



The Harmonious Entanglement of Worlds: Kepler and the Universe



On this 400 year anniversary of the publication of Kepler's *Harmonices Mundi* in which he presented his Third Law, as an astronomer, I'd like to celebrate the truly vast implications of the Third Law in astrophysics.

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- ▶ I can give myself up to the sacred frenzy, I can have the insolence to make a full confession to mortal men that I have stolen the golden vessel of the Egyptians to make from them a tabernacle for my God far from the confines of the land of Egypt. If you forgive me I shall rejoice; if you are angry, I shall bear it; I am indeed casting the die and writing the book, either for my contemporaries or for posterity to read, it matters not which: let the book await its reader for a hundred years; God himself has waited 6000 years for a witness.
 - ▶ *Harmonice Mundi, The Harmony of the World* (1619)
 - ▶

Kepler's epiphany of the Third Law was on 8 March 1618

- ▶ “Those laws [of nature] are within the grasp of the human mind; God wanted us to recognize them by creating us after his own image so that we could share in his own thoughts.”

Defenestration of Prague, 23 May 1618,
the start of the 30 years war.

The Historian Aviva Rothman: Kepler's vision of harmony

- ▶ Kepler was less interested in astronomy than in working for harmony among people.

“If God would set me free from astronomy, I could pursue my interest to work on harmony in the world”

Harmony unites elements that were different. Copernican revolution shook the foundation of harmony. How can one understand harmony if the earth is not longer in the center. The Third Law provides that harmony and truth in the interconnection of all things and people

Politics will not end with one truth, while we will achieve that in astronomy.

Harmonious proportions are built into the cosmos, but not in politics.

The Scientific Legacy of his Third Law

- ▶ Quantitative and testable proof of heliocentricity: the earth behaves like a planet.
- ▶ Quantitative demonstration of the harmony in the motions of planets and moons.
- ▶ The interconnectedness in the solar system and beyond.
- ▶ First example of a power law ($N = r^D$) in nature of which there are many.
- ▶ Demonstration of the “magic” of beautiful mathematics, the elegantly simple $\{a^3 = p^2\}$ why should mathematics conceived in the mind apply to the real world? Why exactly 2 and 3?

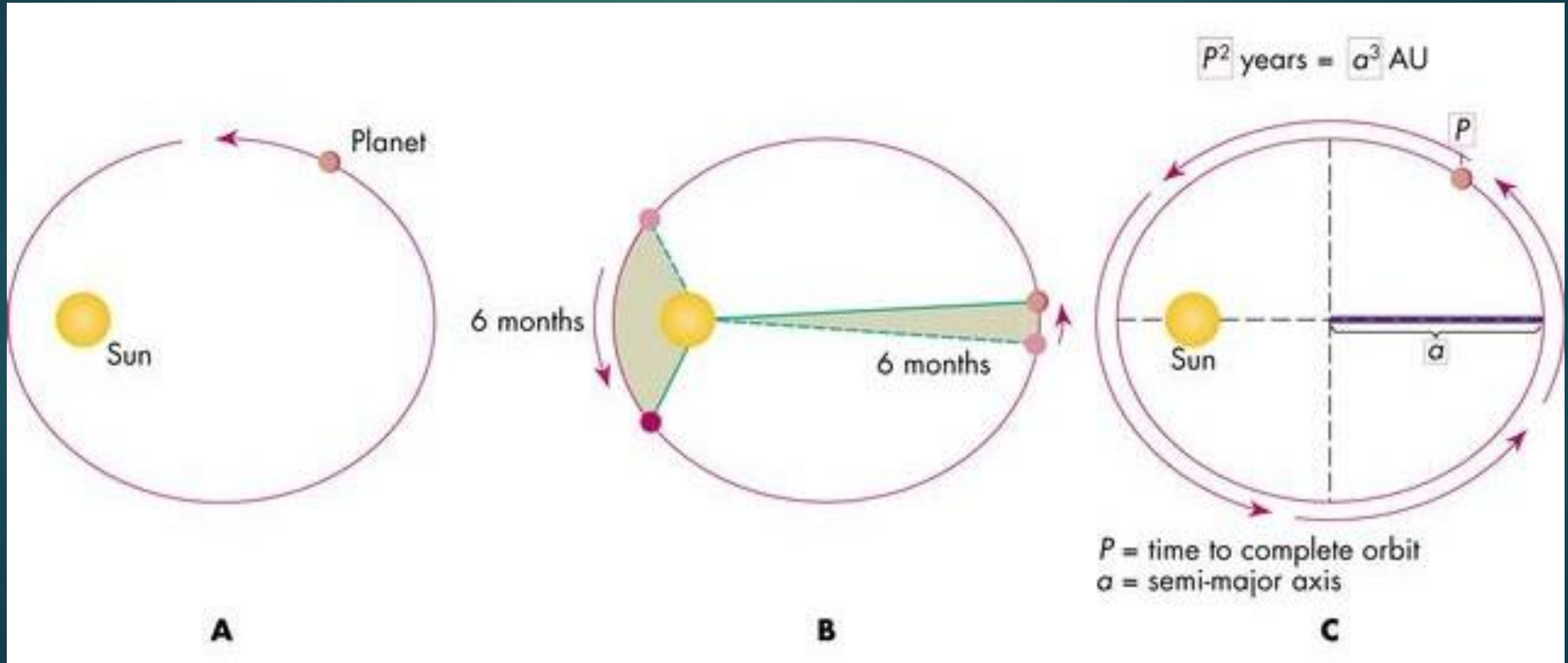
The mathematician straddles two worlds, similar to those of Kepler!

- ▶ *“It is more important to have beauty in one’s equation than to have them fit experiment”. “...the mathematician plays a game in which he himself invents the rule while the physicist plays a game in which the rules are provided by Nature, but as time goes on it becomes increasingly evident that the rules which the mathematician finds interesting are the same as those which Nature has chosen.”*

-Paul Dirac

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- ▶ 1. Celestial objects do not follow perfect, circular orbits around the earth-breaking the Aristotelian duality of celestial and terrestrial realms.
 - ▶ 2. The second law challenges the belief that a celestial object moves with unchanging speed. Furthermore, this is a quantifiably testable prediction. Their speed is greatest closest to the sun and slowest at aphelion. This is a daring prediction based on scant data
 - ▶ 3. The third law is the most powerful and continues, to an extraordinary degree, to reveal the nature of the universe. This is the most magical of them all.

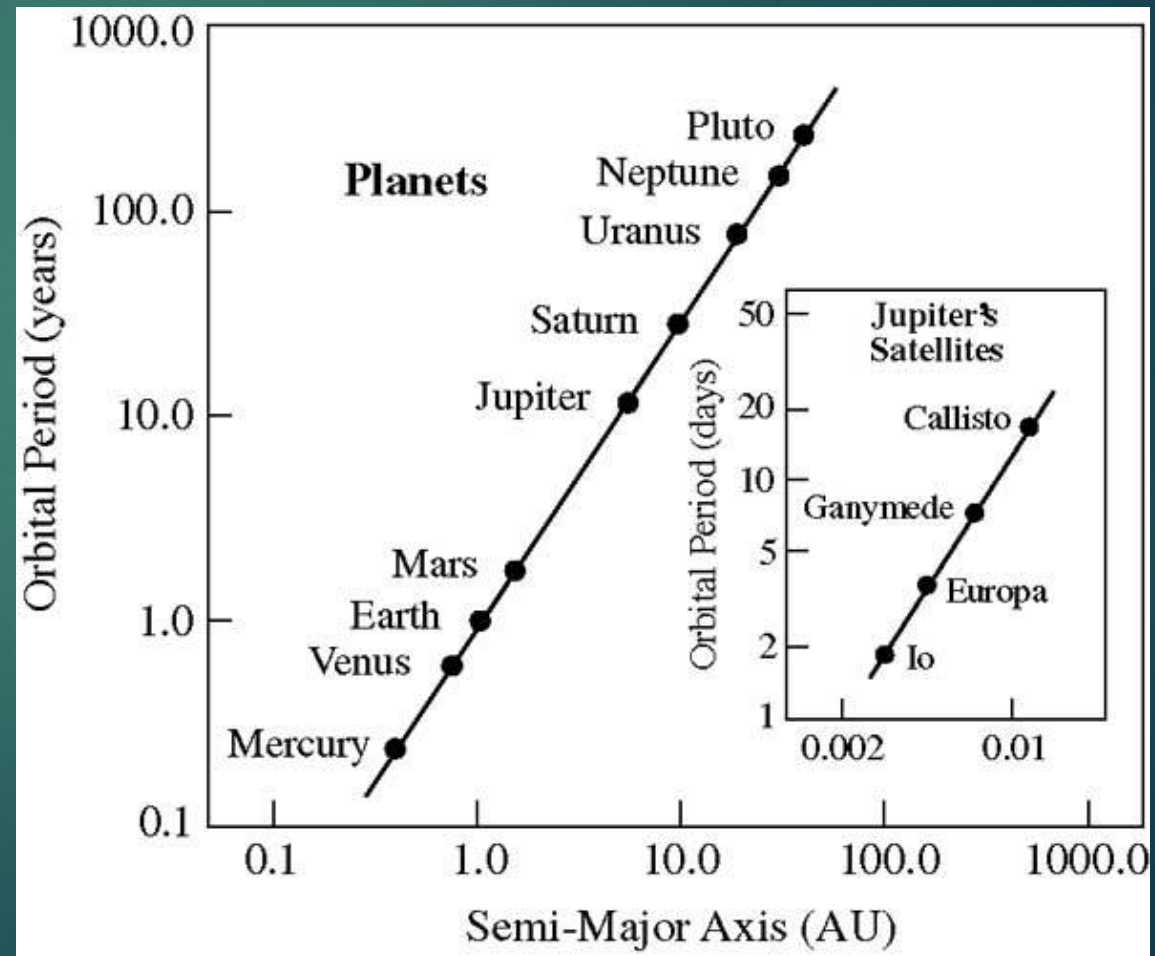
Kepler's Three Laws of Planetary Motion



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- ▶ The Third Law applies across an immense span of space, from moon, planets, galaxies, black holes, clusters of galaxies. All these phenomena are intertwined, interconnected, and entangled, through the process captured by that law.
 - ▶ The idea of interconnectedness certainly did not start with Kepler, but he was the first to propose it in a quantitative and testable form.

A characteristic of power laws is scale invariance: the same phenomena is repeated at different scales

- ▶ $P = a^{3/2}$
- ▶ Change scale to ka
- ▶ $P = (ka)^{3/2}$
- ▶ $P = a^{3/2} k^{3/2}$
- ▶ The same 3/2 power law works regardless of the scale. There is no fundamental length!
- ▶ Kepler discovered that aspect when he showed it worked for the moons of Jupiter which had been discovered by Galileo just 8-9 years earlier.



$$a^3 = p^2$$

	A(AU)	P(years)	a^3	p^2
Mercury	.387	.241	.058	.058
Venus	.723	.615	.378	.378
Earth	1	1	1	1
Mars	1.524	1.881	3.537	3.537
Jupiter	5.203	11.862	140.8	140.7
Saturn	9.534	28.456	867.9	867.7

Ultima Thule

Most distant object visited by space craft. New Horizons passed it in on January 1, 2019, after leaving Pluto. It obeys Kepler's Third Law.



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ULTIMA THULE	44.539	298	88353	88804

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- ▶ Kepler was incredibly close to being able to compare the mass of Jupiter to that of the sun. He also knew the period and distance of the moon and could have compared those with the mass of the Earth.
 - ▶ But, he was held back by his belief that planets were pushed around the sun by magnetism

$$k = a^3/p^2$$

Galilean Moons of Jupiter

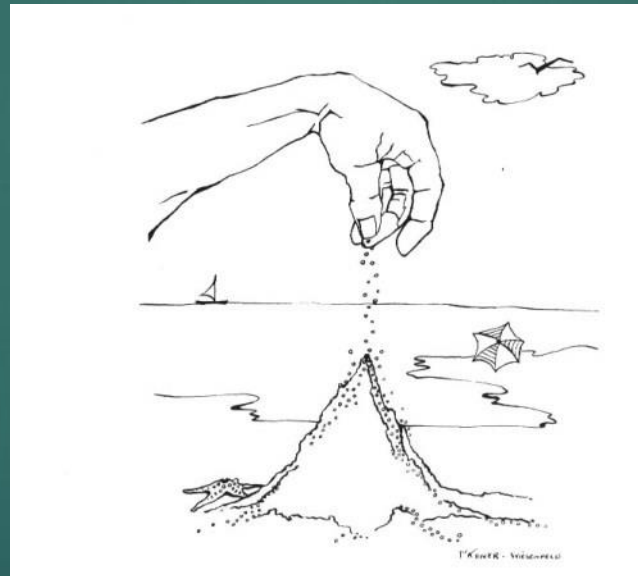
MoonE	a (km)		a ³	P (days)		p ²	a ³ /p ²
Io	421800	0.00282	2.24156E-08	1.769	0.004847	2.34893E-05	9.54292 E-04
Europa	671100	0.004486	9.02803E-08	3.551	0.009729	9.46489E-05	9.53844 E-04
Ganymede	1070400	0.007155	3.66328E-07	7.155	0.019603	0.000384267	9.53314 E-04
Callisto	1882700	0.012585	1.99331E-06	16.69	0.045726	0.00209087	9.5331 E-04
#64	23,396,269	0.156395	0.00382535	731	2.00274	4.01096641	9.53723 E-04
#79	28,570,410	0.190983	0.006965957	981.55	2.689178	7.231678758	9.63256 E-04

Earth's Moon	384400	0.00257	1.69661E-08	27.32	0.074849	0.00560242	3.02835 E-06

Kepler could almost have weighed Jupiter and the Earth

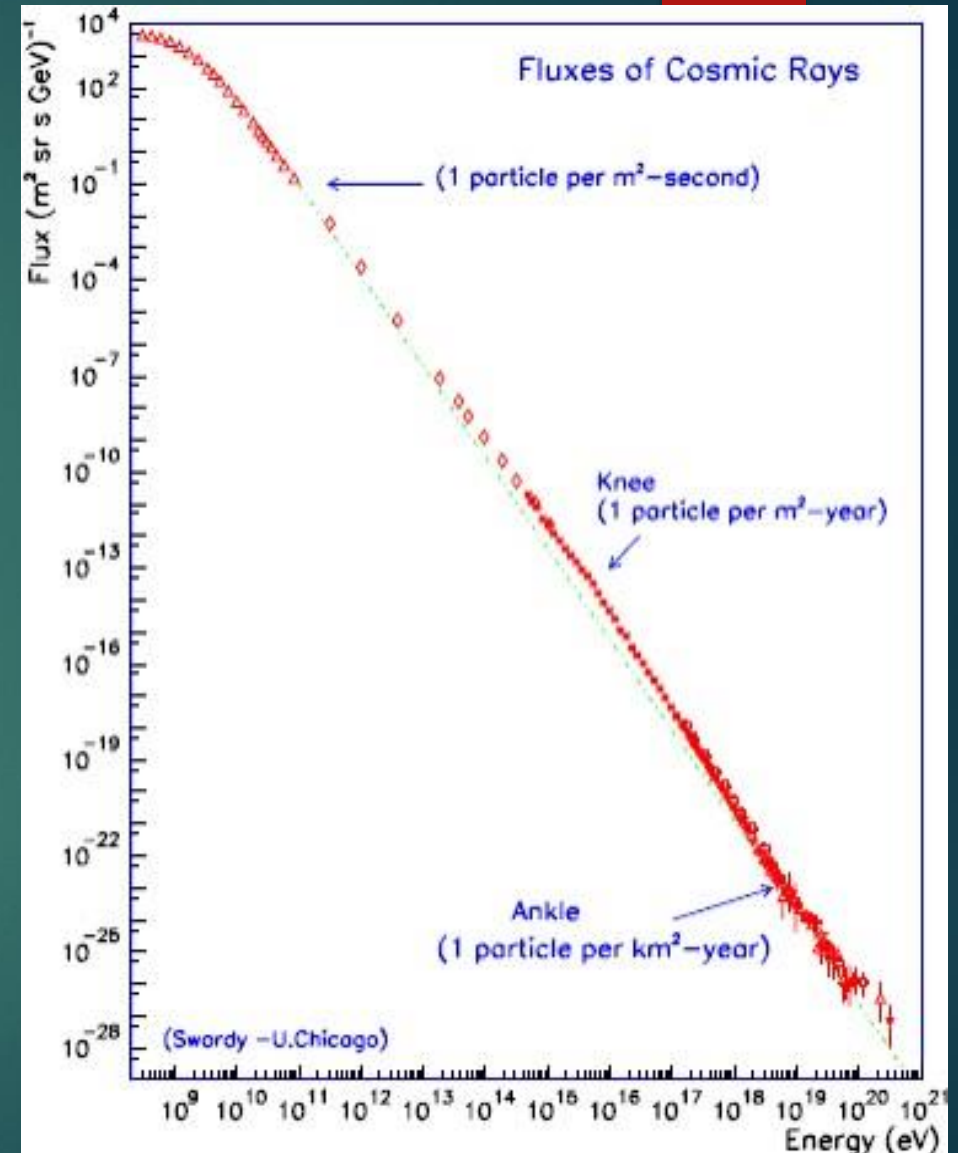
- ▶ Mass of Jupiter = 9×10^{-4} Mass of sun $\sim 1/1000$ Mass of sun. The sun's mass is 1000 times that of the sun.
- ▶ Mass of Earth = 3×10^{-6} Mass of sun. The sun's mass is 333,000 that of the earth

Sand Piles and power laws self-organized criticality organized avalanches



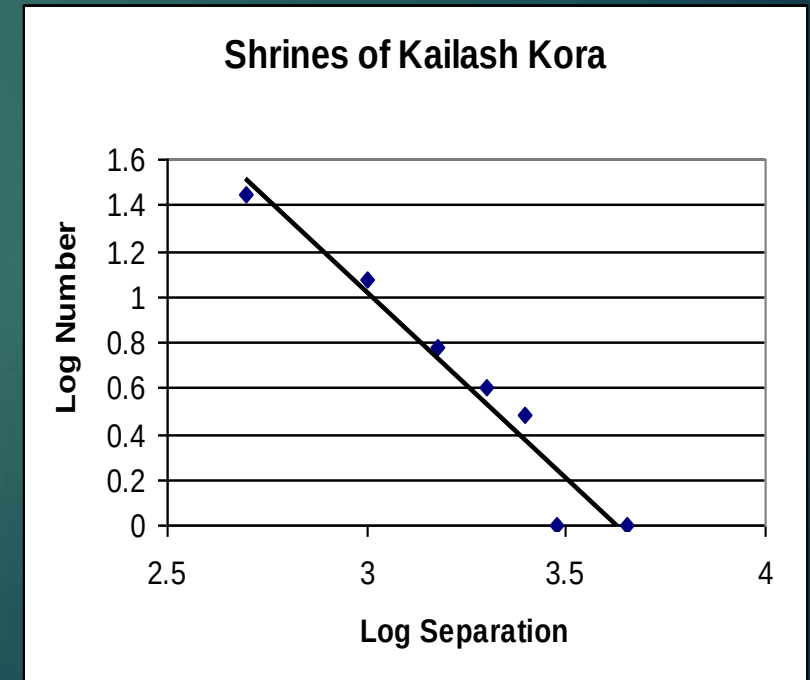
Power Law Distributions

- ▶ Avalanches
- ▶ earthquakes
- ▶ craters on the moon
- ▶ solar flares
- ▶ Fragments of a broken brick
- ▶ Meteorites
- ▶ Cosmic rays
- ▶ Distributions of the sizes of trees in a mature forest
- ▶ Shrines around Mt. Kailash and Varanasi
- ▶ Pilgrims bathing in the Ganga
- ▶ the frequencies of words in most languages
- ▶ frequencies of family names
- ▶ the sizes of power outages,
- ▶ criminal charges per convict
- ▶ volcanic eruptions



Power Law Distributions

- ▶ craters on the moon
- ▶ solar flares
- ▶ Fragments of a broken brick
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- ▶ Distributions of the sizes of trees in a mature forest
- ▶ Shrines around Mt. Kailash
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- ▶ criminal charges per convict
- ▶ volcanic eruptions
- ▶ earthquakes



The next step was Newton who utilized the third law to propose the law of gravity

- ▶ He also made a leap of faith by proposing that the force of gravity varied as $1/r^2$ and that mass was the source of that force.
- ▶ The third law became $M = a^3/p^2$, in solar system units
- ▶ This form of Kepler's Law has huge consequences for astronomy and enabled astronomers to determine the mass of astronomical objects.

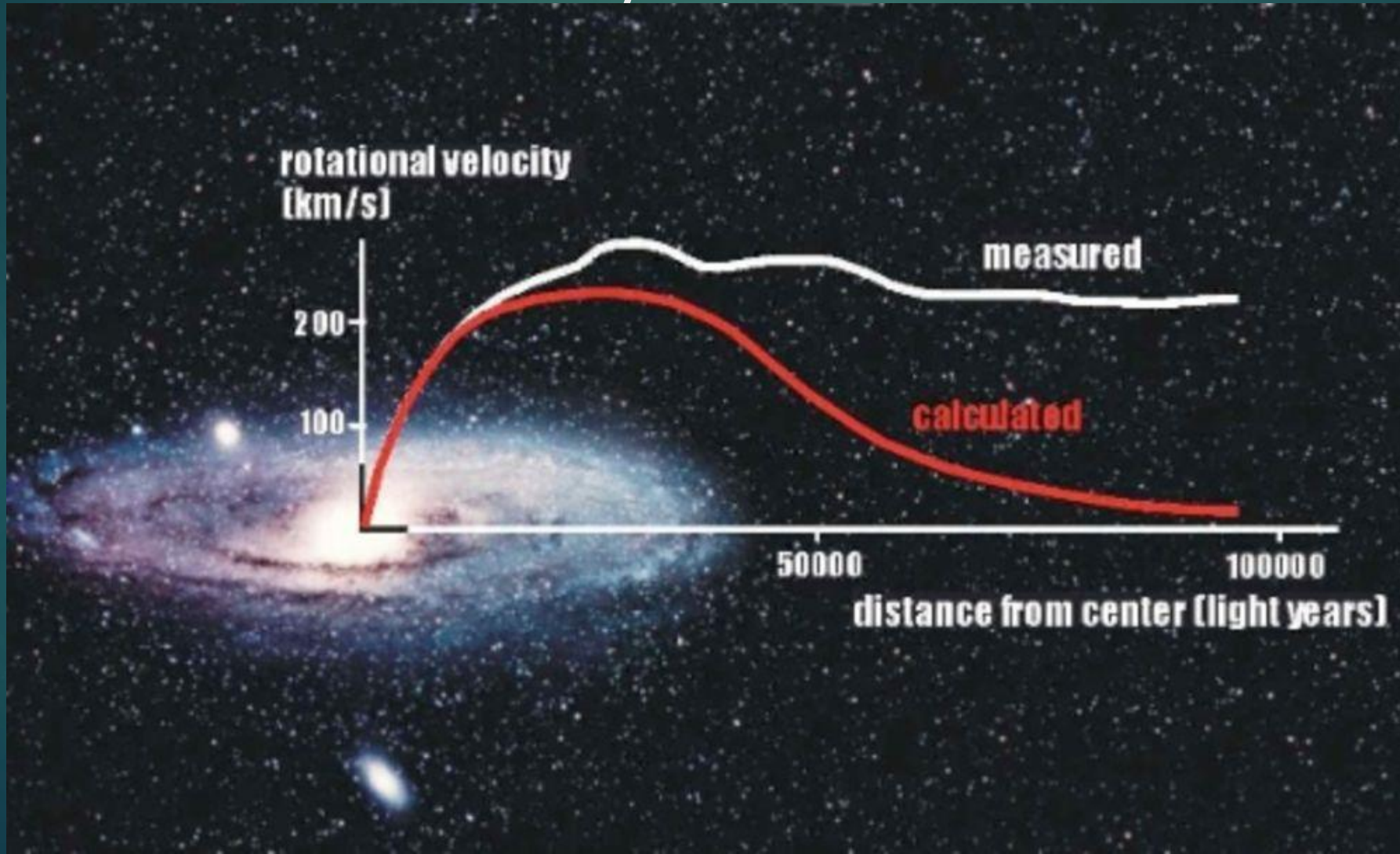
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- ▶ “Kepler's laws, although not rigidly true, are sufficiently near to the truth to have led to the discovery of the law of attraction of the bodies of the solar system. The deviation from complete accuracy is due to the facts, that the planets are not of inappreciable mass, that, in consequence, they disturb each other's orbits about the Sun, and, by their action on the Sun itself, cause the periodic time of each to be shorter than if the Sun were a fixed body, in the subduplicate ratio of the mass of the Sun to the sum of the masses of the Sun and Planet; these errors are appreciable although very small, since the mass of the largest of the planets, Jupiter, is less than 1/1000th of the Sun's mass.”

— **Isaac Newton**, [The Principia: Mathematical Principles of Natural Philosophy](#)

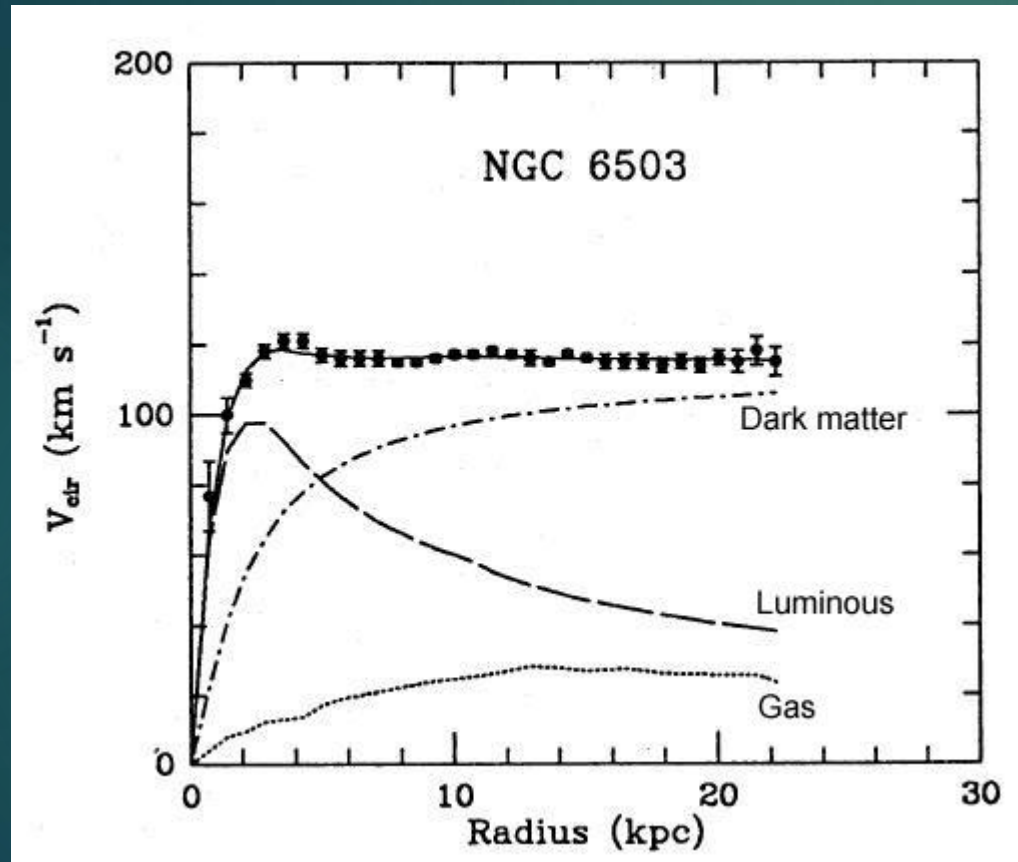
Kepler's Third Law has given us

- ▶ The mass of planets with moons
- ▶ The mass of asteroids with moons
- ▶ The mass of the galaxy
- ▶ The mass of dark matter
- ▶ The mass of the black hole in the center of our galaxy
- ▶ The mass of black holes in other galaxies

Rotation of galaxies: $P = a^{3/2}$
 $v = 2\pi/a^{1/2}$

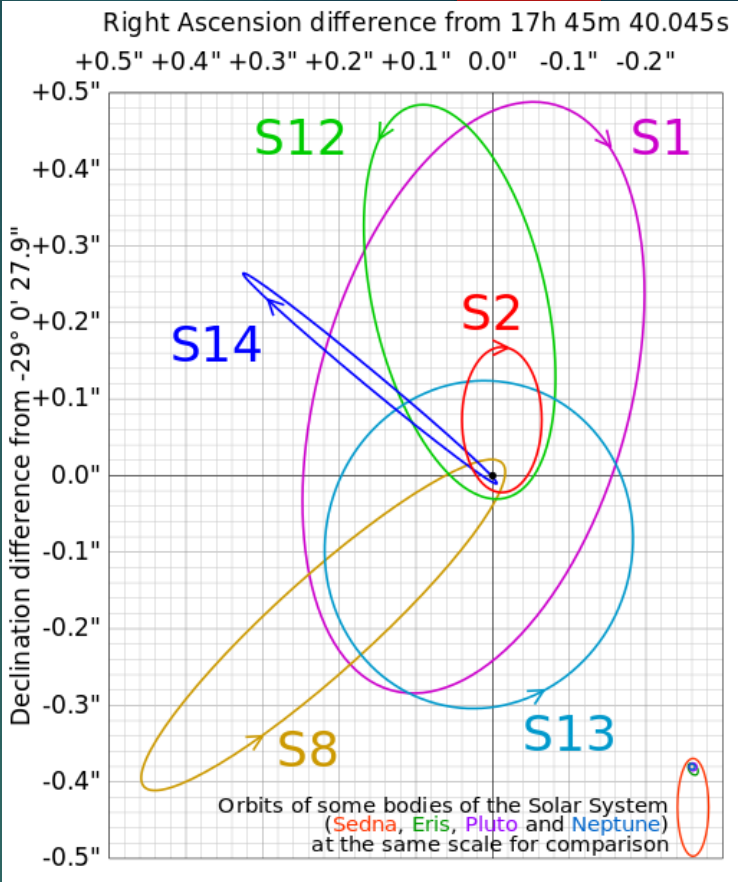


Kepler's Third Law together with Vera Rubin reveals dark matter

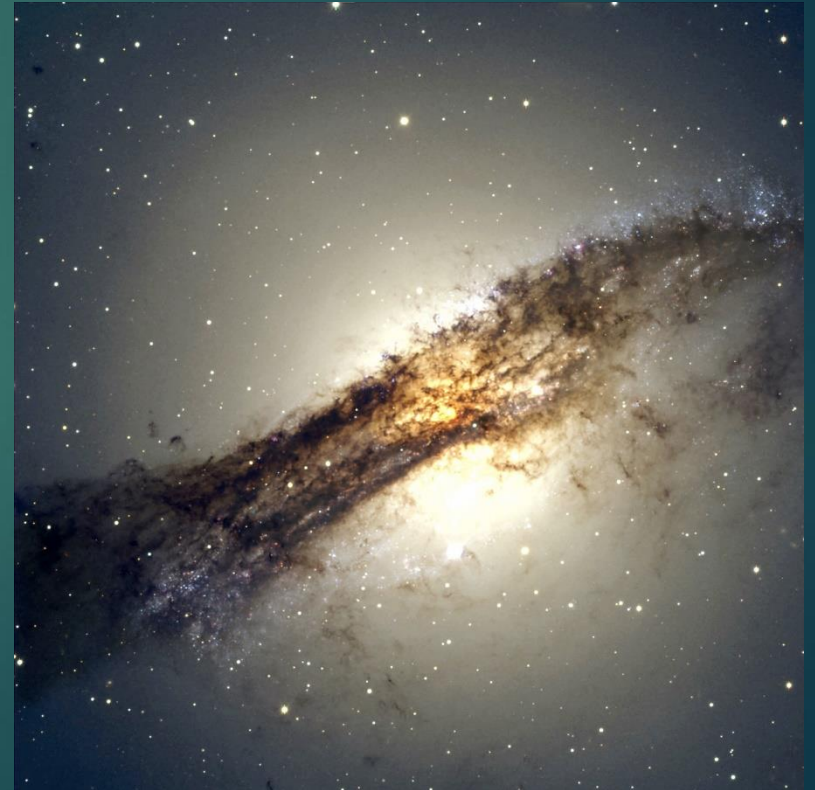
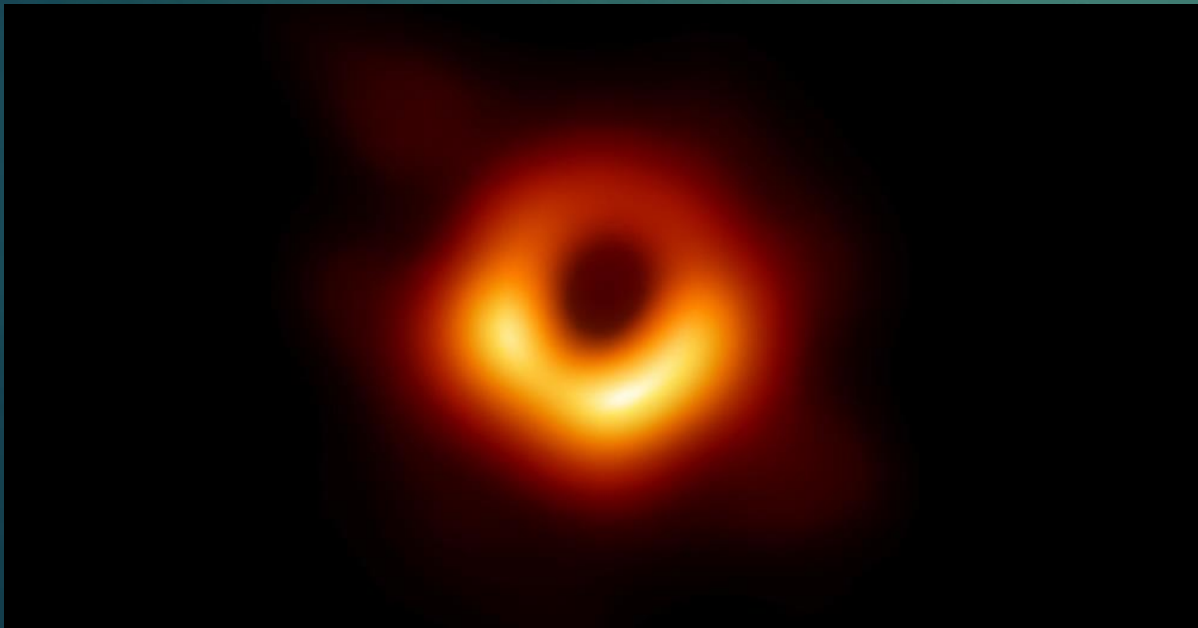


The center of the galaxy contains a black hole with a mass of 4 million suns. Determined from the orbits of stars near the center of the galaxy.

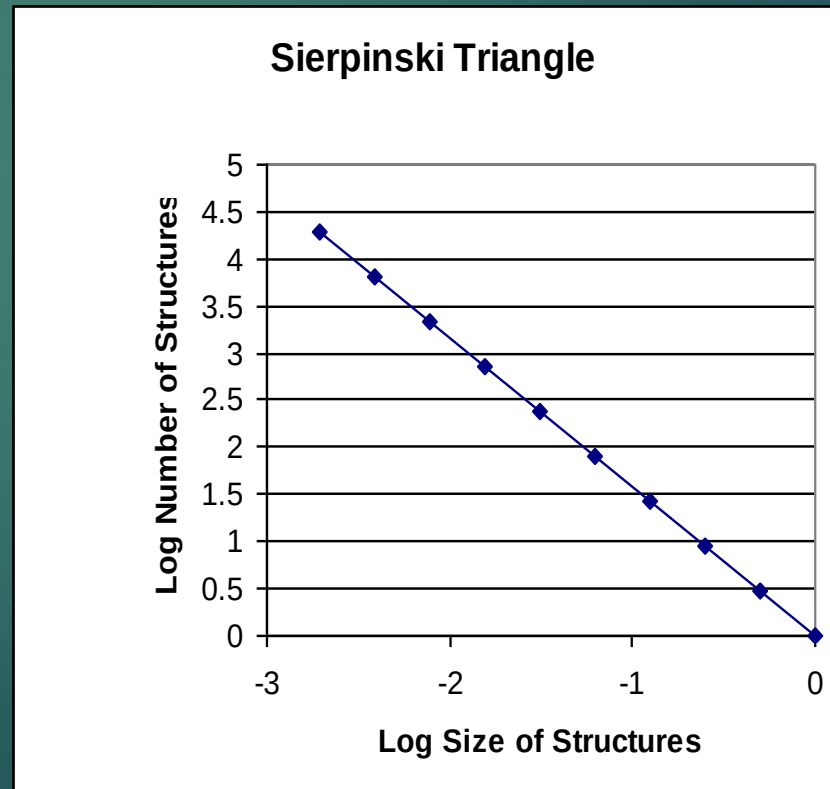
Star	a (AU)	P (years)	a ³	p2	Mass
S1	3300	94.1	35937000000	8854.81	4058472.175
S2	980	15.24	941192000	232.2576	4052362.549
S8	2630	67.2	18191447000	4515.84	4028363.937
S12	2290	54.4	12008989000	2959.36	4057968.277
S13	1750	36	5359375000	1296	4135320.216



M87- black hole M87- black hole in its center-
Measurements in 2011 of the orbits of stars
around its center gave a mass of 6.6 billion suns.
The recent photograph of the black hole yields
a mass of 6.5 billion suns



Fractal Entanglement:
Scale invariance and power laws are
fundamental characteristics of fractals.
Every feature is repeated at different
scales



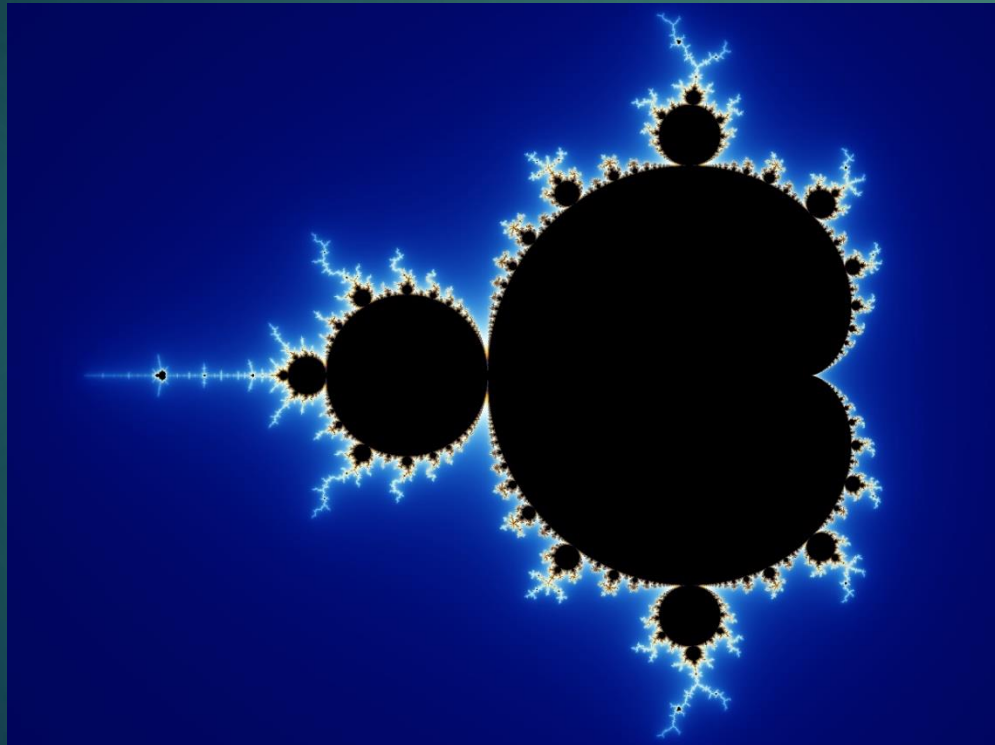
$$N = r^{-1.585}$$

The Universe may be one gigantic fractal: *an ontological vision for the modern world*

- ▶ Phenomena occurring at one scale are related to those occurring at other scales (as demonstrated in Kepler's Third Law).
- ▶ One can not cut out one piece of the universe without destroying its fractal nature.
- ▶ Knowledge of phenomena at one scale can lead to a knowledge of the whole.
- ▶ We are all interconnected and interdependent

Fractals: the parts are interconnected,
inseparable, non-independent and *entangled*.

Mandelbrot Fractal



Cauliflower



Entanglement and Emptiness

- ▶ Nagarjuna, who lived around 150 CE, known for his expositions on emptiness, *śūnyata*. Nagarjuna's philosophy was a watershed in the history of Indian philosophy as he challenged the assumption the absolute existence of anything in the universe.



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- ▶ Emptiness- *sunata*- the assertion that everything every person is empty of absolute existence but is dependent on others.
 - ▶ it is not a nihilistic statement that nothing exists, but rather an assertion about the interweaving of things in the cosmos.
 - ▶ We are all dependent phenomena; there is no phenomenon that arises completely independently of anything else and is therefore unchanging.



After this journey across the universe of planets, stars, and galaxies we return to one of Kepler's main interests that of political harmony, at the beginning of the 30 years war. The Third Law threads the universe, ties it together. We are all interconnected in some way. Perhaps this was the harmony he was seeking, inspired by the orderly behavior of the planets. If only we could learn from those planets how to behave!

