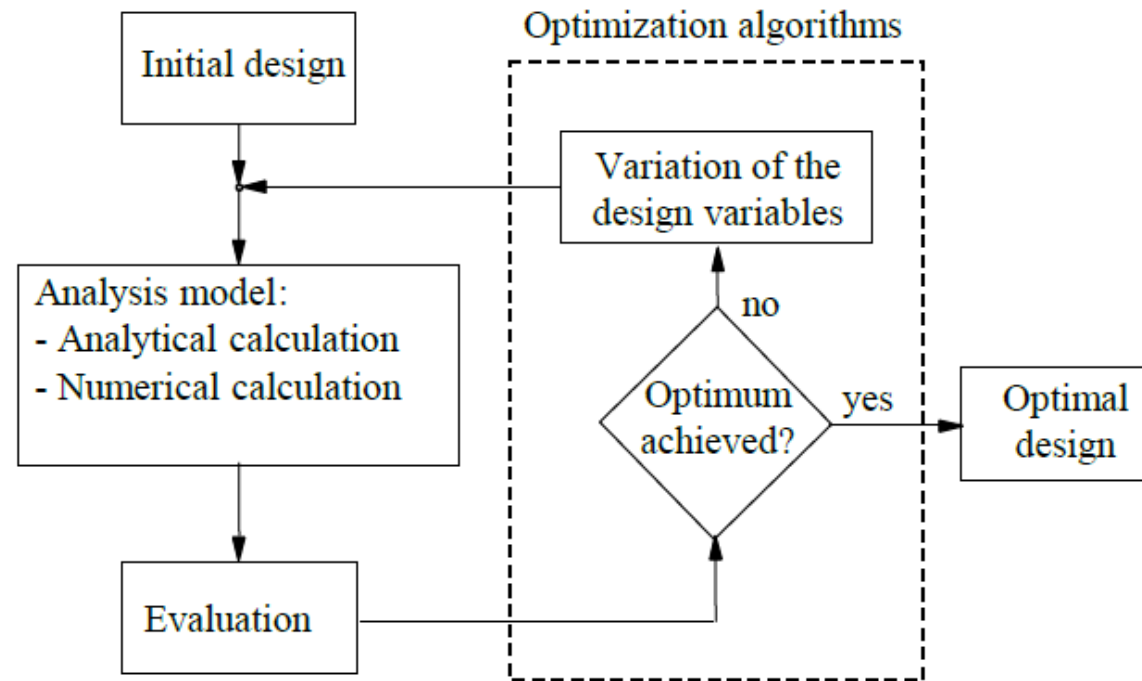


What is Optimisation?

“the greatest degree or best result obtained or obtainable **under specific conditions**” – Dictionary definition

- Variables
- Objectives
- Constraints



Variables

- Material properties
 - Topology
 - Package space
 - Designable material v Non-designable material
 - Panel thickness
 - Laminate lay-up
 - etc
-
- Discrete v continuous

Objectives

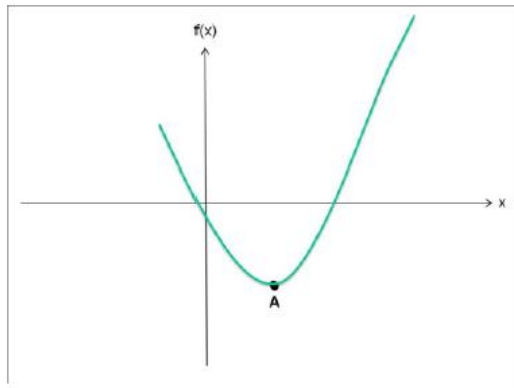
- Measurable response
- Min or max
- Mass
- Mass fraction
- Compliance
- Frequency
- Displacement
- Stress
- strain
- etc
- Multi Objective Optimisation?

Constraints

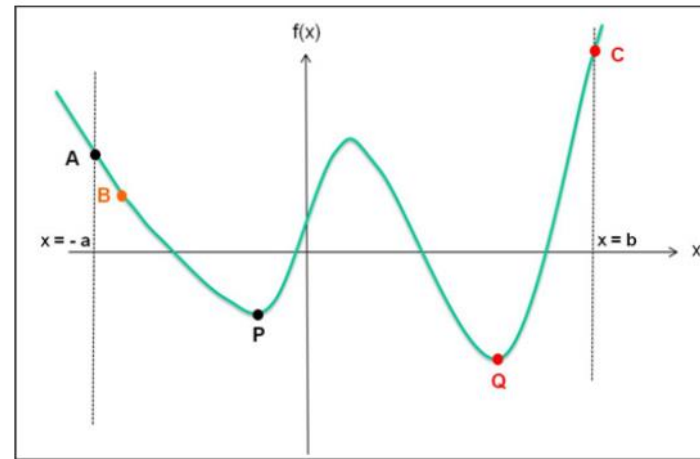
- Only use 30% of the material
 - Limit deflection to x mm
 - Mode $\#y > z$ Hz
 - Modal alignment chart
 - Limit stress or strain
 - Material behaviour must be known
 - Etc
-
- Feasible v infeasible solution

Minimising the Function – Gradient based optimisation

- Minimise a function (if you are using maximise you can simply minimise the inverse)
- A function that has one minimum is a Convex function
 - If quadratic – Differentiate and equate to zero – one answer
 - Local minima = global minima (global optimum is the best of the local optimums)
 - If Higher order multiple minima will exist
 - Local min is MAY NOT be the global minimum
- A minimum can only be GUARANTEED if the optimisation problem is convex



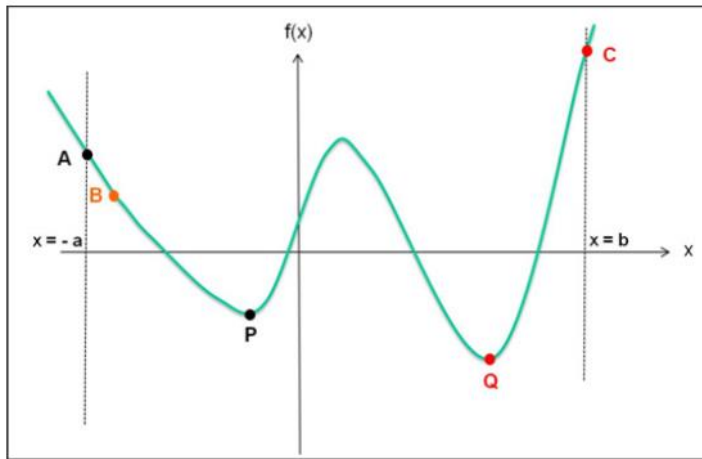
Convex Function, $f(x)$



Non-Convex Function, $f(x)$

Increasing the chance of finding your Minimum

- Most engineering problems can not be shown to be convex
- Different algorithm types alter your chances of finding a more global optimum AT A COMPUTATIONAL COST
- Gradient based solvers will find the minimum relative to the starting point.
- “Multiple starting points optimisation”

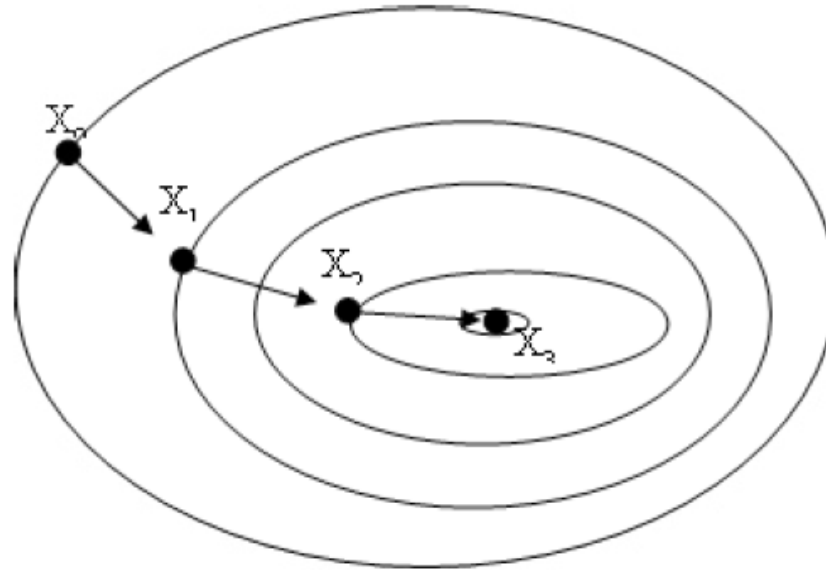


Non-Convex Function, $f(x)$

- Start at “A” = Local optimum “P”
- Start at “B” = Local optimum “P”
- Start at “C” = Global optimum “Q”

Example: Gradient Search Methods

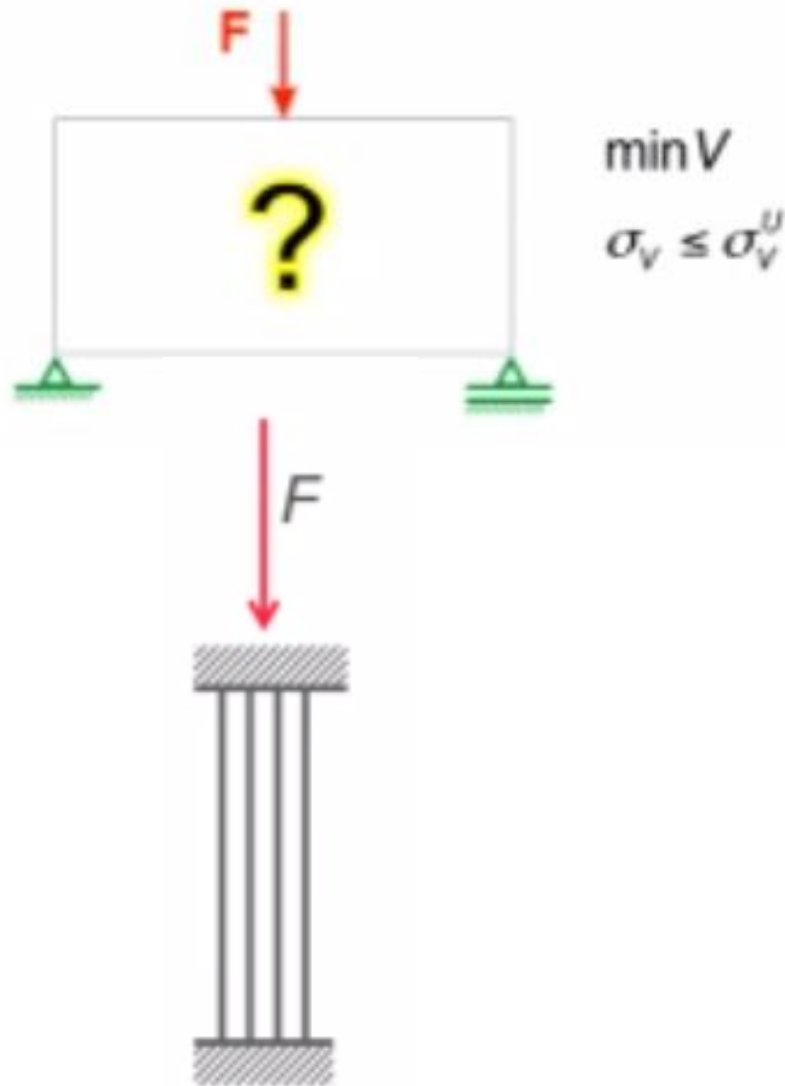
- The Gradient decent method
 - Start at a point X_0
 - The direction in which the objective function decreases most rapidly is calculated
 - Convergence is reached then the gradient is zero
- Iterative solver



Regular/Hard v Soft Convergence

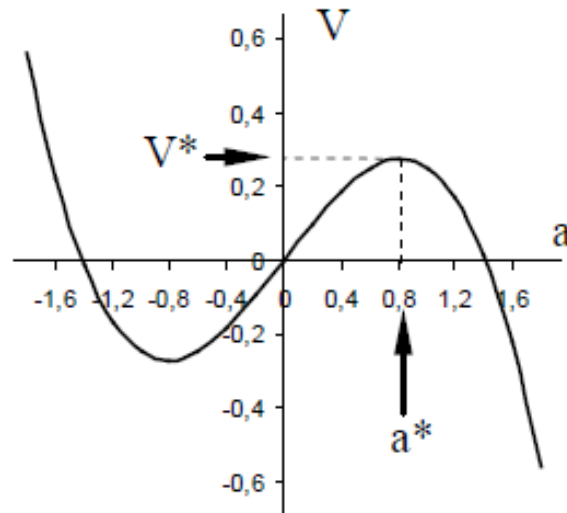
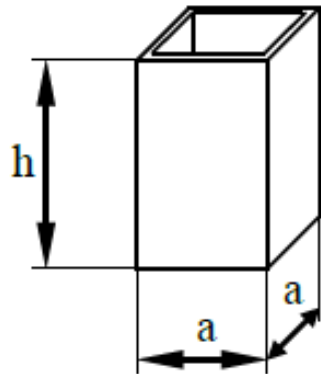
- Regular/Hard convergence
 - Convergence criteria are satisfied for two consecutive iterations
 - Constraint violations are less than certain tolerance
- Soft convergence
 - Little or no change in design variable for two consecutive iterations

Pitfalls in optimisation



- **Unbounded problems**
 - E.g. Minimise volume with stress constraint only
 - OPTIMISED SOLUTION = NO MATERIAL
 - RESOLUTION: Add constraint – e.g. add a displacement constraint
- **Over constrained problems**
 - E.g. Constraint of compliance and constraint on mass – not possible with given variables!
 - RESOLUTION: Change constraints
- **Manifold solutions**
 - E.g. Buckling of struts – Response has multiple modes with identical or similar variables.
 - RESOLUTION: add constraints to particular mode
- **Problems with no solution**
 - Minimise stress when force is applied at a point (stress=force/area)
 - RESOLUTION: Exclude region from constraint
- **Variables are at their max/min**
 - Change the bounds of your variables

A simple mathematical example



- **Objective**
 - Maximise volume
- **Constraint**
 - Use 2 square meters of material
- **Variables**
 - h and a

Notes adapted from/Further reading

- Practical Aspects of Structural Optimisation, Altair University, 2014