



Prifysgol Cymru
Y Drindod Dewi Sant
University of Wales
Trinity Saint David

Examination Answer Books

12 Pages

Instructions to Candidates

1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
4. Write your answers in ink.
5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
6. No part of this book is to be torn out; it must be handed in complete.
7. If more books than one are used, fasten them together using the tags available from the invigilator(s).
8. Every book must be handed in. In no circumstances may answer books, used or unused, be taken out of the examination venue.

Complete, fold and seal
(see note 2 below)

SURNAME: Webb

FORENAME(S): Thomas

1	Student Number	WEB25887285
2	Title of Examination Paper	Mathematics Test 2
3	Module Code	GOEN4001
4	Date and Time of Examination	18 May 2026 14:00
5	Examination Venue	Main Hall
6	Seat Number	K4
7	Number of Answer Books handed in	1

List the numbers of the questions you have attempted	Office Use
Q1	
Q2	
Q3	
Q1	30 ✓
Q2	30 ✓
Q3	~~33~~ 29 LZ ✓
Total	~~93~~ /100

(Gweler yr ochr arall am y fersiwn Cymraeg)

89/100 ✓

93%

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$$Q1) a) 1) Z_1 + Z_2 = 3 + 7j + 4 + 5j = 7 + 12j \quad \checkmark \quad \underline{\underline{1}}$$

$$2) Z_2 + 3Z_3 = 4 + 5j + 3(2 + 6j) = 4 + 5j + 6 + 18j \\ = 10 + 23j \quad \checkmark \quad \underline{\underline{2}}$$

$$3) Z_2^2 + Z_3 = (4 + 5j)^2 + 2 + 6j = 16 - 25 + 40j + 2 + 6j \\ = -7 + 46j \quad \checkmark \quad \underline{\underline{3}}$$

$$4) Z_1 Z_2 Z_3 = (3 + 7j)(4 + 5j)(2 + 6j)$$

$$Z_1 Z_2 = 12 - 35 + 28j + 15j = -23 + 43j \quad \checkmark$$

$$Z_1 Z_2 Z_3 = (-23 + 43j)(2 + 6j) = -46 - 258 + 86j - 138j$$

$$= -304 - 52j \quad \checkmark \quad \underline{\underline{4}}$$

$$5) \frac{Z_1}{Z_2} = \frac{3 + 7j}{4 + 5j} \times \frac{4 - 5j}{4 - 5j} = \frac{12 + 35 + 28j - 15j}{16 + 25 - 20j + 20j} \\ = \frac{47 + 13j}{41} = \frac{47}{41} + \frac{13}{41}j \quad \checkmark \quad \underline{\underline{5}}$$

$$6) \frac{Z_1^2 - Z_2^2}{Z_1^2 - Z_1 Z_2} = \frac{(Z_1 + Z_2)(Z_1 - Z_2)}{Z_1(Z_1 - Z_2)} = \frac{Z_1 + Z_2}{Z_1}$$

$$= \frac{Z_1}{Z_1} + \frac{Z_2}{Z_1} = 1 + \frac{Z_2}{Z_1} \quad \checkmark \quad \underline{\underline{3}}$$

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b) $2x^2 - 6x + 9 = 0$

Let $a=2$, $b=-6$, $c=9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{6 \pm \sqrt{36 - (4 \times 2 \times 9)}}{4}$$

$$= \frac{6 \pm \sqrt{-36}}{4} = \frac{6 \pm 6i}{4} = \frac{3}{2} \pm \frac{3}{2}i$$

$\therefore x = \frac{3}{2} + \frac{3}{2}i$ or $x = \frac{3}{2} - \frac{3}{2}i$

In polar form: $r = \sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2} = 2.12$

$\theta = \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{3}{2}}\right) = 45^\circ$ or $\theta = \tan^{-1}\left(\frac{-\frac{3}{2}}{\frac{3}{2}}\right) = -45^\circ$
 $= 180 - 45 = 135^\circ$

$\therefore x = 2.12 \angle 45^\circ = 2.12 (\cos 45 + i \sin 45)$

or $x = 2.12 \angle 135^\circ = 2.12 (\cos 135 + i \sin 135)$

Verify

$$\begin{aligned} 2\left(\frac{3}{2} + \frac{3}{2}i\right)^2 - 6\left(\frac{3}{2} + \frac{3}{2}i\right) + 9 &= 0 \\ &= 2\left(\frac{9}{4} - \frac{9}{4} + 9i\right) - (9 + 9i) + 9 = 0 \\ &= 9i - 9 - 9i + 9 = 0 \end{aligned}$$

$$= \frac{9}{2} - \frac{9}{2} + 9i - 9 - 9i = 0$$

$0 = 0 \checkmark$

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Q1b

7/9

$$c) \quad Z^n = r^n (\cos(n\theta) + i\sin(n\theta))$$

$$(5+3i)^6$$

$$r = \sqrt{5^2 + 3^2} = 5.83$$

$$\theta = \tan^{-1}\left(\frac{3}{5}\right) = 30.96^\circ$$

show workings

$$(5+3i)^6 = 5.83^6 (\cos(6 \times 30.96) + i\sin(6 \times 30.96))$$

$$= 39304 (\cos 185.76 + i\sin 185.76)$$

$$= -39105.6 - 3944.6i$$

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Q1c

5/6

Q2a) 1) $\text{Det } |A| = 1(6 \times 5 - 4 \times 2) - 3(0 \times 6 - 4 \times 5) + 3(0 \times 2 - 5 \times 5)$
 $= 22 + 60 - 75 = 7$

Q2

2) $\text{Det } |C| = (5 \times 6) - (4 \times -2) = 38$

30/32

3) $\begin{bmatrix} 1 & 3 & 3 \\ 0 & 5 & 4 \\ 5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 2 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} (3+6+3) & (5-6+3) \\ (0+10+4) & (0-10+4) \\ (15+4+6) & (25-4+6) \end{bmatrix}$

$= \begin{bmatrix} 12 & 2 \\ 14 & -6 \\ 25 & 27 \end{bmatrix}$

Q2a

17/17

4) $\begin{bmatrix} 1 & 3 & 3 \\ 0 & 5 & 4 \\ 5 & 2 & 6 \end{bmatrix} \begin{bmatrix} 0 & 2 & 2 \\ 3 & 5 & 1 \\ 5 & 1 & 4 \end{bmatrix} = \begin{bmatrix} (0+9+15) & (2+15+3) & (2+3+12) \\ (0+15+20) & (0+25+4) & (0+5+16) \\ (0+6+30) & (10+10+6) & (10+2+24) \end{bmatrix}$

$= \begin{bmatrix} 24 & 20 & 17 \\ 35 & 29 & 21 \\ 36 & 26 & 36 \end{bmatrix}$

5) Not possible with $2 \times 2 \times 3 \times 3$
 since $2 \neq 3$

6) If $\text{DET } |w| = 4$,

$4 = x(4(x+2) - 2 \times 5) - 3(4(x+2) - 2 \times 2) + -4(5 \times 4 - 4 \times 2)$

$4 = x(4x - 2) - 3(4x + 12) - 112$

$4 = 4x^2 - 2x - 12x - 36 - 112 = 4x^2 - 14x - 148$

$\Rightarrow 2x^2 - 7x - 76 = 0$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{7 \pm \sqrt{49 - 4 \times 2 \times -76}}{2 \times 2}$

$\therefore x = \frac{7 + 3\sqrt{73}}{4} \approx 8.16$ or $x = \frac{7 - 3\sqrt{73}}{4} \approx -4.66$

b) ~~Q2b~~ $A = \begin{bmatrix} 6 & 3 & 1 \\ 2 & 0 & 5 \\ 4 & 2 & 3 \end{bmatrix}$

Q2b10/10

$$\text{DET } |A| = 6(0 \times 3 - 5 \times 2) - 3(2 \times 3 - 5 \times 4) + 1(2 \times 2 - 0 \times 4) \\ = -60 + 42 + 4 = -14$$

Minors of $A = \begin{bmatrix} (0 \times 3 - 5 \times 2) & (2 \times 3 - 5 \times 4) & (2 \times 2 - 0 \times 4) \\ (3 \times 3 - 1 \times 2) & (6 \times 3 - 1 \times 4) & (6 \times 2 - 3 \times 4) \\ (3 \times 5 - 1 \times 0) & (6 \times 5 - 1 \times 2) & (6 \times 0 - 3 \times 2) \end{bmatrix}$

$$= \begin{bmatrix} -10 & -14 & 4 \\ 7 & 14 & 0 \\ 15 & 28 & -6 \end{bmatrix}$$

Applying signs rule: $\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$$= \begin{bmatrix} -10 & 14 & 4 \\ -7 & 14 & 0 \\ 15 & -28 & -6 \end{bmatrix}$$

Transposition for $\text{Adj}|A| = \begin{bmatrix} -10 & -7 & 15 \\ 14 & 14 & -28 \\ 4 & 0 & -6 \end{bmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|} \times \text{Adj}(A) = \frac{-1}{14} \begin{bmatrix} -10 & -7 & 15 \\ 14 & 14 & -28 \\ 4 & 0 & -6 \end{bmatrix}$$

$$\begin{aligned} \text{c) } 2x - y - 2z &= -2 \\ -x + y + z &= 0 \\ x - 2y + z &= 8 \end{aligned}$$

$$\begin{bmatrix} 2 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -1 & -2 \\ -1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -2 \\ 0 \\ 8 \end{bmatrix}$$

$$\begin{aligned} \text{DET} &= 2(1 \times 1 + 2 \times 1) + 1(-1 \times 1 - 1 \times 1) - 2(2 \times 1 - 1 \times 1) \\ &= 6 - 2 - 2 = 2 \end{aligned}$$

Minors:

$$\begin{bmatrix} (1 \times 1 + 2 \times 1) & (-1 \times 1 - 1 \times 1) & (2 \times 1 - 1 \times 1) \\ (-1 \times 1 - 2 \times 2) & (2 \times 1 + 2 \times 1) & (-2 \times 2 + 1 \times 1) \\ (-1 \times 1 + 2 \times 1) & (2 \times 1 - 2 \times 1) & (2 \times 1 - 1 \times 1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -2 & 1 \\ -5 & 4 & 5 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{Signs} \rightarrow \begin{bmatrix} 3 & 2 & 1 \\ 5 & 4 & -5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\text{Transposition} \rightarrow \begin{bmatrix} 3 & 5 & 1 \\ 2 & 4 & 0 \\ 1 & -5 & 1 \end{bmatrix} \rightarrow \text{Inverse} = \begin{bmatrix} \frac{3}{2} & \frac{5}{2} & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & -\frac{5}{2} & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{3}{2} & \frac{5}{2} & \frac{1}{2} \\ 1 & 2 & 0 \\ \frac{1}{2} & -\frac{5}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -2 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} (-3 + 0 + 4) \\ (-2 + 0 + 0) \\ (-1 + 0 + 4) \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$$\therefore x = 1, y = -2, z = 3$$

Correct solution
but you have to
use Cramer's Rule
(-3 marks)

Q2C

3/6

$$\begin{aligned} \text{Q3)a)1)} \quad & \int (2x^4 + 3x^3 - 3x^2 + 5x) dx \\ & = \frac{2}{5}x^5 + \frac{3}{4}x^4 - x^3 + \frac{5}{2}x^2 + C \quad \underline{\underline{3}} \end{aligned}$$

Q333/34

$$2) \quad (3x+1)(4x+3) = 12x^2 + 9x + 4x + 3 = 12x^2 + 13x + 3$$

$$\begin{aligned} \Rightarrow & \int (\sin 3x + \cos 5x + 12x^2 + 13x + 3) dx \\ & = -\frac{1}{3} \cos(3x) + \frac{1}{5} \sin(5x) + 4x^3 + \frac{13}{2}x^2 + 3x + C \quad \underline{\underline{3}} \end{aligned}$$

Q3a20/2016/20

$$\begin{aligned} 3) \quad & \int_0^3 (4x^3 + 3x^2 + 2x - 3) dx \\ & = [x^4 + x^3 + x^2 - 3x]_0^3 \end{aligned}$$

$$\begin{aligned} & = [3^4 + 3^3 + 3^2 - (3 \times 3)] - [0^4 + 0^3 + 0^2 - (3 \times 0)] \\ & = 108 \quad \underline{\underline{3}} \end{aligned}$$

$$4) \quad \int_1^2 (e^{2x} + 2x^3 + 7) dx$$

$$\begin{aligned} & = \left[\frac{1}{2}e^{2x} + \frac{1}{2}x^4 + 7x \right]_1^2 \quad \underline{\underline{3}} \\ & = \left[\frac{1}{2} \times e^4 + \frac{1}{2} \times 2^4 + (7 \times 2) \right] - \left[\frac{1}{2}e^2 + \frac{1}{2} + 7 \right] \\ & = 38.1 \end{aligned}$$

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$$5) \int_1^4 (\sqrt{x} - 3)(\sqrt{x} + 5) dx = \int_1^4 (x + 5x^{\frac{1}{2}} - 3x^{\frac{1}{2}} - 15) dx$$

$$= \int_1^4 (x + 2x^{\frac{1}{2}} - 15) dx$$

$$= \left[\frac{1}{2}x^2 + \frac{4}{3}x^{\frac{3}{2}} - 15x \right]_1^4$$

$$= \left[\frac{1}{2}4^2 + \frac{4}{3}4^{\frac{3}{2}} - (15 \times 4) \right] - \left[\frac{1}{2} + \frac{4}{3} - 15 \right]$$

$$= -28.2$$

$$6) \int_0^2 \frac{x+1}{x^2+3x+2} dx = \left[\ln(x^2+3x+2) \right]_0^2$$

$$= \ln(2^2 + (3 \times 2) + 2) - \ln(2)$$

$$= 0.693$$

$$b) \int x(4x^2-5)^3 dx, \quad \text{let } u = 4x^2 - 5$$

$$\Rightarrow \frac{du}{dx} = 4x \rightarrow dx = \frac{du}{4x}$$

$$\Rightarrow \int x(4x^2-5)^3 = \int x u^3 \frac{du}{4x} = \int \frac{1}{4} u^3 du$$

$$= \frac{1}{16} u^4 + C = \frac{1}{16} (4x^2-5)^4 + C$$

E.C.F

Correct method
E.C.F

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Q3b

5/6

c) $\int x^2 e^x dx$

let $u = x^2$ and $v = e^x$

$$\int uv dx = u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

$$\int x^2 e^x dx = x^2 \int e^x dx - \int \left(\frac{dx^2}{dx} \int e^x dx \right) dx$$

$$= x^2 e^x - \int 2x e^x dx + C$$

~~is distribute and~~

Take $\int 2x e^x dx$ and apply 'by parts' logic:

$$\int 2x e^x dx = 2x \int e^x dx - \int \left(\frac{d2x}{dx} \int e^x dx \right) dx$$

$$= 2x e^x - \int 2 e^x dx + C$$

$$= 2x e^x - 2e^x + C$$

Substitute back in:

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$= e^x (x^2 - 2x + 2)$$

$$\int_0^1 x^2 e^x dx = [e^1 (1 - 2 + 2)] - [e^0 (0 - 0 + 2)]$$

$$= 0.718$$

Q3C

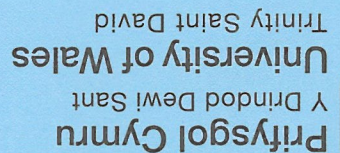
8/8



Question No.

Examiner Only





12 Tüdalen

1. Rhwch yr holl wybodaeth y gofynnir amdani yn y blychau isod.
2. Rhwch eich enw yn lle a ddarparwyd ar y dde, plygwch ef a'i selio i lawr (gan ddefnyddio'r label a ddarparwyd) i guddio'ch enw.

3. Rhaid i'r atebion fod newn llawysgrifen ddarlennadwy. Gall yr Arholwyr naill ai anwybyddu sgrïpt annarllennadwy, neu drefnu iddi gael ei theipio a chael yr ymgeisydd i dalu.

4. Ysgriftennwch eich atebion mewn inc.

5. Dechreuwch bob ateb ar dudalen newydd. Gwnewch unrhyw fras nodiadau yn y llyfr atebion a thynnu llinell drwyddynt.

6. Ni ddylid rhwgo unrhyw ran o'r llyfr hwn; rhaid ei gyflwyno yn gyfan.

7. Os defnyddir mwy nag un llyfr, rhwymwch hwy gyda'i gilydd gan ddefnyddio'r tagiau sydd ar gael oddi wrth y gwylwyr/gwylwyr.

8. Rhaid dychwelyd pob llyfr. Ni chaniateir mynd â llyfrau atebion, wedi eu defnyddio neu heb eu defnyddio, allan o'r lleoliad arholi o dan unrhyw amgylchiadau.

1	Rhif Myfyriwr	
2	Teitl y Papur Arholiad	
3	Cod y Modwl	
4	Dyddiad ac Amser yr Arholiad	
5	Lleoliad yr Arholiad	
6	Rhif Sedd	
7	Nifer y Llyfrau Atebion a gyflwynwyd	

Rhestrwch nifau'r cwestiynau a ymgestwyd genych	Cyfanswm
At Ddbentiaon y Swyddfa	

(See the other side for English version)

Llenwch, plygwh a seliwh
(gweler nodyn 2 isod)

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ENW(AU) CYNTAF:

ENW(AU) CYNTAF: