

Consider finally some equations involving fractions. Recall that the fraction a/b is not defined if $b = 0$. If $b \neq 0$, however, then $a/b = 0$ is equivalent to $a = 0$.

EXAMPLE 3 Solve the following equations:

$$(a) \frac{1 - K^2}{\sqrt{1 + K^2}} = 0 \quad (b) \frac{45 + 6r - 3r^2}{(r^4 + 2)^{3/2}} = 0 \quad (c) \frac{x^2 - 5x}{\sqrt{x^2 - 25}} = 0$$

Solution:

- (a) The denominator is never 0, so the fraction is 0 when $1 - K^2 = 0$, that is when $K = \pm 1$.
- (b) Again the denominator is never 0. The fraction is 0 when $45 + 6r - 3r^2 = 0$, that is $3r^2 - 6r - 45 = 0$. Solving this quadratic equation, we find that $r = -3$ or $r = 5$.
- (c) The numerator is equal to $x(x - 5)$, which is 0 if $x = 0$ or $x = 5$. At $x = 0$ the denominator is $\sqrt{-25}$, which is not defined, and at $x = 5$ the denominator is 0. We conclude that the equation has no solutions. ■

PROBLEMS FOR SECTION 2.5

Solve the equations in Problems 1–2:

$$\begin{array}{lll} 1. (a) x(x + 3) = 0 & (b) x^3(1 + x^2)(1 - 2x) = 0 & (c) x(x - 3) = x - 3 \\ (d) \sqrt{2x + 5} = 0 & (e) \frac{x^2 + 1}{x(x + 1)} = 0 & (f) \frac{x(x + 1)}{x^2 + 1} = 0 \end{array}$$

$$\begin{array}{ll} \text{SM } 2. (a) \frac{5 + x^2}{(x - 1)(x + 2)} = 0 & (b) 1 + \frac{2x}{x^2 + 1} = 0 \\ (c) \frac{(x + 1)^{1/3} - \frac{1}{3}x(x + 1)^{-2/3}}{(x + 1)^{2/3}} = 0 & (d) \frac{x}{x - 1} + 2x = 0 \end{array}$$

SM 3. Examine what conclusions can be drawn about the variables if:

$$\begin{array}{ll} (a) z^2(z - a) = z^3(a + b), a \neq 0 & (b) (1 + \lambda)\mu x = (1 + \lambda)y\mu \\ (c) \frac{\lambda}{1 + \mu} = \frac{-\lambda}{1 - \mu^2} & (d) ab - 2b - \lambda b(2 - a) = 0 \end{array}$$

REVIEW PROBLEMS FOR CHAPTER 2

In Problems 1–2, solve each of the equations.

$$\begin{array}{lll} 1. (a) 3x - 20 = 16 & (b) -5x + 8 + 2x = -(4 - x) & (c) -6(x - 5) = 6(2 - 3x) \\ (d) \frac{4 - 2x}{3} = -5 - x & (e) \frac{5}{2x - 1} = \frac{1}{2 - x} & (f) \sqrt{x - 3} = 6 \end{array}$$

SM 2. (a) $\frac{x-3}{x-4} = \frac{x+3}{x+4}$ (b) $\frac{3(x+3)}{x-3} - 2 = 9\frac{x}{x^2-9} + \frac{27}{(x+3)(x-3)}$
 (c) $\frac{2x}{3} = \frac{2x-3}{3} + \frac{5}{x}$ (d) $\frac{x-5}{x+5} - 1 = \frac{1}{x} - \frac{11x+20}{x^2-5x}$

3. Solve the following equations for the variables specified:

(a) $x = \frac{2}{3}(y-3) + y$ for y (b) $ax - b = cx + d$ for x
 (c) $AK\sqrt{L} = Y_0$ for L (d) $px + qy = m$ for y
 (e) $\frac{\frac{1}{1+r} - a}{\frac{1}{1+r} + b} = c$ for r (f) $Px(Px + Q)^{-1/3} + (Px + Q)^{2/3} = 0$ for x

4. Consider the macro model

$$(i) Y = C + \bar{I} + G, \quad (ii) C = b(Y - T), \quad (iii) T = tY$$

where the parameters b and t lie in the interval $(0, 1)$, Y is the gross domestic product (GDP), C is consumption, $\bar{I}$ is total investment, T denotes taxes, and G is government expenditure.

- (a) Express Y and C in terms of $\bar{I}$, G , and the parameters.
 (b) What happens to Y and C as t increases?

SM 5. Solve the following equations for the variables indicated:

(a) $3K^{-1/2}L^{1/3} = 1/5$ for K (b) $(1 + r/100)^t = 2$ for r
 (c) $p - abx_0^{b-1} = 0$ for x_0 (d) $[(1 - \lambda)a^{-\rho} + \lambda b^{-\rho}]^{-1/\rho} = c$ for b

6. Solve the following quadratic equations:

(a) $z^2 = 8z$ (b) $x^2 + 2x - 35 = 0$ (c) $p^2 + 5p - 14 = 0$
 (d) $12p^2 - 7p + 1 = 0$ (e) $y^2 - 15 = 8y$ (f) $42 = x^2 + x$

7. Solve the following equations:

(a) $(x^2 - 4)\sqrt{5 - x} = 0$ (b) $(x^4 + 1)(4 + x) = 0$ (c) $(1 - \lambda)x = (1 - \lambda)y$

8. Johnson invested \$1500, part of it at 15% interest and the remainder at 20%. His total yearly income from the two investments was \$275. How much did he invest at each rate?

9. If $5^{3x} = 25^{y+2}$ and $x - 2y = 8$, then what is $x - y$?

HARDER PROBLEM

10. Solve the following systems of equations:

(a) $\frac{2}{x} + \frac{3}{y} = 4$ (b) $3\sqrt{x} + 2\sqrt{y} = 2$ (c) $x^2 + y^2 = 13$
 $\frac{3}{x} - \frac{2}{y} = 19$ (d) $2\sqrt{x} - 3\sqrt{y} = \frac{1}{4}$ (e) $4x^2 - 3y^2 = 24$