

Q1. CALCULATING INDUCTIVE REACTANCE AND THEN CURRENT

(a) Calculate the inductive reactance of a 3.00 mH inductor when 60.0 Hz and 10.0 kHz AC voltages are applied.

(b) What is the rms current at each frequency if the applied rms voltage is 120 V?

Strategy

The inductive reactance is found directly from the expression $XL = 2\pi fL$. Once XL has been found at each frequency, Ohm's law as stated in the equation $I = V/XL$ can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and inductance into the equation $XL = 2\pi fL$ gives $XL = 2\pi fL = 6.28(60)(3.00 \text{ mH}) = 1.13 \Omega$ at 60 Hz.

Similarly, at 10 kHz,

$$XL = 2\pi fL = 6.28(10 \text{ kHz})(3.00 \text{ mH}) = 188 \Omega \text{ at } 10 \text{ kHz.}$$

Solution for (b)

The rms current is now found using the version of Ohm's law in Equation $I = V/XL$, given the applied rms voltage is 120 V. For the first frequency, this yields

$$I = V / XL = 120 \text{ V} / 1.13 \Omega = 106 \text{ A at } 60 \text{ Hz}$$

Similarly, at 10 kHz,

$$I = V / XL = 120 \text{ V} / 188 \Omega = 0.637 \text{ A at } 10 \text{ kHz}$$

Discussion

The inductor reacts very differently at the two different frequencies. At the higher frequency, its reactance is large and the current is small, consistent with how an inductor impedes rapid change. Thus high frequencies are impeded the most. Inductors can be used to filter out high frequencies; for example, a large inductor can be put in series with a sound reproduction system or in series with your home computer to reduce high-frequency sound output from your speakers or high-frequency power spikes into your computer.

Q2. CALCULATING CAPACITIVE REACTANCE AND THEN CURRENT

- (a) Calculate the capacitive reactance of a 5.00 mF capacitor when 60.0 Hz and 10.0 kHz AC voltages are applied.
- (b) What is the rms current if the applied rms voltage is 120 V?

Strategy

The capacitive reactance is found directly from the expression in $X_C = 1/(2\pi fC)$. Once X_C has been found at each frequency, Ohm's law stated as $I = V/X_C$ can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and capacitance into $X_C = 1/(2\pi fC)$ gives
 $X_C = 1/(2\pi fC) = 1/(16.28(60)(5.00 \mu\text{F})) = 531 \Omega$ at 60 Hz

Similarly, at 10 kHz,

$$X_C = 1/(2\pi fC) = 1/(16.28(10 \text{ kHz})(5.00 \mu\text{F})) = 3.18 \Omega \text{ at } 10 \text{ kHz}$$

Solution for (b)

The rms current is now found using the version of Ohm's law in $I = V/X_C$, given the applied rms voltage is 120 V. For the first frequency, this yields

$$I = V * X_C = 120 \text{ V} * 531 \Omega = 0.226 \text{ A at } 60 \text{ Hz}$$

Similarly, at 10 kHz,

$$I = V * X_C = 120 \text{ V} * 3.18 \Omega = 3.37 \text{ A at } 10 \text{ Hz}$$

Discussion

The capacitor reacts very differently at the two different frequencies, and in exactly the opposite way an inductor reacts. At the higher frequency, its reactance is small and the current is large. Capacitors favor change, whereas inductors oppose change. Capacitors impede low frequencies the most, since low frequency allows them time to become charged and stop the current. Capacitors can be used to filter out low frequencies. For example, a capacitor in series with a sound reproduction system rids it of the 60 Hz hum.

Q3. CALCULATING IMPEDANCE AND CURRENT

An RLC series circuit has a $40.0\ \Omega$ resistor, a 3.00 mH inductor, and a $5.00\ \mu\text{F}$ capacitor.

- (a) Find the circuit's impedance at 60.0 Hz and 10.0 kHz ,
- (b) If the voltage source has $V_{\text{rms}} = 120\text{ V}$, what is I_{rms} at each frequency?

Strategy

For each frequency, we use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ to find the impedance and then Ohm's law to find current. We can take advantage of the results of the previous two questions rather than calculate the reactances again.

Solution for (a)

At 60.0 Hz , the values of the reactances were found in question 1 to be $X_L = 1.13\ \Omega$ and in question 2 to be $X_C = 531\ \Omega$. Entering these and the given $40.0\ \Omega$ for resistance into:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 513\ \text{ohm at } 60\text{ Hz}$$

Similarly, at 10.0 kHz , $X_L = 188\ \Omega$ and $X_C = 3.18\ \Omega$, so that:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 190\ \text{ohm at } 10\text{ kHz}$$

Discussion for (a)

In both cases, the result is nearly the same as the largest value, and the impedance is definitely not the sum of the individual values. It is clear that X_L dominates at high frequency and X_C dominates at low frequency.

Solution for (b)

The current I_{rms} can be found using the AC version of Ohm's law in the equation $I_{\text{rms}} = V_{\text{rms}}/Z$:

$$I_{\text{rms}} = V_{\text{rms}}/Z = 120\text{ V} / 513\ \Omega = 0.226\text{ A at } 60.0\text{ Hz}$$

Finally, at 10.0 kHz , we find

$$I_{\text{rms}} = V_{\text{rms}}/Z = 120\text{ V} / 190\ \Omega = 0.633\text{ A at } 10.0\text{ kHz}$$

Discussion for (b)

The current at 60.0 Hz is the same (to three digits) as found for the capacitor alone in question 2 from. The capacitor dominates at low frequency. The current at 10.0 kHz is only slightly different from that found for the inductor alone in question 1. The inductor dominates at high frequency.

Q4. CALCULATING RESONANT FREQUENCY AND CURRENT

For the same RLC series circuit having a $40.0\ \Omega$ resistor, a $3.00\ \text{mH}$ inductor, and a $5.00\ \mu\text{F}$ capacitor:

- (a) Find the resonant frequency.
- (b) Calculate I_{rms} at resonance if V_{rms} is $120\ \text{V}$.

Strategy

The resonant frequency is found by using the expression in $f_0 = 1/(2\pi\sqrt{LC})$. The current at that frequency is the same as if the resistor alone were in the circuit.

Solution for (a)

Entering the given values for L and C into the expression given for f_0 in $f_0 = 1/(2\pi\sqrt{LC})$, yields

$$f_0 = 1/(2\pi\sqrt{LC}) = 1/(2\pi\sqrt{(3.00 \times 10^{-3}\ \text{H})(5.00 \times 10^{-6}\ \text{F})}) = 1.30\ \text{kHz}$$

Discussion for (a)

We see that the resonant frequency is between $60.0\ \text{Hz}$ and $10.0\ \text{kHz}$, the two frequencies chosen in earlier questions. This was to be expected, since the capacitor dominated at the low frequency and the inductor dominated at the high frequency. Their effects are the same at this intermediate frequency.

Solution for (b)

The current is given by Ohm's law. At resonance, the two reactances are equal and cancel, so that the impedance equals the resistance alone. Thus,

$$I_{\text{rms}} = V_{\text{rms}}/Z = 120\ \text{V} / 40.0\ \Omega = 3.00\ \text{A}$$

Discussion for (b)

At resonance, the current is greater than at the higher and lower frequencies considered for the same circuit in the preceding example.