

LOCATION: SWANSEA

LLEOLIAD: ABERTAWE

LEVEL 6

LEFEL 6

EXAMINATION PERIOD
JANUARY 2018

CYFNOD ARHOLIADAU
IONAWR 2018

MODULE CODE: M6X00420
ADVANCED STRESS ANALYSIS AND FEA (PART TIME)

TIME: 2 HOURS

ANSWER THREE OUT OF FOUR QUESTIONS

Equipment or materials required – None

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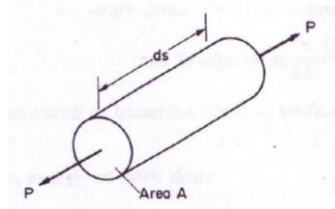
A formula sheet is provided for reference.

Calculators are permitted.

Assume 'g' = 9.81 m/s².

ONE

- a) (i) Consider the bar below subjected to tension. Prove from first principles that the extension, δ , due to the load is given by; $\delta = \frac{Pds}{AE}$, where P is the load, ds is the length of the bar, A is the cross sectional area and E is the Youngs Modulus. [2 Marks]



- (ii) Hence prove that the strain energy, U , for a bar in Tension is given by;
$$U = \frac{P^2 L}{2AE}$$
 [4 Marks]
- (iii) What is meant by the term Resilience? Hence from prove from $U = \frac{P^2 L}{2AE}$,
that resilience may be defined as Resilience = $\frac{\sigma^2}{2E}$ [4 Marks]

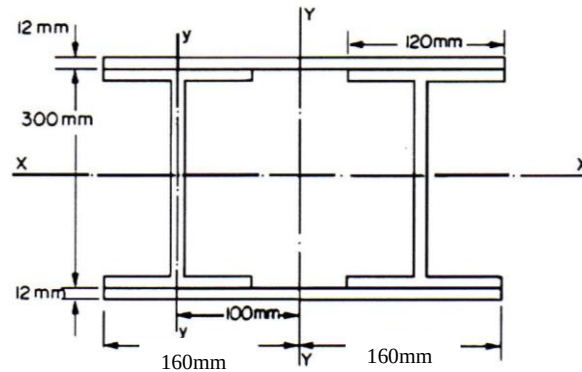
- b) (i) A steel rod has a square cross section 20mm x 20mm and a length of 4m. Calculate the strain energy when a stress of 300MPa is produced by stretching it. ($E=200\text{GPa}$) [3 Marks]
- (ii) The rod then has a metal collar fitted at the lower end, and a mass, m , is dropped onto the collar. Show, by considering the energy transferred from the falling weight to the rod, that the height, h , is related to the stress, σ , by the following relationship;

$$h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$$

- Where V is the volume of the bar, and L is the length. Assume that the extension of the bar is significant in the energy transfer. [4 Marks]
- (iii) If the stress must not exceed 75MPa, from what height could 200kg fall onto the collar, without exceeding this? [3 Marks]

TWO

Two 300mm x 120mm I-section joists are united by 12mm thick plates as shown below to form a 7m long stanchion. Given that the Young's Modulus $E = 200 \text{ GPa}$, compressive yield stress is 320 MPa and a constant a of $1/7500$, assuming pinned ends, determine the following;



Properties of ONE joist;

$$I_{xx} = 96 \times 10^{-6} \text{ m}^4$$

$$I_{yy} = 4.2 \times 10^{-6} \text{ m}^4$$

$$A = 6 \times 10^{-3} \text{ m}^2$$

- Which axis will the strut buckle about?
[5 Marks]
- What is the radius of gyration about this axis?
[2 Marks]
- What is the slenderness ratio of this strut?
[2 Marks]
- According to Euler theory, what load will this strut buckle at?
[3 Marks]
- According to Rankine-Gordon theory, what load will this strut buckle at?
[3 Marks]
- Comment on the results of (d) and (e), and with the use of a diagram explain the difference between experimental results and buckling loads according to Euler theory.
[5 Marks]

THREE

FEA can be used to make the design process more efficient. Discuss this by considering the following;

- a) How this influences the building of physical prototypes and testing in product development. [6 marks]
- b) How both conceptual designs and designs taken through to production are influenced. [8 marks]
- c) Using FEA as a verification tool and a design tool. [6 marks]

FOUR

Discuss the steps that you would undertake when carrying out some FE analysis. Your discussion should include the following;

- a) What 6 steps you should consider using the acronym 'S.I.M.P.L.E, and what you should consider in each of these steps. [12 marks]
- b) What is meant by pre-processing, processing and post-processing [6 marks]
- c) How you would decide what type and size of element you should use. [6 marks]

FORMULA SHEET

Energy

$$U = \frac{P^2 L}{2AE}$$

Strain energy in tension

$$U = \frac{Q^2 L}{2AG}$$

Strain energy in shear

$$U = \frac{M^2 L}{2EI}$$

Strain energy in bending

$$U = \frac{T^2 L}{2GJ}$$

Strain energy in torsion

$$\text{P.E.} = mgh$$

Potential energy

$$\text{K.E.} = \frac{1}{2} mv^2$$

Kinetic energy

Bending/Torsion

$$I_{NA} = \frac{BD^3}{12}$$

Second moment of area of a rectangle

$$I_{NA} = I_G + Ah^2$$

Parallel axis theorem

$$I = \frac{\pi d^4}{64}$$

The second moment of area of a circular shaft.

$$J = \frac{\pi D^4}{32}$$

The polar second moment of area of a circular shaft

Buckling

$$\frac{L}{k}$$

Slenderness Ratio

$$k = \sqrt{\frac{I}{A}}$$

Radius of gyration

$$P_e = \frac{\pi^2 EI}{L^2}$$

Buckling: pinned-pinned

$$P_e = \frac{\pi^2 EI}{4L^2}$$

Buckling: fixed-free

$$P_e = \frac{4\pi^2 EI}{L^2}$$

Buckling: fixed-fixed

$$P_e = \frac{2\pi^2 EI}{L^2}$$

Buckling: fixed-pinned

$$P_R = \frac{\sigma_y A}{1 + a(L/k)^2}$$

Rankine Load

Diwedd/End



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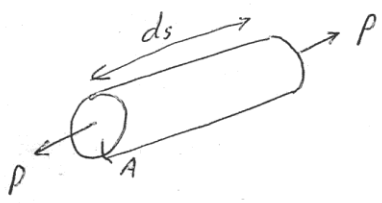
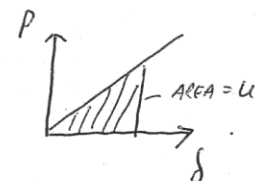
EXAMINATION SOLUTIONS: SESSION 2017/2018

School:	School of Engineering	
Programme:	Engineering Portfolio	Year: 3
Module/Unit:	Advanced Stress and FEA	
Author:	KELVIN LAKE	Page: 1 of

Question No	Question	Marks
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Continuation Sheet

Module/Unit:	ADVANCED STRESS ANALYSIS	Page:	of
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Question No	Question	Marks
(1a)	 $E = \frac{\sigma}{\epsilon}$ $= \frac{P}{A} \div \frac{\delta}{ds} = \frac{P ds}{A \delta} \quad \therefore \delta = \frac{P ds}{AE}$	(2)
(b)	 FOR SMALL SECTION $U = \frac{1}{2} P \delta$ $= \frac{1}{2} P \times \frac{P ds}{AE} = \frac{P^2 ds}{2AE}$ $\therefore$ FOR TOTAL BAR $U = \int_0^L \frac{P^2 ds}{2AE} = \frac{P^2 L}{2AE}$	(4)
(c)	RESILIENCE = STRAIN PER UNIT VOLUME $U = \frac{P^2 L}{2AE} \quad \therefore \frac{U}{AL} = \frac{\sigma^2}{2E}$ $= \frac{\sigma^2 A^2 L}{2AE}$ $= \frac{\sigma^2 AL}{2E}$ = RESILIENCE	(4)

Continuation Sheet

Module/Unit:	ADVANCED STRESS ANALYSIS	Page:	of
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Question No	Question	Marks
(di)	$\sigma = \frac{F}{A} \therefore F = \sigma \times A = 300 \times 10^6 \times (20 \times 20) \times 10^{-6}$ $= \underline{\underline{120 \text{ kN}}}$ $AREA = 20 \times 20 \times 10^{-6} = \underline{\underline{4 \times 10^{-4} \text{ m}^2}}$ $U = \frac{P^2 L}{2AE} = \frac{120,000^2 \times 4}{2 \times 4 \times 10^{-4} \times 210 \times 10^9} = \underline{\underline{342.9 \text{ J}}}$	(3)
(d) ii	P.E. = mgh = mg(h + δL) ① $E = \frac{\sigma}{\epsilon} = \sigma \div \frac{\delta L}{L} = \frac{\sigma L}{\delta L} \therefore \delta L = \frac{\sigma L}{E}$ FROM ① P.E. = mg(h + $\frac{\sigma L}{E}$) ② THAT ENERGY IS ABSORBED BY SHAFT. $U = \frac{P^2 L}{2AE} = \frac{\sigma^2 A^2 L}{2AE} = \frac{\sigma^2 AL}{2E} = \frac{\sigma^2 V}{2E} \text{ ③}$ EQUATING ② & ③ $\frac{\sigma^2 V}{2E} = mg(h + \frac{\sigma L}{E})$ $\frac{\sigma^2 V}{2Emg} = h + \frac{\sigma L}{E} \therefore h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$	(4)

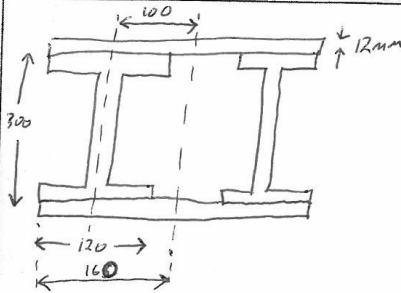
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Question No	Question	Marks
d. iii	$h = \frac{\sigma^2 V}{2Emg} - \frac{\sigma L}{E}$ $= \frac{(75 \times 10^6)^2 \times (4 \times 10^{-4} \times 4)}{2 \times 210 \times 10^9 \times 150 \times 9.81} - \frac{75 \times 10^6 \times 4}{210 \times 10^9}$ $= \frac{9 \times 10^{12}}{6.18 \times 10^{14}} - \frac{300 \times 10^6}{210 \times 10^9}$ $= 14.56 \times 10^{-3} - 1.43 \times 10^{-3}$ $= \underline{\underline{13.13 \text{ mm}}}$	(3)

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Module/Unit:	ADVANCED STRESS ANALYSIS	Page:	
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Question No	Question	Marks
2	 FOR EACH JOIST $I_{xx} = 96 \times 10^{-6} \text{ m}^4$ $I_{yy} = 4.8 \times 10^{-6} \text{ m}^4$ $A = 6 \times 10^{-3} \text{ m}^2$ (a) XX: JOISTS $I_{xx} = 2 \times 96 \times 10^{-6} = \underline{192 \times 10^{-6} \text{ m}^4}$ PLATES: $I_{xx} = \square - \square$ $= \left[\frac{320 \times 324^3}{12} - \frac{320 \times 300^3}{12} \right] \times 10^{-12}$ $= \underline{187 \times 10^{-6} \text{ m}^4}$ $I_{\text{TOTAL}} = 187 \times 10^{-6} + 192 \times 10^{-6}$ $= \underline{379 \times 10^{-6} \text{ m}^4}$	

Continuation Sheet

Module/Unit:	ADVANCED STRESS ANALYSIS	Page:	
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Question No	Question	Marks
2 (a) CONT	YY NOW $I_G = I_{NA} + Ah^2$ JOISTS: $I_{YY} = 2 (4.2 \times 10^{-6} + 6 \times 10^{-3} \times 0.1^2)$ $= 128.4 \times 10^{-6} \text{ m}^4$ PLATES $I_{YY} = \frac{BD^3}{12}$ $= 2 \left(\frac{12 \times 320^3}{12} \right) \times 10^{-12}$ $= 6.55 \times 10^{-5} \text{ m}^4$ $\therefore I_{YY \text{ total}} = 128.4 \times 10^{-6} + 6.55 \times 10^{-5}$ $= 193.9 \times 10^{-6} \text{ m}^4$ SO, ~~IT WILL BUCKLE ABOUT XX~~ $I_{XX} = 379 \times 10^{-6} > I_{YY} = 193.9 \times 10^{-6}$ $\therefore$ IT WILL BUCKLE ABOUT YY.	(5)

Continuation Sheet

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Question No	Question	Marks
(b)	$K = \sqrt{\frac{I}{A}}$ TOTAL AREA = 2 × TUBES + 2 × PLATES $= 2 \times 6 \times 10^{-3} + 2 (320 \times 12) \times 10^{-6}$ $= \underline{\underline{0.01968 \text{ m}^2}}$ $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{193.9 \times 10^{-6}}{0.01968}}$ $= \underline{\underline{0.1 \text{ m}^2}}$	(2)
(c)	SLENDERNESS RATIO = $\frac{L}{K}$ $= \frac{7}{0.1}$ $= \underline{\underline{70}}$	(2)

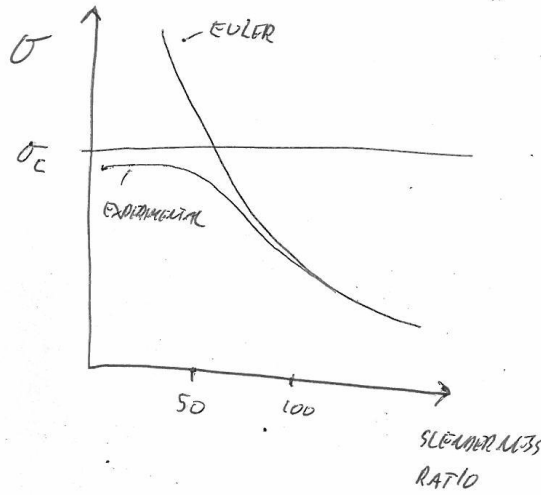
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Question No	Question	Marks
d	$P_e = \frac{\pi^2 EI}{L^2}$ $= \frac{\pi^2 \times 200 \times 10^9 \times 193.9 \times 10^{-6}}{7^2}$ $= \cancel{7.8 \times 10^6} = \underline{\underline{7.8 \times 10^6 \text{ N}}}$	
e	$P_R = \frac{\sigma_y A}{1 + a \left(\frac{L}{r} \right)^2}$ $= \frac{320 \times 10^6 \times 0.01968}{1 + \frac{1}{7500} \left(\frac{7}{0.1} \right)^2}$ $= \underline{\underline{3.8 \times 10^6 \text{ N}}}$	3 3

Continuation Sheet

Module/Unit:	ADVANCED STRIPS	Page:	
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Question No	Question	Marks
(F)	FOR THIS STRUT, WHICH IS APPROACHING THE VALIDITY LIMIT FOR EULER'S THEORY, THERE IS AN APPRECIABLE DIFFERENCE BETWEEN THE RESULTS GIVEN BY THE TWO THEORIES. 	

THREE

Discussion could include, but is not limited to, items such as the following:

- a) The use of FEA can be used to significantly reduce the number of prototypes needed in product development. Instead of building physical prototypes, virtual prototypes can be built and tested. This significantly reduces the time and the expense of building prototypes. However, some physical prototypes will probably still be needed to correlate the FE models to gain confidence in these models and to 'sign-off' the final design. In an ideal world, no prototype tooling would be necessary, instead the manufacturer would go straight to production tooling.
- b) FEA can be used at a conceptual stage to judge the performance of a design without incurring the expense of having to produce 3D CAD within a CAD package. This means that the performance and the cost of the proposed design can be understood by both the supplier and the potential customer when making a decision as to which conceptual design to develop further, this is particularly relevant to suppliers who are often producing concepts to 'quote for business' at their own expense. The use of FEA at a conceptual stage significantly reduces the cost of producing these as discussed in part (a). Once a conceptual design has been decided upon this can be used to take the design through to production. It is at this stage that other analytical tools such as moldflow analysis would need to be considered further, however, the original geometry from the conceptual design can still be used to inform the design.
- c) The use of FEA has traditionally been used as a verification tool. That is to say that a component has been produced using experience and judgement of a designer and the performance of this component is assessed in an FE environment and this is then iterated until the performance of the component meets the design specification. A better use of FEA is to use it as a design tool, using optimisation techniques (such as topology), to inform the design thus ensuring, for example, that no major loadpaths are missed. By doing this the design will be a more efficient solution. This is a particularly powerful tool when applied to conceptual designs as discussed in part (b).

FOUR

Discussion could include, but is not limited to, items such as the following:

- a) When carrying out some FEA the basic steps can be considered using the following acronym,
- **Split.** The first step that needs to be undertaken is to split the geometry down into finite elements (i.e. to create elements and nodes) so that mathematical assumptions can be made from one element to the next. As long as the distances between the nodes is small enough (i.e. the distance over which the assumption is being made), are small enough, then a close approximation can be obtained. At this stage then the element type (1D, 2D or 3D), order (1st or 2nd order) and density (size) needs to be considered.
 - **Interpret.** In order to be able to obtain a reasonable result in a reasonable amount of time the geometry needs to be approximated. Small details that add complexity to the analysis, but would not have an adverse effect on the analysis being undertaken in terms of run-time should be simplified or ignored. That is to say that the part needs to be 'idealised'
 - **Material.** Once the part has been simplified and meshed, then material information needs to be added. This may determine if a linear or a non-linear solver may be used (depending on if the material is linear or non-linear). Careful consideration needs to be given as to if the material actually used is the same as the material in the FE model. This may be a cause for poor correlation.
 - **Physical Property.** If 1D or 2D elements are used, then physical property information needs to be added to the model. That is to say, what is the section of any 1D elements (for example second moment of area, polar second moment of area, area and orientation)? If 2D elements are used then thickness of the elements needs to be considered. Again, this may be a cause for poor correlation to testing, for example, what is the manufacturing tolerance on a 2mm thick sheet of material? If 3D elements are being used, then this information is defined by the geometry of the elements.
 - **Loading.** Once the above information has been added to the FE model the solver will be able to calculate the stiffness matrix for the structure. From this, a set of simultaneous equations can be obtained. In order to solve these, then the loading and boundary conditions needs to be known. That is to say, how is the part loaded, in what direction, and how is it constrained? This can also be a source of poor correlation (probably the main cause for poor correlation). For example over what area is the force being applied (if the part is being loaded at a single node, then in theory the stress in this location would be infinite ($\text{stress} = \text{force} / \text{area}$)), in what direction? It is very easy to say a face is fixed in 6 degrees of freedom (i.e. it can not move at all), but this suggests that the part that the component is attached to is of infinite stiffness which will not be the case.
 - **Evaluate.** Once the model has been solved the analyst needs to evaluate their results. I.e. have any of the previous steps influenced the results? Does any of the information in the previous steps need to be updated? Using a computer to carryout FEA can only give approximate results. How appropriate is it to quote the

results to many significant figures given the approximations that have been made?
The results are computer generated, and like any computer generated information
'garbage in = garbage out'!

- b) Pre-processing describes preparing the FE model before analysis can be carried out (that is to say, steps 'SIMPL', discussed in part (a)). Processing, describes the computer actually carrying out calculations (using matrix algebra), in order to solve the equations generated from the stiffness matrix and the boundary conditions. Post-processing describes interpreting the results (i.e. step 'E' discussed in part (a)).
- c) In order to determine the type and order of elements to use, the original geometry may be considered, (i.e. is it 1D, 2D or 3D?), the time and the computing power available to get a solution, and the accuracy required from the results. Things that may be considered include the error norm, changes in stress from one element to the next, differences when viewing 'nodal' or 'elemental' results, and the minimum number of elements across the thickness of the part when using 3D elements. Meshes can either be refined manually or this process can be automated by using H or P mesh refinement. If what element size to use is unclear, then the analyst may try different element sizes and then see if the results being obtained are converging on a solution.