

Capsule Note



From I_x we get $I_x \ddot{\theta}$ about the CoG also an $mh^2 \ddot{\theta}$ term for the offset to contact point so $(I_x + mh^2) \ddot{\theta}$

This is balanced by $mgh \sin \theta$ pulling down. So we have

$$(I_x + mh^2) \ddot{\theta} - mgh \sin \theta = 0$$

Or when vertical:

$$(I_x + mh^2) \ddot{\theta} - mgh \theta = 0$$

Ass if:

$$\theta = e^{st}$$

$$\ddot{\theta} = s^2 e^{st}$$

So

$$(I_x + mh^2) s^2 - mgh = 0 \quad e^{st} \neq 0$$

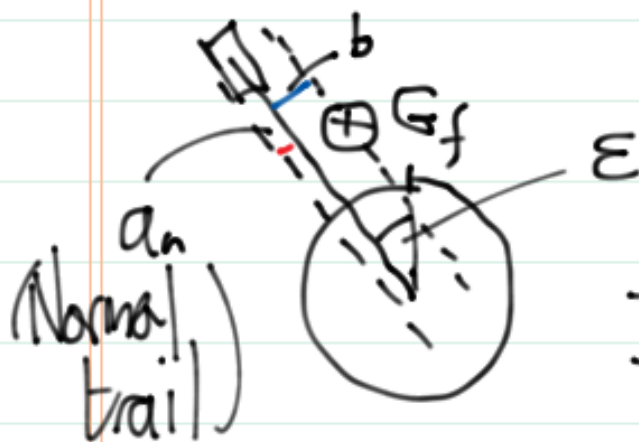
Solve

$$s = \sqrt{\frac{mgh}{I_x + mh^2}}$$

Refer to
ODE Notes

This is the eigenvalue, so frequency..
Time is the inverse of frequency, so $\tau = \frac{1}{s}$
$$\tau = \sqrt{\frac{I_x + mh^2}{mgh}}$$

Wobble Notes:



$$I_A = I_G + M_f b^2$$

I_G = Inertia about C of F

I_A = Inertia about Steer Axis

C = Damping Constant

F_{sf} = lateral tyre force



So equating moments:

$$I_A \ddot{\delta} + C \dot{\delta} + F_{sf} a_n = M_f g b_f \delta \sin \epsilon + N_f a_n \delta \sin \epsilon$$

So

$$I_A \ddot{\delta} = -C \dot{\delta} - F_{sf} a_n + M_f g b_f \delta \sin \epsilon + N_f a_n \delta \sin \epsilon$$

Damping

Restoring Torque

Turning Torque due to M_f

Turning Torque due to N_f

Both the N_f and M_f terms are small compared to the lateral force so we ignore them.

$$I_A \ddot{\delta} + C \dot{\delta} + F_{sf} a_n = 0$$

Now, we would rather the equation be in terms of V , rather than F_{sf} , so use

$$F_{sf} = K_{sf} \lambda_f \quad \text{where}$$

$$\lambda_f = \frac{\dot{\delta} a_n}{V} + \delta \cos \epsilon$$

has a V in it

So $F_{sf} = K_{sf} \left(\frac{\dot{\delta} a_n}{V} + \delta \cos \epsilon \right)$

Put this into

$$I_A \ddot{\delta} + C \dot{\delta} + F_{sf} a_n = 0$$

to get

$$I_A \ddot{\delta} + C \dot{\delta} + K_{sf} \left(\frac{\dot{\delta} a_n}{V} + \delta \cos \epsilon \right) a_n = 0$$

$$\Rightarrow I_A \ddot{\delta} + C \dot{\delta} + K_{sf} \frac{a_n^2}{V} \dot{\delta} + K_{sf} a_n \delta \cos \epsilon$$

$$\Rightarrow I_A \ddot{\delta} + \left(C + \frac{K_{sf} a_n^2}{V} \right) \dot{\delta} + K_{sf} a_n \delta \cos \epsilon$$

get terms together

$$I_A \ddot{\delta} + \left(C + \frac{K_{sf} a_n^2}{V} \right) \dot{\delta} + K_{sf} a_n \cos \epsilon \delta = 0$$

Now, as before

$$\delta = e^{st}$$

Now

$$s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S = \frac{-\left(C + \frac{K_{\lambda} a_n^2}{V}\right) \pm \sqrt{\left(C + \frac{K_{\lambda} a_n^2}{V}\right)^2 - 4 I_A K_{\lambda} a_n \cos \epsilon}}{2 I_A}$$

$$S = -\left(\frac{C}{2 I_A} + \frac{K_{\lambda} a_n^2}{2 I_A V}\right) \pm \sqrt{\frac{C}{2 I_A} + \frac{K_{\lambda} a_n^2}{2 I_A V} - \frac{4 I_A K_{\lambda} a_n \cos \epsilon}{4 I_A^2}}$$

add fractions

$$S = \frac{-CV + K_{\lambda} a_n^2}{2 I_A V} \pm \sqrt{\left(\frac{CV + K_{\lambda} a_n^2}{2 I_A V}\right)^2 - \frac{K_{\lambda} a_n \cos \epsilon}{I_A}}$$

This will be complex when the discriminant is Negative

Discriminant $\Rightarrow$ if $ax^2 + bx + c = 0$

$$\text{Discriminant } \Delta = b^2 - 4ac$$

just the
bit under the
square root

When the solution is complex, we get oscillations (Wobble!)

This occurs when

$$V > \frac{1}{2} \sqrt{\frac{K_{\lambda} a_n^3}{I_A \cos \epsilon}}$$

With a frequency of:

$$\nu = \frac{1}{2\pi} \sqrt{\frac{K_{\lambda} a_n}{I_A} \cos \epsilon} \sqrt{(1 - \zeta^2)}$$

where ζ (zeta) is the damping Ratio

$$\zeta = \frac{CV + K_{\lambda} a_n^2}{2V \sqrt{I_A K_{\lambda} a_n \cos \epsilon}}$$

Weave Notes

$$I_A = I_G + M_r l^2$$

Moment of inertia at A

C is damping contact,
 F_{sr} = lateral force at tyre

From this we get

$$I_A \ddot{\psi} = -c \dot{\psi} - F_{sr} l$$

$$F_{sr} = K_{\lambda} \lambda_r \quad \text{and} \quad \lambda_r = \frac{\dot{\psi} l}{V} + \psi \cos \epsilon$$

Meaning: $F_{sr} = K_{\lambda} \left(\frac{\dot{\psi} l}{V} + \psi \cos \epsilon \right)$

So: $I_A \ddot{\psi} + c \dot{\psi} + \frac{K_{\lambda} \dot{\psi} l^2}{V} + K_{\lambda} l \cos \epsilon \psi = 0$

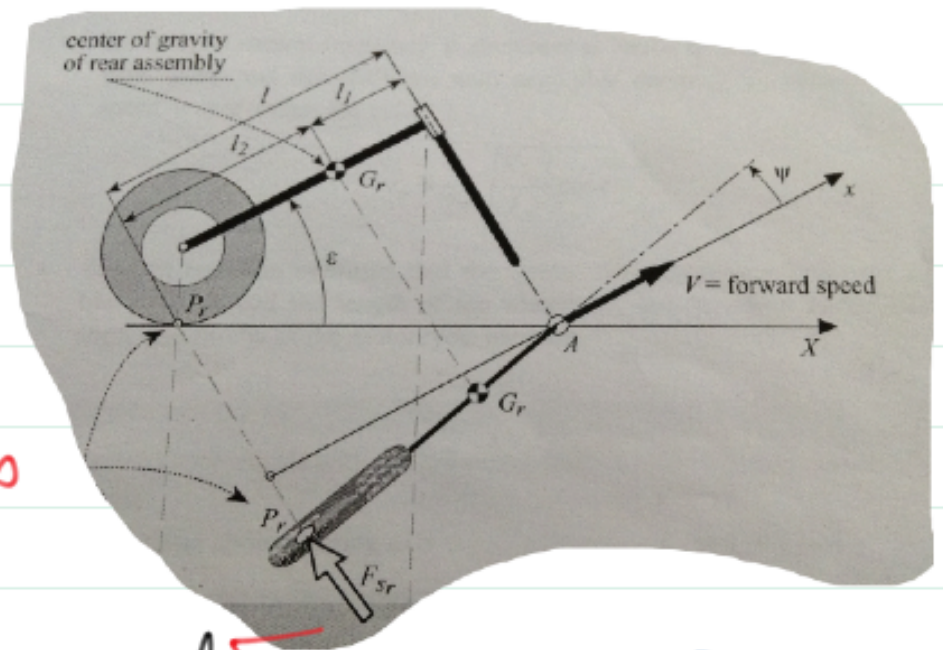
Which simplifies to:

$$I_A \ddot{\psi} + \left(c + \frac{K_{\lambda} l^2}{V} \right) \dot{\psi} + K_{\lambda} l \cos \epsilon \psi = 0$$

Make the $\psi = e^{st}$ substitution again $\rightarrow$ giving

$$I_A s^2 + \left(c + \frac{K_{\lambda} l^2}{V} \right) s + K_{\lambda} l \cos \epsilon = 0$$

Again, use quadratic formula: $\rightarrow$



Now, as before:

$$S = \frac{-\left(C + \frac{K_{xr} l^2}{V}\right) \pm \sqrt{\left(C + \frac{K_{xr} l^2}{V}\right)^2 - 4 I_A K_{xr} l \cos \epsilon}}{2 I_A}$$

$$S = -\left(\frac{C}{2 I_A} + \frac{K_{xr} l^2}{2 I_A V}\right) \pm \sqrt{\left(\frac{C}{2 I_A} + \frac{K_{xr} l^2}{2 I_A V}\right)^2 - \frac{4 I_A K_{xr} l \cos \epsilon}{4 I_A^2}}$$

Giving:

$$S = \frac{C V + K_{xr} l^2}{2 I_A V} \pm \sqrt{\left(\frac{C V + K_{xr} l^2}{2 I_A V}\right)^2 - \frac{K_{xr} l}{I_A} \cos \epsilon}$$

As before, this solution becomes complex when $(b^2 - 4ac)$ is Negative. We then see Oscillations $\rightarrow$ Instability

This occurs when the velocity

$$V > \frac{1}{2} \sqrt{\frac{K_{xr} l^3}{I_A \cos \epsilon}}$$

giving a frequency:

$$V = \frac{1}{2 \pi} \sqrt{\frac{K_{xr} l}{I_A} \cos \epsilon} \sqrt{(1 - \zeta^2)}$$

The damping ratio falls rapidly as speed increases

$$\zeta = \frac{C V + K_{xr} l^2}{2 V \sqrt{I_A K_{xr} l \cos \epsilon}}$$

$\uparrow$
Zeta