

1) Write the S-matrix for a two port network, with two ports are perfectly matched.

Soln ::

$$[S] = \begin{vmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{vmatrix}$$

As the ports are perfectly matched.

there is no reflection at Port 1 and Port 2.

$$S_{01}, S_{11} = S_{22} = 0$$

$$[S] = \begin{vmatrix} 0 & S_{12} \\ S_{21} & 0 \end{vmatrix}$$

2) A two port network has the following scattering matrix.

$$[S] = \begin{bmatrix} 0.15 \angle 0^\circ & 0.85 \angle -40^\circ \\ 0.85 \angle 40^\circ & 0.3 \angle 0^\circ \end{bmatrix}$$

Determine the following

- Verify if the network is Reciprocal
- Verify if the network is a lossless network.
- Calculate the return loss at port 1 when Port 2 is terminated with a load that matches the network impedance.

Soln.

$$S_{11} = 0.15 \angle 0^\circ$$

$$S_{12} = 0.85 \angle -40^\circ$$

$$S_{21} = 0.85 \angle 40^\circ$$

$$S_{22} = 0.3 \angle 0^\circ$$

a) Condition for Reciprocal.

$$S_{ij} = S_{ji}$$

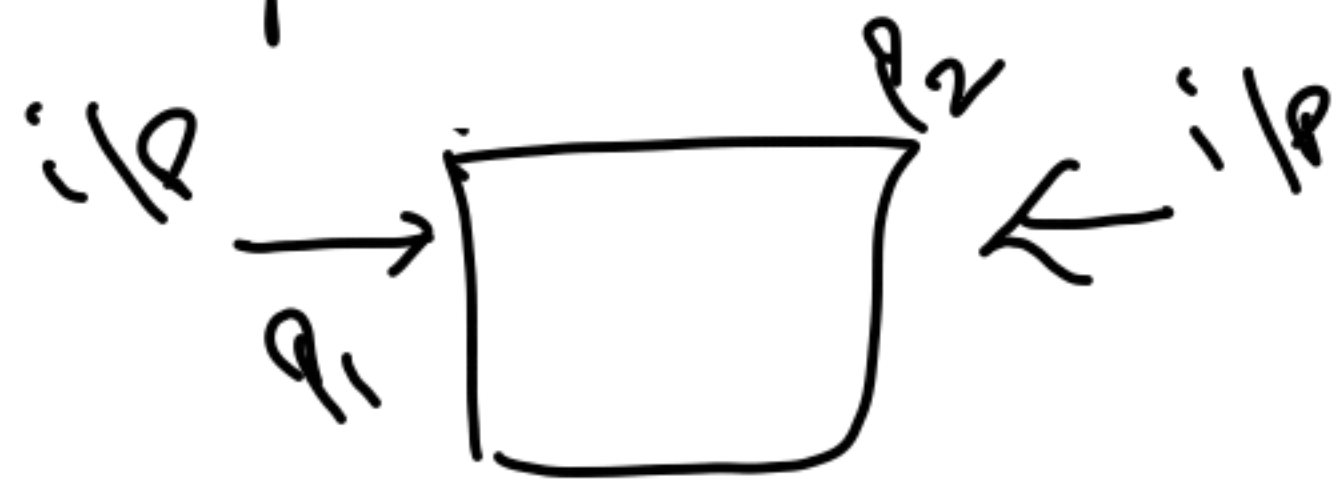
$$S_{21} = 0.85 \angle 40^\circ$$

$$S_{12} = 0.85 \angle -40^\circ$$

$$S_{21} \neq S_{12}$$

As $S_{21} \neq S_{12}$, the given network is not a Reciprocal Network.

b) Verify if it is a lossless network.



$$S_{11}, S_{21}$$

$$S_{22}, S_{12}$$

When the i/p is at port 1

$$|S_{11}|^2 + |S_{21}|^2 = 1, \text{ Condition for lossless network.}$$

$$= |0.15|^2 + |0.85|^2$$

$$= 0.745 \neq 1$$

This network is a lossy network.

c) Return loss

$$\text{Return loss} = -20 \log_{10}(|S_{11}|)$$

$$= -20 \log_{10} |0.15|$$

$$= 16.478 \text{ dB}$$

3) A 3 port RF network is given by the following S-matrix.

$$[S] = \begin{vmatrix} 0.3 \angle 0^\circ & 0.4 \angle -20^\circ & 0.5 \angle 40^\circ \\ 0.4 \angle -20^\circ & 0.25 \angle 0^\circ & 0.6 \angle -50^\circ \\ 0.5 \angle 40^\circ & 0.6 \angle -50^\circ & 0.35 \angle 0^\circ \end{vmatrix}$$

- Verify if it is a Reciprocal network.
- Verify if the network is lossless
- Calculate the Return loss at port 1, Provide port 2 and 3 are matched terminated

Soln:

$$S_{11} = 0.3 \angle 0^\circ$$

$$S_{12} = 0.4 \angle -20^\circ$$

$$S_{13} = 0.5 \angle 40^\circ$$

$$S_{31} = 0.5 \angle 40^\circ$$

$$S_{32} = 0.6 \angle -50^\circ$$

$$S_{33} = 0.35 \angle 0^\circ$$

$$S_{21} = 0.4 \angle -20^\circ$$

$$S_{22} = 0.25 \angle 0^\circ$$

$$S_{23} = 0.6 \angle -50^\circ$$

a) Verify if reciprocal

$$S_{12} = S_{21} \quad S_{13} = S_{31}$$

$$S_{23} = S_{32}$$

$$\underline{S_{12} = S_{21}}$$

$$0.4 \angle -20^\circ = 0.4 \angle -20^\circ$$

$$\underline{S_{13} = S_{31}}$$

$$0.5 \angle 40^\circ = 0.5 \angle 40^\circ$$

$$S_{23} = S_{32}$$

$$0.6 \angle -50^\circ = 0.6 \angle -50^\circ$$

All the above conditions are satisfied,
Hence it is a Reciprocal network.

b) Verify if it is a lossless network.

Input at port 1

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 = 1$$

$$= |0.3|^2 + |0.4|^2 + |0.5|^2$$

$$\boxed{0.5 \neq 1}$$

Input at port 2

$$|S_{22}|^2 + |S_{12}|^2 + |S_{32}|^2 = 1$$

$$= |0.4|^2 + |0.25|^2 + |0.6|^2$$

$$\boxed{0.58 \neq 1}$$

Input at port 3

$$|S_{13}|^2 + |S_{33}|^2 + |S_{23}|^2 = 1$$

$$|0.5|^2 + |0.6|^2 + |0.35|^2 = 0.7$$

$$\boxed{0.7 \neq 1}$$

It is lossy network.

c) Return loss ∴

$$\text{Return loss} = -20 \log_{10} |S_{11}|$$

$$= -20 \log_{10} (0.3)$$

$$\text{Return loss} = 10.458 \text{ dB}$$

4) A 2 port Reciprocal network is given by.

$$|S| = \begin{vmatrix} 0.4 \angle 0^\circ & 0.6 \angle -20^\circ \\ 0.6 \angle 20^\circ & 0.3 \angle 0^\circ \end{vmatrix}$$

- Calculate the insertion loss from port 1 to port 2.
- Calculate the gain from port 2 to port 1.
- Calculate the isolation between port 1 and port 2.

(a) Insertion loss from port 1 to port 2.

$$\hat{I}_L = -20 \log_{10} |S_{21}|$$

$$= -20 \log_{10}(0.6)$$

$$\hat{I}_L = 4.437 \text{ dB}$$

b) Calculate the gain from port 2 to port 1

$$G = 20 \log_{10} S_{12}$$

$$= 20 \log_{10} (0.6)$$

$$G = -4.44 \text{ dB}$$

c) Isolation between port 1 and port 2.

$$I = -20 \log_{10} (S_{21})$$

$$= -20 \log_{10} (0.6)$$

$$I = 4.44 \text{ dB}$$

$$[S] = \begin{vmatrix} 0 & 0 \\ 0.5 \angle 20^\circ & 0 \end{vmatrix}$$

$$[S] = \begin{vmatrix} 0 & 0 \\ S_{21} & 0 \end{vmatrix}$$

$$[S] = \begin{vmatrix} 0 & 0 & 0.5 \angle 0^\circ \\ S_{21} & 0 & 0 \\ 0 & S_{32} & 0 \end{vmatrix}$$

