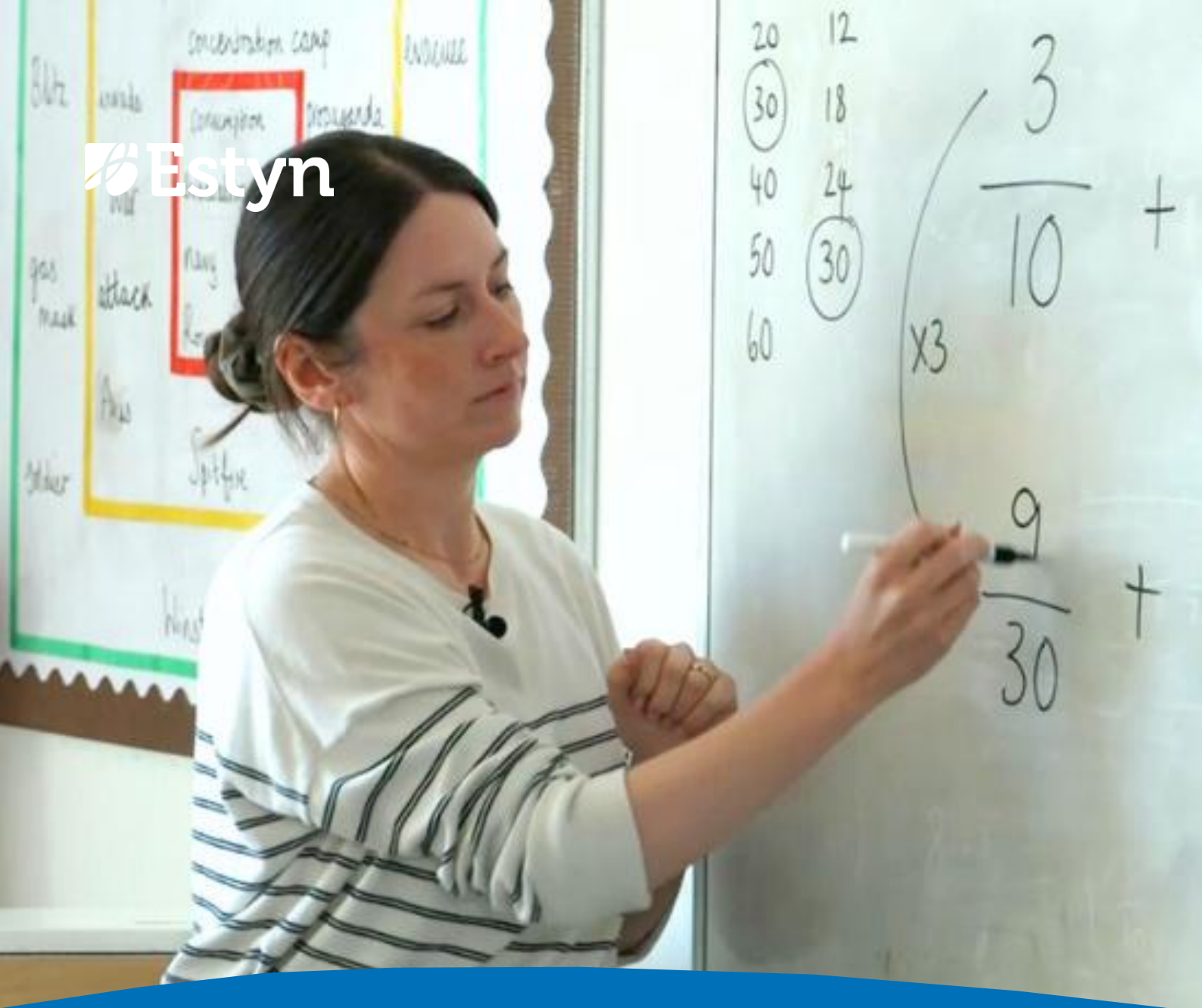




Unlocking potential: Insights into improving teaching and leadership in mathematics education

June 2025

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Executive summary

This thematic report focuses on the quality of teaching, learning, and leadership in mathematics in schools across Wales. It outlines effective practice and areas for improvement in teaching, curriculum, leadership, and assessment. Whilst the report identifies aspects of effective practice, it is important to note that, overall, successful mathematics teaching and leadership combines many of the aspects of effective practice identified in this report.

We visited a cross-section of secondary, primary and all-age schools in autumn 2024. In each school, we visited mathematics lessons, spoke with senior leaders, leaders and teachers of mathematics and pupils, and scrutinised a range of documentation. We also drew on evidence from school inspections undertaken over the last three years and the results of two school surveys developed for this report. We surveyed a sample of local authorities and school improvement partners and met with a smaller sample of these to discuss some of the aspects in greater detail. The findings in the report are based on a combination of inspection evidence since February 2022, and the evidence gained on the thematic visits.

Whilst we observed aspects of effective teaching and leadership of mathematics, overall, we found that the quality of teaching of mathematics was too variable and pupils' standards in mathematics were too low. We believe the variability in the quality of teaching of mathematics is partly linked to a reduction in subject-specific support across Wales. Our findings revealed that there is too little focus on the importance of teachers' subject knowledge and subject pedagogy and, as a result, too often teachers have a limited understanding of the best ways to support pupils' learning.

The report emphasises the importance of effective teaching methods, high-quality questioning, a supportive learning environment, professional learning, and parental involvement in enhancing mathematics education. These elements collectively contribute to improving pupil engagement, understanding, and outcomes in mathematics.

Where teaching was most effective and pupils made good progress, teachers:

- had high expectations of what pupils could achieve and challenged all pupils well
- made effective use of a range of teaching and assessment techniques and used their findings from assessments to guide their teaching, adjusting the pace and level of challenge responsively
- ensured that pupils understood what they were learning rather than just following instructions to get the correct answer

- questioned pupils effectively to deepen their understanding of mathematical concepts
- skilfully exposed and addressed pupils' misconceptions
- ensured that pupils were constantly thinking and that the level of challenge was appropriate, stretching pupils without overwhelming them
- helped pupils to make relevant connections to other areas of mathematics and, where appropriate, to everyday life

Where teaching was less effective and pupils did not make enough progress, teachers:

- did not focus closely enough on what they wanted pupils to learn but instead planned tasks and activities that kept pupils busy
- did not plan well enough to teach the understanding of mathematical concepts and identify and address misconceptions; they often taught procedural shortcuts rather than helping pupils understand mathematical concepts and make connections between topics
- did not practice responsive teaching within mathematics classrooms, often due to a lack of confidence in their subject knowledge or pedagogy
- did not question pupils well enough to encourage them to deepen their thinking or improve their understanding of aspects of mathematics
- did not use questioning well enough to address pupils' misconceptions, often because they did not have the subject knowledge to explore aspects around and within topics effectively
- limited the opportunities for pupils to develop independent learning skills due to over-directed teaching, lack of challenge, or ineffective questioning
- did not challenge pupils, particularly the most able and pupils at transition points well enough

We recognise the importance of designing a mathematics curriculum that is comprehensive and coherent. In schools that designed a successful mathematics curriculum they had a number of strong features, such as:

- they were structured to build on pupils' learning
- appropriate time allocated for pupils to acquire mathematical knowledge, understanding, and skills
- the provision of opportunities for pupils to apply their knowledge in authentic contexts and develop their mathematical independence
- high-quality teaching that involved a balance of explicit teaching and time for pupils to explore concepts, collaborate with others, and develop their mathematical independence

- teachers collaborated in their topic planning and framed what effective progression looked like across the topic area, including identifying and addressing misconceptions; where pupils made the most progress, teachers anticipated what came later and taught the pupils accordingly

In general, there were common factors that limited the effectiveness of curriculum planning. These included:

- a lack of understanding of Curriculum for Wales requirements and guidance; furthermore, teachers and leaders often did not know who to approach to clarify and support curriculum-related questions, and this added unhelpfully to teachers' confusion and cognitive overload
- an inconsistent understanding of the effective use of the mathematical proficiencies
- a lack of clarity in national guidance to support teachers and leaders in understanding the minimum expectations of pupils' understanding and progress in mathematics; often, this meant that teachers and leaders did not have high enough expectations of how pupils should progress in their mathematical understanding
- limited support to improve the quality of mathematics teaching, which was focused enough on pedagogy or how teachers should use their subject knowledge effectively to help pupils learn
- support and professional learning for teachers of mathematics that was too generic, not effective, or wide-ranging enough and many teachers were unaware of the professional learning opportunities available to them externally
- too many teachers who relied heavily on bought-in schemes of work, using them without considering pupils' needs well enough or without considering how to adapt and use a selection of resources to achieve the best learning outcomes

We make a number of recommendations to support the Welsh Government, local authorities and leaders and staff within schools to strengthen mathematics teaching across Wales.

Introduction

This thematic report is written in response to a request for advice from the Welsh Government in their remit letter to Estyn for 2024-2025. The report provides a national overview of the strengths and areas for improvement in the teaching of mathematics within the 7 to 16 age range. It identifies good practice in the teaching of mathematics based on the impact it has on pupils' progress in mathematics and numeracy. It focuses particularly on effective practice in primary, secondary and all-age schools with regards to teaching, curriculum, leadership, and assessment.

This report draws on findings from inspection evidence as well as from visits to a sample of 30 primary, secondary and all-age schools. The evidence includes feedback from interviews with relevant stakeholders, the review of relevant documentation, online surveys, and observations of teaching. Further details of the evidence base can be found in the 'Methodology and evidence base' section [here](#).

The intended audience for this report is the Welsh Government, headteachers, leaders and teachers of mathematics in schools, local authorities, school improvement services, and partnerships for initial teacher education. It aims to support the system to secure improvements in this important aspect of teaching to improve the experiences of pupils studying mathematics and support them to reach their full potential.

The report explores and highlights effective practice in the teaching of mathematics, which is exemplified through videos and spotlights. It provides a resource to support teachers and leaders of mathematics to evaluate and improve their own practice with respect to its impact on pupil progress. Furthermore, this report considers the support offered to schools to improve the quality of mathematics teaching and increase pupil outcomes.

Background

The importance of being numerate:

It is widely accepted that having a good standard of numeracy is important to increase young people's life chances. Andreas Schleicher, OECD noted that "Good numeracy is the best protection against unemployment, low wages and poor health". However, National Numeracy notes 'it is culturally acceptable in the UK to be negative about maths. We don't talk about other life skills in this way, but we hear 'I can't do maths' so often it doesn't seem a strange thing to say (Kowsun, 2008)'. It is also accepted that young people entering adulthood with poor standards of numeracy or weak skills in mathematics have a

significant effect on the economy. According to research from Pro Bono Economics, it is estimated that 58% of adults in Wales have low numeracy skills (Franklin, 2021). Within an increasingly digital and technical age, the need for appropriate standards of numeracy is more important than ever.

Elements of successful mathematics teaching and learning

Inspection evidence indicates successful mathematics education relies on an effective curriculum that is both comprehensive and coherent. A high-quality curriculum is carefully structured to provide:

- pupils with the time to acquire mathematical knowledge, understanding and skills
- opportunities to deepen mathematical understanding and develop a surety in applying methods
- opportunities for pupils to apply their mathematical knowledge and skills to solve problems in relevant contexts

In partnership with high quality curriculum, successful mathematics teaching generally contains the following elements:

- teachers with good subject knowledge who are confident in their approach to teaching
- teachers who understand how pupils learn best in mathematics lessons and plan their teaching accordingly
- high quality teaching that includes an understanding of effective teaching methodologies for mathematics
- a balance of explicit teaching and time for pupils to explore concepts, talk and think about them, collaborate with others and develop their mathematical independence
- placing a high importance on teaching pupils to understand what they are doing rather than simply following a method; this helps to reduce pupils' cognitive load and will free up space for pupils to be more successful when solving problems
- celebrating success and embracing mistakes, planning for, and addressing, misconceptions effectively

Furthermore, pupils' attitudes to learning mathematics will have an impact on their progress. Our evidence indicates that pupils are more likely to develop a positive attitude to mathematics if they enjoy success in it. It is important therefore that mathematics education helps pupils to:

- enjoy what they are learning whilst ensuring that provision is challenging and effectively develops pupils' knowledge and skills

- where relevant, understand the application of what they are learning in other subjects across the curriculum and in life
- experience success and failure in mathematics; this helps pupils to learn – time must be given to slower learners to gain the understanding of important concepts and knowledge; however, it is important to guard against other learners over-rehearsing while they wait, or covering work they already know
- build on their particular starting points and past experiences in the home or in education
- understand that it is acceptable to make mistakes in mathematics so that they can learn from them and build their resilience

In 2013, Estyn published a good practice in mathematics at Key Stage 4 thematic [report](#). Recommendations of the review concluded that:

Mathematics departments should:

- R1 make sure that pupils develop secure number, algebraic and problem-solving skills at key stage 3
- R2 improve the quality of teaching and learning in mathematics lessons by making sure that lessons are well structured, engaging and challenging, and link to other topics and subjects, and number and algebraic skills are developed and applied in new contexts
- R3 use assessment to inform pupils about how they are doing and what they need to do to improve
- R4 minimise early entry for GCSE in mathematics and ensure that pupils follow courses of study that allow them to achieve the highest grades
- R5 base self-evaluation and improvement planning on evidence from observing pupils' standards in mathematics lessons and scrutiny of their work, and
- R6 share best practice within and between schools and use it to support teachers' professional development

Local authorities and regional consortia should:

- R7 provide support, advice and professional development opportunities for mathematics teachers, including facilitating professional networks to share best practice

The Welsh Government should:

- R8 support schools and regional consortia in raising standards in mathematics for all pupils
- R9 review National Curriculum level descriptors at Key Stage 3 with the view to raising levels of expectation at level 5 in number and algebraic skills

In 2015, Estyn published a further good practice in mathematics thematic report, which examined standards in mathematics at Key Stage 3 and the factors affecting achievement.

The recommendations were to improve standards of mathematics at Key Stage 3:

Mathematics departments should:

- R1 monitor the performance of pupils eligible for free school meals and offer targeted interventions as necessary
- R2 meet the needs of pupils who experience difficulties or are more able
- R3 increase the level of challenge for all pupils by making sure that:
 - lessons are structured to engage, motivate and stretch all pupils
 - mathematical problem-solving skills are developed and applied to a wide range of real-life contexts
- R4 ensure that assessment and tracking procedures are robust
- R5 improve departmental self-evaluation and improvement planning
- R6 share best practice across the school and evaluate new ways of working

Local authorities and regional consortia should:

- R7 facilitate networks for sharing best practice between mathematics departments
- R8 provide support, challenge and professional development opportunities for mathematics departments and individual teachers

The Welsh Government should:

- R9 investigate the difference in outcomes between boys and girls in mathematics
- R10 address shortages in the supply of qualified mathematics teachers

Many of the same challenges within the teaching of mathematics remain. For example, during our engagement with school leaders, they noted that there are increasing challenges recruiting specialist teachers of mathematics and as a result there is a growing number of non-specialist teachers. The growing retention issue in Welsh education and failure to recruit maths specialists into ITE continue to compound this issue.

Development of the Curriculum for Wales (CfW) and the mathematics and numeracy area of learning and experience (AoLE)

In 2014, the Welsh Government commissioned a review of the curriculum in Wales. The resulting Curriculum for Wales, consisted of six areas of learning and experience, one of which was mathematics and numeracy. Mathematics and Numeracy [guidance](#) (Welsh Government, 2020) was developed to provide support for schools to develop their own mathematics and numeracy curriculum to help pupils improve their mathematical skills and develop towards the four purposes of the curriculum.

Progress within mathematics is predicated on pupils' understanding of the topics they study, building on their prior knowledge, and applying these skills in authentic contexts. To help with this, progression in this AoLE involves the development of five non-hierarchical, connected yet independent mathematical proficiencies:

- Conceptual understanding
- Communication using symbols
- Fluency
- Logical reasoning
- Strategic competence

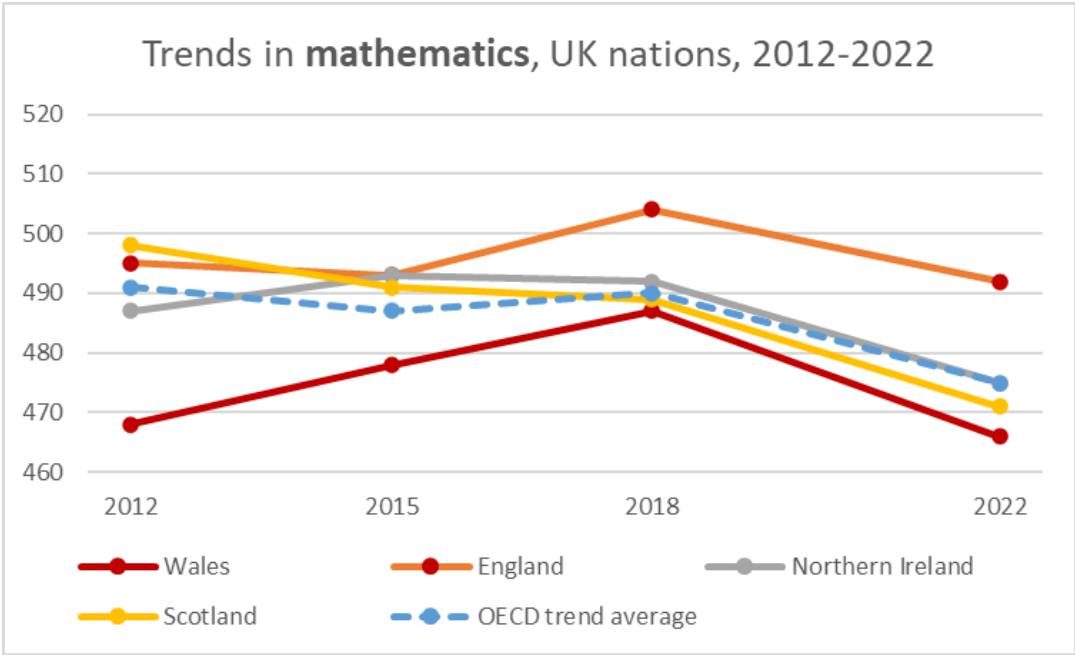
These tools for teaching when used well, help pupils to develop their mathematical understanding successfully.

Standards in mathematics in Wales

Overall, we know that standards in mathematics are not sufficiently high. Recent inspection reports from Estyn, results from PISA and information about GCSE outcomes support this. Overall, the quality of teaching of mathematics is not effective enough. Too often teachers have a limited understanding of effective teaching methodologies for mathematics, and do not use their subject knowledge well enough to foresee what pupils will find difficult and plan purposefully to help them overcome this.

Chart 1, below, shows that between 2012 and 2018, the performance of Welsh pupils in the Program of International Student Assessment (PISA) mathematical assessments improved so that it was closer to the OECD average. However, in 2022 Wales's overall performance fell and the overall mean average score in mathematics was significantly lower than the OECD average. While a decline was seen across many countries, Wales's results declined more than other countries in the UK (Estyn, 2023).

Chart 1: Trends in mathematics



Whilst the performance of pupils in GCSE examinations in 2023-2024 was designed to be comparable with that of 2018-2019, an analysis of the data for the performance of groups of pupils in mathematics examinations shows some concerning trends.

GCSE performance of males and females

GCSE outcome data in Wales demonstrates that females’ performance has deteriorated considerably since 2018-2019.

In Table 1, below, it is clear that whilst the performance of males has deteriorated slightly, the performance of females has seen a greater fall. In 2023-2024, females on average performed about a third of a grade worse in mathematics and numeracy than in 2018-2019.

Table 1: Number of pupils and GCSE numeracy indicator, by sex, 2018/19 and 2023/24

Year	Sex	Number of pupils	Numeracy Indicator points
			score
2018/19	Male	16,126	36.5
	Female	15,222	37.9
2023/24	Male	18,110	35.9
	Female	16,961	35.8

Source: Welsh Government, Examination results: September 2023 to August 2024

Table 2: Change in numeracy indicator scores, by sex, 2023/24 compared to 2018/19

Sex	Change in points	% Change (grade per pupil)
Male	-0.6	-10.0
Female	-2.1	-35.0

Source: Welsh Government, Examination results: September 2023 to August 2024

GCSE performance of pupils eligible for free school meals (eFSM).

The data shows a very worrying trend for this group of pupils. The gap between the performance of pupils eligible for FSM and their counterparts has widened since the pandemic.

For the numeracy indicator, pupils who were eligible for free school meals on average, performed a quarter of a grade worse in 2023-2024 than in 2018-2019.

Table 3: Number of pupils and numeracy indicator scores, by free school meal status, 2018/19 and 2023/24

Year	FSM status	Number of Pupils	Numeracy Indicator points score
2018/19	eFSM	4,597	29.5
	nFSM	25,660	39.7
2023/24	eFSM	6,304	28.0
	nFSM	27,481	39.0

Source: Welsh Government, examination results: September 2023 to August 2024

Table 4: Change in numeracy indicator scores, by free school meal status, 2023/24 compared to 2018/19

Sex	Change in Points	% Change (grade per pupil)
eFSM	-1.5	-25.0
nFSM	-0.7	-11.7

Source: Welsh Government, examination results: September 2023 to August 2024

The Estyn annual [report](#) for 2022/2023 stated that:

During the 2022/2023 academic year, our inspectors noted that the teaching of mathematics and numeracy was ineffective in a large minority of lessons, with too much emphasis on procedural fluency and insufficient focus on developing pupils’ mathematical proficiencies (Estyn, 2024a).

The 2023-2024 early insights into the Estyn annual [report](#) for mathematics highlighted a lack of challenge in primary schools and deficiencies in provision in secondary schools

leading to pupils' poor application of their numeracy skills across the curriculum. It states that in primary schools:

Too often, teachers plan independent activities for pupils that do not challenge them sufficiently and do not help them to progress with their learning. As a result, particularly with numeracy, pupils do not apply their skills in work across the curriculum at a high enough level'. Similarly in secondary schools 'In the majority of schools, shortcomings in provision mean that pupils do not develop their numeracy skills well enough across the curriculum (Estyn, 2024b).

Recent developments

In 2023, the Welsh Government established a mathematics and numeracy plan (Welsh Government, 2023) which acknowledges the 'fragility around mathematics and numeracy progress in both primary and secondary schools'. The plan details the additional aims, actions and timescales that Welsh Government will take:

to further support schools and settings to engage, enthuse and strengthen practitioners' confidence with the provision of mathematics and numeracy across the curriculum and achieve our long-term aims.

This thematic report is structured into three sections.

The first section evaluates how schools are developing a strong foundation of mathematical understanding.

The second section unpicks effective curriculum organisation and the wider principles of effective mathematics education. In addition, it considers effective practice in preparing teachers to teach mathematics.

The third section explores the subject of improving teaching and incorporates ongoing support, challenge and professional learning

At the end of each chapter there are a series of questions to support self-evaluation and planning for improvement.

Recommendations

The Welsh Government should:

- provide schools with more detail of expectations for pupil progress in mathematics and numeracy to support teachers with their curriculum planning
- ensure that local authorities and school improvement partnerships provide effective support to improve the quality of teaching and leadership in mathematics
- consider how they can promote a positive attitude to learning mathematics for pupils, parents and carers
- provide clarity for schools about how proficiencies should support the curriculum design process
- Consider approaches to promoting mathematics teaching as a career

Local authorities and school improvement partnerships should:

- improve the quality of support for the development of high-quality teaching and leadership of mathematics
- increase the level of subject specific support for schools
- target practical support to help teachers and leaders of mathematics understand effective curriculum design and delivery including the effective use of the five mathematical proficiencies
- ensure that professional learning opportunities are planned and evaluated effectively to ensure high quality impact

Senior leaders in schools should:

- ensure that **all** staff and pupils develop and exemplify a positive attitude towards mathematics and understand its value
- ensure that teachers are given adequate time to improve their subject knowledge and pedagogy
- ensure that assessment practices and processes support the effective teaching of mathematics
- ensure that professional learning is tailored to the specific needs of mathematics education
- ensure that assessment policies are flexible enough to be effective in different subjects, particularly in mathematics and numeracy

Schools should:

Ensure that mathematics subject leaders:

- maintain a clear focus on addressing the strengths and areas for improvement in the teaching of mathematics
- improve the quality of teaching of mathematics and ensure that the proficiencies are understood and used appropriately
- ensure a secure knowledge of all staff of minimum expectations of pupils' understanding and progress in mathematics
- ensure that curriculum provision builds effectively upon prior learning and between primary and secondary school

Ensure that teachers of mathematics:

- teach for pupils' understanding rather than focusing solely on performance
- develop their own confidence in subject knowledge and subject pedagogy to get the best out of the pupils
- develop a range of strategies to support the checking for understanding / responsive teaching

Chapter 1: Developing a strong foundation of mathematical understanding through highly effective teaching

The details in the tiles below do not solely define effective teaching in mathematics. However, along with other features discussed in this chapter, they are the common features that we observed in the best examples of effective teaching and learning.

Features of effective teaching	Expectation	There is a highly effective consistent approach to teaching and learning of mathematics within the school	Teachers have high expectations of what pupils can do
Qualities of effective teachers of mathematics	Teachers draw on their understanding of the mathematical proficiencies well to ensure pupils deepen their understanding of mathematics and make connections between topics and subjects	Teachers have a comprehensive understanding of appropriate mathematical pedagogy	Teachers have a secure subject knowledge and model their thinking and subject specific language effectively
Planning for learning	Teachers plan to expose and address misconceptions. They treat misconceptions as opportunities for learning rather than treating them as wrong answers	Teachers teach for understanding rather than setting tasks for completion	Teachers break learning down into manageable chunks which supports the development of pupils' understanding and reduces cognitive load
Teachers ensure that pupils have regular opportunities for retrieval practice	Teachers choose carefully the examples they use with pupils, building in small steps and exposing misconceptions	Teachers provide pupils with opportunities to apply their learning in context and solve problems	Even when pupils are taught in mixed attainment groups, teachers differentiate the work appropriately
Facilitating learning	Teachers provide pupils with appropriate challenge in their learning	Teachers provide pupils with clear explanations. They use methods and resources effectively so that pupils make good progress in their learning	Teachers place a strong emphasis on teaching the 'why' as well as the 'how'
Teachers utilise good quality questioning that probes understanding and informs planning	Teachers ensure that pupils are always thinking throughout the lesson	Teachers encourage pupils to set and test hypothesis.	Teachers assess pupils learning effectively and use it to inform the direction of learning and future planning

Our evidence indicated that the quality of teaching of mathematics was too variable. In a minority of cases teaching was effective and ensured that pupils made sufficient progress. However, in a majority of cases, pupils' progress was limited by aspects of poor teaching and leadership did not ensure teachers of mathematics continually developed their practice. Based on the observations we made during thematic visits and inspections, this chapter considers the most effective approaches to the teaching of mathematics and provides helpful prompts for teachers to consider when planning for pupils' learning.

Characteristics of effective teaching

Where teaching was most effective, teachers had **high expectations** of what pupils could achieve. They **challenged** pupils appropriately, ensuring that the work was at the upper limit of what pupils could comfortably handle. These lessons included effective **differentiation** to ensure that all pupils were working at their highest potential and beyond. This was when we saw pupils learning most effectively.

Teachers with **strong subject knowledge** focused on helping pupils understand mathematical concepts rather than just teaching them steps that prepared them to answer similar questions correctly. These teachers prioritised pupils' **understanding** over teaching for performance. They planned lessons by starting with the desired learning outcomes and worked backwards to design lessons that helped pupils achieve those outcomes. These teachers did not plan tasks just to keep pupils busy. Instead, they planned activities that built on pupils' learning. These teachers provided **clear explanations** of mathematical concepts, anticipated and addressed misconceptions, and taught both the 'why' and the 'how'. When relevant, they encouraged pupils to set and test their own questions, which deepened their understanding and helped them make connections between different concepts. These teachers responded to pupil contributions and progress to develop the learning throughout lessons.

Effective teachers were **reflective practitioners** who knew their own strengths and areas for improvement. They continuously refined their practice in response to pupils' learning. These teachers used the **mathematical proficiencies** naturally as teaching tools to help pupils deepen their understanding and make connections across different areas of mathematics and other subjects. They did not plan lessons to address a specific proficiency but developed these skills in their pupils so that they could draw on them organically when working mathematically in different contexts. For example, pupils used facts they already knew, drew diagrams, or reflected on similar experiences when tackling new challenges

The very best teachers had extensive subject knowledge and understood well how pupils learn best in mathematics. They confidently **questioned, probed, and challenged** pupils effectively. These teachers did not accept pupils' first answers. Instead, they asked questions to ascertain pupils' understanding of the 'why' things worked or presented them with situations where the method may not work. These teachers regularly **assessed** pupils' understanding and adapted their teaching, accordingly, ensuring that pupils did not wait or struggle for too long, thus making effective progress. In essence they taught pupils how to fly and then provided them with opportunities to demonstrate and apply their skills.

Classroom climate

Building **positive relationships** with pupils is essential for fostering enthusiasm for mathematics. Teachers who cultivated positive attitudes in pupils typically shared their love of the subject and helped pupils understand that making mistakes was a positive learning feature in mathematics. Pupils in these classes were comfortable to have a go even if they did not always get the correct answer. These were seen positively as learning points.

In a few classrooms, teachers provided pupils with opportunities to develop their **independent learning skills** and encouraged them to set and test hypotheses. For example, Year 4 pupils in **Ysgol Bryn Hedydd** were free to choose how they conducted investigations and considered the best way to display their data. This encouraged them to challenge themselves purposefully and consider alternative hypotheses during the course of a mathematics investigation and predict how this might affect outcomes. In these best classrooms, pupils felt safe to make mistakes and embraced the learning that came from their errors.

Where teachers helped pupils to develop their mathematical reasoning best, they ensured that they created the expectation that pupils explained their reasoning and developed their confidence by justifying their choices instead of just providing a numerical answer. Although in a few cases teachers effectively used real-world contexts to make learning more relevant, too many teachers lacked a deep understanding of how to develop pupils' mathematical reasoning and merely set simple mathematical problems in context.

Effective teachers served as excellent **language role models**, accurately using mathematical vocabulary and helping pupils to develop their terminology purposefully. These teachers had high expectations of pupils' precise use of **mathematical vocabulary**. Consequently, pupils used mathematical terminology correctly both within and outside of mathematics lessons. For example, in Year 1 at **Ysgol Felinfach**, pupils confidently and

precisely used terms like 'more than,' 'less than,' and 'equal to'. Pupils in Years 3 and 4 accurately used language related to place value and calculation methods.

Pupils with **positive attitudes** towards mathematics generally made stronger progress than those with less positive attitudes. When we observed pupils with positive attitudes towards learning mathematics, it was often because:

- teachers presented work at the appropriate, but challenging level, enabling pupils to experience success.
- pupils were **challenged** appropriately, with suitable scaffolding and stretch; they were not held back waiting for other pupils to catch up, nor were they left without the understanding of what they needed to do
- **starters** were used effectively to ensure pupils were regularly retrieving and deepening their learning; in the best cases the starters were efficient, time appropriate, and supported learning well
- the use of mini whiteboards encouraged pupils to view **mistakes** as opportunities to learn and improve
- teachers used **assessment** points and data to adapt their teaching rather than as a vehicle for measuring value added
- pupils were encouraged to think deeply about the subject and developed their **understanding** through timely and highly effective questioning
- teachers ensured that pupils had access to appropriate **resources** to support their learning; for example, in **Ysgol Corn Hir**, pupils referred to the 'Wal Fathemateg' (Maths Wall) when they needed help with vocabulary, suitable methods, or to access additional resources including instructional videos

In these schools, teachers inspired pupils to have a love for learning mathematics through high expectations and positive reinforcement, contributing to a **culture of success** where pupils felt encouraged to achieve

Other pedagogy and resources

More effective teachers of mathematics had a comprehensive understanding of a variety of **mathematical methodologies and pedagogies**. They used these judiciously depending on pupils' progress, the topic, and the intended learning.

A few teachers used practical resources particularly effectively and dispelled the myth that practical resources are only effective with lower attaining pupils. For example, in **Ysgol Bryn Hedydd**, Year 4 pupils were working on writing a formula to calculate the n th term of a number sequence and were using cubes effectively to help them 'see' the pattern before moving on to the abstract application.

Other effective resources and teaching approaches are outlined below:

The use of mini whiteboards.

When used well, **mini whiteboards** were effective tools in the mathematics classroom. They were used successfully to help pupils visualise their way through a problem when **solving problems** with multiple steps. Pupils often felt more confident to ‘have a go’ when using a mini whiteboard and they told us that they were not afraid to make mistakes when using them. When used well, mini whiteboards fostered **collaboration** between pupils during activities when they firstly think and then share their thinking with a learning partner. They were also used purposefully as an **assessment** aid for teachers. Effective use of the mini whiteboard helped teachers to assess pupils’ understanding and identify individual or widespread misconceptions as soon as they formed.

Year 6 lesson at **Ysgol Gymraeg Bro Morgannwg**

In Year 6, the teacher used mini whiteboards skilfully to:

- monitor the response of every pupil
- identify and address misconceptions quickly
- provoke discussions about the mathematics and develop pupils’ conceptual understanding, their logical reasoning and their ability to convey their ideas using sophisticated mathematical terminology
- create an environment where pupils were willing to ‘have a go’ and not be afraid to make mistakes
- adjust the pace of the lesson and the level of challenge according to the pupils’ responses

Previously, the pupils had been learning about equivalent fractions. In this lesson the pupils were learning how to add and subtract fractions that had different denominators. During the first part of the lesson the teacher revisited pupils’ understanding of equivalent fractions to ensure that they had the pre-requisite knowledge needed to be able to understand how to add fractions.

The lesson started with the teacher asking pupils to simplify the fraction $\frac{6}{10}$ writing it in its lowest term. Pupils were asked to discuss the problem with their partner and then note the answer on their mini whiteboards. The teacher also asked pupils to signify on their mini whiteboards if they did not understand. Pupils engaged in a lively discussion and most noted a possible answer on their mini whiteboards. The teacher gave pupils a countdown ‘Tri, dau, un, show me your answers’. She checked pupils’ answers and then

asked one pupil to explain how they had obtained their answer. She encouraged the pupil to use the correct mathematical terminology, such as ‘denominator’, ‘numerator’ and ‘equivalent’. The teacher then asked pupils to simplify slightly more difficult fractions (where the numerator and denominator had to be divided by 3 and 5) writing them in their lowest terms. Weaker pupils had been strategically placed where the teacher and teaching assistant could provide further support as required. Pupils quickly grasped the idea, and the teacher adjusted the pace of the lesson skilfully in response. The teacher then emphasised the meaning of the concept ‘equivalent’ by showing that two equivalent fractions led to pupils having the same amount of a bar of chocolate, just that the pieces were of a different size/shape.

She then asked pupils to simplify $\frac{24}{30}$ writing it in its lowest term. Pupils were again asked to discuss the problem with their partner and then note the answer on their mini whiteboards. The teacher checked pupils’ responses and then chose three pupils’ boards which included the answers $\frac{12}{15}$, $\frac{8}{10}$ and $\frac{4}{5}$. She asked pupils to explain whether $\frac{8}{10}$ and $\frac{12}{15}$ were incorrect and which of the three was the best answer. Again, pupils were encouraged to include correct terminology in their explanations. The teacher probed pupils’ thinking by challenging their ideas, asking other pupils whether they agreed or not and asking them to explain their reasoning. She thanked the pupils who had provided the answers $\frac{8}{10}$ and $\frac{12}{15}$ calling these ‘camgymeriadau campus’ (brilliant mistakes) as they provided really good opportunities for discussion, and reinforced that everybody makes mistakes when they are learning new concepts. This successfully created an environment where pupils were not afraid of making mistakes. The teacher provided pupils with further practice, addressing any misconceptions swiftly.

The lesson continued with the teacher asking pupils to note on their whiteboard a fraction that was equivalent to $\frac{4}{7}$ which had a denominator of 21. After providing pupils with a few further similar questions, the teacher asked pupils whether it would be possible to have a fraction that was equivalent to $\frac{3}{10}$ with a denominator of 15. This provoked a lively discussion.

Now that all pupils fully understood the idea of equivalent fractions, the teacher knew that the pupils had the pre-requisite understanding they needed and continued with the lesson by introducing the idea of adding fractions with different denominators.

At **Ebbw Fawr Learning Community**, pupils were proficient in using mini whiteboards as part of their daily learning activities. The mathematics department has effectively integrated mini whiteboards as an assessment tool, enabling teachers to promptly address and explore class and individual issues.

In Year 9, pupils built well on their prior knowledge to learn about the volume of a cone. The lesson began with a review of the volume of a prism, and pupils revisited essential knowledge and skills. Effective lesson planning ensured that pupils could successfully build upon their previous learning. The teacher then introduced the topic and used a video resource to demonstrate the relationship between the volume of a cone and the volume of a cylinder. With minimal instruction, pupils documented their observations.

Pupils were asked to derive their own formula for the volume of a cone based on their understanding of the volume of a cylinder. All pupils engaged well using the mini whiteboards, reflecting the teacher's high expectations for participation and proper use of these tools. The positive rapport the teacher had established with the class was evident in the mature way in which pupils shared the whiteboards, contributing to high participation and the expectation of 'no opt-out.' This fostered a positive learning culture.

The mini whiteboards were used effectively by the teacher to scaffold the learning process. By displaying a selection of the boards, the teacher demonstrated a comprehensive subject knowledge through a range of open and closed questions, challenging pupils' understanding in depth. This process allowed other pupils to reflect on their answers and address any misconceptions.

Subsequently, the class teacher modelled the calculation of the volume of a cone, clearly illustrating how to substitute values into the formula, with a focus on key elements such as layout, accuracy, and units.

As the lesson progressed, the teacher used the mini whiteboards at appropriate intervals to check for understanding. For example, the teacher checked pupils' deeper understanding when they used the inverse operation to calculate a length from the volume of the cone. The teacher also identified mistakes and explored written calculations and the appropriateness of units.

The lesson underscored the importance of having clear and systematic routines in place, coupled with high expectations regarding the use of resources in lessons. The class teacher considered the mini whiteboards as a key resource for checking understanding during the lesson. The teacher then used their expertise in teaching methods and their mathematical subject knowledge to address and correct these misconceptions effectively.

Effective questioning

In many cases teachers asked a range of questions to help pupils **recall prior learning** suitably. However, too often teachers answered their own questions or targeted questions at those pupils who were willing to take part and answer correctly.

In the few best examples, teachers went beyond questioning for recall and asked questions to encourage critical and reflective thinking and probe and **deepen pupils' understanding**. In these examples, teachers encouraged pupils to expand their thinking and offer detailed explanations of their findings. As a result, pupils were inquisitive about mathematical concepts and used appropriate methods, such as trial and error, to test hypotheses.

Radyr Comprehensive School have had a strong focus on improving the quality of teaching. The head of mathematics has worked effectively with his department to plan a clear and coherent curriculum with a strong focus on mathematical understanding. To support this, teachers have worked together to develop the use of questioning, to probe and deepen pupils' understanding. They think carefully about important misconceptions and plan and target their questioning well to expose these. As a result, pupils make effective progress in their mathematics lessons and have a positive attitude towards the subject.

In a very few cases, teachers with strong subject knowledge observed learning carefully and prompted pupils to deepen their understanding through timely and effective questioning. In these cases, teachers and pupils considered that mistakes and misconceptions were learning opportunities.

Effective teachers used questioning skilfully to assess pupils' understanding and progress. They planned their questioning carefully and ensured that pupils were encouraged to think deeply. Teachers used a range of questioning techniques including clear and unambiguous multiple-choice questions that were carefully designed to help teachers to identify and understand pupils' misconceptions, or to test a particular skill. If pupils held a misconception, they would not be able to calculate the correct answer unless by lucky guess. Each of the answers would identify a different mistake or misconception.

At **The Bishop of Llandaff CiW High School**, pupils effectively use ABCD cards with diagnostic questions to demonstrate their understanding and misconceptions in mathematics. For instance, Year 9 pupils used these cards while learning about trapezia. The cards were placed in the middle of the table before the lesson started, making them accessible to all pupils.



The lesson began with a recall of prior knowledge, followed by the introduction of learning objectives. The ABCD cards were then used to assess the class's understanding of a trapezium. Pupils were asked to indicate, by showing the correct card, which of the shapes

A, B, C, or D was a trapezium. The shapes were carefully chosen by the teacher to reveal misconceptions. For instance, they included a five-sided shape and a regular shape with 4 sides and 2 pairs of parallel sides. Their answers prompted discussions, where misconceptions were addressed.

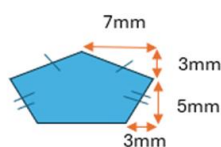
Next, pupils were introduced to the definition and formula for the area of a trapezium. They were given opportunities to think and discuss in groups and with the teacher, identifying for themselves possible misconceptions with the use of the formula. Pupils then completed a short deliberate practice task to find the area of trapezia, with questions increasing in complexity. The teacher circulated the class, providing immediate feedback through live marking and challenging faster learners with more complex questions.

At an appropriate point before the task was completed, the teacher instigated a planned feedback progress check to ascertain progress against lesson objectives. Through carefully constructed multiple choice questions, designed to highlight key misconceptions, pupils used the ABCD cards to indicate their answers. Discussions took place to elicit misunderstanding and eradicate misconceptions. As only a very small number of pupils were still making mistakes the lesson moved on but with those pupils receiving individual and targeted support.

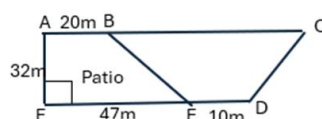
Throughout the lesson, the teacher conducted three planned feedback progress checks to assess the class's progress against the lesson objectives. Pupils used the ABCD cards to answer carefully constructed multiple-choice questions designed to highlight key misconceptions.

In the final part of the lesson, pupils worked on finding the area of compound shapes that included a trapezium. They initially used mini whiteboards and received appropriate scaffolding if necessary to break down the compound shapes into more recognisable forms. Pupils were then challenged to collaborate and apply their knowledge to solve more complex problems, such as:

1. Calculate the area of this shape:



2. A garden ACDF is a trapezium. The garden has two sections – a patio ABFE and a lawn BCDE. Given the ratio area of patio : area of lawn = 4:5, calculate the length of BC.



The lesson exemplified the effective use of diagnostic questions to address and eradicate misconceptions. The teacher used the ABCD cards to gauge pupils' understanding, ensuring that wrong answers were appropriately addressed. This approach allowed the teacher to quickly determine whether pupils understood the content, enabling them to move at an effective pace through the lesson. As a result, pupils made good progress.

Variation theory

When used well, variation theory ensured that pupils understood the essential features of a concept rather than resorting to following a set of predetermined steps. The questions were not just more of the same, but one thing was changed at a time so that pupils had the opportunity to focus on and understand one part of the concept. For example.

1.	1,	3,	6,	10,	10	Mean =
2.	2,	4,	7,	11,	11	Mean =
3.	0,	2,	5,	9,	9	Mean =
4.	0,	2,	5,	9,	4	Mean =
5.	10,	2,	5,	9,	4	Mean =

Use of variation theory to ensure that pupils continue thinking while practising a method in **Ysgol Penglais School**

The effective use of variation theory was one of a number of notable features seen in this Year 8 lesson. The department use the first part of each lesson to provide valuable retrieval practice opportunities for pupils. Pupils settled quickly and worked well on the starter, which provided pupils with opportunities to revisit areas of mathematics that they had studied previously. The pupils coped well, showing that they had good recall of prior learning. The teacher provided answers so that pupils were able to self-mark their work and chose two of the questions to go through in depth, using questioning to ensure that pupils explained their reasoning in depth.

The learning goal of the lesson was that pupils would learn how to calculate the mean of a dataset that was given in a table and how this connected with their prior knowledge around averages. In the previous lesson, the class had been working on calculating measures of average and spread of data given in the form of a list. They had explored the idea of what was meant by ‘average’, which was good preparation for the latter part of this lesson. The teacher started the formal part of the lesson by asking pupils to calculate the mean of a list of numbers. Pupils displayed their method and answers on their mini whiteboards and the teacher checked their answers. She saw that all had good recall and swiftly posed the pupils with the question:

The following list shows the number of goals scored by a hockey team during the first twenty games of the season. Calculate the mean average number of goals per game.

2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 5, 5, 5, 5

Rather than giving the numbers on the whiteboard, she read them aloud and asked pupils to write them on their mini whiteboards. Nearly all pupils achieved the correct answer, and many realised that they could group the 2s together and use multiplication rather than

repeated addition in a more efficient method. The teacher chose one pupil's mini whiteboard who had used the multiplication and asked him to explain what they had done. She annotated the board as the pupil spoke so that all pupils could understand this efficient method.

The teacher then asked the pupils to answer the following question. She encouraged them to discuss their ideas and not to worry if they were not sure where to start – she would provide clues shortly.

The following table shows the number of goals scored by a hockey team during the first twenty games of the season. Calculate the mean average number of goals per game.

<i>Number of goals</i>	<i>Number of games</i>
2	8
3	5
4	3
5	4

A few pairs of pupils realised quickly what needed to be done. After allowing pupils to think about the problem for around a minute, the teacher asked the pupils to think about how this dataset would look like in the form of a list and to think how many games there were where 2 goals were scored. This was enough guidance to allow many of the pupils then to understand how to find the answer. The teacher then circulated the class, helping the pairs of pupils who were still unclear. In a short period, nearly all pupils understood how to calculate the mean from the table. The teacher asked pupils to answer a similar question independently on their whiteboards, and then asked the class to glue in a ready prepared example so that they had notes in their books.

She then provided the pupils with time to practise the skill of calculating the mean from a table. The class were familiar with the idea of variation theory and understood the idea that, after answering the first question, they needed to look for any patterns that linked questions 1 and 2 and use what they had noticed to predict the answer to question 2, providing reasons for their prediction. They would then use the formal method to answer question 2, before comparing their prediction with the answer they had obtained. The process would be repeated by comparing question 2 and 3 and using the answer for question 2 as a basis for predicting the answer to question 3, and so on. The questions provided were as follows:

Answer Q1 by calculating the mean number of goals per game

For all questions that follow:

(a) Look at **what is the same and what is different** between this question and the question you just answered. Use this to **predict what the mean** for this new question will be. Explain why you made this prediction.

(b) Use the method you learnt to **calculate the mean average** number of goals per game from this table

(c) Does your **prediction agree with your answer**? If not, can you see why not?

Q1

Number of goals	Number of games
2	2
3	3
4	4
5	1

Q2

Number of goals	Number of games
3	2
4	3
5	4
6	1

Q3

Number of goals	Number of games
3	4
4	6
5	8
6	2

Q4

Number of goals	Number of games
6	4
8	6
10	8
12	2

Q5

Number of goals	Number of games
1	4
3	6
5	8
7	2

Q6

Number of goals	Number of games
1	2
3	8
5	6
7	4

Q7

Number of goals	Number of games
1	22
3	8
5	6
7	4

The worksheet provided pupils with beneficial practice of the method for calculating the mean from a table. The variation theory element ensured that pupils carried on thinking throughout this period, looking for patterns and thinking about the underlying mathematics – what effect did the change have on numerator and denominator of the sum, and what effect this would have on the mean. Pupils worked hard throughout this period of the lesson, discussing their predictions and comparing their findings with their partners. Their predictions and ability to justify their choices showed that pupils had gained a very good understanding of the concept of the mean as a measure of the central location of a set of data. For the last part of the lesson, the teacher went through the answers, discussing what pupils had discovered about their predictions and how any differences between predictions and answers could be explained.

Direct teaching

This is the explicit teaching of knowledge and skills. Where teaching was most effective, teachers understood the importance of direct teaching. They ensured that **pupils were actively engaged** in this stage of their learning, by involving them in the process, rather than them simply listening to the teacher talk. Teachers modelled effectively their own thinking using appropriate resources and manipulatives to support pupils to move towards the abstract form. They then encouraged pupils to share their own thinking and asked relevant questions. Pupils explored the material, found solutions, and solved problems. The teacher moved around the classroom, discussing with small groups or individual pupils to identify correct answers and misconceptions, which were then addressed in the main class discussion. This process was flexible and adapted to the pupils' understanding rather than following a rigid formulaic approach to lessons. In these lessons, progress was measured by pupils' understanding rather than their performance on specific questions.

Dialogic techniques

Dialogic teaching is not just about talking; it's about **using conversation as a tool** to foster a deeper, more engaged learning experience. This approach involves various talk structures, with **effective questioning** being the most crucial element. It aims to spark interest, encourage thinking, deepen understanding, broaden ideas, and help pupils develop arguments, ultimately supporting lifelong learning.

Where dialogic teaching was most effective, schools developed a **whole-school approach** to developing dialogic and interactive techniques. It was important that techniques such as the turn-and-talk approach and cold-calling ensured that every pupil was involved. Alongside high-quality questioning, dialogic teaching included **live marking** and **verbal feedback** to support pupils' understanding.

Teachers play a vital role in creating a classroom culture that ensures that mistakes and misconceptions are seen as learning opportunities. Effective teachers used **probing questions** successfully to discuss misconceptions, and pupils responded positively by suggesting how they might amend their ideas or improve their work. However, we saw instances where not enough time or importance was given to dialogic techniques. In such cases, teachers did not use these techniques effectively. For instance, they always advocated for a silent classroom, or accepted simple one-word answers without further probing.

Ysgol Gymraeg Bro Morgannwg – an example of planning carefully for opportunities to ask highly effective questions

During previous lessons, pupils had learned about rules of indices and how to express numbers in standard form. During this lesson, the teacher had planned a lesson where pupils would learn to add and multiply numbers expressed in standard form without a calculator. In order to assess pupils' pre-requisite fluency with multiplying decimals, the teacher skilfully designed a starter activity that would expose common misconceptions and any lack of fluency in this area of mathematics. The starter included the following three questions (amongst others):

(a) 6×1.5

(b) 8.2×2.5

(c) 7.2×1.6

While pupils were working on the starter, the teacher circulated the classroom to identify which pupils were using different methods. After providing pupils with time to work independently, she held a whole-class discussion around the different methods used by pupils. For question (a) she asked a pupil who she had seen using a multiply 6 by 15 and then divide by 10 methods to provide their answer and explain their method. When the pupil stated:

'I multiplied 6 with 15 and then moved the decimal point',

the teacher asked the pupil to correct himself, explaining that moving the decimal point was not an acceptable explanation. The pupil corrected himself to state that they had divided by 10.

The teacher then asked another pupil, pupil X, who she had seen using a more efficient method, to explain how they had arrived at their answer. Pupil X explained that they had converted 1.5 into one and a half and worked out 1×6 and a half of 6 and added their answers. The teacher prompted the pupil to justify their choice of method and explain why they had used this method rather than the method used by the first pupil.

The teacher then asked a third pupil, pupil Y:

'Pupil Y, I noticed that you also got the second answer correct by using long multiplication and then dividing by 10 twice. Can you tell me, how you could use pupil X's method to answer question (b) more efficiently'?

Pupil Y was then able to use the model provided by pupil X to explain how they could work out double 8.2, half of 8.2 and then add the answers and find the answer very quickly.

The teacher then asked the class to state on mini whiteboards which method would be used for question (c). Nearly all pupils noted that they would need to use long multiplication then divide by 100. The teacher asked pupil Z to justify their reasons for choosing that method and why the more efficient method used for question (a) and (b) would not be better in this instance.

The careful choice of examples, and use of skilful questioning, meant that pupils were guided to think about the underlying mathematical structures, were required to justify their choice of methods and were building their fluency in this area of mathematics.

In the best cases, teachers ensured that pupils were, where relevant, given access to a wide range of **resources** and methods to support their learning. Where this was most effective, pupils had a good understanding of what resources were available and selected them independently when they were needed. However, too often teachers did not consider well enough the learning that they wanted to happen, and which resources would best support pupils' understanding.

Pupils were better supported to develop their understanding when teachers broke learning down into **manageable chunks**. This reduced **cognitive load** where pupils tried to 'remember' too much all at once and as a result information was lost and learning was more limited. Instead, teachers choose examples carefully to elicit understanding and vary the focus and complexity of the work. Where teachers used manipulatives, both concrete and representational, to develop pupils' understanding of abstract concepts, they chose these carefully to ensure that they maximised the intended learning. Where this happened, we saw pupils able to explain confidently how they used the resources to help them solve problems and understand concepts. However, too many teachers did not consider the use of manipulatives well enough. Often teachers used manipulatives for pupils' enjoyment and not when they needed them. As a result, pupils made less progress than they could. The effectiveness of the use of concrete and representational manipulatives varied in supporting pupils' understanding of simple and abstract mathematical concepts.

Using representations to teach for understanding and build pupils' long-term learning: mathematics lesson at **Lewis School Pengam**

In the previous lesson, this middle ability Year 8 class had been learning how to use bar models to divide a total in a specific ratio. When pupils arrived at the lesson they settled quickly and started working on the retrieval starter:

In a class, the ratio of boys to girls is 5:2 respectively.

- How many boys and girls would there be if the total amount of pupils was 28?
- Would it be possible to have a class with this ratio of boys to girls if there were 17 boys?

Pupils worked well on the starter, and nearly all were able to answer the first part correctly, showing good recall of prior learning. The teacher quickly went over the answer, asking pupils to explain each step they had taken and why they had taken those steps. The

teacher then questioned pupils for their opinions on the second part of the question. She ensured that pupils justified the detailed reasons for their answers. During the discussions, she emphasised the use of correct terminology and ensured that pupils understood what the words ‘total’ and ‘respectively’ meant in the context of this question.

The teacher then presented the pupils with two new questions and asked the pupils to discuss in pairs what was the same about the questions and what was different. They were encouraged to draw bar models for each if they thought that would be helpful.

1. *A sum of money is to be shared in the ratio 5:3 between Alan and Bethan. If £40 is to be shared, how much would each get?*
2. *A sum of money is to be shared in the ratio 5:3 between Alan and Bethan. If Alan receives £40, then how much money was shared?*

Pupils discussed the two questions, and many drew bar models to visualise the problems. Nearly all saw that the ratio was the same in both questions and a majority saw that, in the first question, the £40 was the total to be shared, while, in the second question, the £40 represented one part – the money Alan would receive.

The teacher held a classroom discussion on these ideas, and questioned pupils skilfully to ensure that they explained their reasoning. While pupils were explaining their ideas, she annotated the white board so that other pupils could visualise the ideas being discussed. She then re-explained the differences between the two questions to ensure that all pupils understood. Pupils listened carefully to her explanations. She then posed a further question:

3. *A sum of money is to be shared in the ratio 5:3 between Alan and Bethan. If Alan gets £40 more than Bethan, how much would each get?*

Pupils were given the opportunity to think, in pairs, about how this question differed from the previous questions. Pupils then explained that the £40 represented the difference between Alan’s share and Bethan’s share, which was 2 blocks in their bar models. The teacher then showed how the bar models could be used to answer each of the questions, questioning pupils throughout her explanations.

The teacher then provided the pupils with a short practice in identifying whether the question gave the total, one part, or the difference. She circulated the classroom to monitor progress and provided the correct answers so that the pupils were able to self-mark their work. Most pupils succeeded in identifying the type of question correctly.

Most of the remainder of the lesson was spent on pupils practising and consolidating their newly learnt skills and knowledge. As a plenary, the teacher showed how, in the next

lesson, they would be applying their understanding of ratio to solve a variety of ratio questions which were set in context.

In this lesson, pupils made strong progress in developing their conceptual understanding of ratio. The way in which the teacher ensured that pupils understood how ratio could be represented using models meant that pupils were constantly thinking. The requirement to explain their reasoning and justify their answers ensured that pupils built long-term learning of the concept of ratio during this lesson.

Misconceptions

Where teaching was most effective, teachers built on pupils' understanding carefully and sometimes in small steps. They planned carefully to **expose and address misconceptions**. Teachers with a stronger understanding of how pupils learn mathematics often had the expertise to anticipate and plan for these misconceptions confidently. They designed their questions to expose and address misconceptions. In these cases, pupils were encouraged to discuss errors and plan together how to improve them.

In a few cases, teachers and pupils discussed confidently any misconceptions and teachers supported pupils to explore alternative strategies. This relied on effective planning and strong subject knowledge and understanding. For example, teachers used diagrams to discuss alternative methods and address common whole-class misconceptions.

In **Ysgol Corn Hir**, leaders have developed effective strategies for addressing pupil misconceptions. Staff promote a positive learning environment where making mistakes is accepted as a natural part of the learning process. Teachers make effective use of 'discussion mats' at the beginning of units of work. For example, at the beginning of the unit on using standardised units of measurement, Year 1 and 2 pupils discuss pictorial images of measuring equipment and equations with a partner or in a small group. Effective professional learning and 'how to' guides enable staff to assess pupils' understanding and identify misconceptions early as they facilitate these discussions. Teachers use this information skilfully to adapt provision and provide purposeful supporting resources and activities. For example, pupils use a dedicated area, the 'Hwylfan Heriau' to practise activities that develop pupils' conceptual understanding and their understanding of why their original thoughts were a misconception. At the end of the unit, pupils revisit the discussion mats to summarise what they have learned and what the images and concepts now mean to them.

In too many cases, particularly those teachers with weaker pedagogical understanding, there was insufficient planning to **address misconceptions**. These teachers often lacked awareness of common misconceptions and often reinforced these misconceptions

themselves. For example, they used incorrect methods like 'timesing' or 'add a zero'. Furthermore, when pupils displayed misconceptions, they were treated as wrong answers and teachers did not embrace them as opportunities for pupils to deepen their understanding.

Often, teachers with weaker mathematical understanding taught methods that worked temporarily, such as telling pupils to 'move the decimal point' or always use FOIL when expanding brackets (which doesn't apply when there are more than two terms in a bracket). They did not consider what pupils would need to do at a later stage or how to teach the topic effectively so that they would not have to unteach and reteach concepts later.

Assessment and feedback

In a minority of cases where assessment was effective, it was thoughtfully planned, purposeful, and was adapted well to support pupils' needs. Teachers carefully planned the assessment approaches to identify what they wanted to find out. **Purposeful feedback** was provided following assessment points and teachers provided pupils with sufficient time to understand what they did well, how to improve, and to act on the feedback. Leaders closely **monitored and tracked** pupils' progress using the information to respond to individual learning needs and well-being, shaping the direction of learning and informing them about the quality of teaching.

At **Ysgol Uwchradd Aberteifi**, pupils appreciate the regular feedback they receive from their maths teachers on how they can improve the quality of their work. Where appropriate, teachers within the department use mini white boards regularly during lessons to monitor how well pupils have understood what has been explained and provide formative feedback, which ensures that misconceptions are addressed quickly. Pupils are provided with valuable opportunities to practise methods independently and teachers provide pupils with the solutions at the end of lessons so that pupils can self-assess their progress. Where pupils identify that they have not completely understood the concept, they seek advice from the teacher.

The school assessment policy provides departments with the flexibility to choose strategies that will maximise pupil progress in that subject. To accompany regular verbal feedback, the mathematics department formally assesses pupils' understanding at the end of each unit of work. After each test, teachers provide whole-class feedback on the most common mistakes seen, followed by valuable opportunities for pupils to attempt questions that were similar to those that they found difficult. Teachers re-visit pupils' responses to ensure that their feedback has been acted upon and has led to improvements to pupils' work. Teachers use the information from these assessments well to inform

future homework so that pupils have regular retrieval practice on topics that they found difficult.

In the best cases, assessment was seamlessly **integrated** into every lesson, helping teachers gauge pupils' understanding and progress. It identified misconceptions and ensured that the pace of the lesson matched the pace of pupils learning well. Teachers used this information effectively to adjust the direction and pace of the lesson.

Where this was most effective, teachers employed a variety of appropriate techniques that helped assess pupils' recall of prior learning, encouraged critical and creative thinking, and clearly identified the next steps in learning. Some strategies included:

- skilful use of starter / recall activities to expose and address misconceptions
- mini whiteboards and diagnostic questions to gauge pupils' understanding and application of new learning.
- identification of pupils' understanding at timely points in a lesson by exposing them to answers that were derived from misconceptions to identify the next steps in learning
- effective questioning to check pupils' understanding
- using assessment information derived from a lesson to inform future lesson starters
- providing clear success criteria that enabled older pupils to successfully assess their progress
- offering instant and/or verbal feedback, circulating the class to give immediate corrections, direction, and pose probing questions; these teachers understood that written feedback in mathematics can often be too late

Teachers at **Llanishen High School** have revised their approach to assessment and feedback. Teachers in the mathematics department have implemented a strategy that focuses on circulating the classroom, engaging in 'live marking', and providing timely feedback to pupils. This approach has proven effective as it allows the teachers to identify and address misconceptions immediately. For instance, they may pause the lesson to clarify common misunderstanding, ensuring that all pupils can progress confidently. This method demonstrates the importance of responsive teaching approaches in mathematics and the proactive role of teachers in responding to pupil needs in real time.

During a Year 10 lesson, pupils demonstrated significant progress in their understanding of number and algebra skills. The lesson commenced with a well-planned recap of rational and irrational numbers, square numbers and factors of numbers, which served as a foundation for combining these skills. The teacher assessed the pupils' initial understanding through a series of open and closed questions, effectively gauging their grasp of these skills.

The teacher's subject knowledge was evident during the explanation phase, where they skilfully conveyed complex ideas before allowing pupils to work independently on a range of meticulously crafted exercises designed to develop fluency. Throughout the independent work phase of the lesson, the teacher actively circulated the classroom, “live marking”, providing precise verbal feedback tailored to each pupil's strengths and areas for improvement.

The teacher's proactive approach included addressing key learning points based on real-time observations. For example, they clarified the concept of the difference of two squares and addressed misconceptions related to simplifying surds involving negative values. This responsive teaching method enabled these pupils to reflect on their learning and make strong progress over time.

However, in too many cases, teachers used assessment as an end-of-topic task, gathering scores rather than as a diagnostic tool to measure progress and inform current and future learning. These teachers did not circulate the class or practise instant feedback techniques. As a result, the pace of learning in these lessons was too slow and pupils often practised what they already knew instead of acquiring new knowledge understanding or skills. In a few cases, teachers attempted to assess against the mathematical proficiencies, which was ineffective and did not help pupils to make sufficient progress.

Questions to evaluate the effectiveness of teaching and learning to support mathematical understanding

When planning your lessons, do you:

- start with what you want pupils to learn as the basis for planning your lesson?
- consider each pupil individually and differentiate the lesson accordingly? Do you challenge each pupil just above where they are comfortable?
- have high expectations of what pupils can do?
- plan for progression and to build on prior learning? Do you know what pupils have already experienced and plan to build on this rather than start from scratch?
- understand the topic fully and plan for the why questions that pupils may ask?
- consider the resources that may help pupils develop a deeper understanding?
- consider how you will teach for understanding as well as the tasks you will set for the pupils?
- consider what pupils will find difficult / potential misconceptions when planning the lesson or series of lessons? Do you plan specifically to expose and address these to ensure effective pupil progress?
- plan for the assessment of pupils' learning? Including regular questioning and opportunities to adjust the lesson dependent on pupil progress and understanding?
- plan purposeful opportunities for pupils to apply mathematical concepts when they are ready?
- plan for pupils to apply their knowledge and skills independently?
- consider how you will ensure that pupils are thinking throughout the lesson?

During lessons do you:

- use questioning skilfully to regularly assess how well pupils are making progress and adjust the lesson accordingly?
- see mistakes as learning points?
- circulate the class providing pupils with real time feedback?
- adapt the lesson depending on pupils' response?
- make effective use of resources to help pupils make the strongest progress?

Following lessons do you:

- use assessment information to inform future teaching and set the start point of future lessons?
- reflect on what went well and what needs to be improved both with pupils' progress and understanding and with the quality of teaching?

Chapter 2: Principles of an effective curriculum

Where curriculum was effective:



The curriculum is everything a pupil experiences when in school. This includes aspects of what they learn, where they learn and how they learn.

Pupils who had positive attitudes and enjoyed learning mathematics generally made the best progress. Where pupils enjoyed learning mathematics, the quality of teaching was high, and the curriculum challenged them appropriately. In these cases, the curriculum was designed to be relevant to the pupils' experiences, which stimulated both their **engagement and curiosity**. Teachers modelled learning well and possessed strong subject knowledge. They inspired pupils to develop a love for learning mathematics through high expectations and positive reinforcement. This contributed to a culture of success that permeated all aspects of school and home life, where pupils felt encouraged to achieve.

To strengthen the mathematics curriculum, in the few best cases, schools involved **parents** and the **community** in pupils' learning. This ranged from classes for parents to help them support their children more effectively, to speakers from the community demonstrating how they use their mathematical skills in work and everyday life.

In **Ysgol Uwchradd Aberteifi**, all teachers in the department tell pupils that they are not allowed to say that they are weak at maths. This message has also been shared with teachers and parents. There is a policy at the school that all teachers must talk positively about maths.

Last year, the Head of Mathematics put messages on social media advertising a Maths for Adults course enabling parents to learn some of the key skills in order to help their children at home. The school has also provided information evenings for parents with strategies to support their children. These were popular with the hall full of parents. Teachers try to share these messages and offer support to parents when they meet them in parents' evening.

In **Olchfa School**, Parental workshops focused on the use of manipulatives.

In September 2022, within the framework of Curriculum for Wales, the mathematics department at Olchfa School trialled a Concrete-Pictorial-Abstract approach with pupils in lower attaining groups. The aim was to support pupils in developing their strategic competence and understanding of mathematical concepts, improve their confidence, and foster more positive attitudes towards mathematics. They introduced the use of specific concrete manipulatives to develop pupils' arithmetic and number skills, and double-sided counters to improve their understanding of directed numbers. While the impact on pupils' progress was noticeable from both teachers' and pupils' perspectives, this approach was not fully understood by parents who were more familiar with a traditional approach to teaching and learning.

Research into the impact of parental engagement strategies on pupils' learning prompted the idea of creating workshops for parents to introduce and explain the use of manipulatives in the learning of mathematics. They hoped that these workshops would help create a more positive attitude towards learning mathematics and provide tools for parents to positively support their children at home.

In the Spring term of 2023-2024, the school selected the pupils and parents who they felt would benefit the most from this initiative. They targeted parents/carers who had requested help supporting their children, those who had questions about the teaching approach, and those who had hinted at negative experiences when learning mathematics themselves. Importantly, they collaborated with the pastoral team to include families who the school was particularly interested in reaching out to. The school sent individual invitations to the chosen families and received a very positive response. In preparation for the workshops, the school engaged with a programme aimed at helping adults improve their confidence and competence with their own numeracy skills.

The school ran two workshops for parents and carers and encouraged the children to attend with them where appropriate. At each workshop, the school welcomed around 25 adults and children.

The theme of the first workshop in May 2024 was how to gain a deeper understanding of directed number calculations by using double-sided counters. The interactive session guided parents and carers through short activities designed to share how teachers use the counters to help pupils visualise and add and subtract directed numbers. The theme of the second workshop in June 2024 was how to gain fluency and confidence in mental calculations with the use of rods. This workshop was again interactive, and participants were provided with activities to practise efficiently adding and subtracting numbers by bridging through 10, as well as demonstrating other uses of this manipulative. Families were provided with their own set of counters and rods to take away. They were also shown how to access and use interactive resources if they didn't have the physical resources. The aim was to maximise the opportunities for parents and carers to support their children at home.

Both workshops were conducted through a very positive atmosphere with plenty of eureka moments from parents and carers who had struggled with particular concepts or methods previously. Following the first workshop, many parents were keen to attend the second workshop and other workshops as appropriate. Parents with a language barrier were supported successfully by the school's English as an Additional Language staff.

The positive feedback collected from participants following the workshops supports the school's view that the workshops had a great impact on the parents' and carers'

confidence in their own mathematics and their knowledge of how to support their children. This has resulted in a more positive attitude from pupils in class.

In schools where pupils made good progress and sustained this, curriculum organisation was considered carefully and designed to meet the needs of all learners. Where pupils made the best progress, there was often a **consistent approach** to teaching and learning of mathematics within the school.

Designing a curriculum

Features of effective curriculum organisation included:

- Senior leaders who understood the importance of planning the curriculum around the **distinct nature** of mathematics. In these cases, senior leaders provided space and time for curriculum leaders to interpret policies and plan an effective curriculum. These senior leaders continued to challenge and monitor the quality and impact of the curriculum.
- Middle leaders who considered carefully the **deployment** of their staff, ensuring that staff were assigned according to their skills. They did not always place their better teachers with higher attaining classes, recognising that teachers with the strongest subject knowledge and expertise supported students of all abilities effectively.
- Middle leaders who ensured that their staff gained a wide range of **experience**, such as teaching younger pupils or those studying for higher-level qualifications. However, they considered the risks and ensured that they mitigated carefully for any risks.
- Effective **differentiation**. In the best cases, even where pupils were taught in attainment groupings the teachers differentiated the work in those classes well.
- Teachers who regularly **discussed mathematics and pedagogy**. They indicated appropriate pedagogy in schemes of work and developed resources collaboratively.
- Departments who developed a **consistent approach to teaching and learning**. In these schools, teachers taught for understanding and planned work carefully to avoid teaching tricks and step-by-step methods. They thought about what was coming later for pupils and addressed these things by teaching properly in the first place.
- Staff who were responsive to pupils' needs and used **assessment information** well when designing the curriculum. They ensured an inclusive experience for all pupils.

- Leaders who **closely monitored and tracked pupils' progress**. They used the information wisely to respond to individual learning needs and well-being, shape the direction of learning and inform the quality of teaching.
- The provision of regular opportunities for staff and pupils to **review and update** the curriculum planning topics and documentation.

When developing an effective curriculum, the best schools:

- planned a **progressive curriculum** that built effectively on pupils' prior learning; they ensured that each aspect of the curriculum built appropriately on pupils' prior learning, particularly at points of transition, allowing pupils to revisit mastered skills without relearning them
- adopted a **cyclical approach** to curriculum development, ensuring that pupils regularly encountered different aspects of the curriculum
- designed a curriculum that **balanced** direct teaching methods to introduce and reinforce new concepts with engaging and independent tasks to deepen and solidify previous knowledge
- **collaboratively designed** schemes of work and learning plans that highlighted suitable opportunities for using concrete and representational manipulatives, along with other relevant **resources and methodologies**
- developed a comprehensive strategy for addressing misconceptions
- developed a curriculum that helped build understanding rather than a reliance on teaching tricks or using step-by-step guides
- utilised the five proficiencies as teaching tools, providing pupils with opportunities to develop these proficiencies themselves so that they could use them naturally in new and challenging situations
- planned for **appropriate assessment points** in addition to regular in-class assessments, using this data to inform purposeful planning
- created opportunities to develop pupils' **reasoning skills** and their approaches to problem-solving, ensuring that these were mathematically rich and not contrived
- provided pupils and parents with helpful learning journey documentation so that they understood well what would be learned during the year

Where we saw the strongest curriculum arrangements:

In schools with primary aged pupils, teachers:

- understood how pupils best learn mathematics and these teachers developed firm foundations for mathematics

- utilised high quality resources and used manipulatives well to help pupils understand the concepts they were learning
- provided opportunities for pupils to apply their mathematical skills appropriately across the curriculum

In schools with secondary aged pupils:

- the leadership of mathematics was well-considered and highly effective
- teachers regularly demonstrated high quality teaching
- where teaching was less effective, there were comprehensive systems in place, including professional learning opportunities, to rapidly address concerns and improve the quality of teaching well.
- teachers understood and used the proficiencies effectively to help pupils develop their understanding and make important connections between aspects of their work
- transition was seamless and pupils built on prior knowledge well

In **Ysgol Cwm Banwy**, leaders have a clear vision for developing pupils' mathematics skills. Staff collaborate effectively to plan enriching, authentic learning experiences that engage pupils purposefully, fire their imaginations and nurture their love for learning. Activities meet the needs of pupils successfully and reflect the school's local context well. Pupils in Years 1,2 and 3, for example, fulfil an 'apprenticeship' as they work with a local joiner to design and build imaginative bird boxes to be located within the grounds of the school's memorial garden. They apply a wide range of mathematical skills and concepts, try out and refine ideas and use appropriate tools successfully. During the apprenticeship, they consider aspect such as sustainability and conservation and develop useful leadership and teamworking skills. Over time, teachers plan for progressive development of pupils' skills effectively via these authentic experiences and, as a result, pupils make strong progress over time.

Where the curriculum was not as effective, schools:

- often relied too heavily on published schemes of work to plan their learning and teaching; they did not consider well enough what they wanted pupils to learn, and which aspects of the scheme would support pupils to learn best.; they did not consider well enough whether planned questions addressed misconceptions or supported pupils' progress
- often provided limited or low-level opportunities for pupils to apply their numeracy skills across the curriculum
- provided limited opportunities for pupils to develop their independent learning skills

- did not understand well enough the purpose of the proficiencies and often planned lessons around them, assessed against them and considered how to report on them
- produced schemes of work and learning plans that lacked appropriate guidance regarding prior learning, mathematical vocabulary, misconceptions, pedagogical approaches (concrete and manipulative representations), and guidance on variation
- did not develop subject plans that considered well enough their approach to assessment and feedback, improving the quality of teaching including questioning, identification of how they would consider misconceptions and provide authentic learning experiences
- often overcrowded the curriculum leaving insufficient space to develop pupils' understanding and application of skills when pupils were ready

Initial teacher education

The quality of initial teacher education for the teaching of mathematics was too variable. In some partnerships, the number of student teachers was very small, and they were not always provided with enough relevant opportunities to discuss and improve mathematical pedagogy or explore mathematical subject knowledge well enough. In other partnerships, larger numbers of student teachers facilitated these discussions and explorations more effectively, although the prior knowledge and experiences of some student teachers varied significantly, and within larger cohorts there was too much opportunity for those who were less confident to hide from improving their knowledge, understanding and skills. Often these student teachers were not challenged well enough to improve and responsive teaching by tutors was not prioritised well enough to address these difficulties.

Where practice was strongest:

- programmes supported student teachers well to understand curriculum requirements as well as the importance of high-quality teaching
- programmes supported student teachers to plan mathematics lessons effectively; these student teachers were supported to develop a repertoire of effective pedagogy and strengthen their subject knowledge – these programmes supported student teachers to teach pupils to build their understanding rather than follow a series of steps
- tutors supported student teachers well to plan lessons to meet the needs of all learners
- programmes taught student teachers about planning for and addressing misconceptions that pupils may have in each aspect of mathematics

- programmes placed a strong emphasis on the understanding of curriculum requirements such as the proficiencies, progression, planning, and assessment in mathematics
- tutors engaged student teachers well with relevant academic texts and theories

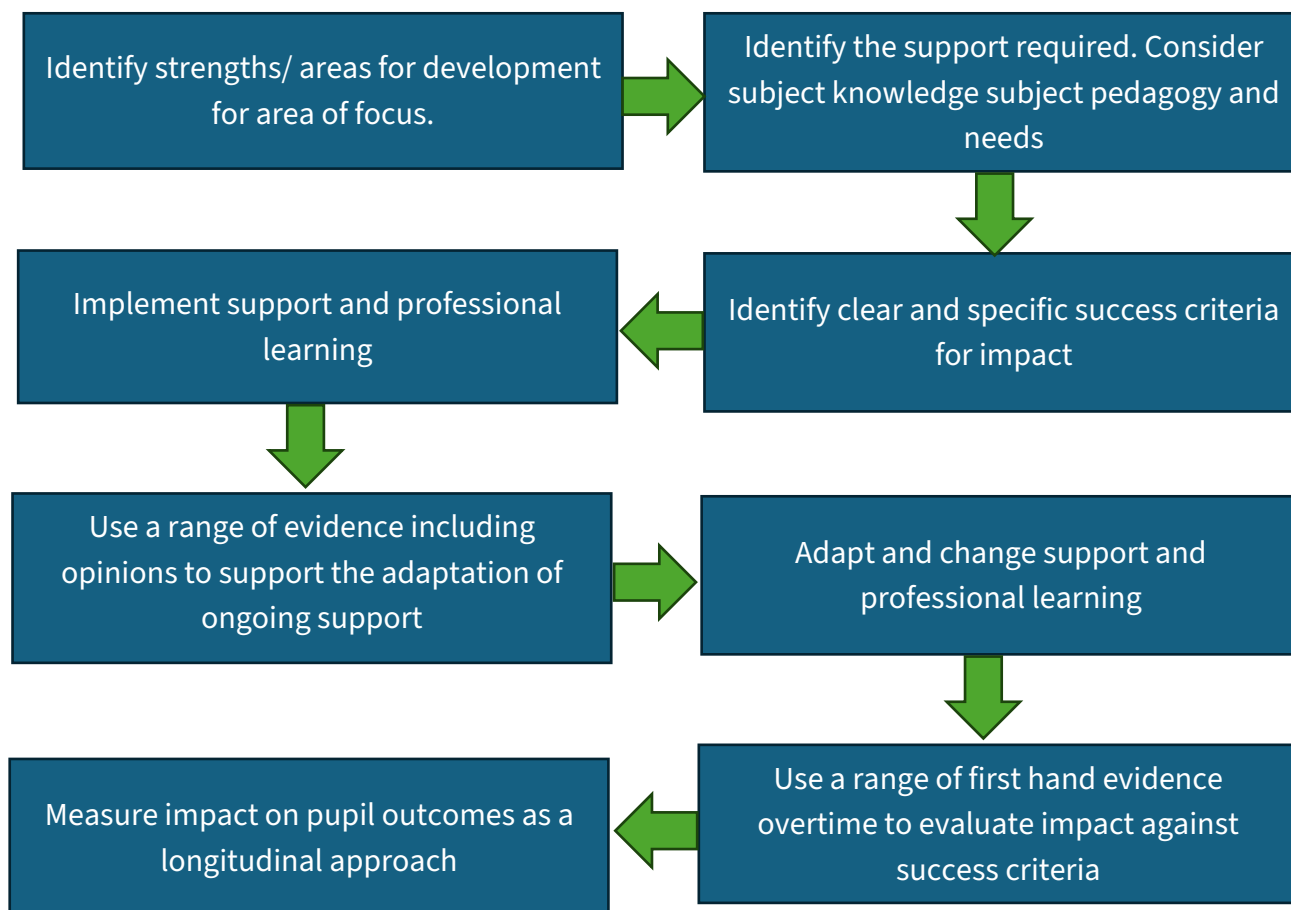
Other features of effective practice that helped student teachers make good progress in their learning:

- tutors had secure subject knowledge and demonstrated a strong understanding of subject pedagogy and confidently helped student teachers to develop these
- tutors drew out student teacher misconceptions around Curriculum for Wales and subject specific misconceptions well
- tutors supported student teachers well to understand and plan to address common pupil misconceptions; this was an important feature
- tutors modelled high quality teaching; they knew the student teachers well and exemplified how to consider the needs of individual students when planning a lesson and a series of lessons relating the learning sensitively but effectively to these
- tutors skilfully targeted student teachers with relevant aspects of the lesson to draw on their strengths, increase their confidence and support the aspects that they were less confident with
- tutors knew the student teachers well and had a good understanding of their strengths and areas for development; they understood how pupils learn mathematics and applied these principles to individually support student teachers effectively

Questions to consider when planning for an effective curriculum:

- How well do teachers inspire pupils to have a ‘love’ for learning mathematics?
- How well do teachers and leaders foster pupils’ positive attitudes to learning and confidence in mathematics?
- How well do we promote positive attitudes and aspirations within the community?
- Is the mathematics curriculum offer effective and progressive? Does it build on prior knowledge successfully particularly at points of transition? Does it include helpful opportunities for pupils to use the five mathematical proficiencies and apply and connect their learning?
- How effective is curriculum organisation for mathematics in our school? This may include aspects such as grouping and staff deployment. How do we know?
- What does effective numeracy across the curriculum look like? How well does our school provide opportunities and support for pupils to apply mathematics in authentic contexts? (This will include the differentiation of numeracy work across the curriculum).
- How well do we track pupils’ progress to inform purposeful planning?
- Does our mathematics department have suitable autonomy with respect to expectations for feedback and marking, assessment, and curriculum design to suit the teaching of mathematics rather than fitting to a whole-school strategy?
- How well do our teachers and leaders of mathematics plan for learning? For example, how well do they:
 - consider what pupils will find difficult / potential pitfalls when planning their lesson or series of lessons?
 - plan to expose and address misconceptions to ensure effective pupil progress?
 - plan for progression and to build on prior learning?
 - plan for the assessment of pupils learning during and after the lesson?
 - use assessment information to inform teaching?
 - plan purposeful opportunities for pupils to apply mathematical concepts when they are ready?
 - plan for pupils to apply their knowledge and skills independently?

Chapter 3: Improving teaching through ongoing support, challenge and professional learning



Professional learning is key to improving the quality of teaching. Teachers and leaders of mathematics generally felt that **internal professional learning** had the greatest impact on their practice. A minority of them felt that the **external professional learning** they had access to was of a high quality, tailored to mathematics and the school's context, and had a positive impact on both the quality of teaching and the degree of pupils learning. Overall, however, the quality and impact of professional learning for the teaching of mathematics was too variable. As a result, the quality of teaching remains too variable, and pupils' mathematical knowledge skills and understanding are not developing well enough.

Generally, Curriculum for Wales documentation was not understood well enough, and schools felt that guidance was not clear enough. Often, schools felt that that this was because there was too much information for them to **interpret** about each AoLE and the guidance was not specific enough about minimum expectations or the progress that pupils should make. As a result, leaders' and teachers' interpretation of the curriculum was

varied. In addition, there was variability in schools' understanding of the intentions of the mathematical proficiencies and how they should be used in the teaching of mathematics. Professional learning to support their understanding was too variable. Engagement with professional associations and groups within Wales that was tailored specifically to the Curriculum for Wales was limited. As a result, schools spend too much time trying to decipher guidance without the confidence that they have correctly interpreted what they have to do.

Where teachers found **school-based professional learning** to be valuable, teachers and leaders noted that they had time to collaborate and plan together. In these cases, they identified the best mathematics teaching methods and explored and expanded their subject knowledge, planning together to ensure consistency across all classes.

Where professional learning worked well:

- strategies were tailored to the needs of the individual subject area
- leaders and teachers knew their own strengths and areas for improvement well; they shared their expertise and planned effectively for improvement by providing targeted professional learning opportunities to enhance staff skills
- leaders created effective plans to address areas that needed improvement, including organising meaningful professional learning activities for staff both internally and from external bodies
- subject leaders and teachers in charge of mathematics delivered relevant professional learning sessions tailored to meet the specific needs of the school's mathematics team, including aspects such as enhancing understanding of metacognition, improving the use of the proficiencies, and some topic based professional learning
- leaders and teachers worked together to develop comprehensive and helpful subject strategies

In **St Joseph's RC High School**, they used collaborative time valuably to develop plans to address misconceptions. Together, they identified the common misconceptions and shared ideas and agreed how these should be addressed effectively. Schemes of work and learning plans reflected the discussions and supported well all team members, in particular, new or non-specialist teachers and those with weaker subject knowledge or pedagogical expertise. These plans helpfully included details about the sequencing of lessons, connections to other skills and topics both within and outside of mathematics, and identified prior learning, subject specific vocabulary and common misconceptions.

Ysgol Penglais School has improved mathematical practice well over time.

In 2017, the new headteacher at Ysgol Penglais focused on improving school standards by prioritising teaching and learning. Recognising the importance of subject-specific pedagogy, the headteacher encouraged curriculum middle leaders to research effective teaching strategies. The head of the mathematics department responded by researching and trialling various strategies to enhance pupils' long-term understanding of mathematical concepts.

After extensive research, various classroom approaches were trialled, many of which successfully improved pupils' engagement and progress in maths. Departmental meetings and professional learning sessions encouraged all members to try these strategies with their classes. Regular, informal teaching observations led to productive discussions about the strategies' merits and impact. This practice, along with regular pupil-voice activities, has helped the department effectively evaluate their provision.

Some of the strategies that the department trialled were:

- **Silent explanations.** This involves the teacher silently annotating the board during the first example, allowing pupils to think about the reasoning behind each step. Subsequent examples are verbally explained, with opportunities for pupils to contribute and ask questions.
- The use of **mini whiteboards** and **diagnostic questions** to assess all pupils' understanding and identify and address misconceptions early. This timely identification prevents misconceptions from becoming embedded. It also improves questioning, as teachers use pupils' responses to initiate rich discussions about misconceptions and their connections to other aspects of mathematics.
- The use of **variation theory** to ensure that pupils are constantly thinking while they practice, looking for patterns, and developing an understanding of the underlying mathematical structures within a concept.
- The regular use of **retrieval practice** so that pupils do not forget important concepts. This includes the use of retrieval homework.
- Use of the '**Turn and Talk**' strategy to encourage pupils to have rich conversations around mathematics, and for teachers to give sufficient time for pupils to think before responding to their questions.
- Re-introduction of **dedicated** lessons to provide opportunities for pupils to think deeply.
- The use of **Framer models** when defining concepts so that pupils can see examples, and non-examples of mathematical ideas. This has led to pupils having a better conceptual understanding of aspects of their work.
- the 'Test Twice' strategy involves giving Key Stage 3 pupils the end of year exam, marking it, and providing feedback. Videos on each topic help pupils understand areas

where they didn't perform well. About two weeks later, pupils are re-examined to see if they have improved and retained the new learning. The department records the average of the two marks for summative assessment tracking. This strategy has increased pupil engagement in the feedback process, and pupils have responded positively to it.

- the use of **visualisers** to expose and address misconceptions.

Over time, this work has led to improvements in the quality of teaching and greater consistency in practice within the department. It has also led to a culture where teachers regularly reflect on their practice, are self-critical and are always seeking ways to improve their teaching.

Since 2017, there have been substantial improvements in the quality of mathematics teaching at Ysgol Penglais. This is reflected in the progress that pupils make, the attitudes they have towards mathematics, and the standards they achieve at the end of Key Stage 4.

Overall, we found that opportunities for leaders and teachers of mathematics to access **external professional learning** to improve the quality of teaching or the leadership of teaching were too variable and often limited. This included the provision of subject-specific support. Often, external professional learning was non-subject specific or did not focus sufficiently on the understanding of mathematical methodologies. Whilst there were more opportunities for external professional learning aimed at enhancing the quality of mathematics teaching for primary-aged pupils than for older pupils, overall, there were limited opportunities for professional learning to elicit understanding of the five proficiencies. In addition, our evidence suggests that there is a lack of professional associations or groups in Wales from which teachers of mathematics could seek advice.

External support for the professional learning of teachers and leaders of mathematics often included the facilitation of the sharing of good practice at subject network meetings for both mathematics and numeracy. Where external support was strongest, professional learning opportunities for mathematics teachers included training for newly qualified teachers and bespoke support for schools interested in, for instance, the concrete, visual, abstract approach to mathematics particularly in the primary sector. Additionally, school improvement services provided some professional learning opportunities that could be delivered at cluster or school level, including support from School Improvement Partners.

The evaluation of the **impact** of professional learning opportunities for mathematics has been limited. Some school improvement partners reported that their efforts have helped schools develop a better understanding of how pupils' mathematical skills develop over time, and improved schools' ability to evaluate progress. However, overall, measurement of impact was limited, with most evaluations based on attendance numbers and participant feedback. A few services stated that they followed up after a period to see how

many new strategies had been adopted, but examples of impact were often operational rather than substantive.

Support for teaching and leadership in mathematics was generally provided through network meetings, toolkits, and resources. In the best cases, effective pedagogy was a regular agenda item at these meetings, and coaching was available for practitioners on an individual basis. Bespoke support was offered to schools where mathematics and/or numeracy had been identified as needing improvement. However, there was limited subject-specific support across Wales, and the quality of teaching remained too variable.

Questions to evaluate the effectiveness of professional learning

For the Welsh Government, local authorities and school improvement partners:

- What specific to mathematics professional learning opportunities are available to teachers of mathematics including opportunities to share good practice?
- Are there effective formal support groups or national associations?
- What support and challenge is provided for the progressive development of effective mathematics teaching for all teachers? Is this support sustained throughout the different stages of a teacher's development?
- Is there sufficient focus on the support to improve teachers' subject knowledge? What does effective support look like?
- What support and challenge is provided for teachers' knowledge of, and effective use of, mathematics subject pedagogy?
- How are mathematics subject leaders supported to develop their own leadership skills, particularly evaluating and improving teaching?
- How are you evaluating the impact of your work on improving leadership and teaching?

For senior leaders:

- Is there sufficient focus on subject pedagogy in the professional learning programme?
- How are mathematics subject leaders developing pedagogy in their departments?
- Is there sufficient flexibility in professional learning to ensure that it is tailored well to developing individual subject pedagogy and expertise?
- Does the school assessment policy provide departments with the flexibility to choose strategies that will maximise pupil progress in that subject?
- Are discussions around teaching methodologies an agenda point on departmental and line management meetings?

For mathematics subject leaders:

- How often do we have 'sharing good practice' or 'top tips' discussions during our departmental meetings?
- As a department, do we discuss pedagogy and different teaching approaches on a regular basis – formally and informally?
- How often do we observe other members of our department teaching? What do or could we learn from this?

- How often do we discuss approaches to formative assessment / assessment for learning in mathematics?
- Which administrative tasks could be completed by an email / bulletin in order to free departmental meeting time for discussions on T&L?
- Do we read books / articles or listen to podcasts around maths teaching and discuss these as a department?
- How often do we, as a department discuss teaching approaches? For example, do we choose a topic, and then consider and discuss:
 - what knowledge and skills pupils will need before commencing the learning of the new topic?
 - the different approaches to teaching that topic?
 - how this topic can be broken into smaller steps?
 - what pupils are going to find hard when learning this topic?
 - what examples will we choose and why?
 - what opportunities this topic provides for pupils to develop their problem-solving skills?
 - how pupils will know how to identify and when to apply this concept when problem solving?
 - what other concepts could have problems set in similar contexts (e.g. what type of questions might be set with right angled triangles?)
 - what opportunities there are during the teaching of this topic to develop pupils' reasoning skills?
 - what misconceptions pupils might make during the learning of this topic?
 - whether the method we are teaching is adaptable for all situation encountered in the study of that topic?
 - what the open questions are that could be asked during the teaching of this concept?
- How often do we, as a department discuss approaches to formative assessment? For example, do we discuss:
 - how we know if all pupils understand the concept and its application?
 - what misconceptions pupils are likely to develop when we teach certain topics and how we will identify the misconceptions that pupils have made?
 - how we ensure that a pupil knows that they are on the right path?
 - what prompts we can use to move pupils' understanding to the next level?
 - how we ensure that the pupil has communicated their answer in a clear fashion?
 - how we know that pupils are following mathematical conventions?

Methods and evidence base

This report draws on findings from inspection evidence since February 2022, interviews with headteachers, leaders and teachers of mathematics, pupils, local authorities and school improvement partners. It includes findings from school visits to a sample of nine primary, 15 secondary and three all-age schools, and additional findings from recent inspections of one primary and two secondary schools. These findings have been drawn from school curriculum documents, meetings, and observations of teaching.

In addition, it considers evidence from online surveys sent to the headteachers of the schools we visited (21 responses), leaders and teachers of mathematics across Wales (181 responses), and local authorities and school improvement partners (24 responses).

Of those who responded to the survey for leaders and teachers of mathematics, 78.5% were from secondary schools, 17.7% were from primary schools, 3.3% were from all-age schools, and 0.6% worked in sixth form provision. The majority (70.7%) of respondents stated that they were teachers of mathematics, whilst 29.3% stated they were leaders of mathematics. The majority (64.4%) of those who responded to a question about length of service (149) indicated that they have been teaching mathematics for over 10 years, 16.1% indicated they had been teaching between 6 and 10 years, 14.8% that they had been teaching for 1 – 5 years, whilst 4.7% had been teaching mathematics for less than a year.

Copies of the questionnaires we used for this thematic review can be found on the publication page.

Estyn would like to thank the following schools that supported this thematic review:

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Caersws CP School	St Joseph's Catholic and Anglican Secondary School
Cefn Hengoed Community School	St Joseph's RC High School
Crickhowell High School	St Julian's Primary School
Crossgates CP school	The Bishop of Llandaff CiW High School
Ebbw Fawr Learning Community	The John Frost School
Guilsfield CP school	Ysgol Bryn Hedydd
Haverfordwest High VC School	Ysgol Corn Hir
Lewis School Pengam	Ysgol Cwm Banwy
Llanfoist Fawr Primary School	Ysgol Felinfach
Llanishen High School	Ysgol Gyfun Ystalafera: Bro Dur
Milford Haven School	Ysgol Gymraeg Bro Morgannwg
Monmouth Comprehensive School	Ysgol Penglais School

Olchfa School	Ysgol Uwchradd Aberteifi
Prestatyn High School	Ysgol Y Creuddyn

Glossary

Glossary of terms

The transition from concrete manipulatives to representational mathematics to abstract mathematics is a teaching approach designed to help pupils understand and internalise mathematical concepts.

Concrete Stage	This is the "hands-on" phase where pupils use physical objects to understand mathematical concepts. This stage helps pupils connect abstract ideas to real-world experiences. For example, they may use blocks, tiles and counters to understand mathematical concepts such as number and expanding brackets and factorisation in algebra.
Representational Stage	In this stage, pupils may use pictures, drawings, or digital images instead of physical objects to understand mathematical concepts. These visual tools help them see the connections between different mathematical ideas and make it easier to move from hands-on learning to more abstract thinking.
Abstract Stage	Finally, pupils use numbers and symbols to understand mathematical concepts. They realise that these symbols can replace the physical objects and pictures they used before. This helps them understand math better and get ready for more difficult problems. It's important to note that this isn't just a straight path from the concrete stage to the representational to the abstract. Instead, it's a flexible system where pupils can move back and forth between stages, making connections and building their understanding as they go.
Manipulatives:	
Concrete manipulatives	Concrete manipulatives in mathematics are physical objects that pupils can see, touch and move to help them understand mathematical concepts. For example, using blocks to represent numbers or shapes can help pupils visualise and grasp mathematical relationships and operations. These tools are especially useful in the early stages of learning, where they can

	help pupils form a solid foundation for more abstract mathematical thinking later. In the concrete phase pupils handle objects like counters and place value cards to understand concepts and their relationship to each other. It's important to choose manipulatives carefully so that pupils don't just play with them, as this reduces their representational value. Concrete manipulatives should be used purposefully and be relevant to the learning goals. Effective teaching, including clear instructions, is vital to ensure these resources support pupils' progress.
Representational manipulatives	As pupils progress, they need to develop an understanding of mathematical concepts beyond just using physical objects. They should be able to imagine and work with mathematical ideas in their minds. This deeper understanding is important for solving more complex problems. This is when they move from using physical objects to using pictures or drawings, for example using a number line to understand difference. However, to do this well, they first need to use physical objects to create a mental image of the math concepts.
Abstract mathematics	Abstract mathematics is about understanding ideas and patterns rather than specific numbers and shapes. It focuses on the logic and rules that define how numbers and operations work, helping us solve problems and prove theories. Too often, mathematics teaching starts with abstract concepts, which can make it hard for pupils to grasp the underlying ideas. When teachers jump into abstract concepts too early, they often end up teaching tricks instead of developing pupils' understanding. The final stage of learning mathematics is applying the skills to new and unfamiliar situations. This is when they begin to reason mathematically.
Cognitive load	Cognitive load is the amount of mental effort used in working memory. In the context of mathematics teaching, it's important not to overload pupils' cognitive load to ensure effective

	learning. Some strategies to avoid cognitive overload in the teaching of mathematics: • Teachers should ensure that pupils are fluent in the basic number skills required for the task. This ensures that pupils' working memory is not overloaded with remembering the basic calculations and as a result they can concentrate on solving mathematical problems. • Teachers should break down complex problems into smaller and more manageable parts. • Teachers should provide clear and simple instructions and use diagrams, charts, and other visual tools appropriately. • Teachers should connect new information to concepts that pupils are already familiar with to make it easier to understand. • Teachers should encourage pupils to write down important information or interim calculations. Teachers should provide pupils with regular retrieval practice to ensure that pupils' prior learning is regularly explored.
Cold calling	Cold Calling in the classroom is a teaching technique where the teacher asks a question to a pupil who has not raised their hand or indicated that they want to answer. This method ensures that all pupils stay engaged and attentive, as they could be called upon at any time. It helps to check for understanding and encourages participation from all pupils, not just those who are eager to answer.
Dialogic teaching approaches	Dialogic teaching is an approach that uses conversation to spark interest, encourage thinking, deepen understanding, broaden ideas, and help pupils develop arguments. This method supports lifelong learning. It involves a variety of talk structures, with the most important being effective questioning. Alexander (2006) outlined five key aspects of dialogic teaching: • Collective: Learning tasks are tackled together by teachers and pupils as a group or class.

	• Reciprocal: Both teachers and pupils listen to each other, share ideas, and consider different viewpoints. • Supportive: pupils can express their ideas freely without fear of making mistakes, and they assist each other in reaching shared understandings. • Cumulative: Ideas are built upon by teachers and pupils, creating coherent lines of thought and inquiry. • Purposeful: Dialogic teaching is planned and guided by teachers with specific educational goals in mind. Mercer and Littleton (2007) identified different types of talk (disputational talk, cumulative talk and exploratory talk), with exploratory talk being the most dialogic. This type of talk is collaborative and investigative, where pupils work together to explore and understand concepts. Dialogic teaching is not just about talking; it's about using talk as a tool to foster a deeper, more engaged learning experience.
Fluency in number	Numerical fluency refers to the ability to understand numbers and use them effectively to solve problems. When pupils are numerically fluent, they can perform calculations accurately and swiftly, grasp number concepts, and apply them to various and new situations. They should understand the reasoning behind the mathematics and be able to select the most appropriate method for the problem at hand. Refer to the mathematical proficiencies below.
FOIL	In mathematics, FOIL is a method used to multiply two binomials*. The term FOIL stands for First, Outside, Inside, Last, which indicates the order in which you multiply the terms in the binomials. First: Multiply the first terms in each binomial. Outside: Multiply the outer terms in the binomials. Inside: Multiply the inner terms in the binomials. Last: Multiply the last terms in each binomial.

	For example, to multiply $(a+b)$ with $(c+d)$ F: $a \times c$ O: $a \times d$ I: $b \times c$ L: $b \times d$ Then, you add all these products together to get the final result. <i>* A binomial is a mathematical expression that consists of two terms connected by a plus or minus sign (often represented as a bracket with 2 terms).</i>
Mathematics pedagogies	The effective methods and practices used to teach mathematics. There are a wide range of techniques, strategies, and approaches that teachers use to help pupils understand mathematical concepts, develop problem-solving skills, and foster a positive attitude towards mathematics. These include aspects such as the use of mini whiteboards, diagnostic questioning, variation theory etc. Effective mathematical pedagogy is built on a secure understanding of how pupils learn mathematics. Lessons should be engaging and challenging yet accessible to all pupils at their pace of learning. Teachers should use a range of effective tools and resources and should be continually assessing pupils' understanding and adapting the lesson accordingly. For example: Mini whiteboards An effective tool that can help with problem-solving by supporting pupils to visualise their way through a problem. In addition, they support pupils to work together for instance with activities such as think, pair, share, and they support teachers to assess and understand pupils' learning and identify misconceptions. Diagnostic questioning Diagnostic questioning involves using multiple-choice questions

	that are carefully crafted to identify and understand pupils' misconceptions and mistakes. These questions are clear and unambiguous, designed to test specific skills such as the rules of angles on a straight line. Pupils should not be able to arrive at the correct answer if they still hold a misconception, unless they make a lucky guess. Each incorrect multiple-choice answer should provide insight into the pupils' misunderstanding or mistake. Variation theory Variation theory focuses on the key parts of a concept by showing what changes and what stays the same. This helps pupils understand the main ideas and make connections. Questions are designed to change one thing at a time while keeping everything else the same, so pupils can focus on that one thing. Teachers encourage pupils to predict the answer to each question based on the previous one, use a taught method to solve it, and think about how their guess matches or differs from the actual answer. For example: Find $\frac{1}{3}$ of 18 Find $\frac{2}{3}$ of 18 Find $\frac{2}{3}$ of 36 Find $\frac{4}{3}$ of 36 Find $\frac{3}{4}$ of 36 For example: Continue these sequences: 2,4,6,8... 1,3,5,7... 21,23,25,27...
Mathematical proficiencies	(The proficiencies are: Conceptual understanding, Communication using symbols, Fluency, Logical reasoning, and Strategic competence).

	Conceptual Understanding: pupils build on, deepen, and connect mathematical concepts as they encounter increasingly complex ideas. Communication Using Symbols: understanding that symbols are abstract representations and developing flexibility with their application and manipulation. Fluency: developing the ability to remember facts, relationships, and techniques, making them firmly established, memorable, and usable. Logical Reasoning: this proficiency involves pupils developing the ability to think logically and critically about mathematical situations. Strategic Competence: strategic competence is about pupils developing the ability to formulate, represent, and solve mathematical problems.
Mathematical reasoning	This is when pupils think about mathematics problems in a logical way to understand and accurately solve them. Examples and non-examples are an important part of mathematical reasoning. Pupils make links and connections between what they already know and the problem they are faced with.
Misconceptions	These are varied and are wide ranging, for example, ‘moving’ the decimal point, the bigger a coin the more it is worth or when measuring, pupils not starting from the end of a ruler. It is important that teachers do not teach misconceptions for example telling pupils that when they multiply by 10, they simply add a zero. In addition, teachers should plan to address misconceptions in their teaching and should plan assessment opportunities to tease out misconceptions held by pupils and address them as soon as possible.
Responsive teaching	Responsive teaching is a way of teaching that involves setting clear goals and planning for learning carefully. It involves the teacher developing an understanding of what pupils have learned and where they are struggling and then adapting the

	teaching methods to help them improve. It focuses on using real-time assessments to adjust lessons based on pupils' success and confidence. Often this is done in the moment or between lessons. Additionally, it aims to meet individual pupil needs by adapting strategies and materials to work with pupils' current understanding and misconceptions, strengths, and interests. When done well, pupils are engaged, they receive timely feedback, and it creates a supportive and productive classroom environment.
Turn and Talk	This is a teaching method where pupils talk to a partner about a topic for a short time. The teacher gives them a question or topic, and they turn to their partner to discuss it. One pupil talks while the other listens, and then they switch roles. They come to an agreement on their thinking. This is then shared with another pair or the whole class.

Numbers – quantities and proportions

nearly all =	with very few exceptions
most =	90% or more
many =	70% or more
a majority =	over 60%
half =	50%
around half =	close to 50%
a minority =	below 40%
few =	below 20%
very few =	less than 10%

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