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Analytical methods

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3pm

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Q1) a)

$$p(x) = \sqrt{\frac{1}{2x^3} + (2x)^3 - \frac{6}{7}}$$

$$\left(\frac{1}{2x^3} + (2x)^3 - \frac{6}{7}\right)^{\frac{1}{2}} \quad \left(2x^{-3} + (2x^3) - \frac{6}{7}\right)^{\frac{1}{2}}$$

$$\left(2x^{-3} + 8x^3 - \frac{6}{7}\right)^{\frac{1}{2}} \quad \frac{1}{2} \left(2x^{-3} + 8x^3 - \frac{6}{7}\right)^{-\frac{1}{2}}$$

$$\frac{1}{2} \left(-1x^{-4} + 24x^2\right)^{-\frac{1}{2}} \quad \boxed{p'(x) = \frac{1}{2} \left(\frac{-1}{x^4} + 24x^2\right)^{-\frac{1}{2}}}$$

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$$b) \quad g(x) = \frac{5x+3}{2x+1}$$

$$1st) \quad \frac{d}{dx} \left[\frac{5x+3}{2x+1} \right]$$

$$\frac{(5 \cdot \frac{d}{dx} [x] + \frac{d}{dx} [3])(2x+1) - (5x+3)(2 \cdot \frac{d}{dx} [x] + \frac{d}{dx} [1])}{(2x+1)^2}$$

$$\frac{5(2x+1) - 2(5x+3)}{(2x+1)^2}$$

$$\boxed{= \frac{5}{2x+1} - \frac{2(5x+3)}{(2x+1)^2} \quad \text{or} \quad -\frac{1}{(2x+1)^2}}$$

$$2nd) \quad -\frac{d}{dx} \left[\frac{1}{(2x+1)^2} \right]$$

$$= -(-2)(2x+1)^{-3} \cdot \frac{d}{dx} [2x+1]$$

$$= \frac{2(2 \cdot \frac{d}{dx} [x] + \frac{d}{dx} [1])}{(2x+1)^3}$$

$$\boxed{= \frac{4}{(2x+1)^3}}$$

Q2)

$$y = \frac{(3x+1) \cdot \sin^3(x)}{e^{2x}}$$

$$1st) \quad \ln y = \ln \left[\frac{(3x+1) \cdot \sin^3(x)}{e^{2x}} \right]$$

$$\ln y = \ln [(3x+1) \cdot \sin^3(x)] - \ln [e^{2x}]$$

$$\ln y = \ln [3x+1] + 3 \ln [\sin(x)] - (2x) \cdot \ln [e']$$

$$\boxed{\ln y = \ln [3x+1] + 3 \ln [\sin(x)] - 2x}$$

$$2nd) \quad \frac{1}{y} \cdot \frac{dy}{dx} = 3 \cdot 0 + 3 \cdot \frac{\sin' x}{\cos x} - 2$$

$$\frac{dy}{dx} = y \cdot \left[3 + 3 \cdot \frac{\sin' x}{\cos x} - 2 \right]$$

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Q3) a) $f(x, y) = x^2 \sin(2x + 3y)$

$$\frac{\partial f(x, y)}{\partial x}$$

b) $\frac{\partial^2 f(x, y)}{\partial y \partial x} = \frac{\partial}{\partial x} \left[\frac{\partial f(x, y)}{\partial y} \right]$

$$\frac{\partial^2 f(x, y)}{\partial y \partial x} = \frac{\partial}{\partial x} \left[\frac{d}{dy} x^2 \sin(2x + 3y) \right]$$

$$\frac{d}{dx} [2x^2 \cos(2x + 3y)]$$

$$\boxed{\frac{\partial^2 f(x, y)}{\partial y \partial x} = 4x \cdot \cos(2x + 3y) - x^2 \cdot \sin(2x + 3y)}$$

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$$Q4) \quad f(x) = -2x^3 + x^2 + 8x - 10$$

$$\text{step 1)} \quad f'(x) = \frac{d}{dx} (-2x^3 + x^2 + 8x - 10)$$

$$f'(x) = \frac{d}{dx} (-2x^3) + \frac{d}{dx} (x^2) + \frac{d}{dx} (8x) - \frac{d}{dx} (10)$$

$$f'(x) = -2 \cdot 3x^2 + 2x + 8 - 0$$

$$\boxed{f'(x) = -6x^2 + 2x + 8}$$

$$\text{step 2)} \quad f'(x) = 0$$

$$-6x^2 + 2x + 8 = 0$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 3 \cdot (-4)}}{2 \cdot 3}$$

$$x = \frac{1 \pm \sqrt{1+48}}{6}$$

$$x = \frac{1 \pm \sqrt{49}}{6}$$

$$x = \frac{1 \pm 7}{6}$$

$$x = \frac{1 \pm 7}{6}$$

$$\boxed{x_1 = \frac{4}{3} \quad x_2 = -1}$$

step 3)

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Step 4)

Step 5)

Step 6)

Step 7)

Step 8)

