

ACMA 5011 Thermo-fluid Mechanics

Examination Content.

1 Heat transfer by Conduction

Fourier's Law $\dot{Q} = -k A \cdot \frac{dT}{dx}$

Composite body problems.

2 Heat Transfer by Convection

(i) Free Convection (ii) Forced Convection.

$$\dot{Q} = h A (T_s - T_f)$$

3 Heat transfer by Radiation

Stefan-Boltzmann Law

$$\dot{Q} = \epsilon A \sigma T^4$$

4 Basic fluid Mechanics

(i) Continuity equation

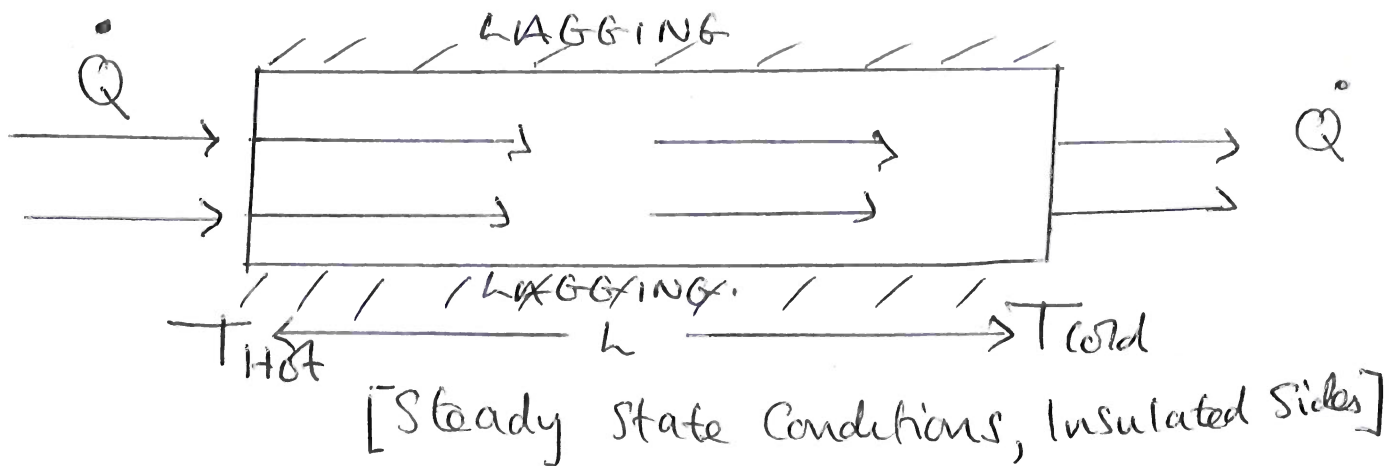
(ii) Bernoulli Equation for Inviscid/Ideal fluids

5 The First Law of thermo-dynamics applied to open systems.

$$\dot{m} \left[u_1 + p_1 v_1 + \frac{1}{2} C_1^2 + g z_1 \right] + \dot{W} + \dot{Q} = \dot{m} \left[u_2 + p_2 v_2 + \frac{1}{2} C_2^2 + g z_2 \right]$$

Fourier's Law of Heat Conduction In Solids.

Fourier discovered the equation governing how heat is transmitted through a solid.



$$\dot{Q} = \frac{dQ}{dt} = -kA \cdot \frac{dT}{dx}$$

k - Thermal Conductivity Constant

A - Cross Sectional Area.

$\frac{dT}{dx}$ - Temperature gradient

- Sign $\Rightarrow$ Temperature decreases as length increases

Usually we use the more practical formula

$$\dot{Q} = \frac{dQ}{dt} = k.A \frac{(T_{Hot} - T_{Cold})}{L}$$

The general Heat equation

$$\frac{\partial T}{\partial t} + \underline{V} \cdot \nabla T = k \cdot \nabla^2 T$$

Time
Dependence

Convection
Terms

Diffusion
Terms.

A Simple Case

Consider Case where there is no time dependence,
no convection and in 1 Dimension. Then the
equation becomes:-

$$\nabla^2 T = 0$$

$$\Rightarrow \frac{d^2 T(x)}{dx^2} = 0 \quad \because \begin{aligned} T(0) &= T_H \\ T(L) &= T_C \end{aligned}$$

Integrating $\Rightarrow$

$$\frac{dT(x)}{dx} = C$$

$$\Rightarrow T(x) = \int C dx$$

$$\underline{T(x) = Cx + D} \quad (\text{General Solution})$$

Now Applying the boundary Conditions:

$$T(x) = Cx + D$$

$$T(0) = C(0) + D$$

$$\underline{T_H = D}$$

$$\therefore T(x) = Cx + T_H$$

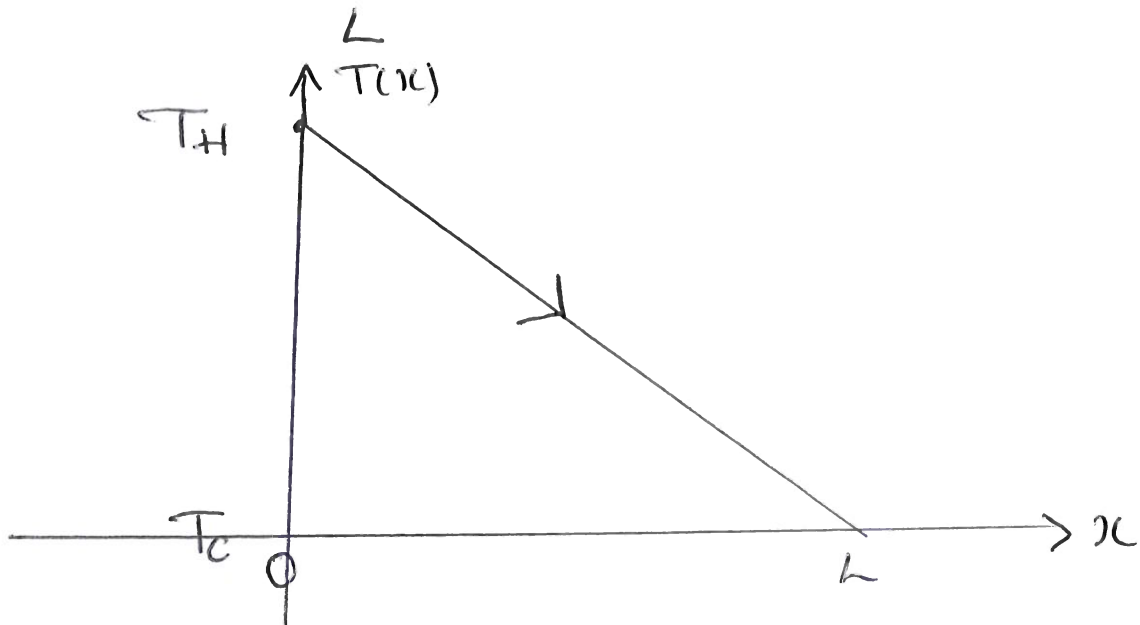
$$T(L) = CL + T_H$$

$$T_C = CL + T_H$$

$$T_C - T_H = C \cdot L$$

$$\underline{\frac{T_C - T_H}{L} = C}$$

$$\therefore T(x) = \frac{(T_C - T_H)}{L} x + T_H \quad \text{particular solution.}$$



Temperature distribution along an insulated bar.

The Diffusion Equation

(Simple Case in 1D).

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

Such that

$$u(0,t) = 0$$

$$u(1,t) = 0$$

$$u(x,0) = 1 \quad 0 < x < 1$$



$u(0,t) = 0$ $u(1,t) = 0$

Solution let $u(x,t) = X(x)T(t)$

$$\Rightarrow XT' = X''T$$

$$\Rightarrow \frac{XT'}{XT} = \frac{X''T}{XT}$$

$$\Rightarrow \frac{T'}{T} = -p^2 = \frac{X''}{X} \text{ for non-trivial solutions}$$

$$\Rightarrow \frac{T'}{T} = -p^2 \text{ and } \frac{X''}{X} = -p^2$$

$$\Rightarrow \frac{1}{T} \frac{dT}{dt} = -p^2 \text{ and } X'' + p^2 X = 0$$

let $X(x) = e^{mx} \Rightarrow$ Aux eqn

$$m^2 + p^2 = 0$$

$$\Rightarrow m = \pm ip$$

$$X(x) = A_1 e^{+ipx} + B_1 e^{-ipx}$$

$$X(x) = A_1 \cos(px) + B_1 \sin(px)$$

$$\ln T = -p^2 t + E_1$$

$$\Rightarrow T = C_1 e^{-p^2 t}$$

$$u(x,t) = C e^{-p^2 t} [A_1 \cos(px) + B_1 \sin(px)]$$

$$\Rightarrow u(x,t) = e^{-p^2 t} [A \cos(px) + B \sin(px)]$$

Applying boundary conditions $\Rightarrow$

$$u(0,t) = 0 = e^{-p^2 t} [A]$$

$$\Rightarrow A = 0$$

$$\therefore u(x,t) = e^{-p^2 t} [B \sin(px)]$$

Also

$$u(1,t) = e^{-p^2 t} [B \sin(p(1))] = 0$$

$$\Rightarrow \sin p = 0 \quad [\text{else trivial solution}]$$

$$\Rightarrow p = \sin^{-1}(0)$$

$$p = n\pi \quad \forall n \in \mathbb{Z}^+$$

$$\Rightarrow u(x,t) = \sum_{n=1}^{\infty} e^{-n^2 \pi^2 t} \cdot B_n \sin(n\pi x)$$

$$u(x,0) = 1 = \sum_{n=1}^{\infty} e^{(0)} \cdot B_n \sin(n\pi x)$$

$$\Rightarrow 1 = \sum_{n=1}^{\infty} B_n \sin(n\pi x)$$

$$\text{Then: } B_n = \frac{2}{1} \int_0^1 1 \cdot \sin(n\pi x) dx$$

$$B_n = 2 \left[-\frac{\cos(n\pi x)}{n\pi} \right]_0^1$$

$$B_n = 2 \left\{ -\frac{\cos(n\pi)}{n\pi} - \left(-\frac{\cos(0)}{n\pi} \right) \right\}$$

$$= 2 \left\{ -\frac{(-1)^n}{n\pi} + \frac{1}{n\pi} \right\}$$

$$= \frac{2}{n\pi} [1 - (-1)^n]$$

$$= 0 \quad \text{if } n \text{ is even}$$

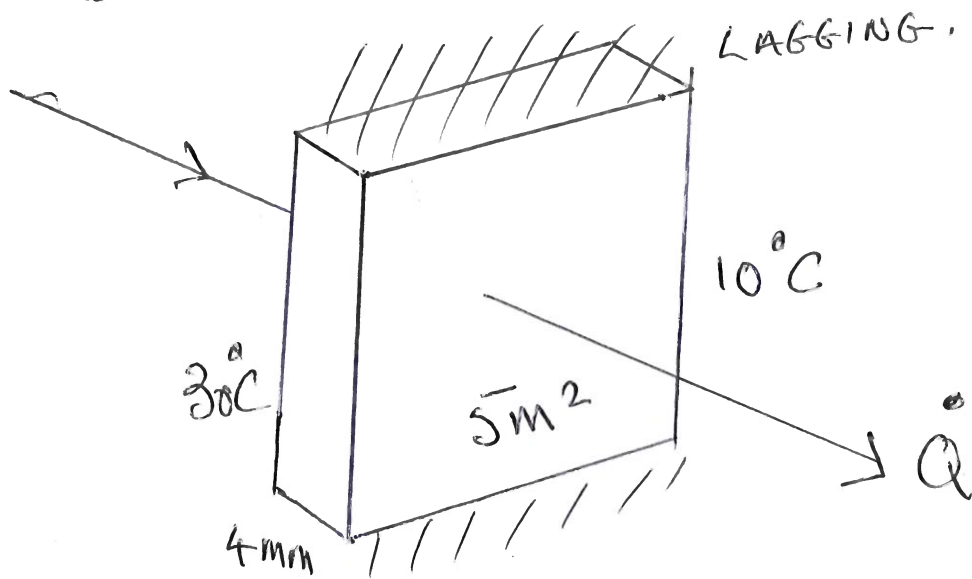
$$= \frac{4}{n\pi} \quad \text{if } n \text{ is odd.}$$

$$\therefore u(x,t) = \sum_{n \text{ odd}} \frac{4}{n\pi} e^{-n^2\pi^2 t} \sin(n\pi x)$$

$$u(x,t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{(2n-1)} e^{-(2n-1)^2\pi^2 t} \sin((2n-1)\pi x)$$

Examples:

Ex.1 Find the steady rate of heat loss through a solid of cross sectional area 5m^2 with thickness 4mm , if the thermal conductivity constant is $1\text{Wm}^{-2}\text{K}^{-1}$ and the temperature of inside surface is 30°C and the outer surface 10°C as shown.



$$\frac{dQ}{dt} = k A \frac{(T_H - T_C)}{L}$$

$$= 1 \times 5 \times \frac{(30 - 10)}{4 \times 10^{-3}}$$

$$= \frac{5 \times 20}{4 \times 10^{-3}} \text{ Watts}$$

$$= \underline{25 \times 10^3 \text{ Watts}}$$

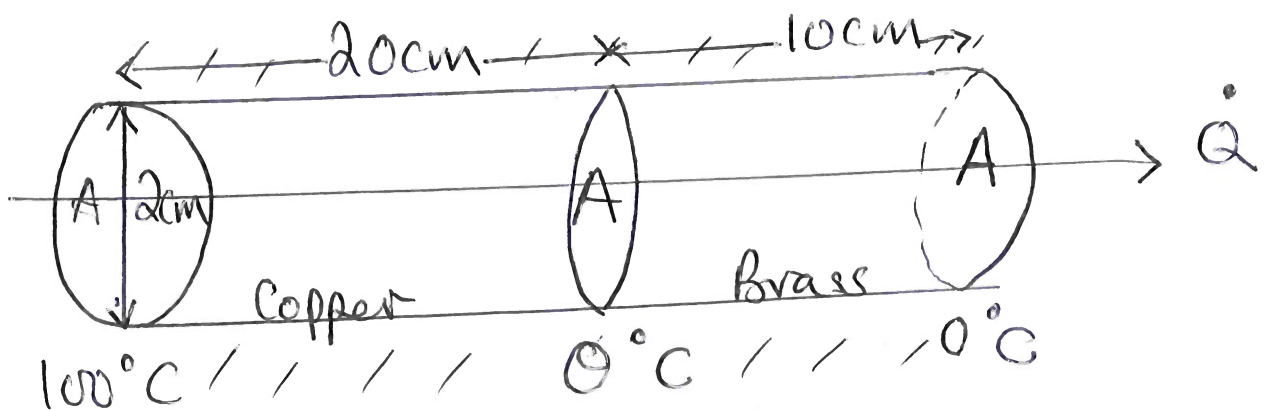
Composite Problems.

A Copper bar of 2cm diameter and length 20cm is joined to a brass rod of the same diameter but only 10cm long. No heat is lost through the sides of the composite bar. One end of the bar is maintained at 100°C and the other at 0°C as shown.

$$\text{If } k_{\text{copper}} = 384 \text{ Wm}^{-1}\text{K}^{-1}$$

$$k_{\text{brass}} = 109 \text{ Wm}^{-1}\text{K}^{-1}$$

Find the temperature of the junction and the steady rate of heat flow through the composite bar.



Applying Fourier's Law to each Section of the bar $\Rightarrow$

Copper Section:

$$\frac{dQ}{dt} = k_{\text{copper}} \times A \times \frac{(100 - \theta)}{20 \times 10^{-2}}$$

$$= \frac{384 \times A \times (100 - \theta)}{20 \times 10^{-2}}$$

Brass Section:

$$\frac{dQ}{dt} = k_{\text{brass}} \times A \times \frac{(\theta - 0)}{10 \times 10^{-2}}$$

$$= \frac{109 \times A \times \theta}{10 \times 10^{-2}}$$

As Steady State Conditions are reached

$$\Rightarrow \frac{384 \times \cancel{A} \times (100 - \theta)}{\cancel{20} \times \cancel{10^{-2}}} = \frac{109 \times \cancel{A} \times \theta}{\cancel{10} \times \cancel{10^{-2}}}$$

$$384(100 - \theta) = 2 \times 109 \times \theta$$

$$38400 - 384\theta = 218\theta$$

$$38400 = 602\theta$$

$$\Rightarrow \frac{38400}{602} = \theta$$

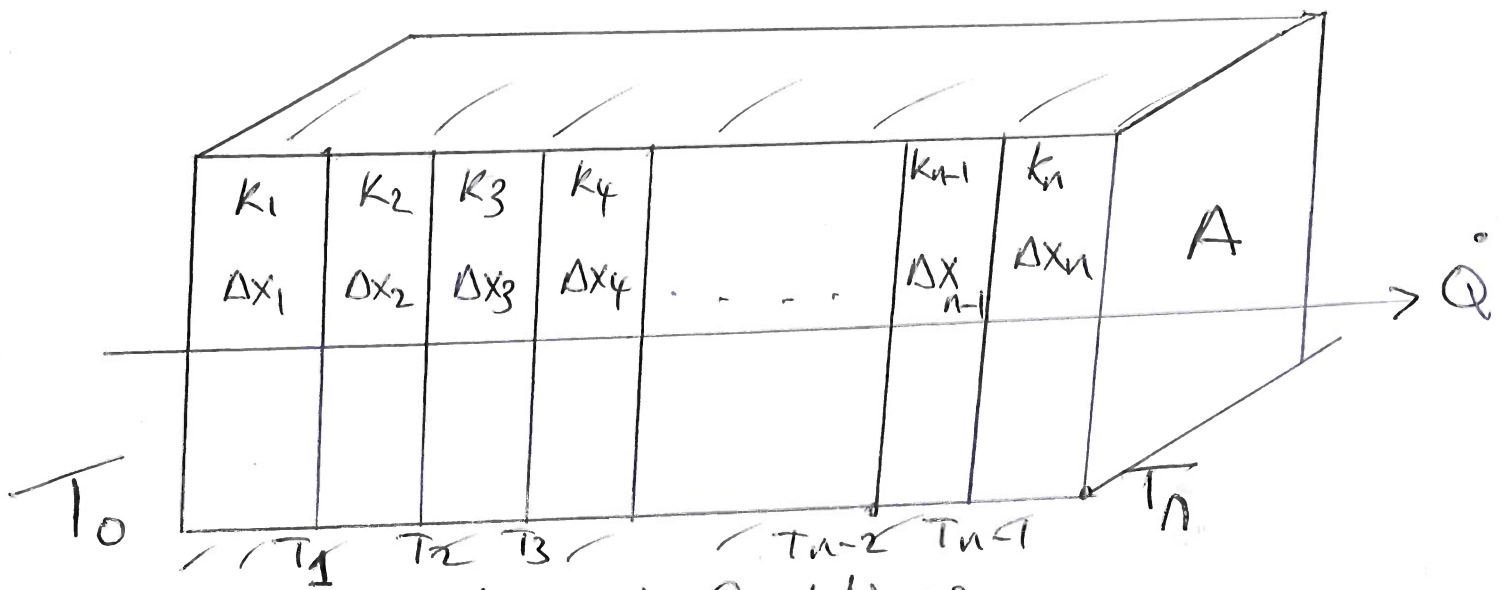
$$\underline{63.79^\circ\text{C} = \theta \text{ (2 d.p.)}}$$

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= \pi \times (1 \times 10^{-2})^2\end{aligned}$$

∴ for Copper Section:-

$$\begin{aligned}\frac{dQ}{dt} &= \frac{384 \times \pi \times 10^{-4} \times (100 - 63.79)}{20 \times 10^{-2}} \\ &= \underline{21.84 \text{ Watts (2dp's)}}$$

The general Composite body



Steady State Conditions.

Applying Fourier's law to each section $\Rightarrow$

$$\frac{dQ}{dt} = \dot{Q} = \frac{k_1 A (T_0 - T_1)}{\Delta x_1}$$

$$\dot{Q} = \frac{k_2 A (T_1 - T_2)}{\Delta x_2}$$

$$\dot{Q} = \frac{k_3 A (T_2 - T_3)}{\Delta x_3}$$

$\vdots$

$$\dot{Q} = \frac{k_{n-1} A (T_{n-2} - T_{n-1})}{\Delta x_{n-1}}$$

$$\dot{Q} = \frac{k_n A (T_{n-1} - T_n)}{\Delta x_n}$$

$$\frac{\dot{Q} \Delta x_1}{k_1 A} = T_0 - T_1$$

$$\frac{\dot{Q} \Delta x_2}{k_2 A} = T_1 - T_2$$

$$\frac{\dot{Q} \Delta x_3}{k_3 A} = T_2 - T_3$$

⋮

$$\frac{\dot{Q} \Delta x_{n-1}}{k_{n-1} A} = T_{n-2} - T_{n-1}$$

$$\frac{\dot{Q} \Delta x_n}{k_n A} = T_{n-1} - T_n$$

ADDING $\Rightarrow$

$$\frac{\dot{Q}}{A} \left[\frac{\Delta x_1}{k_1} + \frac{\Delta x_2}{k_2} + \frac{\Delta x_3}{k_3} \dots \frac{\Delta x_{n-1}}{k_{n-1}} + \frac{\Delta x_n}{k_n} \right] = T_0 - T_n$$

$$\frac{\dot{Q}}{A} \cdot \sum_{i=1}^n \frac{\Delta x_i}{k_i} = T_0 - T_n$$

$$\Rightarrow \dot{Q} = \frac{A [T_0 - T_n]}{\sum_{i=1}^n \frac{\Delta x_i}{k_i}}$$

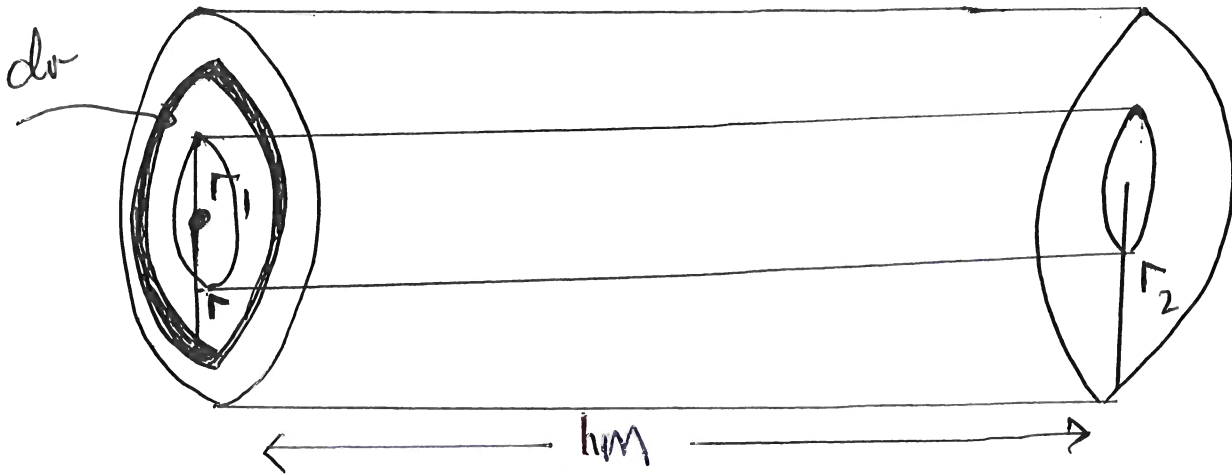
$$\text{or } \dot{Q} = \frac{T_o - T_n}{\frac{1}{A} \sum_{i=1}^n \frac{\Delta x_i}{k_i}}$$

The Quantity $\frac{1}{A} \sum_{i=1}^n \frac{\Delta x_i}{k_i}$ is known as

the Thermal Resistance.

This analogy is similar to that used for Electrical Conductors.

Heat flow through a Cylinder



Consider a Cylinder of Internal radius r_1 and external radius r_2 as shown above.

Let the Inside and outside surface temperatures be T_1 and T_2 respectively.

Consider the heat flow through a small element of thickness ' dr ' at any radius r where the temperature is T . Let ' k ' be the thermal conductivity of the material.

Then from Fourier's law

$$\begin{aligned}\frac{dQ}{dt} &= -k A \frac{dT}{dr} \\ &= -k (2\pi r \times l) \cdot \frac{dT}{dr}\end{aligned}$$

$$\dot{Q} = -kA \frac{dT}{dr}$$

$$\dot{Q} = \text{constant}$$

$$\Rightarrow \dot{Q} = -k 2\pi r h \cdot \frac{dT}{dr}$$

$$\Rightarrow \dot{Q} \int_{r_1}^{r_2} \frac{dr}{r} = -k 2\pi h \int_{T_1}^{T_2} dT$$

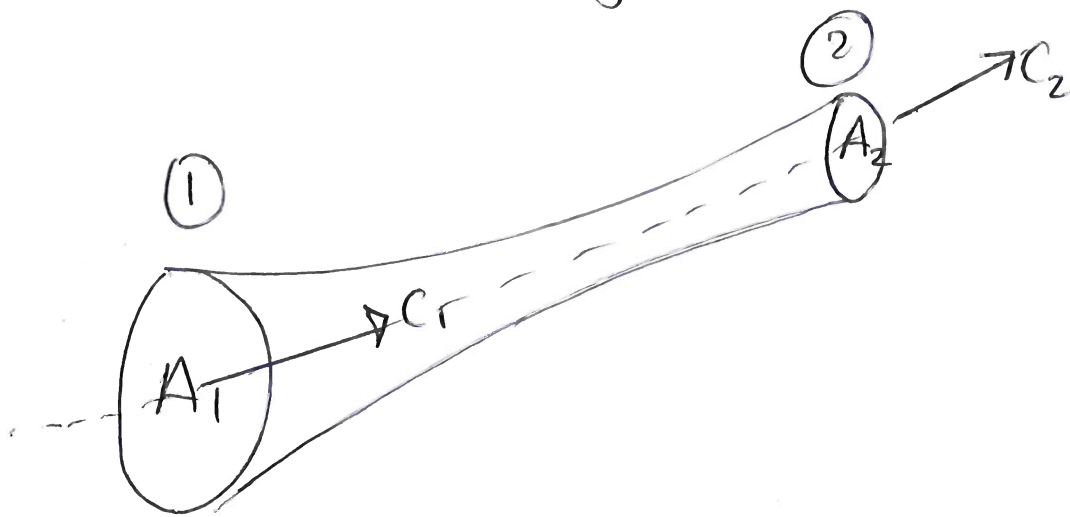
$$\dot{Q} [\ln r]_{r_1}^{r_2} = -k 2\pi h [T]_{T_1}^{T_2}$$

$$\dot{Q} (\ln r_2 - \ln r_1) = -k 2\pi h [T_2 - T_1]$$

$$\dot{Q} \ln \left(\frac{r_2}{r_1} \right) = 2\pi h k (T_1 - T_2)$$

$$\Rightarrow \dot{Q} = \frac{2\pi h k (T_1 - T_2)}{\ln(r_2/r_1)}$$

The Continuity Equation.



Consider an Incompressible fluid undergoing Steady flow i.e. ($\dot{m} = \text{constant}$) through a tube as shown above. The flow is assumed to be Stream-lined and not turbulent.

$$\text{Then as } \dot{m} = \rho A C$$

$$\text{Then at (1) } \dot{m} = \rho A_1 C_1$$

$$(2) \dot{m} = \rho A_2 C_2$$

then as $\dot{m}$ is a constant

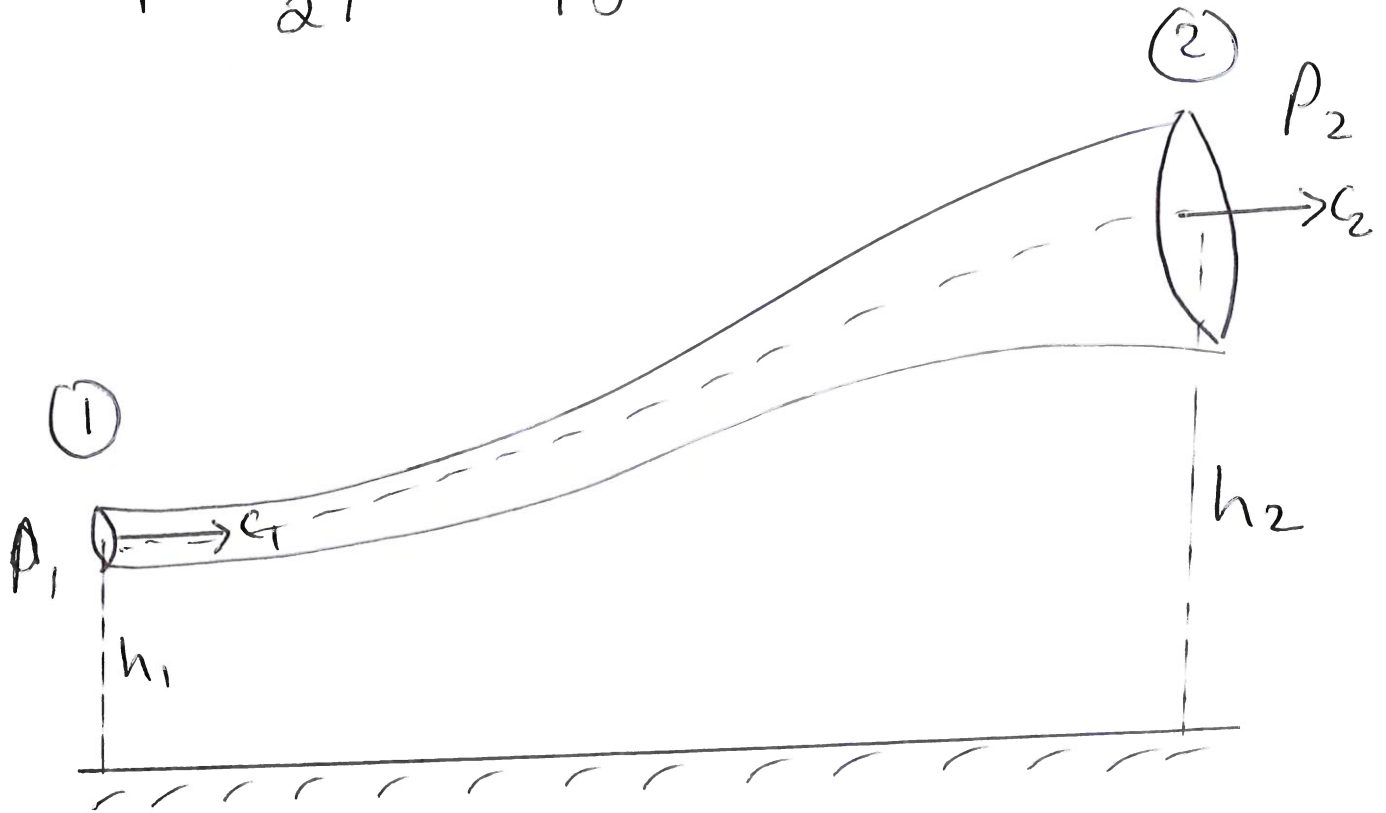
$$\rho A_1 C_1 = \rho A_2 C_2$$

$$\Rightarrow \underline{A_1 C_1 = A_2 C_2}$$

Bernoulli's Equation for an Ideal Fluid

For an Incompressible, non-viscous fluid undergoing steady flow then along a stream Line.

$$P + \frac{1}{2} \rho C^2 + \rho gh = \text{Constant}.$$



Then:-

$$P_1 + \frac{1}{2} \rho C_1^2 + \rho gh_1 = P_2 + \frac{1}{2} \rho C_2^2 + \rho gh_2$$

Bernoulli Equation Examples.

Bernoulli's Equation

$$P_1 + \frac{1}{2} \rho C_1^2 + h_1 \rho g = P_2 + \frac{1}{2} \rho C_2^2 + h_2 \rho g + H_f$$

$$H_f = h \rho g \quad (\text{Pressure difference due to head losses}).$$

Continuity Equation

$$A_1 C_1 = A_2 C_2$$

Mass flow Rate

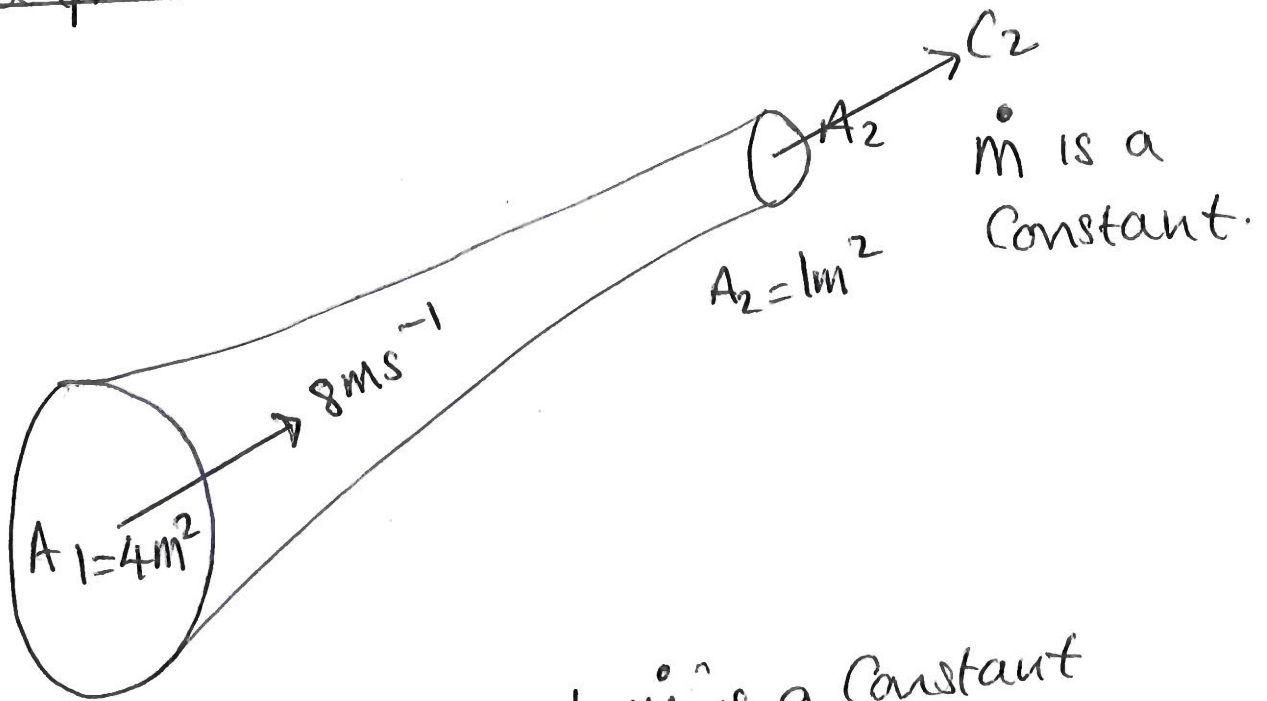
$$\dot{m} = \rho A C$$

$$\dot{m} = \frac{A C}{v} \quad (v - \text{specific volume})$$

Volume flow Rate

$$\dot{Q} = \frac{\dot{m}}{\rho}$$

Example.



as $\dot{m} = \rho AC$ and $\dot{m}$ is a Constant
then if fluid is Incompressible ($\rho = \text{Constant}$)

$$\Rightarrow \cancel{\rho} A_1 C_1 = \cancel{\rho} A_2 C_2$$

$$\Rightarrow A_1 C_1 = A_2 C_2$$

$$\Rightarrow 4 \times 8 = 1 \times C_2$$

$$\underline{32\text{ms}^{-1} = C_2}$$

Example.

$$P_1 = 2 \times 10^5 \text{ Pa}$$

$$A_1 = 6 \text{ m}^2$$

$$h_1 = 4 \text{ m}$$

$$C_1 = 4 \text{ ms}^{-1}$$

$$P_2 = ?$$

$$C_2 = ?$$

$$A_2 = 2$$

$\dot{m}$ is a Constant.

(ρ is Constant)

$$h_2 = 6 \text{ m} \quad \rho = 1000 \text{ kg m}^{-3}$$

Find C_2 and P_2

as $\dot{m}$ is a Constant

$$A_1 C_1 = A_2 C_2$$

$$\Rightarrow 6 \times 4 = 2 C_2$$

$$\underline{12 \text{ ms}^{-1} = C_2}$$

from Bernoulli's equation.

$$P_1 + \frac{1}{2} \rho C_1^2 + h_1 \rho g = P_2 + \frac{1}{2} \rho C_2^2 + h_2 \rho g$$

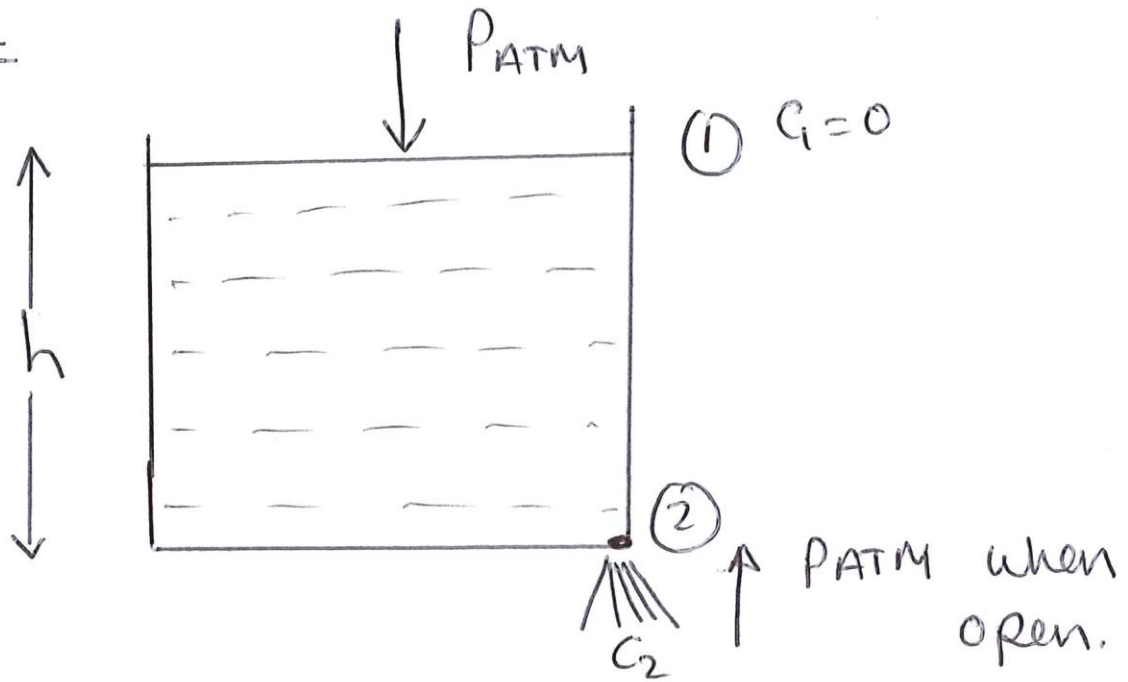
$$2 \times 10^5 + \frac{1}{2} (1000) 4^2 + 4(1000) 9.8 = P_2 + \frac{1}{2} (1000) 12^2 + 6(1000) 9.8$$

$$2 \times 10^5 + 500(16) + 4000(9.8) - 500(144) - 6000(9.8) = P_2$$

$$2 \times 10^5 + 8000 + 39200 - 72000 - 58800 = P_2$$

$$\underline{116,400 \text{ Pa} = P_2}$$

Example



$$P_1 + \frac{1}{2} \rho C_1^2 + h_1 \rho g = P_2 + \frac{1}{2} \rho C_2^2 + h_2 \rho g$$

here $P_1 = P_2 = P_{ATM}$

take $C_1 = 0$ fluid starts from rest

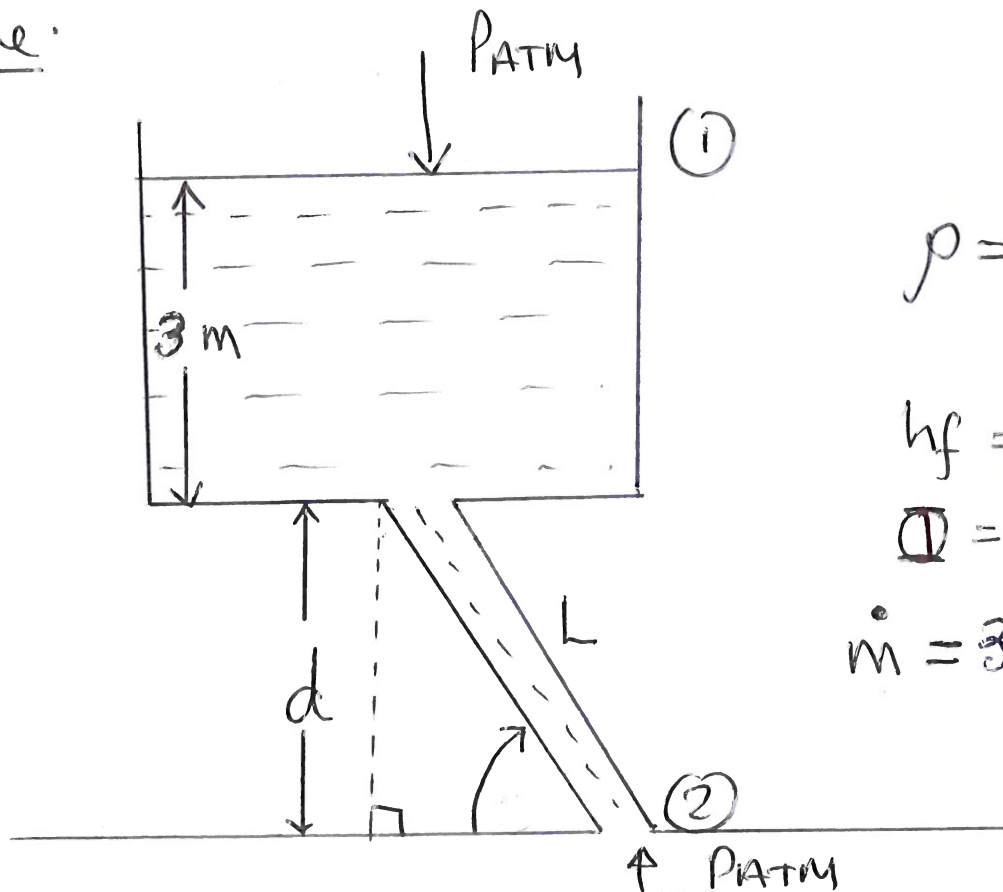
$$h_1 = h, \quad h_2 = 0$$

Then the equation becomes:-

$$h \cancel{\rho} g = \frac{1}{2} \cancel{\rho} C_2^2$$

$$\sqrt{2hg} = C_2$$

Example.



find i) fluid velocity at ②
 (ii) d (iii) L

from Bernoulli's equation

$$P_1 + \frac{1}{2} \rho C_1^2 + h_1 \rho g = P_2 + \frac{1}{2} \rho C_2^2 + h_2 \rho g + 2 \rho g$$

$$P_1 = P_2 = P_{ATM}$$

$$h_2 = 0, C_1 = 0$$

$$\Rightarrow h_1 \cancel{\rho} g = \frac{1}{2} \cancel{\rho} C_2^2 + 4 \cancel{\rho} g$$

$$(3+d)g = \frac{1}{2} C_2^2 + 4g$$

$$\text{Now } \dot{m} = \rho A C_2$$

$$\dot{m} = 30 \text{ tonnes/min}$$

$$= \frac{30 \times 10^3 \text{ kg}}{60 \text{ s}}$$

$$= \underline{0.5 \times 10^3 \text{ kg s}^{-1}}$$

$$\text{Then } 0.5 \times 10^3 = 1000 \times \pi (0.3)^2 \times C_2$$

$$\frac{0.5}{\pi (0.3)^2} = C_2$$

$$\underline{1.7684 \text{ m s}^{-1} = C_2} \quad (4 \text{ dp's})$$

Then in

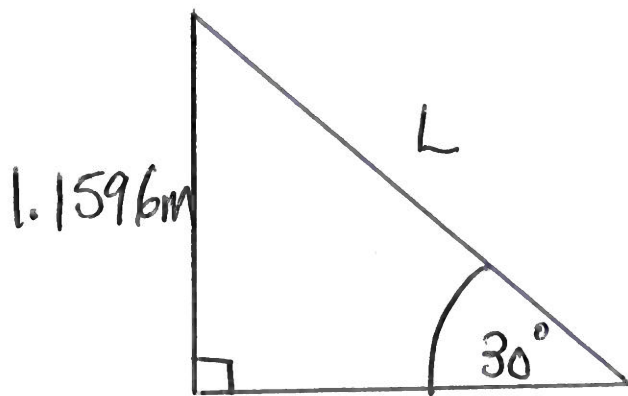
$$(3+d)g = \frac{1}{2} C_2^2 + 4g$$

$$3+d = \frac{1}{2g} C_2^2 + 4$$

$$3+d = \frac{1}{19.6} (1.7684)^2 + 4$$

$$d = \frac{1}{19.6} (1.7684)^2 + 4 - 3$$

$$\underline{d = 1.1596 \text{ m}} \quad (4 \text{ dp's})$$



$$\sin 30^\circ = \frac{1.1596}{L}$$

$$\Rightarrow L = \frac{1.1596}{\sin(30^\circ)}$$

$$L = 2.3192 \text{ m}$$

$$\underline{L = 2.32 \text{ m (2dp's)}}$$

$$\frac{750}{950(\pi)} = C_1$$

$$\underline{0.2513 \text{ ms}^{-1} = C_1 \text{ (4 dp's)}}$$

likewise at (2)

$$\dot{m} = \rho A C_2$$

$$750 = 950 \times \pi (0.5)^2 \times C_2$$

$$\frac{750}{950 \times \pi \times (0.5)^2} = C_2$$

$$\underline{1.0052 \text{ ms}^{-1} = C_2 \text{ (4 dp's)}}$$

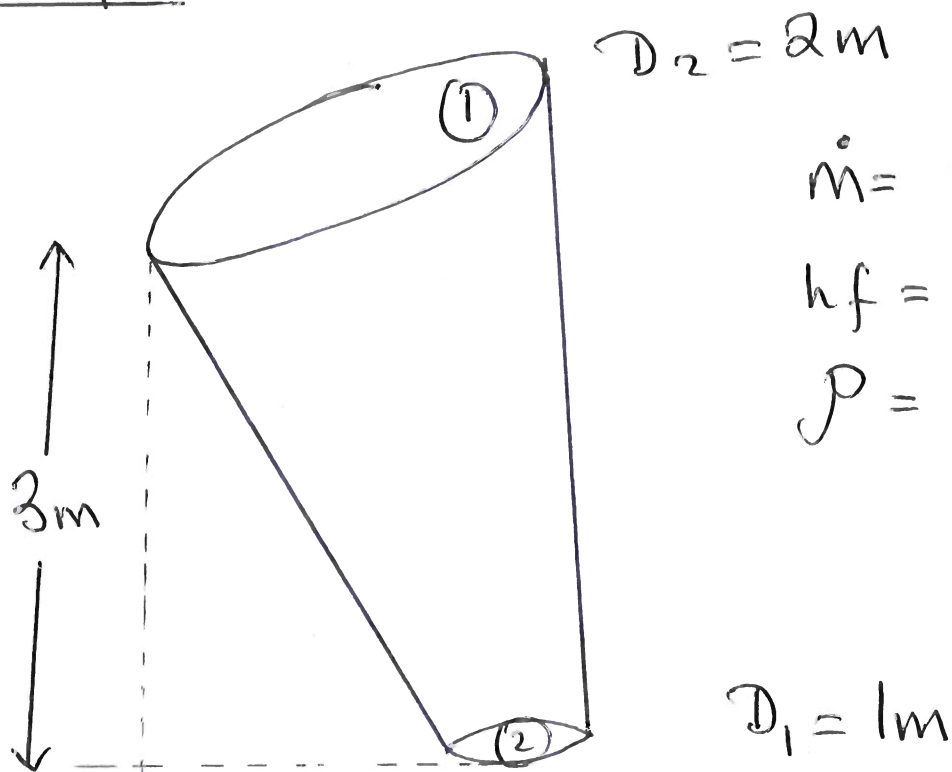
Now using Bernoulli's equation:-

$$P_1 + \frac{1}{2} \rho C_1^2 + h_1 \rho g = P_2 + \frac{1}{2} \rho C_2^2 + h_2 \rho g + h_f \rho g$$

$$\frac{1}{2} \rho C_1^2 + 3 \rho g - \frac{1}{2} \rho C_2^2 - h_f \rho g = P_2 - P_1$$

$$\frac{1}{2} \frac{\cancel{\rho} C_1^2}{\cancel{\rho g}} + 3 \frac{\cancel{\rho g}}{\cancel{\rho g}} - \frac{1}{2} \frac{\cancel{\rho} C_2^2}{\cancel{\rho g}} - \frac{h_f \cancel{\rho g}}{\cancel{\rho g}} = \frac{P_2 - P_1}{\rho g}$$

Example



$$\dot{m} = 45 \text{ tonnes/min}$$

$$h_f = 1.0\text{m}$$

$$\rho = 950 \text{ kg m}^{-3}$$

Find (i) C_1 and C_2 (ii) The Difference in Pressure head between (1) and (2)

$$\dot{m} = \frac{45 \times 10^3 \text{ kg}}{60 \text{ s}}$$

$$\dot{m} = 750 \text{ kg s}^{-1}$$

at (1) $\dot{m} = \rho A C_1$

$$750 = 950 \times \pi (1)^2 \times C_1$$

$$\frac{1}{2g} [C_1^2 - C_2^2] + 3 - h_f = \frac{P_2 - P_1}{\rho g}$$

$$\frac{1}{19.6} [(0.2513)^2 - (1.0052)^2] + 3 - 1 = \frac{P_2 - P_1}{\rho g}$$

$$\underline{\underline{1.9517 \text{ m} = \frac{P_2 - P_1}{\rho g} \quad (4 \text{ dp's})}}$$

The First Law of Thermo-dynamics Applied to Open Systems.

For a closed system the 1st law of thermo-dynamics is $\Delta Q = \Delta U + \Delta W$

ΔQ - heat Added/removed

ΔW - Work done by/on system

ΔU - change in Internal energy.

In a closed system no material can cross the system's boundary.

In an open system, like turbines, material can cross the boundary.

By applying the Law of Conservation of Energy to such systems the following equation can be established.

$$\dot{m} \left[u_1 + P_1 V_1 + \frac{1}{2} C_1^2 + g z_1 \right] + \dot{Q} + \dot{W}$$

$$= \dot{m} \left[u_2 + P_2 V_2 + \frac{1}{2} C_2^2 + g z_2 \right]$$

where $\dot{m}$ is mass flow rate

u_1, u_2 the fluid's internal energy at the start and end points.

PV is the work done for the fluid to enter and exit the system.

$$\frac{1}{2} \dot{m} C^2 = \text{Kinetic Energy / second}$$

$$\dot{m} g z = \text{Potential energy / second.}$$

$\dot{Q}$ = Rate of flow of heat energy either entering or leaving the system.

$\dot{W}$ = The rate of working / Power of the system (Turbine).

The quantity $u + P.V = h$ (enthalpy)

Enthalpy

The Quantity $H = U + P \cdot V$ is defined as the enthalpy of a Substance.

$H \equiv$ The enthalpy

$U \equiv$ The Internal energy

$P \equiv$ The Pressure

$V \equiv$ The Volume

Also many processes occur at a Constant Pressure

Then if $H = U + P \cdot V$

$$H + \Delta H = (U + \Delta U) + P(V + \Delta V)$$

Then

$$\cancel{H} + \Delta H - \cancel{H} = (U + \Delta U) + P(V + \Delta V) - P V - U$$

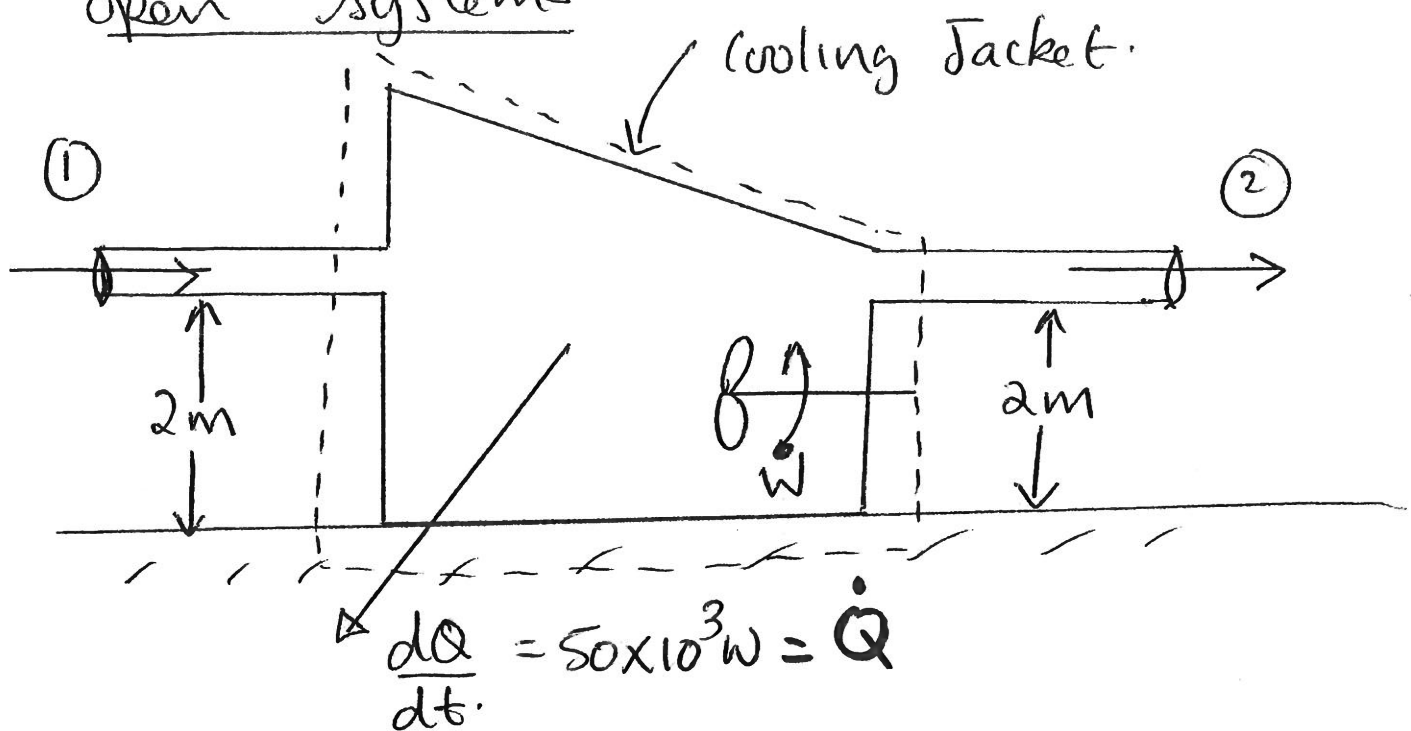
$$\Delta H = \cancel{U} + \Delta U - \cancel{U} + \cancel{P V} + P \Delta V - \cancel{P V}$$

$$\Rightarrow \Delta H = \Delta U + P \cdot \Delta V$$

Cf. $\Delta Q = \Delta U + \Delta W$ 1st Law of Thermodynamics

$$\Delta Q = \Delta H \quad \text{and} \quad \Delta W = P \Delta V.$$

First law of Thermo-dynamics applied to open systems



$$\dot{m} = 1 \text{ kg s}^{-1}$$

$$C_1 = 20 \text{ m s}^{-1}$$

$$P_1 = 1 \times 10^6 \text{ Pa}$$

$$V_1 = 0.8 \text{ m}^3 \text{ kg}^{-1}$$

$$C_2 = 10 \text{ m s}^{-1}$$

$$P_2 = 4 \times 10^6 \text{ Pa}$$

$$V_2 = 0.2 \text{ m}^3 \text{ kg}^{-1}$$

$$\Delta u = 80 \text{ kJ kg}^{-1}$$

(change in internal energy)

The cooling Jacket absorbs heat at a rate of 50 kW.

find (i) $\dot{W}$ (ii) Area of inlet pipe.

Solution:

using :-

$$\dot{m} \left[u_1 + P_1 v_1 + \frac{1}{2} C_1^2 + g z_1 \right] + \dot{W} + \dot{Q} = \dot{m} \left[u_2 + P_2 v_2 + \frac{1}{2} C_2^2 + g z_2 \right]$$

here as $\Delta u = 80 \text{ KJ kg}^{-1}$, let $u_1 = 0$, $u_2 = 80 \times 10^3 \text{ J kg}^{-1}$

Then :-

$$1 \left[(1 \times 10^6 \times 0.8) + \frac{1}{2} (20)^2 + 9.8(2) \right] + \dot{W} + (-50 \times 10^3)$$

$$= 1 \left[80 \times 10^3 + (4 \times 10^6 \times 0.2) + \frac{1}{2} (10)^2 + 9.8(2) \right]$$

$\Rightarrow$

$$800,000 + 200 + \dot{W} - 50,000 = 80,000 + 800,000 + 50$$

$$\dot{W} = 80,000 + 50 + 50,000 - 200$$

$$\dot{W} = \underline{129850 \text{ Watts.}}$$

(ii) Area of Inlet Pipe

$$\text{as } \dot{m} = \frac{A_1 C_1}{V_1}$$

$$1 = \frac{A_1 \times 20}{0.8}$$

$$0.8 = A_1 \times 20$$

$$\frac{0.8}{20} = A_1$$

$$\underline{0.04 \text{ m}^2 = A_1}$$

Non Dimensional Constants

Consider the equation $V = u + at$

where u - The initial velocity of an object

V - The final " " " "

a - The acceleration

t - The time taken

Then V has units ms^{-1}

u " " " ms^{-1}

$$a \cdot t \Rightarrow \text{ms}^{-2} \cdot \text{s} = \text{ms}^{-1}$$

So each Component of the equation has the same consistent units. Now consider what happens by dividing each term by 'u'.

$$\Rightarrow \frac{V}{u} = \frac{u}{u} + \frac{at}{u}$$

$$\frac{V}{u} \Rightarrow \frac{\text{ms}^{-1}}{\text{ms}^{-1}} = 1 \text{ ie no units}$$

$$\frac{u}{u} = \frac{\text{ms}^{-1}}{\text{ms}^{-1}} = 1 \text{ no units}$$

and $\frac{at}{u} = \frac{ms^{-2} \cdot s}{ms^{-1}} = 1$ no units

This is an example of what is known as non-dimensionalising an equation.

Consider The Reynold's number

$$Re = \frac{\rho U L}{\mu}$$

ρ - Density $\Rightarrow kg m^{-3}$

U - Velocity $\Rightarrow ms^{-1}$

L - length $\Rightarrow m$

μ - viscosity $\Rightarrow kg m^{-1} s^{-1}$

Then $Re \Rightarrow \frac{kg m^{-3} \cdot ms^{-1} \cdot m}{kg m^{-1} s^{-1}} = \frac{m}{m} = 1$

ie no units.

Example: The Prandtl Number

$$Pr = \frac{\mu C_p}{k}$$

μ - viscosity $\Rightarrow \text{kg m}^{-1} \text{s}^{-1}$

C_p - Specific heat capacity $\Rightarrow \text{J kg}^{-1} \text{K}^{-1}$

k - Thermal Conductivity $\Rightarrow \text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$

$$\begin{aligned} \text{Then } Pr &= \frac{\cancel{\text{kg m}^{-1} \text{s}^{-1}} \cdot \cancel{\text{J kg}^{-1} \text{K}^{-1}}}{\cancel{\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}}} \\ &= 1 \quad \text{no dimension/units.} \end{aligned}$$

Example: Nusselt Number

$$Nu = \frac{h \cdot L}{k}$$

h - Convective heat transfer coefficient

L - Significant length

k - Thermal Conductivity constant

$$Nu = \frac{h \cdot h}{k}$$

$$\Rightarrow \frac{\cancel{W} m^{-2} \cancel{k} \cdot m}{\cancel{W} m^{-1} \cancel{k}}$$

$$\Rightarrow \frac{m^{-1}}{m^{-1}}$$

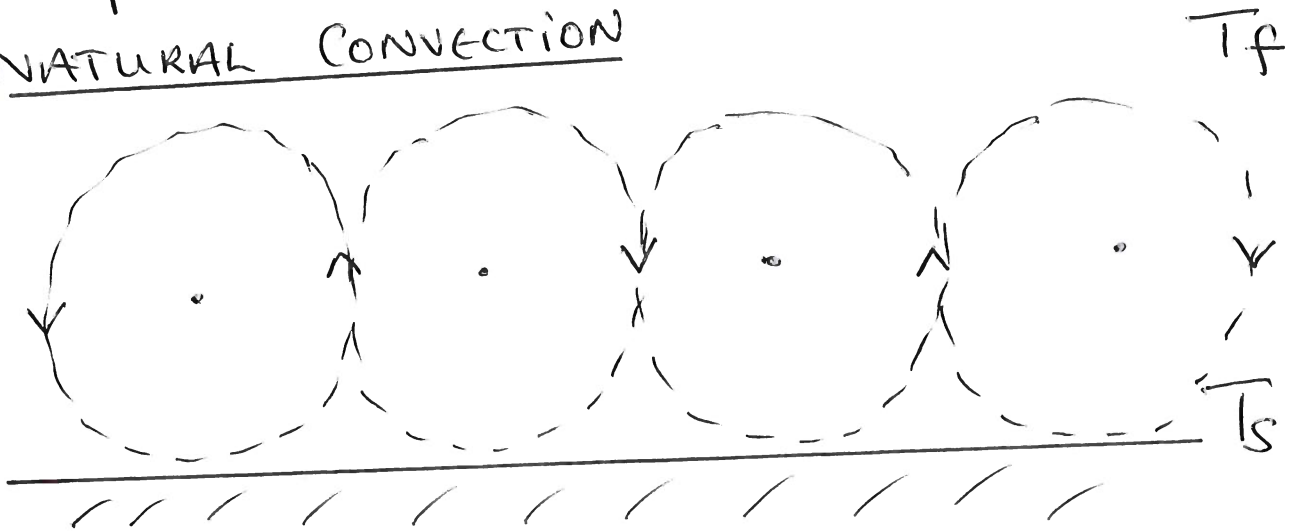
$$= 1 \quad \text{ie. dimensionless.}$$

The use is when an equation can be non dimensionalised only one variable
The Re , Pr ... etc needs to be altered
and not every constituent part of the
equation.

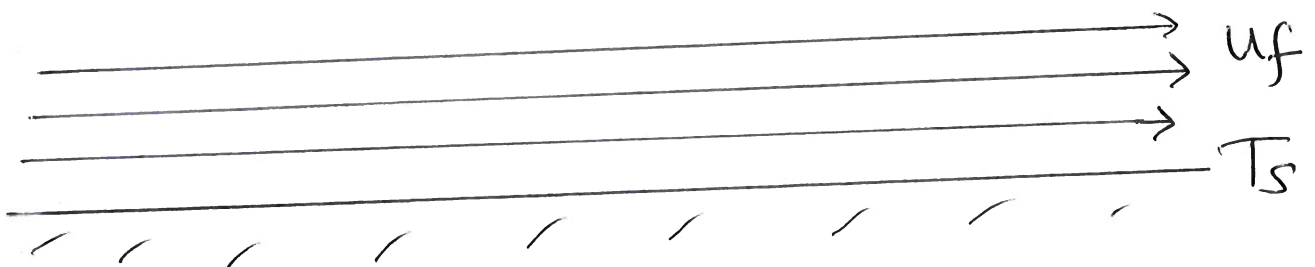
Heat Transfer by Convection

Convection is the process of heat transfer within a fluid (liquid/gas) by mixing one portion of fluid with another. The fluid's motion may be a result of differences in the fluid density caused by temperature differences between a surface and its surroundings. This is known as

NATURAL CONVECTION



The Convection process can also be achieved by Mechanical means, such as a fan heater, the fan blowing air across an electrical element. This process is known as Forced Convection.



The Heat Convection Equation.

For the Convection process it can be shown

that $\dot{Q} = \frac{dQ}{dt} = hA(T_s - T_f)$ Watts

where $\frac{dQ}{dt}$ is the rate of heat transfer (watts)

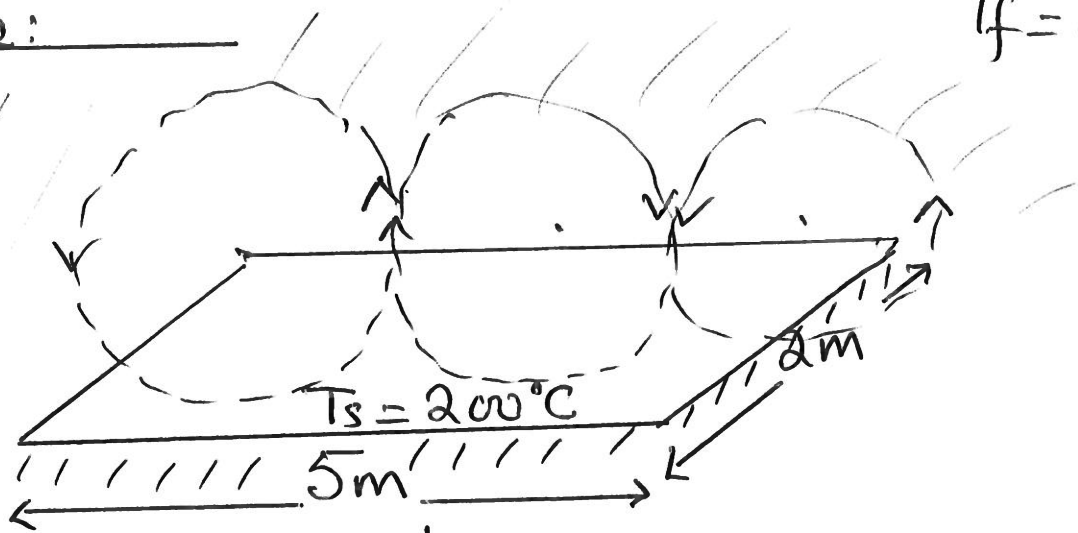
h - is the Convective heat transfer Coefficient
with units $Wm^{-2}K^{-1}$

A - The Surface Area with which the
fluid is in contact.

$T_s - T_f$ - The temperature difference
between the bulk fluid and the
temperature of the surface
(use K or $^{\circ}C$)

Example:

$$T_f = 20^\circ\text{C}$$



$$h = 40 \text{ W m}^{-2} \text{ K}^{-1}$$

$$\frac{dQ}{dt} = h A (T_s - T_f) \quad \left[\begin{array}{l} \text{use either K} \\ \text{or } ^\circ\text{C} \end{array} \right]$$

$$= 40 \times 5 \times 2 \times (473 - 293)$$

$$= 40 \times 10 \times 180$$

$$= \underline{72000 \text{ Watts}}$$

Forced Convection.

$$\dot{Q} = \frac{dQ}{dt} = hA(T_s - T_f) \quad \text{watts}$$

If the Convection process is Caused by a fan or a device moving the air across a surface then this is known as forced Convection.

In such cases it is important to calculate the type of flow which is undertaken; that is laminar (layers) or Turbulent (chaotic), to do this the Calculation of the Reynolds number is important.

$$Re = \frac{\rho v L}{\mu}$$

Then upon determining the flow and geometry a relation of the form

$$Nu = K_f (Re)^\alpha (Pr)^\beta \quad \text{is obtained}$$

along with $Pr = \frac{\mu C_p}{k}$ (Prandtl number)

The Nusselt number, $Nu = \frac{hL}{k}$ also

and thus enables a calculation of

'h' The thermal convective heat coefficient to be obtained as:-

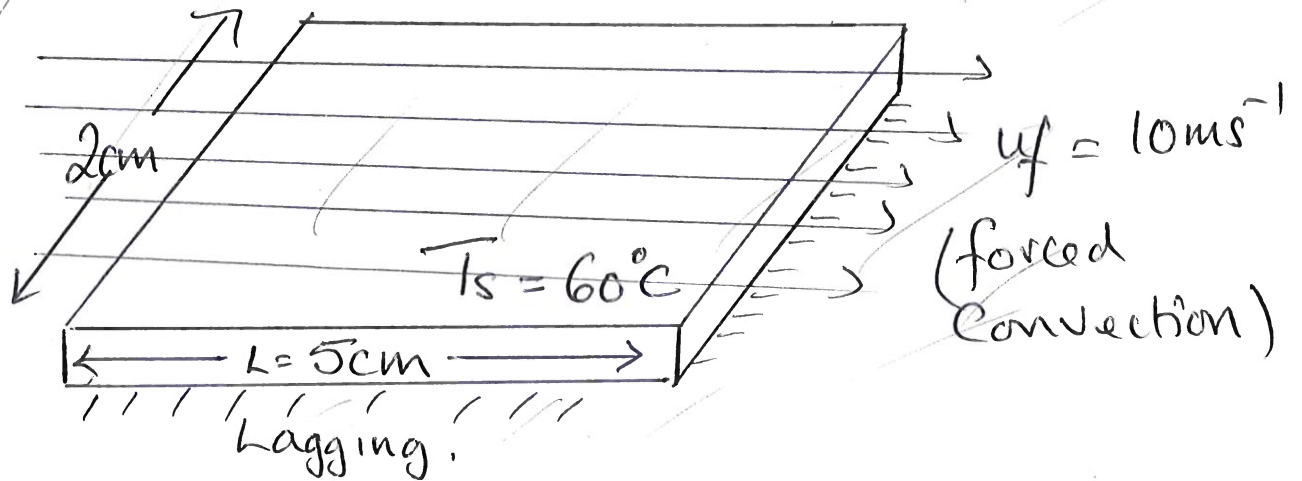
$$Nu = k_1 (Re)^\alpha (Pr)^\beta$$

$$\frac{hL}{k} = k_1 (Re)^\alpha (Pr)^\beta$$

$$\Rightarrow h = \frac{k k_1}{L} (Re)^\alpha (Pr)^\beta$$

Example:

$$T_f = 20^\circ\text{C}$$



$$\rho_{\text{air at } 40^\circ\text{C}} = 1.128 \text{ kg m}^{-3}$$

$$\mu_{\text{air}} = 0.019 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$$

$$k = 0.0272 \text{ W m}^{-1} \text{ K}^{-1}$$

$$C_p = 1.01 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$$

$$\text{then } Re = \frac{\rho U L}{\mu}$$

$$Re = \frac{1.128 \times 10 \times 5 \times 10^{-2}}{0.019 \times 10^{-3}}$$

$$Re = 29684.21053$$

Now for flow over a fixed hot plate
this is less than 50,000 in value so
the flow is laminar and so the
appropriate formula to use is

$$Nu = 0.664 (Re)^{0.5} (Pr)^{0.3}$$

where $Pr = \frac{\mu C_p}{k}$

$$= \frac{0.019 \times 10^{-3} \times 1.01 \times 10^3}{0.0272}$$

$$= \underline{0.7055} \quad (4dp's)$$

$$Nu = 0.664 (29684.21053)^{0.5} (0.7055)^{0.3}$$

$$= 103.0339 \quad 4dp's$$

$$\frac{hL}{k} = 103.0339$$

$$\Rightarrow h = \frac{k}{L} \times 103.0339$$

$$h = \frac{0.0272 \times 103.0339}{5 \times 10^{-2}} = 56.0504 \quad \text{Wm}^{-2}\text{K}^{-1}$$

$$\text{Then } \frac{dQ}{dt} = hA(T_s - T_f)$$

$$= 56.0504 \times 5 \times 10^{-2} \times 2 \times 10^{-2} \times (60 - 20)$$

$$= 56.0504 \times 10^{-3} \times 40$$

$$= \underline{2.2420 \text{ Watts.}}$$

Free or Natural Convection.

Heat transfer by Natural Convection is due to differences in fluid density resulting in natural Circulation patterns and consequently transferring heat energy.

The heat transfer depends upon the Coefficient of Cubical expansion, ' β ', which is given by the formula:-

$$\rho_1 = \rho_2 (1 + \beta \Delta T)$$

where ρ_1 , ρ_2 are the different fluid densities and ΔT the temperature difference between the fluid densities ρ_1 and ρ_2 .

The upthrust per unit volume of fluid is $(\rho_1 - \rho_2)g$ and the velocity of the Convection

Current depends on:

$$(\rho_1 - \rho_2)g = \rho_2 \beta \Delta T \cdot g$$

Furthermore the heat transfer also depends on the fluid's viscosity, the thermal Conductivity of the fluid and the significant length of the geometry Connected with the fluid.

A Suitable non-dimensional number governing thermal Convection incorporating these effects is the Grashof number.

$$Gr = \frac{g \beta \Delta T L^3}{\nu^2}$$

g - Acceleration due to gravity (ms^{-2})

β - Coefficient of Cubic expansion (K^{-1})

ΔT - The temperature difference between the Surface and fluid (K)

L - The Significant length (m)

ν - The kinematic viscosity (m^2s^{-1})

$$\Rightarrow Gr \Rightarrow \frac{\text{ms}^{-2} \cdot \text{K}^{-1} \cdot \text{K} \cdot \text{m}^3}{(\text{m}^2\text{s}^{-1})^2}$$

$$\Rightarrow \frac{\text{m}^4\text{s}^{-2}}{\text{m}^4\text{s}^{-2}}$$

$$\Rightarrow 1 \quad (\text{non dimensional})$$

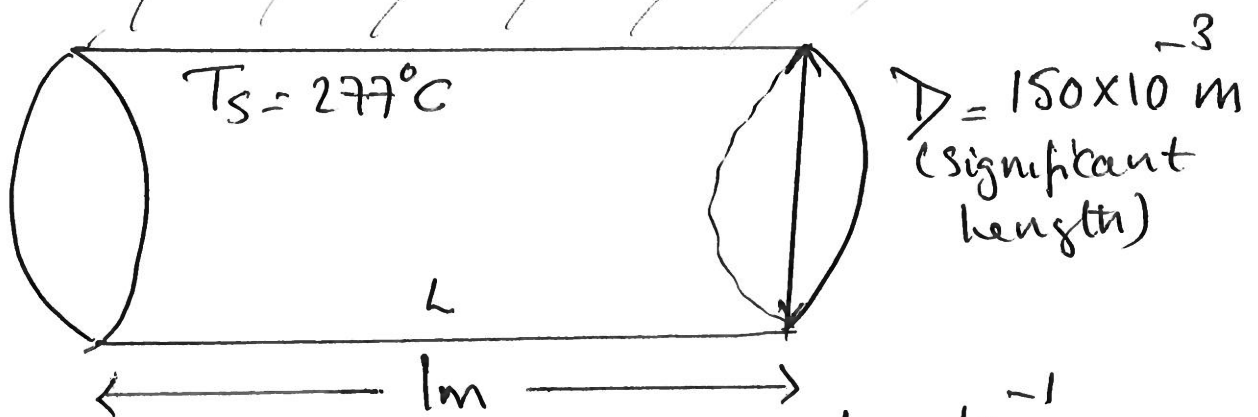
Example:

Calculate the rate of heat loss by Natural Convection per unit length, from a horizontal Pipe of diameter 150mm, the surface of which is 277°C .

Room temperature is 17°C .

($Pr = 0.686$ at 420K)

$$T_f = 17^{\circ}\text{C}$$



$$\beta = \frac{1}{T_f} = \frac{1}{273 + 17} = \frac{1}{290} \text{ K}^{-1}$$

$$\Delta T = (277 - 17) = 260$$

$$L = D = 150 \times 10^{-3}\text{m} \text{ (significant length)}$$

$$\nu = 2.822 \times 10^{-5} \text{ m}^2 \text{ s}^{-1}$$

$$k = 3.635 \times 10^{-5} \text{ W m}^{-1} \text{ K}^{-1}$$

$$\text{Then } Gr = \frac{g \beta \Delta T L^3}{\nu^2}$$

$$Gr = \frac{9.8 \times \frac{1}{290} \times 260 \times (150 \times 10^{-3})^3}{(2.822 \times 10^{-5})^2}$$

$$= 37235842.45$$

$$= 3.723584245 \times 10^7$$

$$< 10^9 \therefore \text{In Laminar Region}$$

$$\Rightarrow Nu = 0.555 (Gr \cdot Pr)^{0.25}$$

$$\text{Mean fluid temperature} = \frac{(273 + 277) + (273 + 17)}{2}$$

$$= \frac{550 + 290}{2}$$

$$= 420\text{K}$$

$$\text{Then from Tables } Pr = 0.686 \text{ (given here)}$$

(or Calculation)

$$\Rightarrow Nu = 0.555 (37235842.45 \times 0.686)^{0.25}$$

$$= \underline{39.4561}$$

Then:

$$Nu = \frac{hL}{k} = 39.4561$$

$$\Rightarrow h = \frac{k}{L} (39.4561)$$

$$= \frac{3.635 \times 10^{-5} \times 39.4561}{150 \times 10^{-3}}$$

$$\underline{h = 9.56153 \times 10^{-3} \text{ W m}^{-2} \text{ K}^{-1}}$$

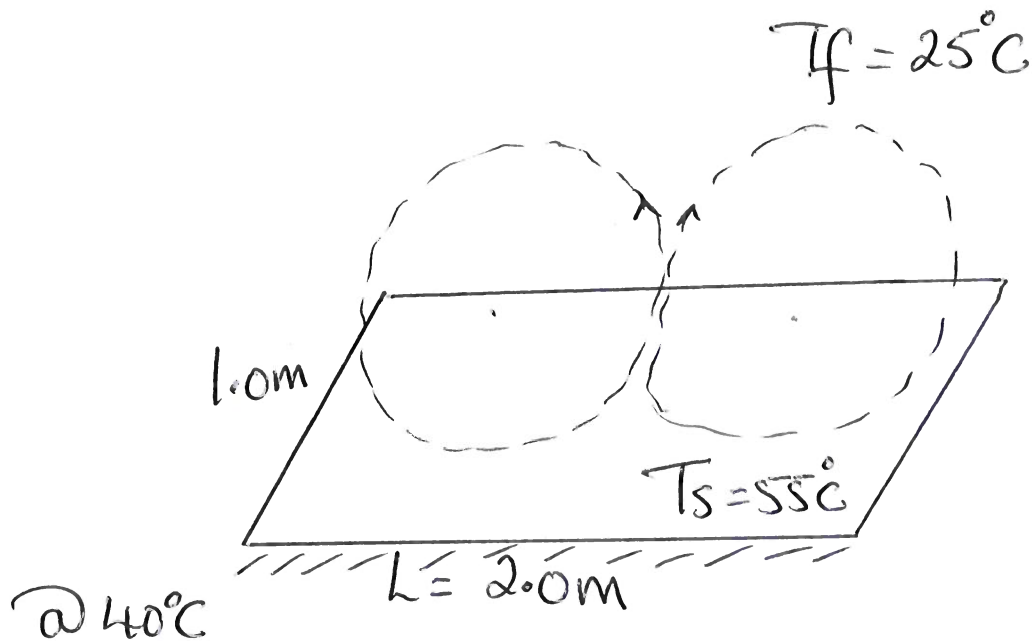
$$\text{Then } \dot{Q} = hA(T_s - T_f)$$

$$= 9.56153 \times 10^{-3} \times [2\pi \times (75 \times 10^{-3}) \times 1] \times [277 - 17]$$

$$= 9.56153 \times 10^{-3} \times 150 \times 10^{-3} \times \pi \times 260 \text{ W}$$

$$= \underline{1.1715 \text{ Watts}} \quad 4 \text{ dp's}$$

EXAMPLE: Free Convection



$$k = 0.0272 \text{ W m}^{-1} \text{ K}^{-1}$$

$$\mu = 0.019 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$$

$$C_p = 1.01 \text{ kJ kg}^{-1} \text{ K}^{-1}$$

$$\rho_{\text{air at } 40^\circ\text{C}} = 1.199 \text{ kg m}^{-3}$$

$$g = 9.8 \text{ m s}^{-2}$$

Solution: $\dot{Q} = hA(T_s - T_f)$

$$\dot{Q} = h(2 \times 1)(55 - 25)$$

$$\dot{Q} = h(2)(30)$$

$$\dot{Q} = 60h \quad \text{watts}$$

to Calculate h

$$Gr = \frac{g \beta \Delta T L^3}{(\mu/\rho)^2}$$

$$Gr = \frac{9.8 \times \frac{1}{(273+40)} (55^\circ - 25^\circ) \cdot (2)^3}{\left(\frac{0.019 \times 10^{-3}}{1.199} \right)^2}$$

$$Gr = \frac{9.8 \times 30 \times 8}{(313)(1.58465 \times 10^{-5})^2}$$

$$= \frac{2.9924 \times 10^{10}}{}$$

flow is in Turbulent Region Thus

$$Nu = 0.14 (Gr \cdot Pr)^{0.33}$$

$$Pr = \frac{\mu C_p}{k} = \frac{0.019 \times 10^{-3} \times 1.01 \times 10^3}{0.0272}$$

$$= \underline{0.7055}$$

$$\therefore Nu = 0.14 \times [2.9924 \times 10^{10} \times 0.7055]^{0.33}$$

$$= \underline{357.453 \text{ 3dp's}}$$

$$\text{Also } Nu = \frac{hL}{k}$$

$$\Rightarrow \frac{h \times 2}{0.0272} = 357.453$$

$$\Rightarrow h = \frac{357.453 \times 0.0272}{2}$$

$$\underline{h = 4.861 \text{ Wm}^{-2}\text{K}^{-1}}$$

Then as from previously

$$\dot{Q} = 60h$$

$$\dot{Q} = 60(4.861)$$

$$\underline{\dot{Q} = 291.68 \text{ Watts}}$$

Thermal Radiation

This is a form of electro-magnetic waves and hence needs no medium to transmit them.

An object which absorbs all radiant energy falling upon it is called a 'black body,' but this does not always imply that the object's colour is necessarily black.

The energy emitted from an object per unit area and per unit time is known as its radiant emittance.

Stefan - Boltzmann's Law

This states that the total energy radiated per unit time per unit surface area of a black body is proportional to the fourth power of the temperature of the body expressed in kelvin.

$$\dot{Q} = \underset{\text{Radiated}}{\text{Energy}} = \sigma T^4$$

Where σ is the Stefan-Boltzman Constant

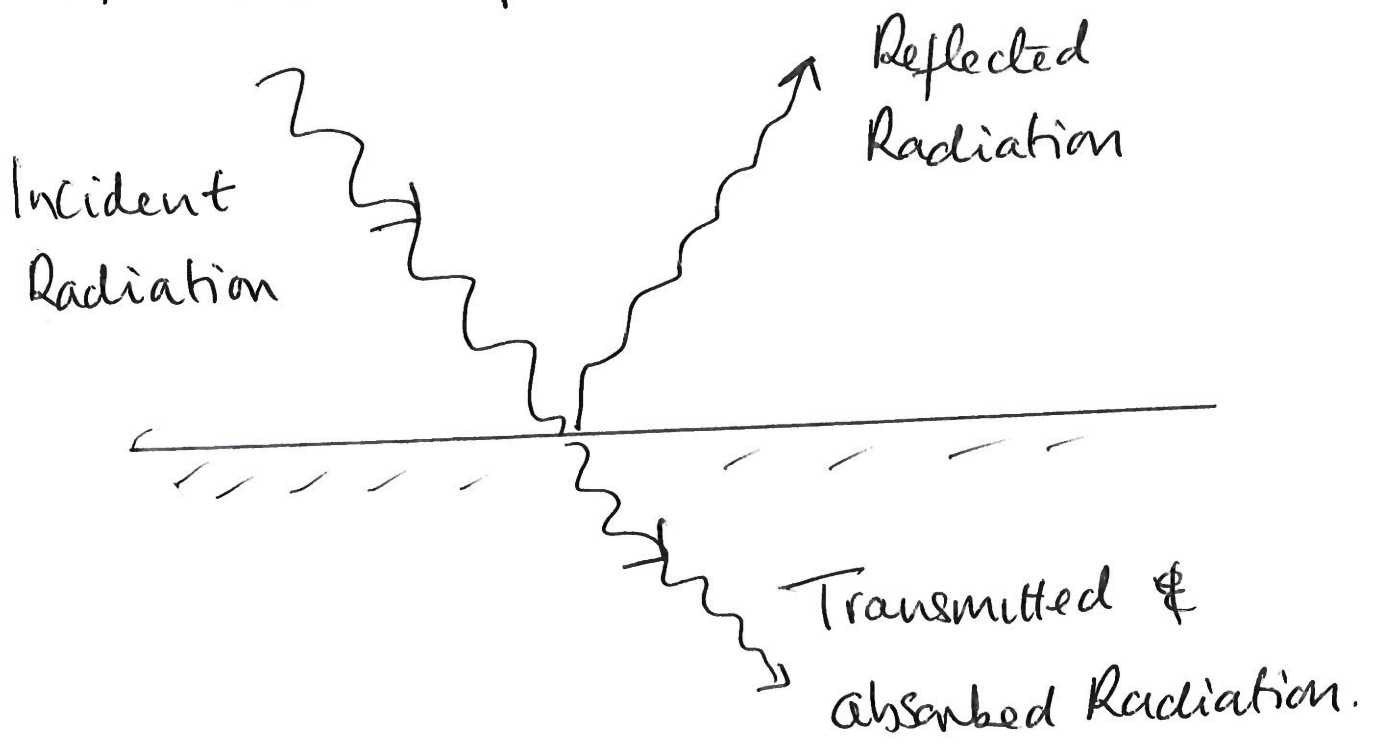
$$\sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{ K}^{-4}$$

For a non black body this law is amended as follows

$$\dot{Q} = \epsilon \sigma A T^4$$

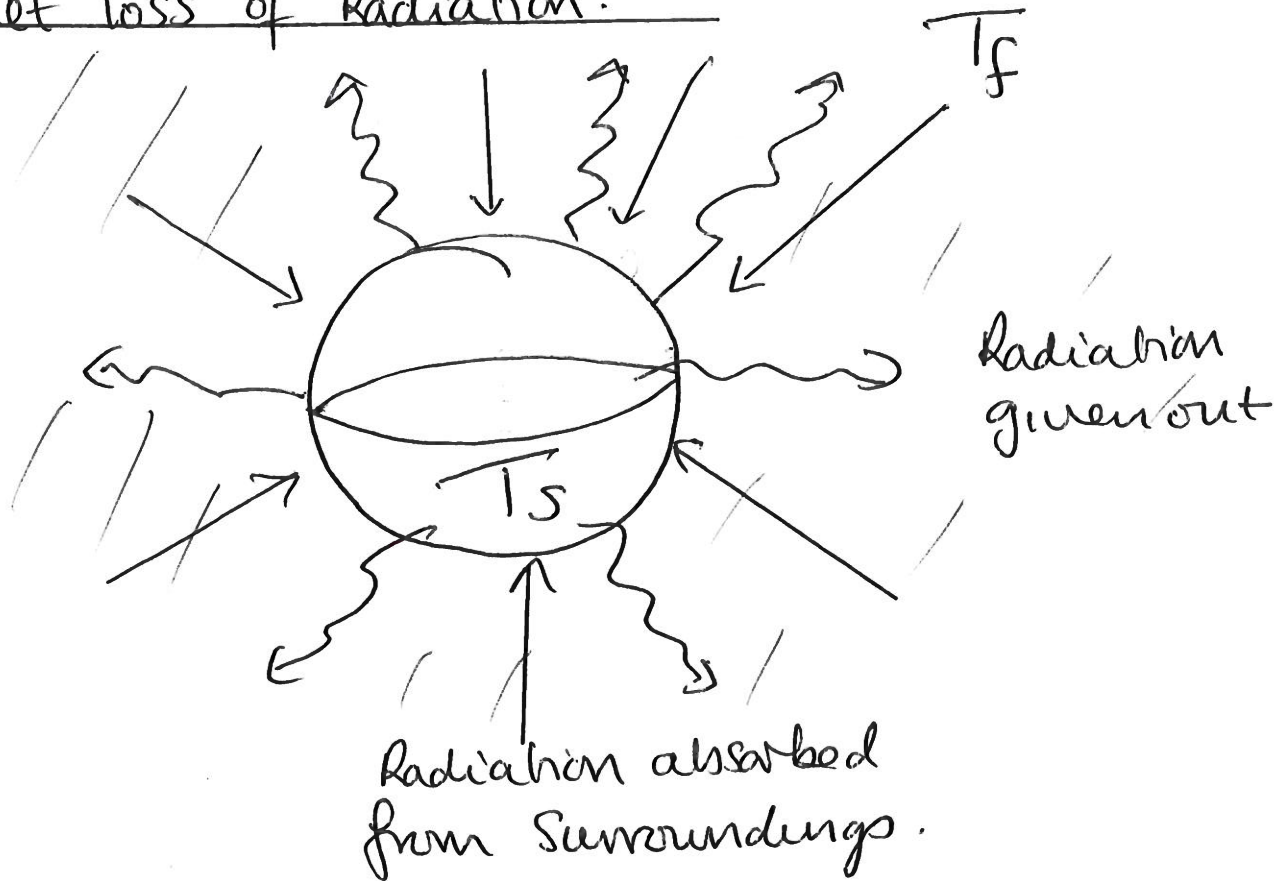
ϵ - A non dimensional Constant usually $0 \leq \epsilon \leq 1$ depending on the material Considered.

A - The Surface Area of the Object.



$$\text{Absorbivity} + \text{Reflectivity} + \text{Transmissivity} = 1,$$

Net loss of Radiation.



$$\text{Then } \dot{Q}_{\text{net}} = \epsilon \sigma A (T_s^4 - T_f^4)$$

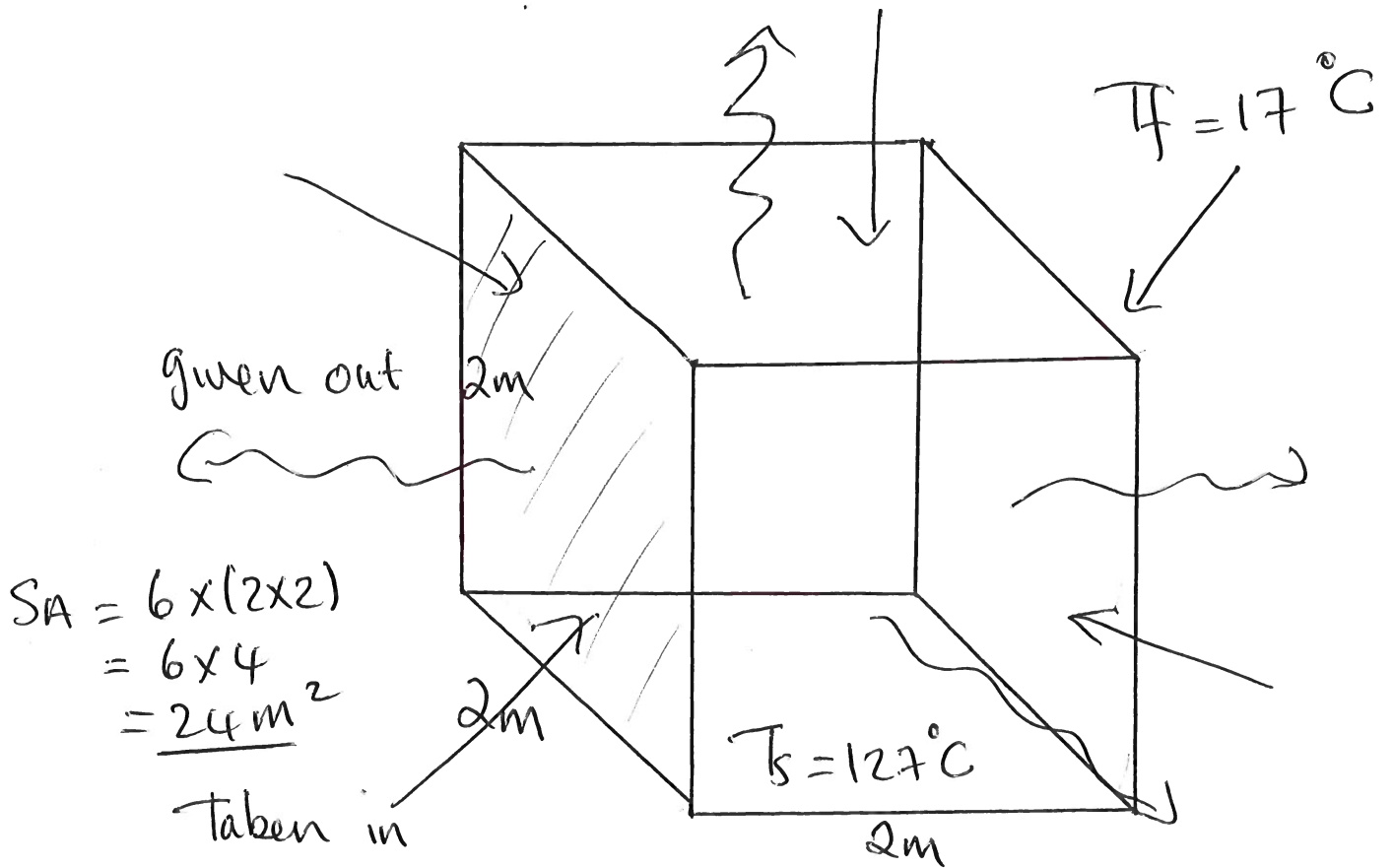
Examples:

Find the energy radiated from a black body of Surface area 2m^2 at a temperature of 27°C

$$\begin{aligned}\dot{Q} &= 1 \times 5.67 \times 10^{-8} \times (27 + 273)^4 \\ \dot{Q} &= 5.67 \times 10^{-8} \times 300^4 \\ &= \underline{459.27} \quad \text{watts.}\end{aligned}$$

Example:

Find the net loss of radiation from a Cube of side 2m at a temperature of 127°C to its surroundings maintained at a constant temperature of 17°C .
The emissivity of the Cube is 0.75



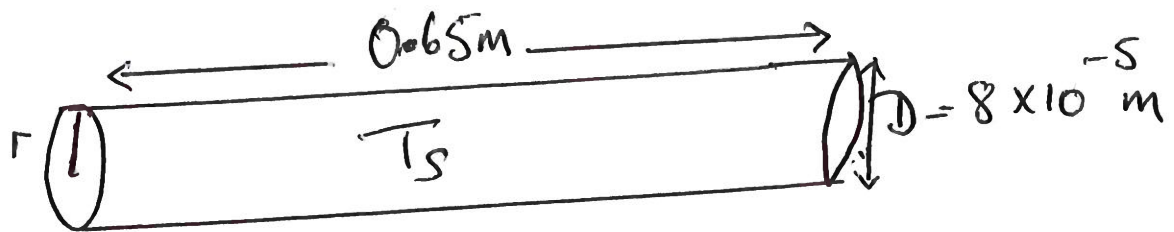
$$\begin{aligned} SA &= 6 \times (2 \times 2) \\ &= 6 \times 4 \\ &= \underline{24 \text{ m}^2} \end{aligned}$$

$$\dot{Q}_{\text{net}} = \epsilon \sigma A (T_s^4 - T_f^4)$$

$$= 0.75 \times 5.67 \times 10^{-8} \times 6 \times 4 \times (400^4 - 290^4)$$

$$= \underline{18.91 \text{ (2dp's) Watts.}}$$

Example: A 100 Watt electric light bulb has a filament which is 0.65m long and a diameter of 8.0×10^{-5} m. Estimate the working temperature of the filament if its total emissivity is 0.6



$$\dot{Q} = \epsilon \sigma A T_s^4$$

$$100 = 0.6 \times 5.67 \times 10^{-8} \times \pi (r^2) \cdot l T_s^4$$

$$100 = 0.6 \times 5.67 \times 10^{-8} \times \pi (4 \times 10^{-5})^2 \times 0.65 \times T_s^4$$

$$100 = 1.1115 \times 10^{-16} T_s^4$$

$$\sqrt[4]{\frac{100}{1.1115 \times 10^{-16}}} = T_s$$

$$\underline{30798.008 \text{ K} = T_s}$$

Example:

The rate at which energy from the Sun 'normally' falls upon the Earth's surface is 1.4 kW/m^2 .

If the radius of the Sun is $7.25 \times 10^5 \text{ km}$ and its distance from the earth is $1.48 \times 10^8 \text{ km}$ estimate the surface temperature of the Sun.

Let the surface temperature of the Sun be T_s
Then treating the Sun and Earth to be black bodies then:-

$$\text{Radiant emittance from the Sun} = \sigma A T_s^4$$

Treating both Earth and Sun to be Perfect Spheres
 $SA = 4\pi r^2$ Then for the Sun

$$\text{Radiant emittance} = 5.67 \times 10^{-8} \times 4\pi \times (7.25 \times 10^5)^2 \times T_s^4$$

Now Consider the Earth to be represented by a small surface on an Imaginary Sphere of radius 14.8×10^7 km, with the Sun at the Centre of this Imaginary Sphere.

Then The Irradiance on Earth's Surface $= \frac{5.67 \times 10^{-8} \times 4\pi \times (7.25 \times 10^8)^2 \times T_s^4}{4\pi \times (14.8 \times 10^{10})^2} \text{ kwm}^{-2}$

and this is equal to 1.4 kwm^{-2}

Thus:

$$1.4 = \frac{5.67 \times 10^{-8} \times 4\pi \times (7.25 \times 10^8)^2 \times T_s^4}{4\pi \times (14.8 \times 10^{10})^2}$$

which upon Rearranging gives.

$$\sqrt[4]{\frac{1.4 \times (14.8 \times 10^{10})^2}{5.67 \times 10^{-8} \times (7.25 \times 10^8)^2}} = T_s$$

$$\underline{5663 \text{ K}} = T_s$$