

Fatigue

Fracture of components due to fatigue is the most common cause of service failure, particularly in shafts, axles, aircraft wings, etc., where cyclic stressing is taking place. The crack is initiated from points of high stress concentration on the surface of the component such as sharp changes in cross-section, slag inclusions, tool marks, etc., and then propagates under the influence of the cyclic loading, until it reaches a critical size when fast fracture of the remaining cross-section takes place.

The surface of a typical fatigue-failed component shows three areas;

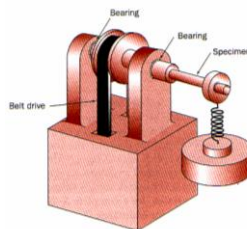
1. The point of initiation
2. A glass-like area containing shell-like markings called “arrest lines” or “conchoidal markings” caused by corrosion when crack is propagating
3. The crystalline area of rupture



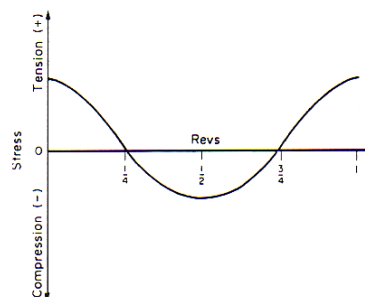
Fatigue failures often occur under conditions where the fluctuating stress is below the tensile strength or even the elastic limit.

The S/N curve

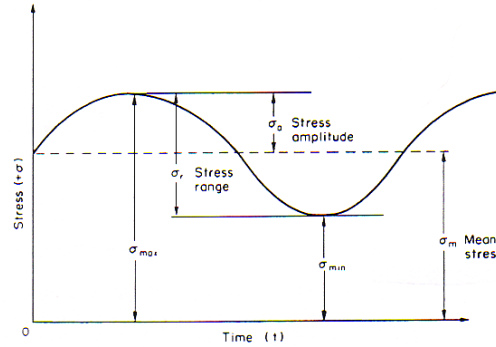
Fatigue test are usually carried out under conditions of rotating – bending and with a zero mean stress as obtained by means of a Wohler machine as shown below;



This puts the top surface into tension and the bottom surface into compression, resulting in the following loading curve;



Tests can be carried out on machines such as the Haigh “push-pull” machine, where the mean stress is positive. This results in the following loading curve;



Where;

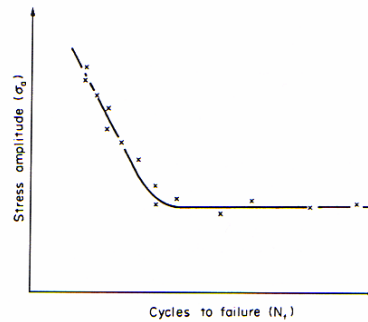
Stress Range, $\sigma_r = 2\sigma_a$

Mean Stress, $\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$

Alternating stress amplitude, $\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$

Stress Ratio, $R_s = \frac{\sigma_{\min}}{\sigma_{\max}}$

The results from many tests can then be plotted on a S/N curve as shown below;

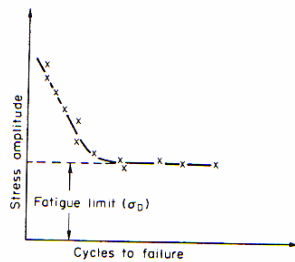


In order to express the relationship between σ_a and N_f the following relationship is often used, providing that the applied load does not produce any plastic deformation;

$$\sigma_r^a N_f = K$$

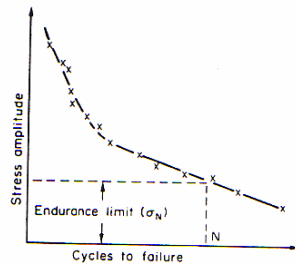
Where a is a constant, which ranges from 8 to 15, and K is another constant depending on the material.

Ferrous metals will typically have the following characteristic S/N curve.



Notice that once the stress falls below a certain value σ_D the part will endure an infinite number of cycles. This stress is called the **Fatigue limit**.

Non-Ferrous metals will typically have the following characteristic S/N curve.



The **Endurance limit** σ_N , is the stress condition under which a specimen would have a fatigue life of N cycles as shown.

The test pieces used to obtain these results are typically plain, polished test pieces. The fatigue life of real components is typically reduced due to differences in;

- Surface finish
- Size
- Type of loading
- The effect of stress concentrations

These factors will reduce the intrinsic fatigue strength thus;

$$\sigma'_N = \frac{\sigma_N}{K_f} [C_a \cdot C_b \cdot C_c]$$

Where σ'_N is the “modified fatigue strength” or “modified fatigue limit”, σ_N is the intrinsic value, K_f is the fatigue strength reduction factor and C_a , C_b , and C_c are factors allowing for size, surface finish, type of loading, etc.

The following types of fatigue loading are commonly seen. The way the part is loaded alters the fatigue life, and should always be quoted;

Axial loading, The material is repeatedly loaded in its axial direction.

Plane Bending, The material is bent around its neutral plane.

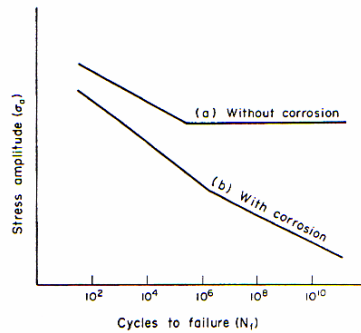
Rotating bending, The material is being rotated at the same time as being subjected to a bending moment.

Torsion, The material is subjected to fluctuating torsional stresses.

Any combination of the above.

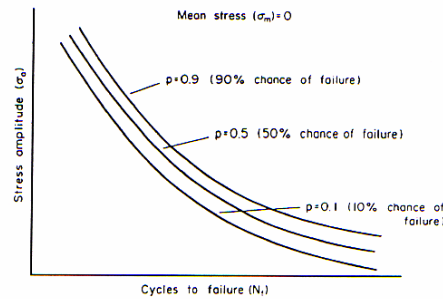
Within a fairly wide range of approximately 100 cycles/min to 6000 cycles/min, the effect of the frequency of load cycling on the fatigue strength of metals is small, however, it may be important, particularly for polymers, so should always be quoted.

Typically, tests are carried out at room temperature. If they are carried out at lower temperatures then the fatigue life of metals will show no deterioration, and may even show a slight improvement. However, if temperature is raised, the fatigue life will decrease due to creep effects. Also, increased temperatures may increase the oxidation process, which in turn will also reduce the fatigue strength as the protective oxide films are cracked allowing further attacks from corrosion.



P/S/N Curves

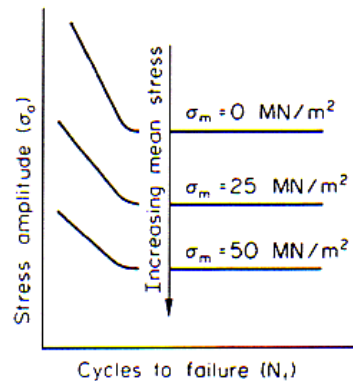
The fatigue life of a component as determined at a particular stress level is a very variable quantity so that seemingly identical specimens may give widely differing results. To overcome this a number of test pieces are tested at the same stress so that the probability of failure can be calculated. Thus the following can be plotted;



Care should be taken when reading S/N curves as to which probability curve is actually plotted.

The Effect of Mean Stress

As the mean stress of a specimen is increased, then the fatigue life is reduced. Or, put another way, if a component is tested with a higher mean stress, in order to see a similar fatigue life to a component tested with a lower mean stress, then the stress amplitude must be reduced.



A number of investigations have been made of the quantitative effect of tensile mean stress, resulting in the following equations;

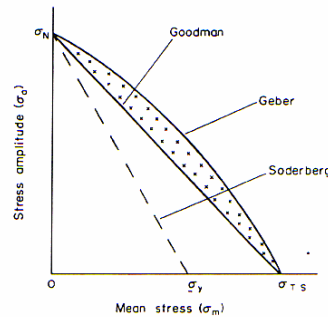
$$\text{Goodman} \quad \sigma_a = \sigma_N \left[1 - \left(\frac{\sigma_m}{\sigma_{TS}} \right) \right] \quad [1]$$

$$\text{Geber} \quad \sigma_a = \sigma_N \left[1 - \left(\frac{\sigma_m}{\sigma_{TS}} \right)^2 \right] \quad [2]$$

$$\text{Soderberg} \quad \sigma_a = \sigma_N \left[1 - \left(\frac{\sigma_m}{\sigma_y} \right) \right] \quad [3]$$

Where σ_N = The fatigue strength for N cycles under zero mean stress conditions
 σ_a = The fatigue strength for N cycles under conditions of mean stress σ_m
 σ_{TS} = Tensile strength of the material.
 σ_y = yield strength of the material.

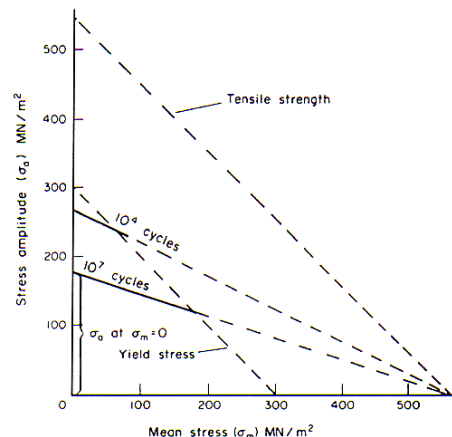
These equations can be plotted as shown below;



In reality most test results fall in the shaded region between the Goodman and Geber curves. However, because Soderberg gives a margin of safety, this is often used. An additional margin of safety is often used, such that equation [3] becomes;

$$\sigma_a = \frac{\sigma_N}{F} \left[1 - \left(\frac{\sigma_m \times F}{\sigma_y} \right) \right] \quad [4]$$

The interrelationship of mean stress and alternating stress amplitude is often shown in diagrammatic form frequently collectively called **Goodman diagrams**. An example of this is the **Haigh Diagram**;

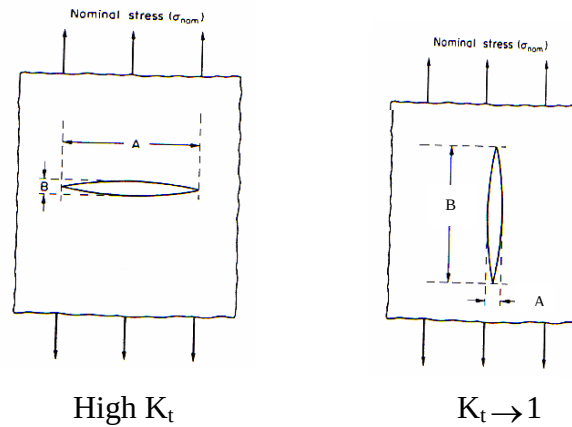


The effect of compressive mean stress upon the life of a component is not so well documented or understood as that of a tensile mean stress, but in general most materials do not become any worse and may even show an improved performance under a compressive mean stress. In calculations it is usual therefore to take the mean stress as zero under these conditions.

Effects of stress concentration

The effect of stress concentration can be illustrated by consideration of an elliptical crack in a plate subjected to a tensile stress. Provided that the plate is large, the “theoretical stress concentration” factor K_t is given by;

$$K_t = 1 + \frac{2A}{B}$$



If the crack is perpendicular to the stress then A is large relative to B, hence K_t will be large. If the crack is parallel to the direction of stress, then A is very small compared with B, hence K_t will be 1.

The effect of a stress concentration on the fatigue limit is indicated by the “*fatigue notch*” or “*fatigue strength reduction*” factor K_f .

K_f is the ratio of the stress amplitude at the fatigue limit of an un-notched specimen, to that of a notched specimen under the same loading conditions. K_f is always less than the static theoretical stress concentration factor because under the ‘compressive’ part of a cycle, the crack is unlikely to grow. The extent to which the stress concentration effect under fatigue conditions approaches that for static conditions is given by the “*notch sensitivity factor*” q , where.

$$q = \frac{K_f - 1}{K_t - 1}$$

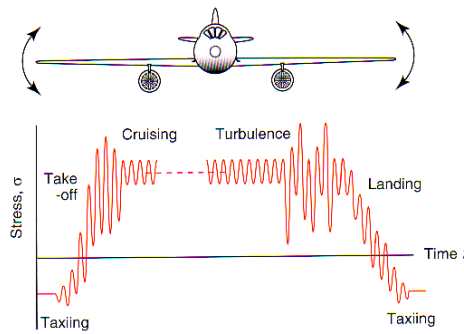
Thus q is always less than 1.

Notch sensitivity depends not only on the material, but also upon grain size, a finer grain size resulting in a higher value of q than a coarse grain size. It also increases with section size, tensile strength, the severity of the notch and the type of loading. In dealing with a ductile material it is usual to apply the factor K_f only to the fluctuating or alternating component of the applied stress. Hence equation [4] becomes;

$$\sigma_a = \frac{\sigma_N}{F \cdot K_f} \left[1 - \left(\frac{\sigma_m \times F}{\sigma_y} \right) \right]$$

Cumulative Damage

In practice stresses seen by components are not sinusoidal inputs of a constant magnitude, but instead will vary. For example, the stress on the underside of a wing might alter as follows during a typical flight.



Several attempts have been made to predict the fatigue strength for such variable stresses using S/N curves for a constant mean stress. The simplest and most well known is “*Miner’s Law*”. Miner stated that

“The fraction of the total damage occurring under one series of cycles at a particular stress level, is given by the ratio of the number of cycles actually endured n to the number of cycles N required to break the component at the same stress level”.

The ratio n/N is called the “*cycle ratio*” and Miner proposed that failure takes place when the sum of the cycle ratios equals 1;

Or

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} + \dots + etc = 1$$

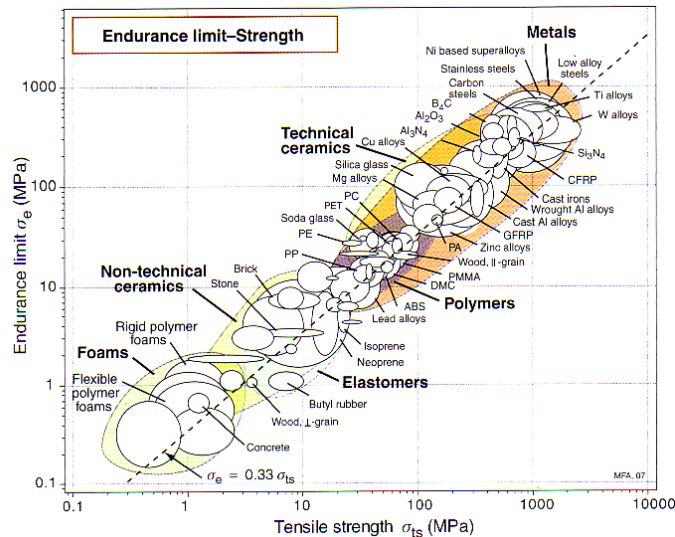
When implementing Miners rule, experience has shown that it is important to consider the higher stress amplitudes first as these have a more damaging effect on fatigue performance than the application of an initial low stress amplitude. Thus the cycle ratios rarely add up to 1, the sum varying between 0.5 and 2.5.

Manipulating resistance to fatigue

Fatigue life is enhanced by choosing materials that are strong, making sure that they contain as few defects as possible, and by giving them a surface layer in which the internal stresses are compressive.

Choosing materials that are strong.

The below chart shows endurance limit at 10^7 cycles at zero mean stress against tensile strength for various materials. Not surprisingly endurance limit and strength are related.

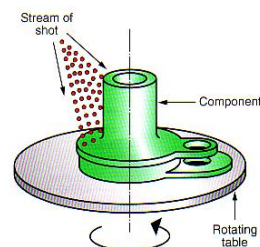


Making sure they contain as few defects as possible.

During processing careful control of the material should ensure that any unwanted particles are removed which could form stress concentrations. Non-destructive testing using techniques such as X-ray imaging or ultrasonic sensing detects dangerous defects, allowing the part to be rejected.

Providing a compressive surface finish.

Cracks only propagate during the tensile part of a stress cycle; a compressive stress forces the crack faces together, clamping it shut. Cracks often initiate from the surface, so putting the surface of the structure into compression will stop these from propagating. One way to do this is to shot peen the surface which plastically compresses the surface, as shown below. Similar techniques include sandblasting. Leaf-springs for cars are shot peened, which can increase their fatigue life by a factor of 5.



Examples

1)

The fatigue behaviour of a specimen under alternating stress conditions with zero mean stress is given by the expression:

$$\sigma_r^a \cdot N_f = K$$

where σ_r is the range of cyclic stress, N_f is the number of cycles to failure and K and a are material constants.

It is known that $N_f = 10^6$ when $\sigma_r = 300 \text{ MN/m}^2$ and $N_f = 10^8$ when $\sigma_r = 200 \text{ MN/m}^2$.

Calculate the constants K and a and hence the life of the specimen when subjected to a stress range of 100 MN/m^2 .

[11.357, 1.356e34, 262e9 cycles]

2)

A steel bolt 0.003 m^2 in cross-section is subjected to a static mean load of 178 kN . What value of completely reversed direct fatigue load will produce failure in 10^7 cycles? Use the Soderberg relationship and assume that the yield strength of the steel is 344 MN/m^2 and the stress required to produce failure at 10^7 cycles under zero mean stress conditions is 276 MN/m^2 .

[685.5kN]

3)

A stepped steel rod, the smaller section of which is 50 mm in diameter, is subjected to a fluctuating direct axial load which varies from $+178 \text{ kN}$ to -178 kN .

If the theoretical stress concentration due to the reduction in section is 2.2 , the notch sensitivity factor is 0.97 , the yield strength of the material is 578 MN/m^2 and the fatigue limit under rotating bending is 347 MN/m^2 , calculate the factor of safety if the fatigue limit in tension-compression is 0.85 of that in rotating bending.

[1.5]

4)

The values of the endurance limits at various stress amplitude levels for low-alloy constructional steel fatigue specimens are given below:

$\sigma_a \text{ (MN/m}^2\text{)}$	$N_f \text{ (cycles)}$
550	1 500
510	10 050
480	20 800
450	50 500
410	1 25 000
380	2 75 000

A similar specimen is subjected to the following programme of cycles at the stress amplitudes stated; $3\,000$ at 510 MN/m^2 , $12\,000$ at 450 MN/m^2 and $80\,000$ at 380 MN/m^2 , after which the sample remained unbroken. How many additional cycles would the specimen withstand at 480 MN/m^2 prior to failure? Assume zero mean stress conditions.

[3598 cycles]