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Test 2

$$\frac{dy}{dx} = y'$$

R 1

$$y = \frac{x^2}{2x+1}$$

$$u = x^2$$

$$v = 2x+1$$

$$u' = 2x$$

$$v' = 2$$

$$y = x^2 (2x+1)^{-1}$$

$$y' = (2x \times 2)$$

$$y' = \frac{(2x \times 2x+1) - (2x \times 2)}{(2x+1)^2}$$

$$y' = \frac{(4x^2 + 2x) - (2x^2)}{(2x+1)^2}$$

$$y' = \frac{(4x^2 + 2x) - (2x^2)}{(2x+1)(2x+1)}$$

$$4x^2 + 4x + 1$$

$$y' = \frac{2x - 2x^2}{4x+1}$$

ii

$$y = \sqrt{x^2 + 1}$$

$$y = (x^2 + 1)^{1/2}$$

$$y' = \frac{1}{2} (x^2 + 1)^{-1/2}$$

iii

$$y = 3x^{3/2} + \frac{3}{x^2} - 6x$$

$$y = 3x^{3/2} + 3x^{-2} - 6x$$

$$\frac{dy}{dx} = 4.5x^{1/2} + -6x^{-3} - 6$$

iv.

$$y = \sec^3 4x$$

$$3 \sec^2(4x) \times u'$$

$$3 \sec^2(4x) \times (\sec(4x))'$$

$$3 \sec^2(4x) \times \sec(4x) \times 4 \times \sec(4x) \times \tan(4x)$$

V.  $y = \overset{u}{x} + \overset{v}{\sin 2x}$

$$\frac{dy}{dx} du = 1$$

$$\frac{dy}{dx} dv = \cos 2x \cdot 2 = 2 \cos 2x$$

$$\frac{dy}{dx} = 2 \cos(2x) + 1$$

vi.  $y = \frac{1 - 10x}{(2x-1)^2}$

$$u = 1 - 10x \quad u' = -10$$

$$v = (2x-1)^2 \quad v' = 2(2x-1)$$

$$y = \frac{(-10(2x-1)^2) - (2(2x-1)(1-10x))}{((2x-1)^2)^2}$$

$$\frac{-10(2x-1)^2 - 4(1-10x)(2x-1)}{(2x-1)^4}$$

$$= \frac{2(10x+3)}{(2x-1)^3}$$

$$\frac{2(10x+3)}{(2x-1)^3}$$

B y

$$2. \int \frac{2x}{(4+3x^2)} dx$$

$$\int \frac{u-4}{3} (u)^{-1}$$

$$\int (2x) \times (4+3x^2)^{-1}$$

$$\int \frac{u-4}{3} (u)^{-1}$$

$$u = 4 + 3x^2$$

$$u-4 = 3x^2$$

$$\sqrt{\frac{u-4}{3}} = x$$

$$\frac{du}{dx} = 6x$$

$$dx = \frac{du}{6x}$$

ii  $\int_4^8 e^{\frac{1}{2}x} dx = 12.778 \quad (3DP)$

2.

C.

$$\int_1^{e^2} \frac{3e^{2x}}{\sqrt{e^x-1}} dx = 20$$

$$\text{ii} \int_1^e \ln x \, dx$$

$$u = \ln x \quad u' = 1/x$$

$$v' = 1/x \quad v = \ln x$$

$$x \ln(x) - \int 1 \, dx$$

$$= x \ln x - x$$

Sub  $e, 1$ 

$$(e \ln e - e) - (1 \ln 1 - 1) = 1$$

$$\text{Q.} \int \frac{4}{x(4 \ln(x) + 1)^2}$$

$$u = 4 \ln(x) + 1 \quad u' = \frac{4}{x}$$

$$\frac{1}{x} dx = \frac{1}{4} du$$

$$\int \frac{1}{u^2} du$$

$$= -\frac{1}{u}$$

$$= -\frac{1}{4 \ln(x) + 1} + C$$

3.

$$\left( \frac{1+i}{1-i} \right)^3$$

$$\frac{1+i}{1-i}$$

$$\frac{1+i}{1-i}$$

$$\frac{1+i}{1-i}$$

$$= 1^3$$

$$= 1^2 i$$

$$= -1 i$$

$$= -i$$