

QUADRATIC EQUATIONS

Question	Answer
1. Solve using Factorisation $x^2 + 5x + 6 = 0$	Two numbers which multiply to 6 and add to 5. These are 2 and 3. $(x + 2)(x + 3) = 0$ $x = -2$ or $x = -3$
2. Solve using Factorisation $x^2 - x - 12 = 0$	Two numbers which multiply to -12 and add to -1. These are -4 and 3. $(x - 4)(x + 3) = 0$ $x = 4$ or $x = -3$
3. Solve using Quadratic Formula $x^2 + 3x - 4 = 0$	$a = 1, b = 3, c = -4$ $x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -4}}{2 \times 1}$ $x = \frac{-3 \pm \sqrt{9 + 16}}{2}$ $x = \frac{-3 \pm \sqrt{25}}{2}$ $x = \frac{-3 \pm 5}{2}$ $x = \frac{-3 + 5}{2} = \frac{2}{2} = 1$ $x = \frac{-3 - 5}{2} = \frac{-8}{2} = -4$ $x = 1$ or $x = -4$
4. Solve using Quadratic Formula $2x^2 + 5x - 3 = 0$	$a = 2, b = 5, c = -3$ $x = \frac{-5 \pm \sqrt{5^2 - 4 \times 2 \times -3}}{2 \times 2}$ $x = \frac{-5 \pm \sqrt{25 + 24}}{4}$ $x = \frac{-5 \pm \sqrt{49}}{4}$ $x = \frac{-5 \pm 7}{4}$ $x = \frac{-5 + 7}{4} = \frac{2}{4} = \frac{1}{2}$ $x = \frac{-5 - 7}{4} = \frac{-12}{4} = -3$ $x = \frac{1}{2}$ or $x = -3$
5. Solve using Quadratic Formula $3x^2 - 7x + 2 = 0$	$a = 3, b = -7, c = 2$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \times 3 \times 2}}{2 \times 3}$

	$x = \frac{7 \pm \sqrt{49 - 24}}{6}$ $x = \frac{7 \pm \sqrt{25}}{6}$ $x = \frac{7 \pm 5}{6}$ $x = \frac{7+5}{6} = \frac{12}{6} = 2$ $x = \frac{7-5}{6} = \frac{2}{6} = \frac{1}{3}$ $x = 2 \text{ or } x = \frac{1}{3}$
6. Solve using Quadratic Formula $4x^2 - 4x - 3 = 0$	$a = 4, b = -4, c = -3$ $x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 4 \times -3}}{2 \times 4}$ $x = \frac{4 \pm \sqrt{16 + 48}}{8}$ $x = \frac{4 \pm \sqrt{64}}{8}$ $x = \frac{4 \pm 8}{8}$ $x = \frac{4+8}{8} = \frac{12}{8} = \frac{3}{2}$ $x = \frac{4-8}{8} = \frac{-4}{8} = -\frac{1}{2}$ $x = \frac{3}{2} \text{ or } x = -\frac{1}{2}$
7. Solve using Quadratic Formula $-2x^2 + 3x + 2 = 0$	$a = -2, b = 3, c = 2$ $x = \frac{-3 \pm \sqrt{3^2 - 4 \times -2 \times 2}}{2 \times -2}$ $x = \frac{-3 \pm \sqrt{9 + 16}}{-4}$ $x = \frac{-3 \pm \sqrt{25}}{-4}$ $x = \frac{-3 \pm 5}{-4}$ $x = \frac{-3+5}{-4} = \frac{2}{-4} = -\frac{1}{2}$ $x = \frac{-3-5}{-4} = \frac{-8}{-4} = 2$ $x = -\frac{1}{2} \text{ or } x = 2$
8. A civil engineer is designing a parabolic arch for a small pedestrian bridge. The height of the arch above the ground is modelled by the equation: $h = -0.5x^2 + 3x + 2$ where: h is the height of the arch in metres. x is the horizontal distance in metres from a reference point on the ground. The engineer needs to determine where the arch meets ground level to establish the span of the arch.	
At ground level, $h = 0$	

$$-0.5x^2 + 3x + 2 = 0$$

$$a = -0.5, b = 3, c = 2$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times -0.5 \times 2}}{2 \times -0.5}$$

$$x = \frac{-3 \pm \sqrt{9+4}}{-1}$$

$$x = \frac{-3 \pm \sqrt{13}}{-1}$$

$$x = \frac{-3 + \sqrt{13}}{-1} = \frac{-3 + 3.606}{-1} = -0.61m$$

$$x = \frac{-3 - \sqrt{13}}{-1} = \frac{-3 - 3.606}{-1} = 6.61m$$

$x = -0.61m$ or $x = 6.61m$ (These are the locations where the arch meets the ground relative to the chosen reference point).

$$\text{Span} = 6.61 - (-0.61) = 7.22m$$

9. A civil engineer is designing a rectangular concrete foundation for a small structure. The length of the foundation is 3 metres greater than its width. The foundation must have an area of $40m^2$. Calculate the width and length of the foundation.

Let the width of the foundation be x metres

Therefore, the length is $(x + 3)$ metres

For a Rectangle:

Area = Length x Width

$$x(x + 3) = 40$$

$$x^2 + 3x = 40$$

$$x^2 + 3x - 40 = 0$$

$$a = 1, b = 3, c = -40$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -40}}{2 \times 1}$$

$$x = \frac{-3 \pm \sqrt{9+160}}{2}$$

$$x = \frac{-3 \pm \sqrt{169}}{2}$$

$$x = \frac{-3 \pm 13}{2}$$

$$x = \frac{-3 + 13}{2} = \frac{10}{2} = 5m$$

$$x = \frac{-3 - 13}{2} = \frac{-16}{2} = -8m \text{ A width cannot be negative so we reject this answer.}$$

The width is $5m$

$$\text{Length} = x + 3 = 5 + 3 = 8m$$

10. A civil engineer is investigating the trajectory of a water jet discharged from a pipe during a hydraulic experiment. The height of the water jet above a reference level is modelled by the equation:

$$h = -0.5x^2 + 4x + 1$$

where h is the height in metres and x is the horizontal distance in metres.

Tasks:

Calculate the horizontal positions where the water jet reaches a height of $5m$.

Explain why there are two possible solutions.

Substitute the known height, $h = 5$

$$5 = -0.5x^2 + 4x + 1$$

$$0 = -0.5x^2 + 4x - 4$$

$$a = -0.5, b = 4, c = -4$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4 \times -0.5 \times -4}}{2 \times -0.5}$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{-1}$$

$$x = \frac{-4 \pm \sqrt{8}}{-1}$$

$$x = \frac{-4 + \sqrt{8}}{-1} = \frac{-4 + 2.82}{-1} = 1.18m$$

$$x = \frac{-4 - \sqrt{8}}{-1} = \frac{-4 - 2.82}{-1} = 6.82m$$

The water jet reaches a height of 5m at approximately 1.18m and 6.82m horizontally from the reference point.

There are two solutions because the water jet passes through the same height twice: once as it rises and again as it falls.

11. A civil engineering contractor is installing a rectangular paved area measuring 8m long and 5m wide. A uniform-width border is added around the outside of the paved area.

The total area, including the border, is $70m^2$

Calculate the width of the border.

Let the border width be x metres

Since the border runs around all four sides

$$\text{Total Length} = 8 + 2x$$

$$\text{Total Width} = 5 + 2x$$

For a Rectangle:

$$\text{Area} = \text{Length} \times \text{Width}$$

$$(8 + 2x)(5 + 2x) = 70$$

$$40 + 16x + 10x + 4x^2 = 70$$

$$4x^2 + 26x + 40 = 70$$

$$4x^2 + 26x - 30 = 0$$

$$a = 4, b = 26, c = -30$$

$$x = \frac{-26 \pm \sqrt{26^2 - 4 \times 4 \times -30}}{2 \times 4}$$

$$x = \frac{-26 \pm \sqrt{676 + 480}}{8}$$

$$x = \frac{-26 \pm \sqrt{1156}}{8}$$

$$x = \frac{-26 + \sqrt{1156}}{8} = \frac{-26 + 34}{8} = \frac{8}{8} = 1m$$

$$x = \frac{-26 - \sqrt{1156}}{8} = \frac{-26 - 34}{8} = \frac{-60}{8} = -7.5m$$

A width cannot be negative so we reject this answer.