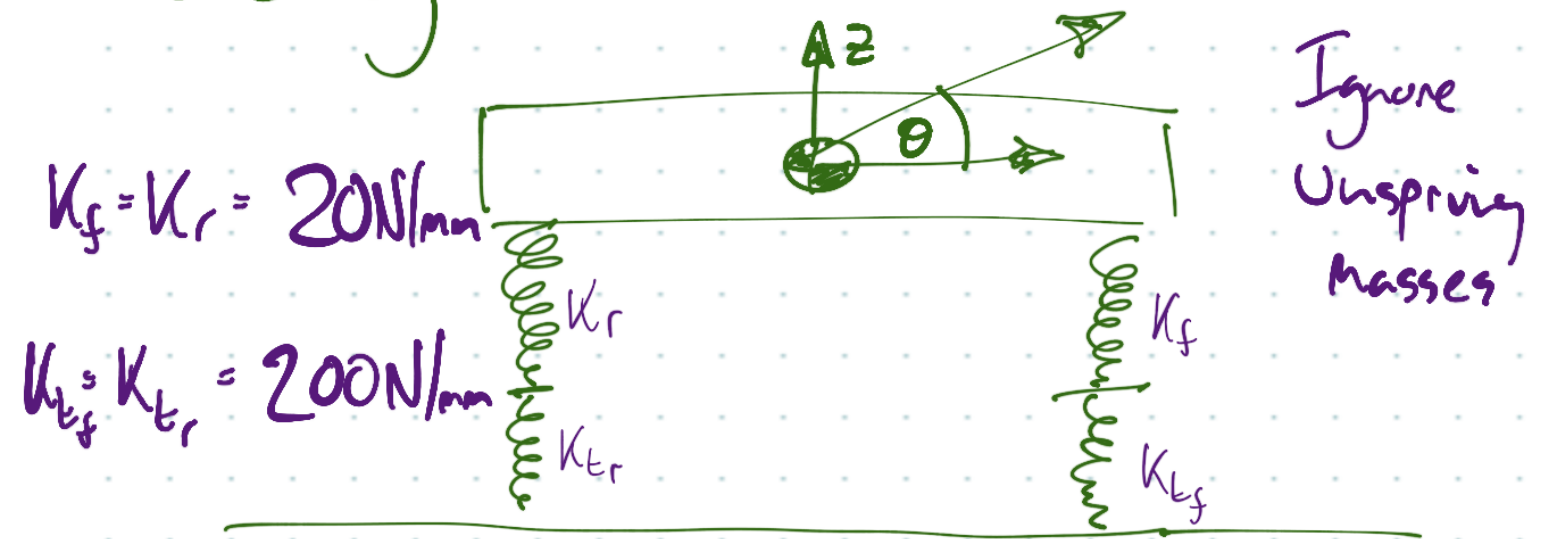


Advanced Suspension Dynamics

One degree of freedom



Pitch and Bounce are
'uncoupled'...

We can combine the spring rates as 'Springs in series'

$$K_f = \frac{K_f K_{t_f}}{K_f + K_{t_f}}$$

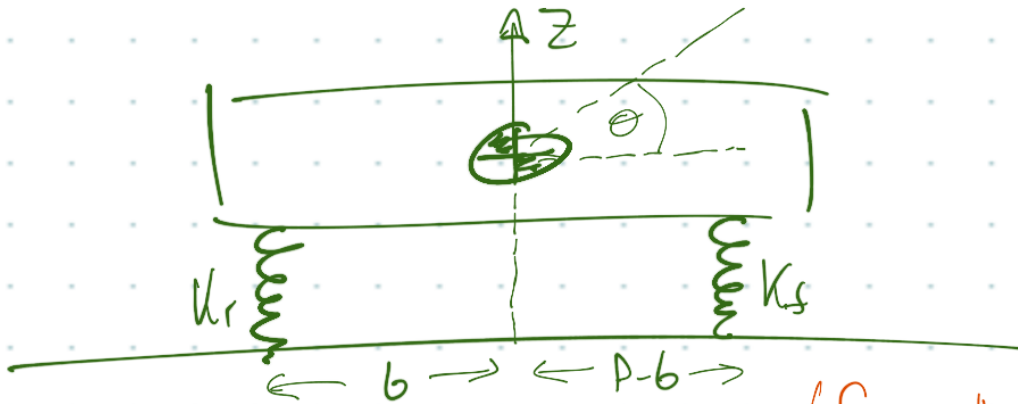
$$K_r = \frac{K_r K_{t_r}}{K_r + K_{t_r}}$$

$$K_f = K_r = \frac{20 \times 200}{220} = 18.18 \text{ N/mm}$$

These come from ---

$$\frac{1}{K} = \frac{1}{K_1} + \frac{1}{K_2} + \dots$$

So we have



Equilibrium of forces :

(Force with which m is decelerating)
= Sum of Kx

$$-M\ddot{z} = K_r z + K_f z$$

$$\text{So } M\ddot{z} + K_r z + K_f z = 0$$

$$M\ddot{z} + (K_r + K_f) z = 0$$

$$M \frac{d^2 z}{dt^2} + (K_r + K_f) z = 0$$

So what about pitch:

We can take moments (we need to think rotationally)

So force gets replaced by torque.

So $F = ma$ becomes $\tau = I\alpha$

So just as we have

$$\boxed{V = r\omega}$$

x	x	$-$	θ	θ
$\dot{x}$	v	$-$	ω	$\dot{\theta}$
$\ddot{x}$	a	$-$	α	$\ddot{\theta}$

So torque = $I\ddot{\theta}$

Now stiffness?

$$\text{force} = kx$$

$$\text{Since } x = r\theta$$

$$\text{force} = kr\theta$$

$$\text{we need torque} = f \times d = kr\theta \times r$$

$$\text{So torque} = kr^2\theta$$

So we need a differential equation:

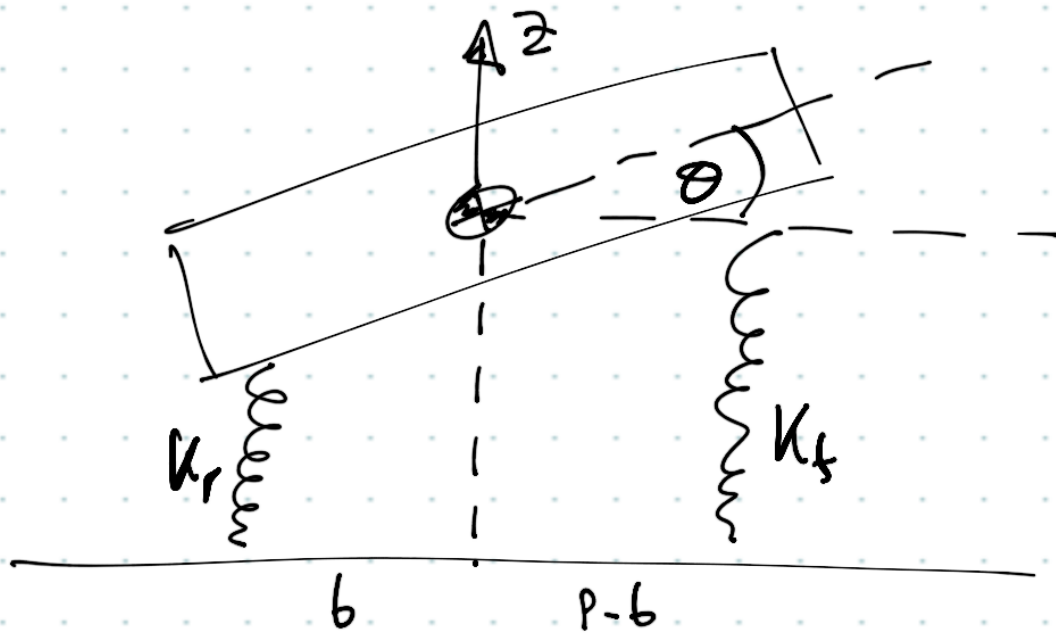
$$I\ddot{\theta} + K_r b^2 \theta + K_f (P-b)^2 \theta = 0$$

$$I\ddot{\theta} + [K_r b^2 + K_f (P-b)^2] \theta = 0$$

This look the same as :

$$M\ddot{z} + (K_f + K_r)z = 0$$





If we consider the situation being 'Coupled'
So pitch and bounce are linked

$$M\ddot{z} + (k_r + k_f)z + k_f(P-b)\theta - k_rb\theta = 0$$

and we get :

$$I\ddot{\theta} + k_f(P-b)z - k_rbz + k_f(P-b)^2\theta + k_rb^2\theta = 0$$

So both equations contain θ and z !

We need to solve these ... together

$$Ma + Kx = 0$$

Diagram of a mechanical system. A horizontal beam of mass M and moment of inertia I is pivoted at its center. A mass m is attached to the beam at a distance l from the pivot. The beam is supported by two springs: a vertical spring with stiffness k_r and a horizontal spring with stiffness k_{ts} . The vertical displacement of the beam is z_r and the horizontal displacement is z_s . The angle of rotation is θ .

$$(3) \quad M_f \ddot{z}_f - K_f z - (p-b)K_f \theta + (K_f + K_{t_f})z_f = 0$$

$$④ \quad M_r \ddot{z}_r - K_r z + b K_r \theta + (K_r + K_{tr}) z_r = 0$$

$$\begin{bmatrix} M & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & m_f & 0 \\ 0 & 0 & 0 & m_r \end{bmatrix} \begin{bmatrix} \ddot{z} \\ \ddot{\theta} \\ \ddot{z}_f \\ \ddot{z}_r \end{bmatrix} + \begin{bmatrix} K_f + K_r & (P-b)K_f - bK_r & -K_f & -K_r \\ (P-b)K_f - bK_r & (P-b)^2 K_f + b^2 K_r & -(P-b)K_f & bK_r \\ -K_f & -(P-b)K_f & K_f + K_{tr} & 0 \\ -K_r & bK_r & 0 & K_r + K_{tr} \end{bmatrix} \begin{bmatrix} z \\ \theta \\ z_f \\ z_r \end{bmatrix} = 0$$

$M \quad a + K \quad x = 0$

fab! What about damping?

$$Ma + Cv + Kx = 0$$

$$[M][a] + [C][v] + [K][x] = 0$$



Hello