = -53.68 \quad y = 145.26 \quad z = 117.89$$

Question Three:

$$x = 1.68 \quad \& \quad x = -7.35$$

Question Four:

i) $\frac{dy}{dx} = 15x^2 - 14x + 3$

ii) $\frac{dy}{dx} = -5x^{-6}$

iii) $\frac{dy}{dx} = \frac{1}{2}e^x - \frac{1}{6}\sin\frac{x}{6} + 2\cos 2x$

iv) $\frac{dy}{dx} = 24x(x^7 + 4)^5(3x^2 - 1)^3 + 35x^6(x^7 + 4)^4(3x^2 - 1)^4$

v)
$$\frac{e^x(5(5x^2+3x+1)^4(10x+3)) - ((5x^2+3x+1)^5)e^x}{(e^x)^2}$$

vi) $(5x - 1)^{\frac{1}{2}}(-\frac{1}{2}(8x + 5)^{-\frac{3}{2}} * 8) + (8x + 5)^{-\frac{1}{2}}(\frac{1}{2}(5x - 1)^{-\frac{1}{2}} * 5$

vii) $5\left(10x + (x^5 + 1)^{\frac{1}{3}}\right)^4 * \left(10 + \frac{5x^4}{3(x^5+1)^{\frac{2}{3}}}\right)$

Question Five:

The second derivative is Acceleration which is velocity over time.

The third derivative is Jerk which is the rate of change of acceleration with respect to time, it's a sudden acceleration or deceleration.

The fourth derivative is Snap which is the rate of change of Jerk with respect to time, is like Whiplash a sudden acceleration followed by instant deceleration.

The fifth derivative is Crackle which is the rate of change of Snap with respect to time.

The Sixth derivative is Pop which is the rate of change of Crackle with respect to time.

Question Six:

- i) $x = 12.84^\circ$ & $x = 167.16^\circ$
- ii) $x = 41.84^\circ, 143.64^\circ, 216.39^\circ, 318.52^\circ$

Question Seven:

- i) $AB = 3.3826\text{cm}$ $AC = 6.3826\text{cm}$ $BC = 8.3826$
- ii) $\text{Area} = 9.618\text{cm}^2$

Please see all workings below

Maths Formative Assessment workings out

Question One:

(i)

$$(x+6)^2 + 2x = x^2 - 5$$

$$(x+6)(x+6) + 2x = x^2 - 5$$

$$x^2 + 12x + 36 + 2x = x^2 - 5$$

$$x^2 + 14x + 41 = x^2$$

$$14x + 41 = \cancel{x^2}^0$$

$$14x = -41$$

$$x = \frac{-41}{14}$$

$$x = \frac{-41}{14} \text{ or } -2.93$$

~~Question Two:~~

(ii) $2^{x+5} = 6^x$

$$\log(2^{x+5}) = \log(6^x)$$

$$(x+5) \log(2) = \cancel{x} \log(6)$$

$$x \log(2) + 5 \log(2) = x \log 6$$

$$x \log(2) - x \log(6) = -5 \log(2)$$

$$x(\log(2) - \log(6)) = -5 \log(2)$$

$$x \log\left(\frac{2}{6}\right) = -5 \log(2)$$

$$x = \frac{-5 \log(2)}{\log\left(\frac{2}{6}\right)}$$

$$x = \frac{5 \log(2)}{\log\left(\frac{4}{3}\right)}$$

$$x = \frac{5 \log(2)}{\log(3)} \text{ or } 3.15$$

iii)

$$2 \log(x+3) = \log(x+5) + \log(x+8)$$

$$\log((x+3)^2) = \log((x+5)(x+8))$$

$$(x+3)^2 = (x+5)(x+8)$$

$$x^2 + 6x + 9 = x^2 + 13x + 40$$

$$6x + 9 = 13x + 40$$

$$-31 = 7x$$

$$x = \frac{-31}{7} \text{ or } -4.43$$

$$x = \frac{-31}{7} \text{ or } -4.43$$

iv)

$$x - \frac{1 - \frac{6x}{5}}{3} = \frac{4 - \frac{x}{3}}{12} + \frac{5}{4}$$

$$x - \frac{1 - \frac{6x}{5}}{3} = \frac{5 - \frac{6x}{5}}{3} = \frac{5 - 6x}{15}$$

$$\frac{4 - \frac{x}{3}}{12} = \frac{\frac{12}{3} - \frac{x}{3}}{12} = \frac{12 - x}{36}$$

$$x - \frac{5 - 6x}{15} = \frac{12 - x}{36} + \frac{5}{4}$$

$$180x - 12(5 - 6x) = 5(12 - x) + 45$$

$$180x - 60 + 72x = 60 - 5x + 45$$

$$252x - 60 = 105 - 5x$$

$$257x = 165$$

$$x = \frac{165}{257}$$

$$x = \frac{165}{257} \text{ or } 0.642$$

$$x = \frac{345}{257} \text{ or } 1.34$$

Question Two:

1) $2x + 4y - 3z = 120$

2) $5x + 2y + z = 140$

3) $x - 3y + 5z = 100$

$$z = 140 - 5x - 2y$$

Eq 1:

$$2x + 4y - 3(140 - 5x - 2y) = 120$$

$$2x + 4y - 420 + 15x + 6y = 120$$

$$17x + 10y = 540$$

Eq 2:

$$x - 3y + 5(140 - 5x - 2y) = 100$$

$$x - 3y + 700 - 25x - 10y = 100$$

$$-24x - 13y = -600$$

$$17x + 10y = 540$$

$$\text{and } -24x - 13y = -600$$

$$13(17x + 10y) = 13(540)$$

$$10(-24x - 13y) = 10(-600)$$

$$221x + 130y = 7020$$

$$240x + 130y = 6000$$

$$(221x + 130y) - (240x + 130y) = 7020 - 6000$$

$$-19x = 1020$$

$$x = \frac{-1020}{19} = -53.68$$

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$$17(-53.68) + 10y = 540$$

$$-912.56 + 10y = 540$$

$$10y = 1452.56$$

$$y = 145.26$$

$$z = 140 - 5(-53.68) - 2(145.26)$$

$$z = 140 + 268.4 - 290.52 = 117.88$$

$$x = -53.68$$

$$y = 145.26$$

$$z = 117.88$$

$$117.89$$

Question Three

$$0.6x^2 + 3.4x - 7.4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3.4 \pm \sqrt{3.4^2 - 4 \times 0.6 \times -7.4}}{2 \times 0.6}$$

~~$$x = \frac{-3.4 \pm \sqrt{3.4^2 - 4 \times 0.6 \times -7.4}}{2 \times 0.6}$$~~

$$x = \frac{-17 + \sqrt{733}}{6} \text{ or } +68 \text{ } 1.68$$

$$x = \frac{-17 - \sqrt{733}}{6} \text{ or } -7.35$$

$$x = 1.68$$

$$x = -7.35$$

Question Four

i) $y = 5x^3 - 7x^2 + 3x - 1$
 $15x^2 - 14x + 3$

$$\frac{dy}{dx} = 15x^2 - 14x + 3$$

ii) $y = -\frac{1}{x^5}$

$$-x^{-5}$$

$$--5x^{-6}$$

$$\frac{dy}{dx} = 5x^{-6}$$

iii) $y = e^{\frac{x}{2}} + \cos \frac{x}{6} + \sin 2x$

$$-\sin \frac{x}{6} \times \frac{1}{6} = -\frac{\sin \frac{x}{6}}{6}$$

$$2 \cos 2x$$

$$\frac{dy}{dx} = \frac{1}{2} e^x - \frac{\sin \frac{x}{6}}{6} + 2 \cos 2x$$

iv) $(x^7 + 4)^5 (3x^2 - 1)^4$

$$u = (x^7 + 4)^5$$

$$u' = 5(x^7 + 4)^4 \times 7x^6$$

$$v = (3x^2 - 1)^4$$

$$v' = 4(3x^2 - 1)^3 \times 6x$$

$$(x^7 + 4)^5 \times 4(3x^2 - 1)^3 \times 6x + (3x^2 - 1)^4 \times 5(x^7 + 4)^4 \times 7x^6$$

$$\frac{dy}{dx} = 24x(x^7 + 4)^5(3x^2 - 1)^3 + 35x^6(x^7 + 4)^4(3x^2 - 1)^4$$

v) $y = \frac{(5x^2 + 3x + 1)^5}{e^x}$

$$u = (5x^2 + 3x + 1)^5 \quad u' = 5(5x^2 + 3x + 1)^4 \times (10x + 3)$$

$$v = e^x$$

$$v' = e^x$$

$$\frac{dy}{dx} = \frac{e^x (5(5x^2 + 3x + 1)^4 (10x + 3)) - (5x^2 + 3x + 1)^5 e^x}{(e^x)^2}$$

Q4 continued

$$\text{Vi) } y = \sqrt{\frac{5x-1}{8x+5}}$$

$$\text{or } y = (5x-1)^{\frac{1}{2}} (8x+5)^{-\frac{1}{2}}$$

$$u = (5x-1)^{\frac{1}{2}}$$

$$u' = \frac{1}{2} (5x-1)^{-\frac{1}{2}} \times 5$$

$$v = (8x+5)^{-\frac{1}{2}}$$

$$v' = -\frac{1}{2} (8x+5)^{-\frac{3}{2}} \times 8$$

$$\frac{dy}{dx} = (5x-1)^{\frac{1}{2}} \left(-\frac{1}{2} (8x+5)^{-\frac{3}{2}} \times 8 \right) + (8x+5)^{-\frac{1}{2}} \left(\frac{1}{2} (5x-1)^{-\frac{1}{2}} \times 5 \right)$$

$$\text{Vii) } y = (10x + \sqrt[3]{x^5+1})^5$$

$$u = 10x + \sqrt[3]{x^5+1}$$

$$y = u^5$$

$$\frac{dy}{dx} = 5u^4 \times \frac{du}{dx}$$

$$5(10x + (x^5+1)^{\frac{1}{3}})^4$$

$$\frac{d}{dx} 10x \Rightarrow 10$$

$$\frac{d}{dx} \sqrt[3]{x^5+1} \Rightarrow (x^5+1)^{-\frac{2}{3}} \times \frac{1}{3} (x^5+1)^{-\frac{2}{3}} \times (5x^4)$$

$$(x^5+1)^{-\frac{2}{3}} = 5x^4$$

$$\Rightarrow \frac{1}{3} (x^5+1)^{-\frac{2}{3}} \times 5x^4 \Rightarrow \frac{5x^4}{3(x^5+1)^{\frac{2}{3}}}$$

$$\frac{du}{dx} = 10 + \frac{5x^4}{3(x^5+1)^{\frac{2}{3}}}$$

$$\frac{dy}{dx} = 5(10x + (x^5+1)^{\frac{1}{3}})^4 \times \left(10 + \frac{5x^4}{3(x^5+1)^{\frac{2}{3}}} \right)$$

Question Six

$$i) 9 \sin x = 2 \quad \sin x = \frac{2}{9}$$

$$x = \sin^{-1}\left(\frac{2}{9}\right)$$

$$x = 12.84^\circ$$

$$180 - 12.84 = 167.16^\circ$$

$$x = 12.84^\circ \text{ and } x = 167.16^\circ$$

$$ii) 5 \sin^2 x - 0.3 \cos x = 2$$

$$5(1 - \cos^2 x) - 0.3 \cos x = 2$$

$$-5 \cos^2 x - 0.3 \cos x + 3 = 0$$

$$5 \cos^2 x + 0.3 \cos x - 3 = 0$$

$$\text{let } \cos x = y$$

$$5y^2 + 0.3y - 3 = 0$$

$$y = \frac{-0.3 \pm \sqrt{0.3^2 - 4 \times 5 \times -3}}{2 \times 5}$$

$$y = 0.745 \quad \text{and} \quad y = -0.805$$

$$\cos^{-1}(0.745) = 41.84^\circ$$

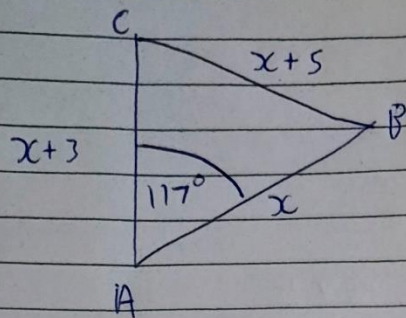
$$\cos^{-1}(-0.805) = 143.61^\circ$$

$$360 - 41.84 = 318.16^\circ$$

$$360 - 143.61 = 216.39^\circ$$

$$x = 41.84, 143.61, 216.39, 318.16$$

Question Seven



i) $a^2 = b^2 + c^2 - 2bc \cos(A)$

$$(x+5)^2 = (x+3)^2 + (x)^2 - 2x(x+3)(x) \cos(117)$$

$$x^2 + 10x + 25 = (x^2 + 6x + 9) + x^2 - 2x(x+3)(x) \cos(117)$$

$$-2(x+3)(x) \cos 117 = 0.908x^2 + 2.724x$$

$$-2x^2 + 6x \Rightarrow 2x^2 + 6x \rightarrow$$

$$x^2 + 10x + 25 = x^2 + 6x + 9 + x^2 + 0.908x^2 + 2.724x$$

$$x^2 + 10x + 25 = 2.908x^2 + 8.724x + 9$$

$$1.908x^2 - 1.724x - 16 = 0$$

$$x = 3.3826 \text{ or } x = -2.479$$

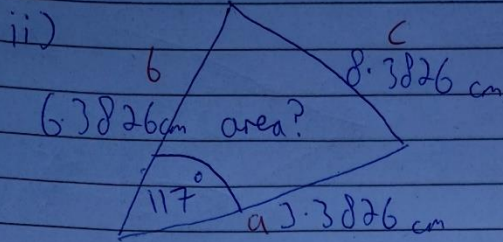
$x = 3.3826$ For x cannot be negative

$$AB = 3.3826 \text{ cm}$$

$$AC = 6.3826 \text{ cm}$$

$$BC = 8.3826 \text{ cm}$$

Q 7 Continued



$$\text{area} = \frac{1}{2} ab \sin(C)$$

$$\frac{1}{2} \times 3.3826 \times 6.3826 \times \sin 117 = 9.618 \text{ cm}^2$$