

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. **Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.**

Complete, fold and seal
(see note 2 below)

SURNAME:

FORENAME(S):

1	Student Number	2026635
2	Title of Examination Paper	Mathematics Exam
3	Module Code	ACDA4008
4	Date and Time of Examination	6/5/22 11:00
5	Examination Venue	FF005
6	Seat Number	
7	Number of Answer Books handed in	1

[illegible]

$$i) y = 6x - 2$$

$$\frac{dy}{dx} = 6 \times 1 x^{1-1} = 2$$

$$= 6$$

$$ii) y = 5x^3 + 17x^{1/2} - \frac{3}{2}$$

$$\frac{dy}{dx} = 15x^2 + 8.5x^{-1/2}$$

$$iii) y = \frac{3}{x^4} + 7x^2$$

$$= \frac{1}{3}x^{-4} + 7x^2$$

$$\frac{dy}{dx} = -\frac{4}{3}x^{-5} + 14x$$

$$iv) y = \sin(6x)$$

$$\frac{dy}{dx} = \cos(6x)$$

$$v) y = \cos(x)$$

$$\frac{dy}{dx} = -\sin(x)$$

$$vii) y = 2x^{-3} + 5x^2$$

$$\frac{dy}{dx} = -6x^{-4} + 10x$$

viii) $y = (\cos + \sin 2x)$

$$\frac{dy}{dx} = 1 + \cos 2x$$

b) $y = \cos 2x$

$$\frac{dy}{dx} = -\sin 2x$$

$$4 \sin(2x)$$

$$\frac{d^2y}{dx^2} = -\cos 2x$$

$$4 \cos(2x)$$

$$\frac{d^3y}{dx^3} = \sin 2x$$

$$8 \sin(2x)$$

$$\frac{d^4y}{dx^4} = \cos 2x$$

$$16 \cos(2x)$$

$$\frac{d^5y}{dx^5} = -\sin 2x$$

$$32 \sin(2x)$$

$$c) y = \frac{(2x+1)^2}{3x^2}$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

$$\frac{dy}{dx} = \frac{2x^{-1} - 6x}{9x^4}$$

$$= \frac{2x^{-1}}{9x^4} - \frac{6x}{9x^4}$$

$$= \frac{2}{9}x^{-5} - \frac{2}{3}x^{-3}$$

$$\frac{dy}{dx} \text{ when } x = -1$$

$$= \frac{2}{9}(-1)^{-5} - \frac{2}{3}(-1)^{-3}$$

$$= \frac{2}{9} \cdot \frac{4}{9}$$

$$\frac{2}{9} \cdot \frac{4}{9}$$

$$i) \int (3x^2 + 4x^3) dx$$

$$= \frac{3x^4}{4} + \frac{4x^4}{4} + C$$

$$= \frac{3x^4}{4} + x^4 + C = 0.75x^4 + x^4 + C$$

$$ii) \int (2x^{332} - 3x) dx$$

$$= \frac{2}{333} x^{333} - \frac{3}{2} x^2 + C$$

$$= 0.006 x^{333} - 1.5x^2 + C$$

$$iii) \int \left(\frac{2}{x} + 4 \right) dx$$

$$\int \left(\frac{1}{2} x^{-1} + 4 \right) dx$$

$$= \frac{1}{2} \ln x + 4x + C$$

$$= \ln \frac{1}{2} x + 4x + C$$

$$vi) \int (2x^2 + 1)(3x - 4) dx$$

$$\int (6x^3 - 8x^2 + 3x - 4) dx$$

$$= \frac{6}{4} x^4 - \frac{8}{3} x^3 + \frac{3}{2} x^2 - 4x + C$$

$$= 1.5x^4 - 2.6x^3 + 1.5x^2 - 4x + C$$

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$$vi) \int_3^6 (3x^2 + x^3) dx$$

$$= 3x^3 + \frac{1}{4}x^4 \Big|_3^6$$

$$\text{upper limit} = (6)^3 + \frac{1}{4}(6)^4$$

$$= 540$$

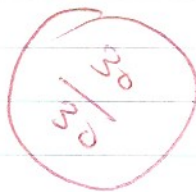
$$\text{lower limit} = (3)^3 + \frac{1}{4}(3)^3$$

$$= 47.25$$

$$\therefore 540 - 47.25 = 492.75$$

$$b) \int (2 \cos x)$$

$$= 2 \sin x + C$$



$$i) \left(\frac{1+i}{1-i} \right)^3$$

$$= \left(\frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^3$$

$$= \left(\frac{(1+i)(1+i)}{(1-i)(1-i)} \right)^3$$

$$= \left(\frac{1+i+i+i^2}{1-i-i+i^2} \right)^3$$

$$= \left(\frac{1+2i+(-1)}{1-2i+(-1)} \right)^3$$

$$= \left(\frac{2i}{-2i} \right)^3$$

$$= \frac{8i^3}{-8i^3} \quad i^3 = -1$$

$$= \frac{8(i^2) \cdot i}{-8(i^2) \cdot i} \quad \text{or} \quad \frac{8(-1)}{-8(-1)}$$

$$= \frac{-8i}{8i} = \frac{-8}{8}$$

$$= -1 \quad -i^3 = -i$$

$$ii) \frac{3}{(1+i)}$$

$$= \frac{3(1-i)}{(1+i)(1-i)}$$

$$1 - i + i - i^2$$

$$= \frac{3-3i}{1-i^2}$$

$$= \frac{3-3i}{2}$$

$$= \frac{3}{2} - \frac{3i}{2}$$

$$iii) \frac{1}{i^{-17}}$$

$$\begin{aligned} & (i)^{17} \\ &= (i^2)^8 \times i \\ &= -1 \times i \end{aligned}$$

$$b) Z_1 = \frac{3-2j}{2+j}$$

$$Z_2 = \frac{3+2j}{1-2j}$$

$$Z_1 = \frac{3-2j}{2+j} \times \frac{2-j}{2-j}$$

$$= \frac{(3-2j)(2-j)}{(2+j)(2-j)}$$

$$= \frac{6-3j-4j-2j^2}{4-2j+2j-j^2}$$

$$= \frac{6-7j-2(-1)}{5}$$

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b) Continued...

$$\frac{8-7j}{5}$$

$$= \frac{8}{5} - \frac{7j}{5}$$

$$Z_2 = \frac{3+2j}{1-2j}$$

$$= \frac{(3+2j)(1+2j)}{(1-2j)(1+2j)}$$

$$= \frac{3+6j+2j+4(j^2)}{1+2j-2j-4(j^2)}$$

$$= \frac{-1+8j}{5}$$

$$= -\frac{1}{5} + \frac{8}{5}j$$

$$b) Z = (2+3i)(3-2i)$$

$$= 6-4i+9i-6i^2$$

$$= 6+5i-6(-1)$$

$$= 12+5i$$

$$R = \sqrt{12^2 + 5^2}$$

$$R = 13$$

$$\theta_0 = \tan^{-1} \frac{y}{x}$$

$$= \tan^{-1} \left(\frac{5}{12} \right)$$

$$\theta_0 = 22.62$$

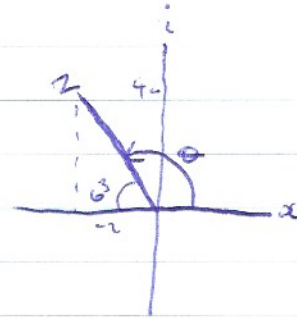
$$\theta_R = \frac{22.62\pi}{180}$$

$$\theta_R = 0.396$$

$$Z = 13e^{j0.396}$$

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$$\begin{aligned}
 c) (1+3i)(1+i) \\
 &= 1 + i + 3i + 3(i^2) \\
 &= 1 + 4i + 3(-1) \\
 &= -2 + 4i
 \end{aligned}$$



$$R = \sqrt{(-2)^2 + (4)^2}$$

$$R = \sqrt{12}$$

$$R = 2\sqrt{3}$$

$$\begin{aligned}
 \theta_D &= \tan^{-1} \frac{4}{-2} \\
 &= -63.43
 \end{aligned}$$

$$= 180^\circ - 63.43^\circ$$

$$\theta_D = 116.57^\circ$$

$$\theta_R = \frac{116.57\pi}{180}$$

$$= 2.034$$

$$Z = 2\sqrt{3} e^{i2.034}$$



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$$a^n = x \quad \log_a x = n$$

a) i)

$$2^{3x-5} = 4^{5x-6}$$

$$3x-5 \log_2 2 = 5x-6 \log_2 4$$

$$3x-5 \log_2 2 = 5x-6 \log_2 4$$

$$3x-5-5x+6 = \log_2 2 - \log_2 4$$

$$-2x+1 = \log_2 2 - \log_2 4$$

$$-2x+1 = \log_2 2 - \log_2 4$$

$$ii) \log(x^2-4) - \log(x+8) = \log x$$

$$\log \frac{x^2-4}{x+8} = \log x$$

$$\log \frac{(x^2-4)(x-8)}{(x+8)(x-8)} = \log x$$

$$x^3 - 8x^2 - 4x + 32$$

b)

i) $\log x^2 y$

$$= \log x^2 + \log y$$

$$= 2 \log x + \log y$$

ii) $\log \frac{x^5 y^3}{z}$

$$= \log x^5 + \log y^3 - \log z$$

$$\frac{5}{2} \log x$$