

Advanced Engine Design Exam (M6X00410)

14th May 2020
13:00pm

Name: Alex Oke

Number: P151012

Question One

* Horizontally mounted in-line twin
* 1st order

$$\text{Piston + Pin} = 240\text{g} = 0.24\text{kg}$$

$$\text{Rod} = 330\text{g} = 0.33\text{kg}$$

$$\text{Shells} = 20\text{g} = 0.02\text{kg}$$

$$\text{Stroke} = 90\text{mm}$$

$$R = 45\text{mm}$$

$$\text{Crank} = 5\text{kg}$$

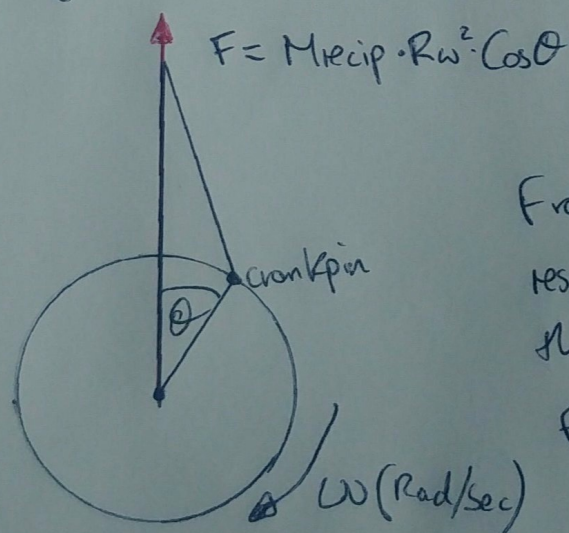
$$\text{Rod length} = 140\text{mm}$$

(Please refer to drawing as rotated 90° as horizontal engine). ^{clockwise}

$$M_{\text{recip}} = (0.24) + \left(\frac{1}{3} \times 0.33\right) = 0.35\text{kg}$$

$$M_{\text{rot}} = (0.02) + \left(\frac{2}{3} \times 0.33\right) = 0.24\text{kg}$$

a.)



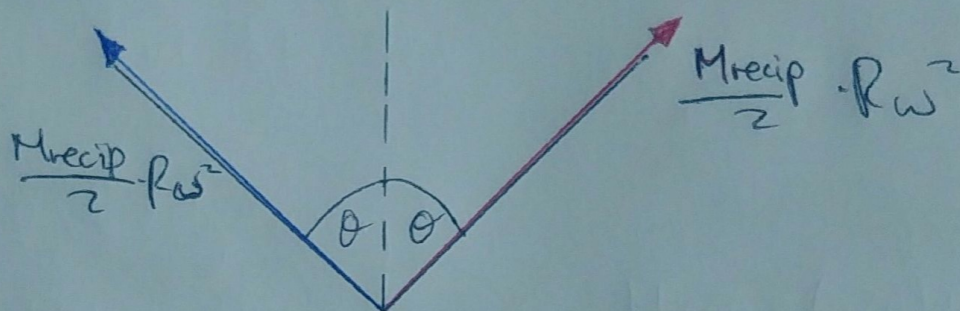
From reciprocating mass, it results in 1st order forces along the vertical cylinder axis.

$$\text{Force} = M_{\text{recip}} \cdot R \cdot \omega^2 \cdot \cos \theta$$

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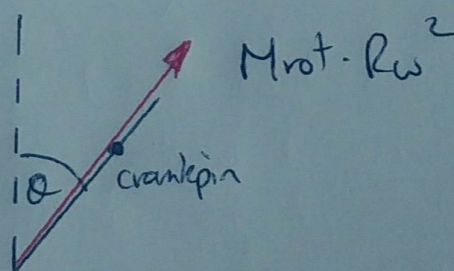
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This diagram above shows the force is split into 2 counter-rotating constant amplitude vectors.

Below is the rotating part creating a centrifugal force:

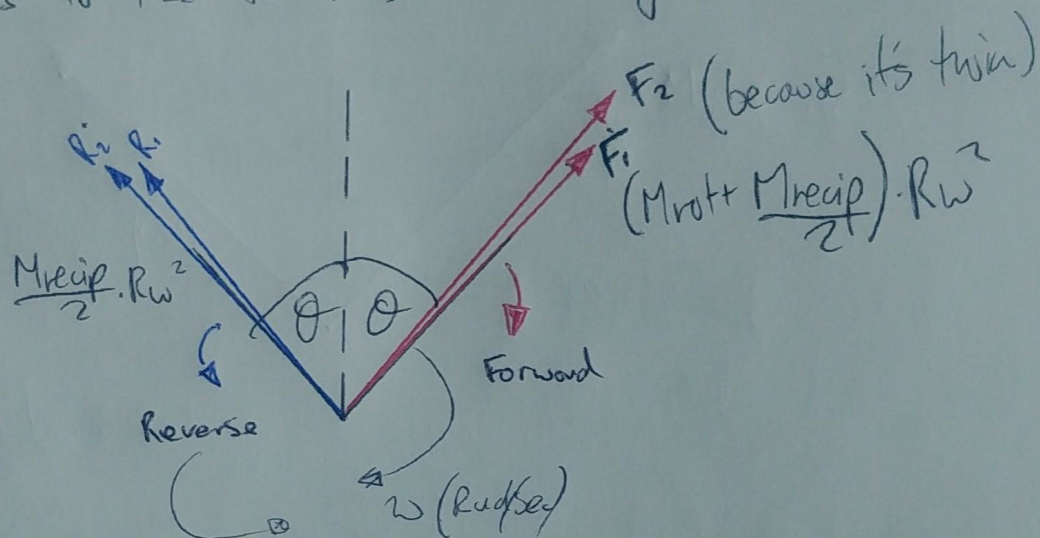


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This combines to the total 1st order force:



So, 1st order forward rotating vector is.

$$= 2 \cdot (M_{rot} + \frac{M_{recip}}{2}) \cdot R \omega^2$$

mass moment

$$= 2 \cdot (M_{rot} + \frac{M_{recip}}{2}) \cdot R$$

$$= 2 \cdot (0.24 + \frac{0.35}{2}) \cdot 45$$

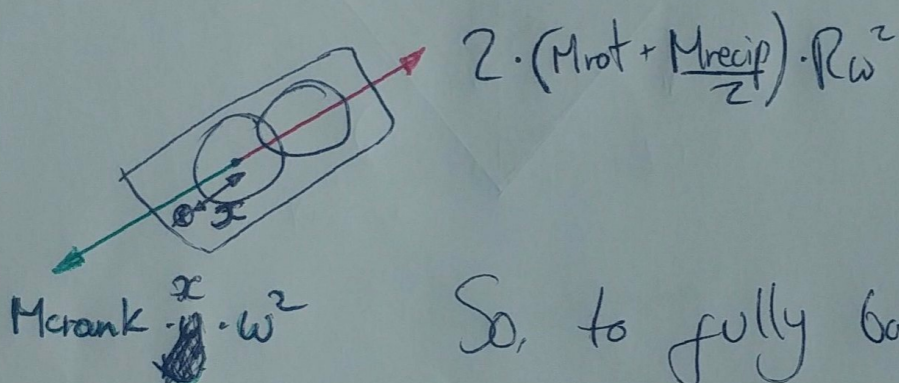
$$= \underline{\underline{37.35 \text{ kg.mmm}}}$$

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- b.) A crankshaft with offset COG creates a centrifugal force to balance the 1st order forward rotating vector.



So, to fully balance =

$$M_{crank} \cdot x \cdot \omega^2 = 2 \cdot \left(M_{rot} + \frac{M_{recip}}{2} \right) \cdot R \cdot \omega^2$$

$$M_{crank} \cdot x = 2 \cdot \left(M_{rot} + \frac{M_{recip}}{2} \right) \cdot R$$

$$M_{crank} \cdot x = 37.35$$

$$x = \frac{37.35}{5}$$

$$x = \underline{\underline{7.47 \text{ mm}}}$$

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C.) So, 1st order reverse vector
$$= 2 \cdot \left(\frac{M_{recip}}{2} \right) \cdot R \omega^2$$

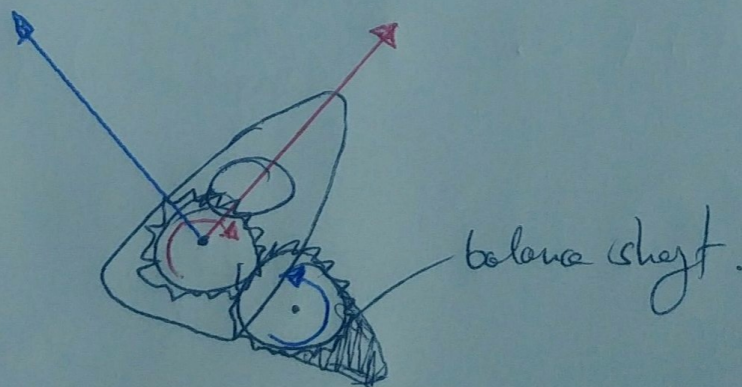
mass moment

$$= 2 \cdot \left(\frac{M_{recip}}{2} \right) \cdot R$$

$$= 2 \cdot \left(\frac{0.35}{2} \right) \cdot 45$$

$$= \underline{\underline{15.75 \text{ kg} \cdot \text{mm}}}$$

d.) To balance 1st order reverse vector force, it require a counter rotating balance shaft. It is best to be gear driven using same size diameter gears to create same rotational speed.



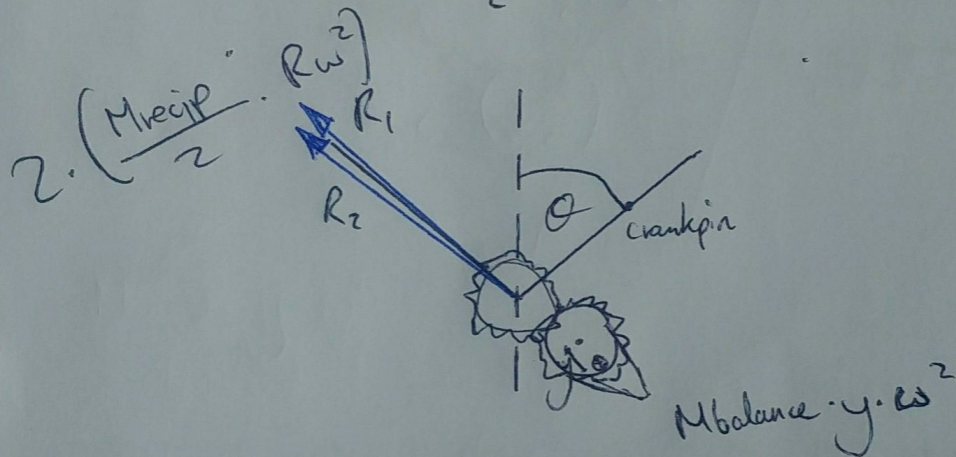
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e.) To fully balance 1st order reverse vector force the offset is calculated as shown:

$$2 \cdot \left(\frac{M_{recip}}{2} \cdot R \omega^2 \right) = M_{balance} \cdot y \cdot \omega^2$$



So,

$$2 \cdot \frac{M_{recip}}{2} \cdot R = M_{balance} \cdot y$$

$$\frac{15.75}{M_{balance}} = y$$

$$y = \frac{15.75}{2.5}$$

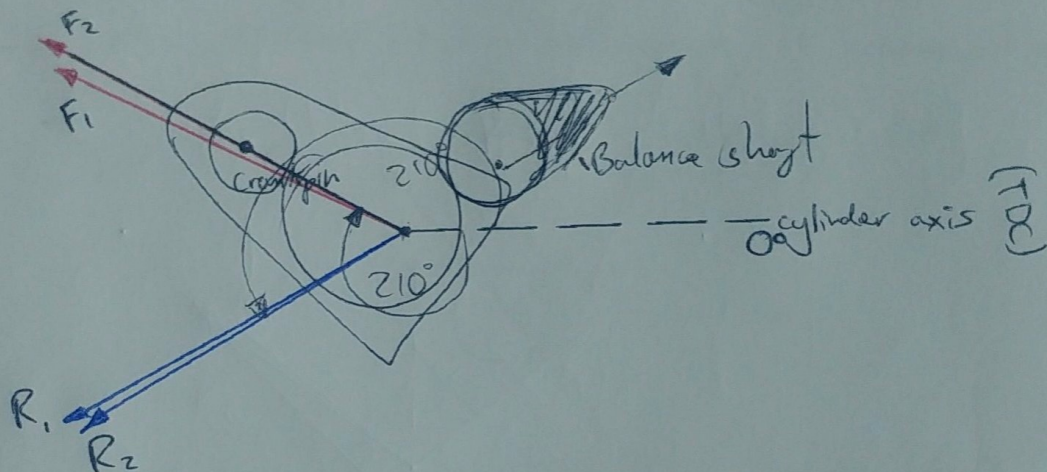
$$y = \underline{\underline{6.3 \text{ mm}}}$$

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F.) Due to engine being horizontal, when the crank is rotated 210° it look like this below:



Eccentric must point opposite to the reverse 1st order vector force.

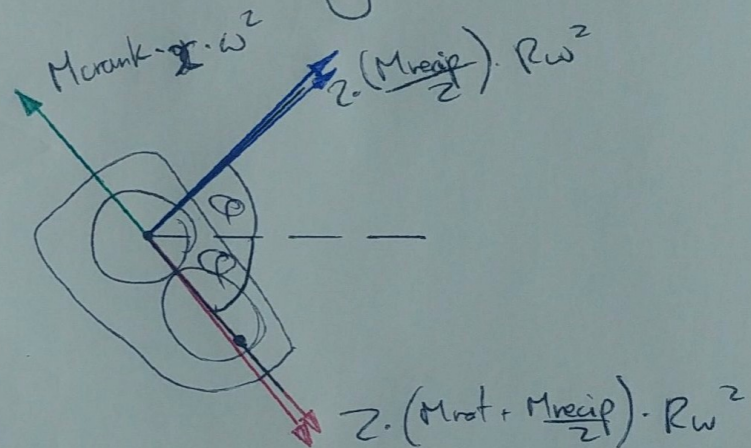
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G.) Being a horizontal engine you would want a resultant force to be ~~the~~ the same as cylinder axis. Therefore below over-balancing is carried out:



$$\left(M_{crank} \cdot x \cdot \omega^2 \right) - \left(2 \cdot \left(M_{rot} + \frac{M_{recip}}{2} \right) \right) = 2 \cdot \left(\frac{M_{recip}}{2} \right) \cdot R \omega^2$$

$$\therefore \left(M_{crank} \cdot x \right) - \left(M_{rot} + \frac{M_{recip}}{2} \right) \cdot R$$

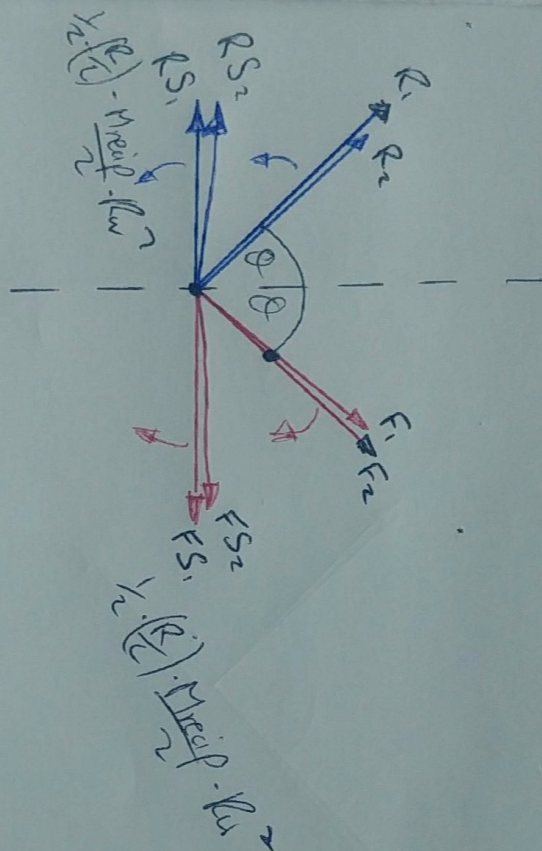
$$\begin{aligned} M_{crank} \cdot x &= \text{mass moment of crank} = 2 \cdot (0.24 + 0.35) \cdot 45 \\ &= \underline{\underline{53.1 \text{ kg} \cdot \text{mm}}} \text{ to overbalance} \end{aligned}$$

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h.)



2nd Order forward

Mass moment

$$= 2 \cdot \frac{1}{2} \cdot \frac{R}{2} \cdot \frac{M_{revip}}{2} \cdot R$$

$$= 2 \cdot \frac{1}{2} \cdot \frac{45}{180} \cdot \frac{0.35}{2} \cdot 45$$

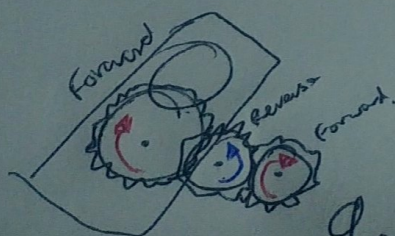
* Mass moment

is same for 6th Forward and Reverse 2nd order

~~2nd Order reverse~~

~~Mass moment~~

~~$$= 2 \cdot \frac{1}{2} \cdot \frac{R}{2} \cdot \frac{M_{revip}}{2} \cdot R$$~~
~~$$= 2 \cdot \frac{1}{2} \cdot \frac{45}{180} \cdot \frac{0.35}{2} \cdot 45$$~~



2nd Order requires use of Secondary gears with half the diameter to create

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I.)

Not enough time to attempt

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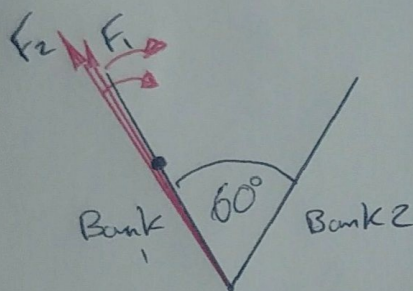
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Question Two

Vee-twin 60° .

a.)



$$\text{piston} = 240\text{g} = 0.24\text{kg}$$

$$\text{conrod} = 360\text{g} = 0.36\text{kg}$$

$$\text{Stroke} = 80\text{mm} = 0$$

$$\text{Rod length} = 140\text{mm}$$

$$R = 40\text{mm}$$

$$M_{\text{recip}} = \left(\frac{1}{3} \times 0.36\right) + 0.24$$

$$= 0.36\text{kg}$$

$$M_{\text{rot}} = \left(\frac{2}{3} \times 0.36\right)$$

$$= 0.24\text{kg}$$

$$\text{So, } F_1 = F_2 = (M_{\text{rot}} + \frac{M_{\text{recip}}}{2}) \cdot R\omega^2$$

* Resultant vector is along bank 1

$$= 2 \cdot \left(M_{\text{rot}} + \frac{M_{\text{recip}}}{2}\right) \cdot R\omega^2$$

1st order mass moment

$$= 2 \cdot \left(0.24 + \frac{0.36}{2}\right) \cdot 40$$

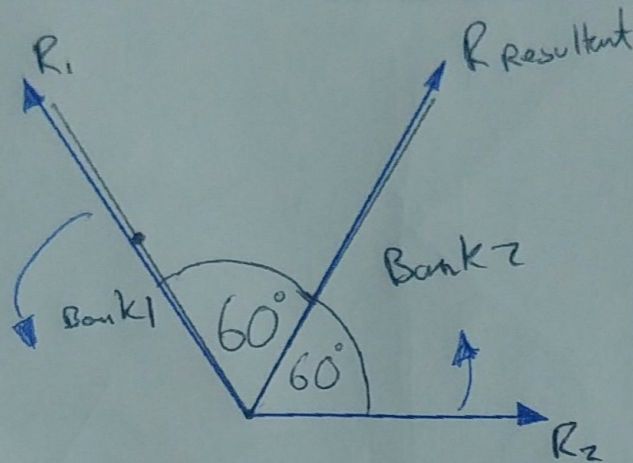
$$= \underline{\underline{33.6\text{kg} \cdot \text{mm}}}$$

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b.)



$$\text{So, } R_1 = R_2 = \frac{M_{\text{recip}}}{2} \cdot R_w^2$$

* direction is along bank 2!

1st order reverse vector

$$\begin{aligned} &= \frac{M_{\text{recip}}}{2} \cdot R_w^2 \cos 60 + \frac{M_{\text{recip}}}{2} \cdot R_w^2 \cos 60 \\ &\therefore = M_{\text{recip}} \cdot R_w^2 \cos 60 \end{aligned}$$

mass moment

$$= M_{\text{recip}} \cdot R \cos 60$$

$$= 0.36 \cdot 40 \cdot \cos 60$$

$$= \underline{\underline{7.2 \text{ kg.m}}}$$

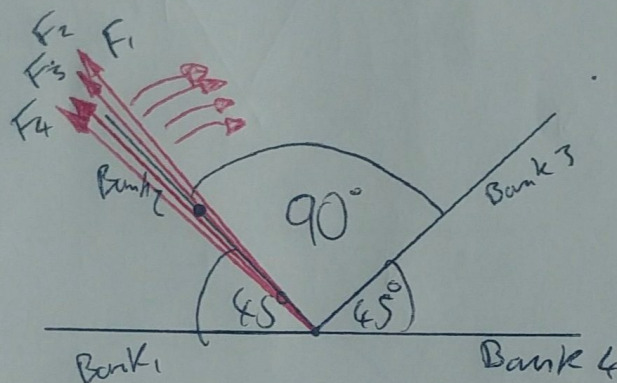
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C.) W4 engine



* Direction is along bank 2 for 1st order vector forward force.

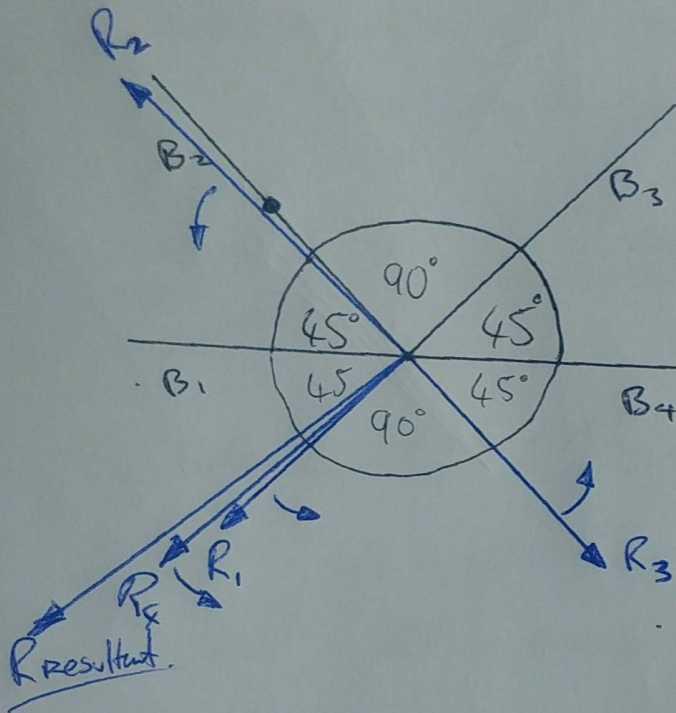
$$\begin{aligned}\text{mass moment} &= 4 \cdot \left(M_{\text{rot}} + \frac{M_{\text{recip}}}{2} \right) \cdot R \\ &= 4 \cdot \left(0.24 + \frac{0.36}{2} \right) \cdot 40 \\ &= \underline{\underline{67.2 \text{ kg. mm}}}\end{aligned}$$

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d.)



$$\text{So, } R = \frac{M_{\text{recip}}}{2} \cdot R$$

$$R_1 + R_4 = 2R$$

$$(R + R) = 2R$$

* Resultant force is opposite bank 3

Mass moment
of 1st order
reverse

$$= 2 \cdot \left(\frac{M_{\text{recip}}}{2} \right) R$$

$$= 2 \cdot \left(\frac{0.36}{2} \right) \cdot 40$$

$$= \underline{\underline{14.4 \text{ kg} \cdot \text{mm}}}$$

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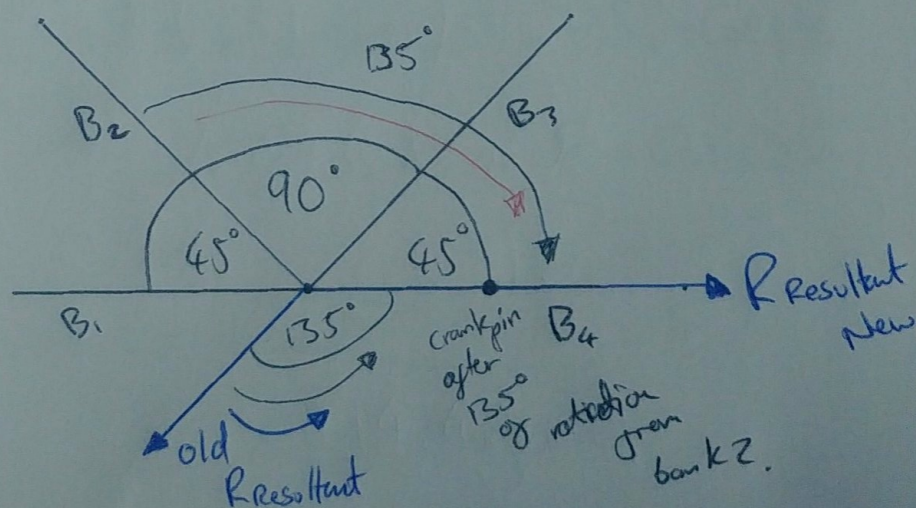
e.) To balance the 1st order forward vector force, you would offset the crank

$$\begin{array}{l} \text{COG} \\ \text{crank} \\ \text{mass} \\ \text{moment} \end{array} = \underline{\underline{67.2 \text{ kg} \cdot \text{mm}}}$$

To balance the 1st order reverse vector force you would use a balance shaft with ~~but~~ a mass moment of

$$= \underline{\underline{14.4 \text{ kg} \cdot \text{mm}}}$$

f.)



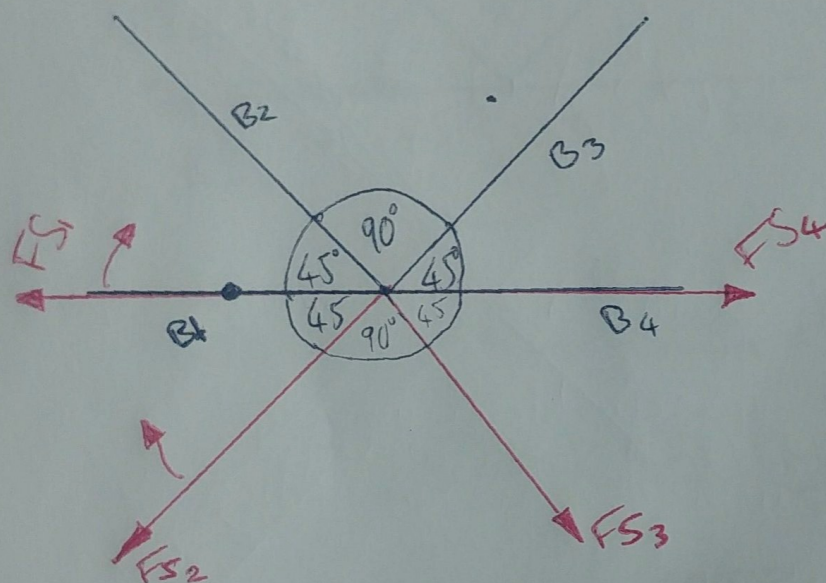
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G.)



$$FS_1 = FS_2 = FS_3 = FS_4 = F$$

$$F = \frac{1}{2} \cdot \left(\frac{R}{L} \right) \cdot M_{recip} \cdot R \omega^2$$

(Horizontal force = 0)

$$\begin{aligned} \text{Resultant} &= FS_2 \times \cos 45 + FS_3 \times \cos 135 \\ \text{vertical force} &= F \times \cos 45 + F \times \cos 135 \end{aligned}$$

$$\text{mass moment} = \boxed{\text{Resultant down}} = F$$

$$= \frac{1}{2} \cdot \left(\frac{R}{L} \right) \cdot M_{recip} \cdot R$$

$$= \frac{1}{2} \cdot \frac{40}{140} \cdot 0.36 \cdot 40$$

$$= \underline{\underline{2.05 \text{ kg} \cdot \text{mm}}}$$

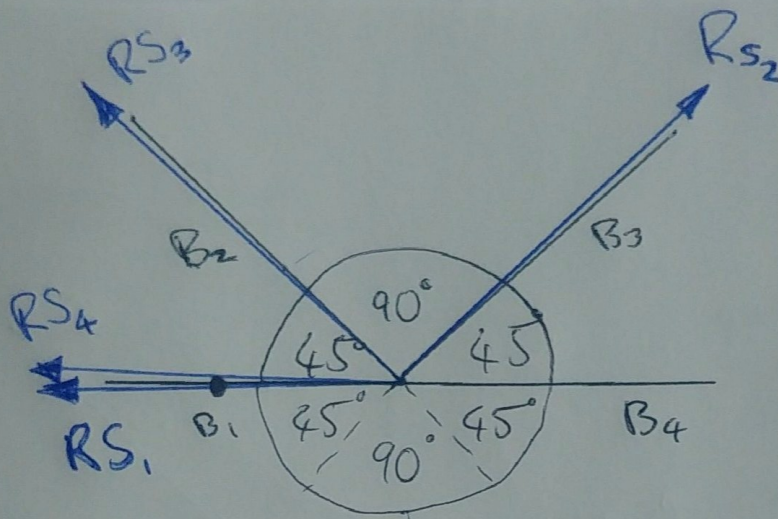
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h.)



$$\text{So, } RS_1 = RS_2 = RS_3 = RS_4 = R$$

$$R = \frac{1}{2} \cdot \left(\frac{R}{L}\right) \cdot \underline{M_{recip}} \cdot R\omega^2$$

Resultant vector
direction = $2R$ along bank 1

$$\begin{aligned} \text{Mass moment} &= 2 \cdot \frac{1}{2} \cdot \left(\frac{R}{L}\right) \cdot \underline{M_{recip}} \cdot R \\ &= 2 \cdot \frac{1}{2} \cdot \left(\frac{40}{140}\right) \cdot 0.36 \cdot 40 \\ &= \underline{4.11 \text{ kg} \cdot \text{mm}} \\ &\text{or} \end{aligned}$$

$$\text{If } R \text{ is not } 2R = \underline{2.05 \text{ kg} \cdot \text{mm}}$$

Run out of time!

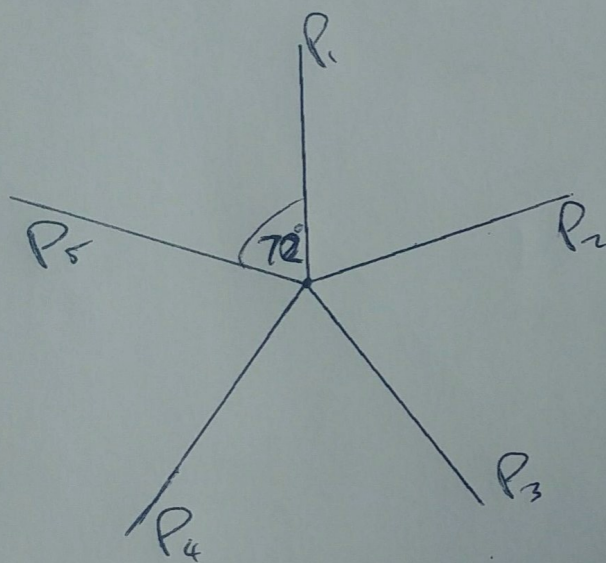
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Question Three

A.)



Piston = 240g = 0.24 kg
Rod = 270g = 0.27 kg
Stroke = 85 mm
R = 42.5 mm
Rod length = 140 mm

$$M_{recip} = \left(\frac{1}{3} \times 0.27\right) + 0.24 = 0.33 \text{ kg}$$

$$M_{rot} = \left(\frac{2}{3} \times 0.27\right) = 0.18 \text{ kg}$$

So, 1st order forward vector

$$= \frac{(M_{rot} + M_{recip})}{2} R \omega^2$$

For each span

mass moment

$$= \left(0.18 + \frac{0.33}{2}\right) \cdot 42.5$$

$$= \underline{\underline{7.19 \text{ kg} \cdot \text{mm per span}}}$$

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b.) M crank each span = 3kg

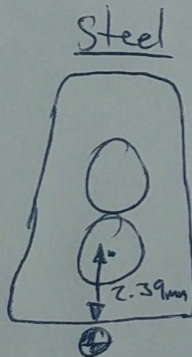
So, $M_{crank} \cdot x \cdot \omega^2 = (M_{rot} + \frac{M_{recip}}{2}) \cdot R \omega^2$

$$M_{crank} \cdot x = M_{rot} + \frac{M_{recip}}{2} \cdot R$$

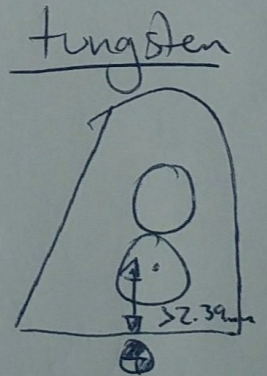
$$x = \frac{7.19}{3}$$

$$x = \underline{\underline{2.39 \text{ mm per span.}}}$$

c.)



Steel density
~ = 7800 kg/m³



Tungsten density
~ = 19,300 kg/m³

The conditions are same so,

$$M_{tungsten} \cdot x_{tungsten} = M_{steel} \cdot x_{steel}$$

* Tungsten increases mass proportionally reducing (x). The moment of inertia is therefore reduced which in racing is good to allow more torque to be used to accelerate the vehicle.