

# PSDE5009 Assignment 1

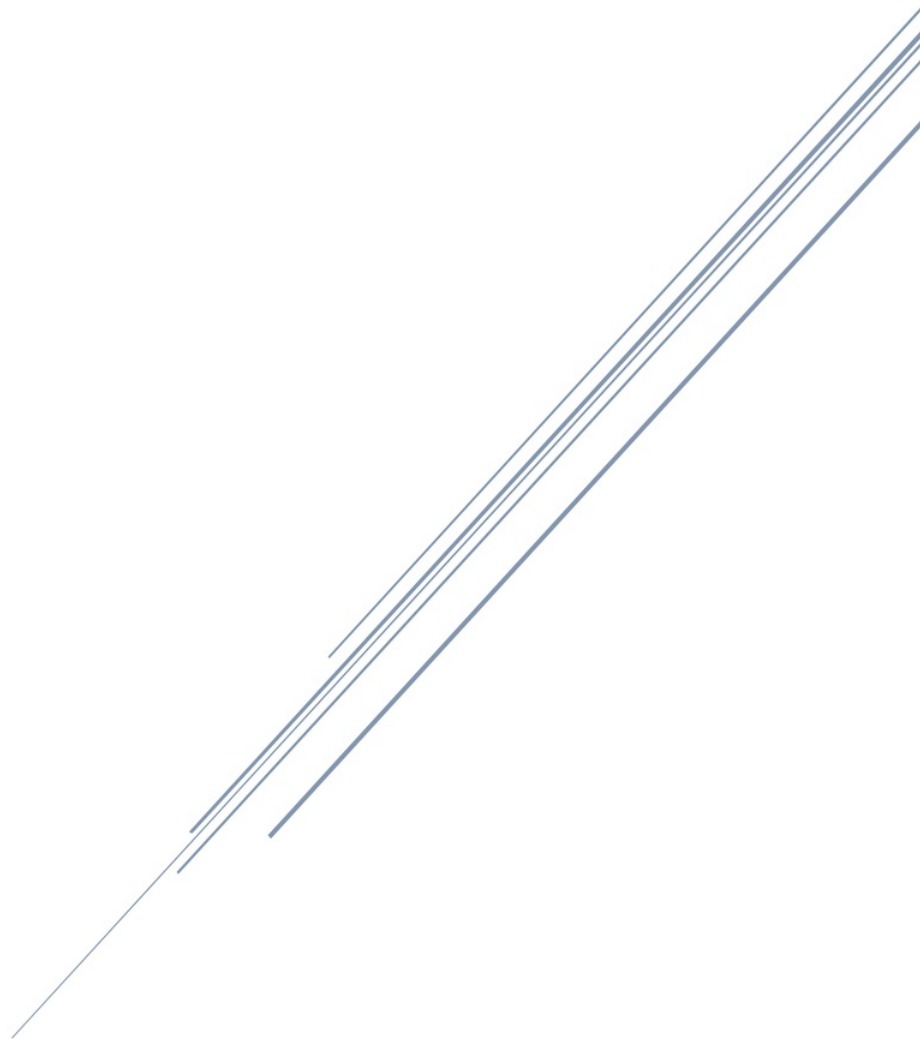
*by* Daniel Ince

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# ASSIGNMENT 1

PSDE5009



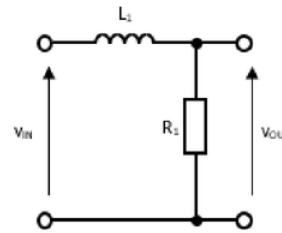
Daniel Ince

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## Analysis of First Order Electric Circuits

### Low pass filter RL



The first part of this task involves the RL circuit shown to the right

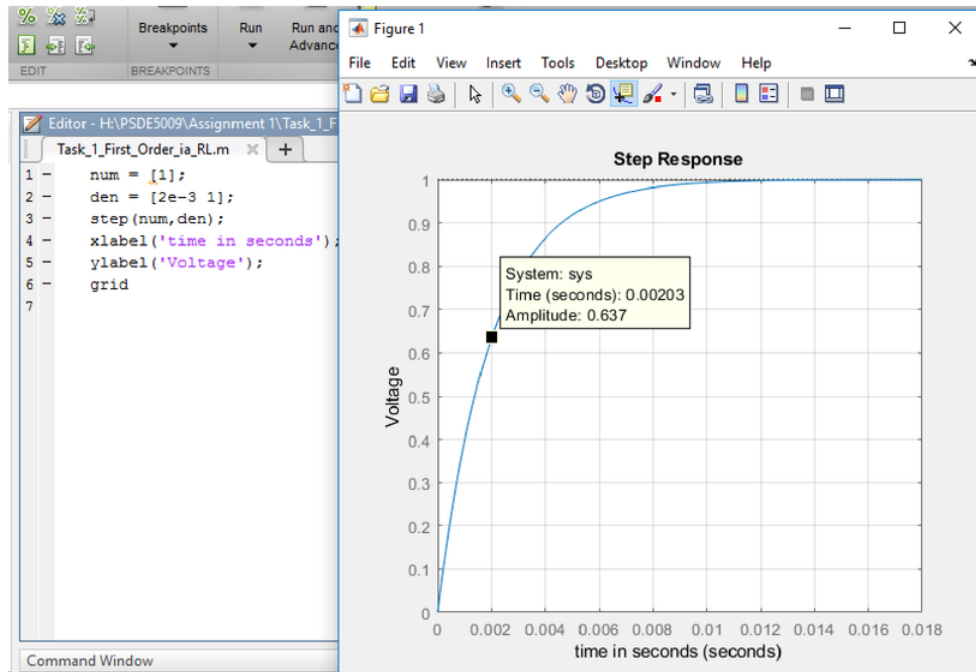


Figure 1: First order RL

The chosen values for the RL low pass filter circuit are as follows:

$$L1 = 0.06H \quad R1 = 30\Omega \quad \tau = \frac{L}{R} = \frac{0.06}{30} = 2 \times 10^{-3}$$

These values can be seen input into the Matlab analysis below.

The Laplace model and transfer function for the following RL circuit.

Using the potential divider rule:  $\frac{Y(s)}{U(s)} = \frac{R}{sL+R}$  therefore:  $\frac{1}{\frac{sL}{R}+1} = \frac{1}{\tau s+1} \rightarrow \frac{1}{0.002s+1}$  The

resulting transfer function can be seen in the Matlab analysis in Figure 1 with a step input for the transient analysis. It can be seen that at 0.637V, the time elapsed is 0.00203 seconds. This ties in with the calculated time constant of 0.002 seconds.

The cut-off frequency will now be derived from the time constant, as:  $\omega c = \frac{R}{L} \rightarrow \omega c = \frac{1}{\tau}$

Therefore,  $\omega c = \frac{1}{0.002} = 500\text{rad/s}$

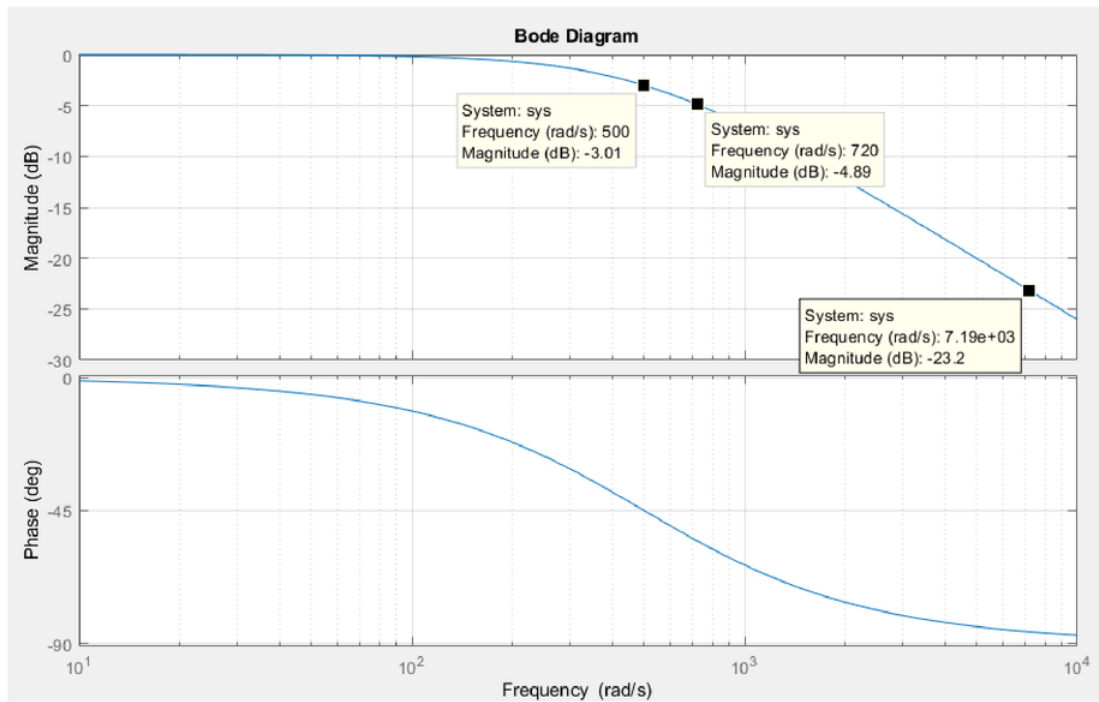


Figure 2: Bode plot RL

In order to determine the cut-off frequency using Matlab, the transfer

function must be derived and then bode plotted in the software.  $\frac{Y(s)}{U(s)} =$

$$\frac{R}{sL+R} \rightarrow \frac{30}{0.06s+30} \rightarrow H(s) = \frac{500}{s+500}$$

in terms of the transfer function, because:

$$\frac{1}{0.002s+1} \text{ with top and bottom multiplied by } \omega c \rightarrow \frac{500}{s+500}$$

```
2 - den = [1 500];
3 - bode(num,den);
4 - grid
```

This transfer function was input into Matlab and ran using the bode plotter to provide the following shown in Figure 2. It can be seen that at the cut-off frequency of 500rad/s, the magnitude is at -3dB, this corresponds to the expected and calculated value.

The decade change from 720 to 7200rad/s drops roughly by 20dB as expected due to theory.

### High pass filter RC

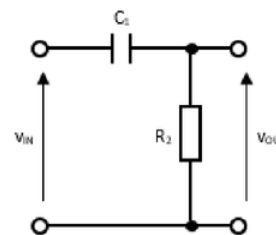
The next part of this task involves analysing the RC circuit shown right.

For the high pass filter, the following values were chosen:

$$R_2 = 2K\Omega \quad C_1 = 2\mu F$$

To derive the transfer function, the potential divider rule is used,

$$\text{resulting in: } Y(s) = \frac{R}{\frac{1}{sC}+R} \times U(s)$$



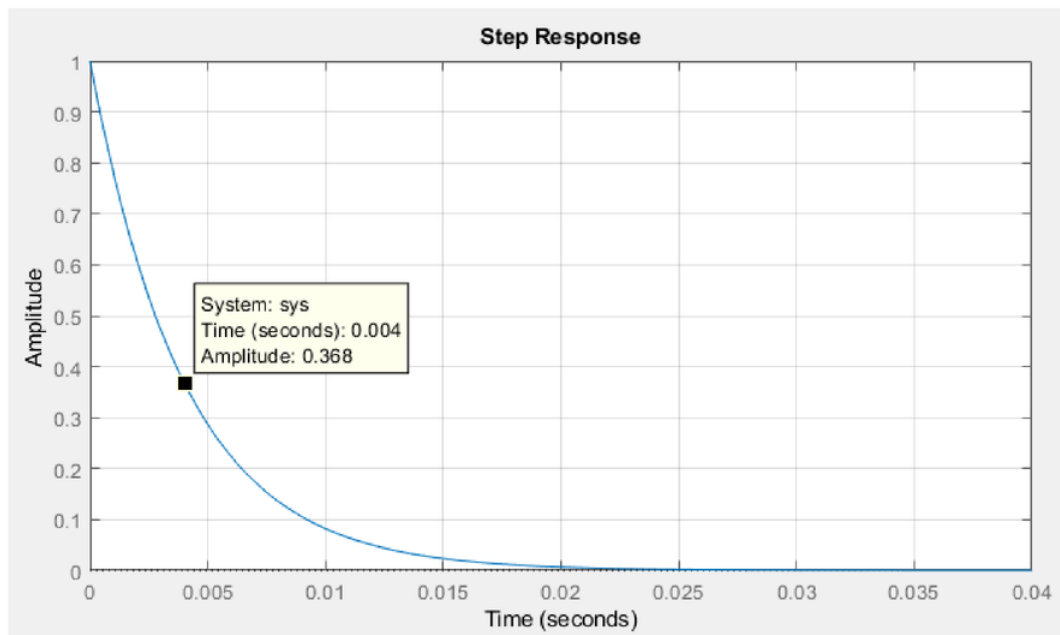
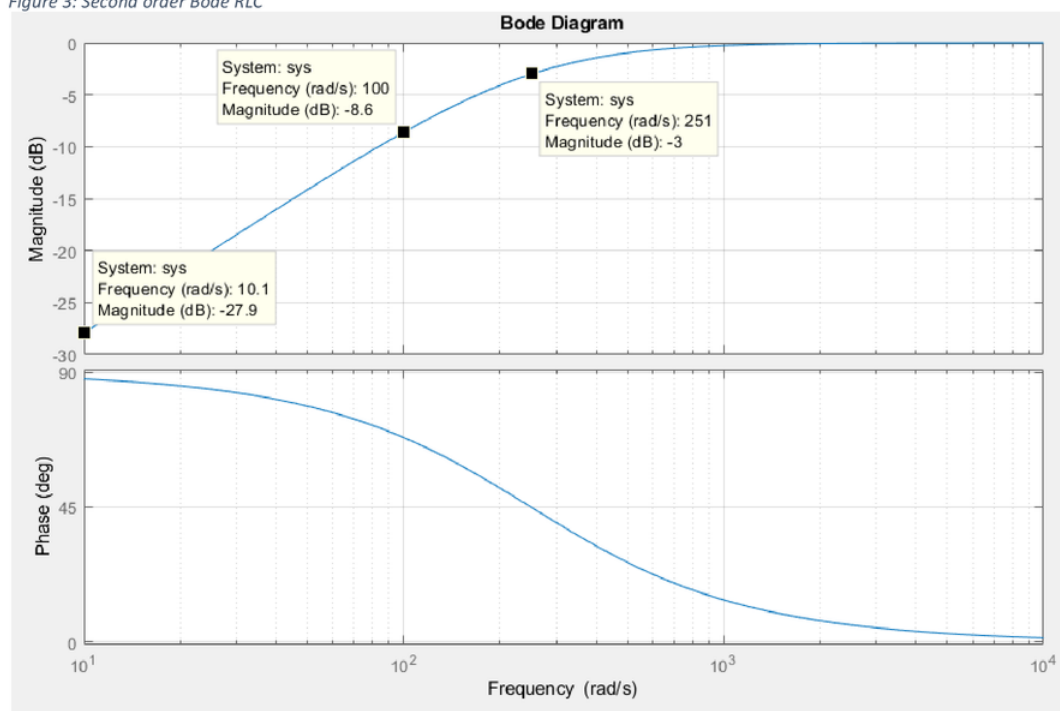


Figure 4: Step RC

```
1 - num = [0.004 0];
2 - den = [0.004 1];
3 - step(num,den);
4 - grid
```

Figure 3: Second order Bode RLC



This becomes  $Y(s) = \frac{RC}{RCs+1}$  with regards to a step input, and because  $\tau = RC$ . The transfer function

can be written as  $H(s) = \frac{\tau s}{\tau s + 1}$ . So  $\tau = 2000 \times 2 \times 10^{-6} = 0.004 \text{ seconds}$ .

Shown in Figure 3, the values were entered into a step input in Matlab.

It can be seen that at 0.368 amplitude voltage, at which point the elapsed time should be equal to the time constant, the time is 0.004s. This ties in with the calculated and expected value for the time constant.

If we then multiply the top and bottom of the transfer function by the cut-off frequency, we will get the function in terms of frequency, allowing easier interpretation of the bode plot.

$$\omega c = \frac{1}{RC} = \frac{1}{0.004} = \frac{250 \text{ rad}}{s} \rightarrow H(s) = \frac{s}{s + \omega c} = \frac{s}{s + 250}$$

Figure 4 shows the cut off frequency at approximately 250rad/s occurring at -3dB as expected. The decade between 10 and 100rad/s also has a difference of 20dB as expected with regards to the calculations and theory.

## Analysis of Second Order Electric Circuits

### Low pass RLC

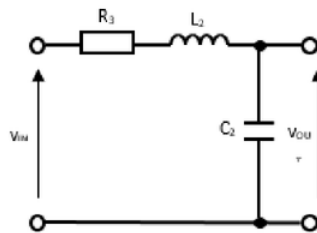
To the right is a second order low pass filter, using the potential divider rule the transfer function can be derived through taking the Laplace transforms of each component.

$Y(s) = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}} X(s)$  This can be converted when rearranged and multiplied by (sL) on the top and bottom to end up with the following.

$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$  Leaving the second order transfer function for this circuit.

Values for the circuit are as follows:  $R_3 = 200$ ,  $L_2 = 0.06\text{H}$  and  $C_2 = 1\mu\text{F}$ . So when put into the equation, the following

results are given.  $H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$



The resonant frequency  $\omega n$  is worked out as follows:  $\omega n = \sqrt{\frac{1}{LC}} = \sqrt{16.67} \times 10^6 = 4082.5 \text{ rad/s}$

The damping factor (Z) is worked out with the equation to give:  $Z = \frac{R}{2} \sqrt{\frac{C}{L}} \rightarrow \frac{200}{2} \sqrt{\frac{1\mu\text{F}}{0.06}} = 0.408$  as Zeta is less than 1, it suggests that this circuit is underdamped and would need to be equal to 1 to be critically damped.

When the transfer function is simulated on matlab with the values, it produces the step response shown in figure 5 which is not critically damped and is underdamped. This ties in with the calculation of the damping factor at 0.408 being underdamped.

```

- num = [16666666.7 ];
- den = [1 3333.3 16666666.7];
- step(num,den);
- grid

```

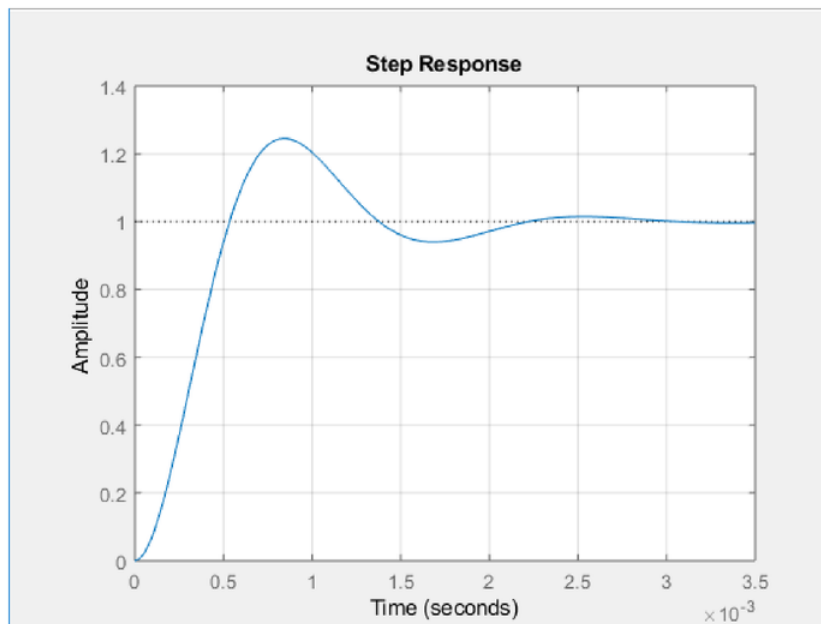


Figure 5: Step Response

In order to change this, the value of R can be altered in order to achieve critical damping.  $Z =$

$$\frac{R}{2} \sqrt{\frac{C}{L}} \rightarrow \frac{489}{2} \sqrt{\frac{1\mu F}{0.06}} = 1.0 \text{ in this case the value was increased from } 200\Omega \text{ to } 489\Omega \text{ as the following}$$

$$\text{calculation proves that for the damping factor to be 1, } R = 2 \sqrt{\frac{L}{C}} \rightarrow 2 \sqrt{\frac{1\mu F}{0.06}} = 489.89$$

An example of the effect this has on the step response is evident in figure 6, as the curve is critically damped.

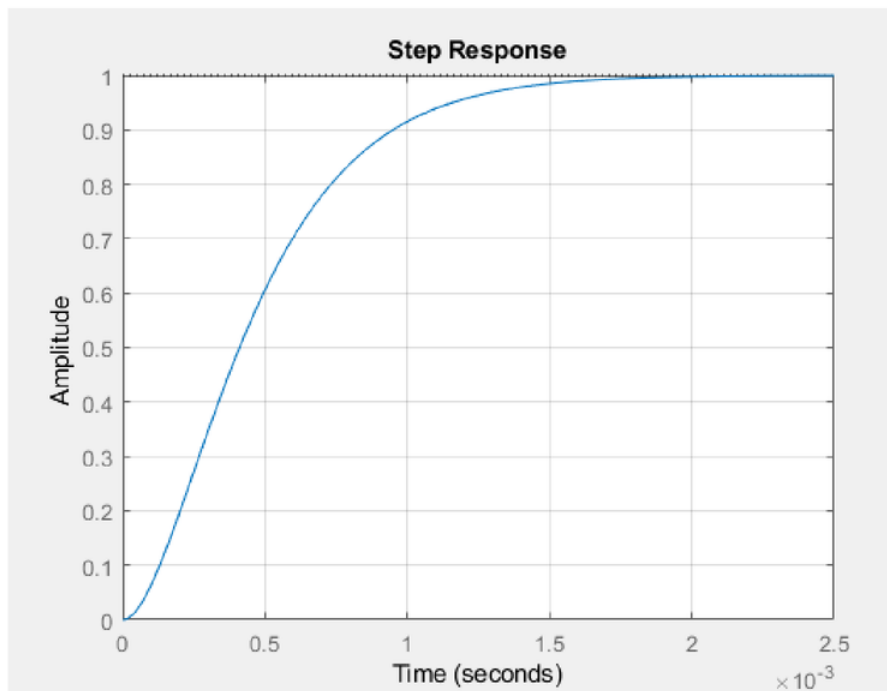
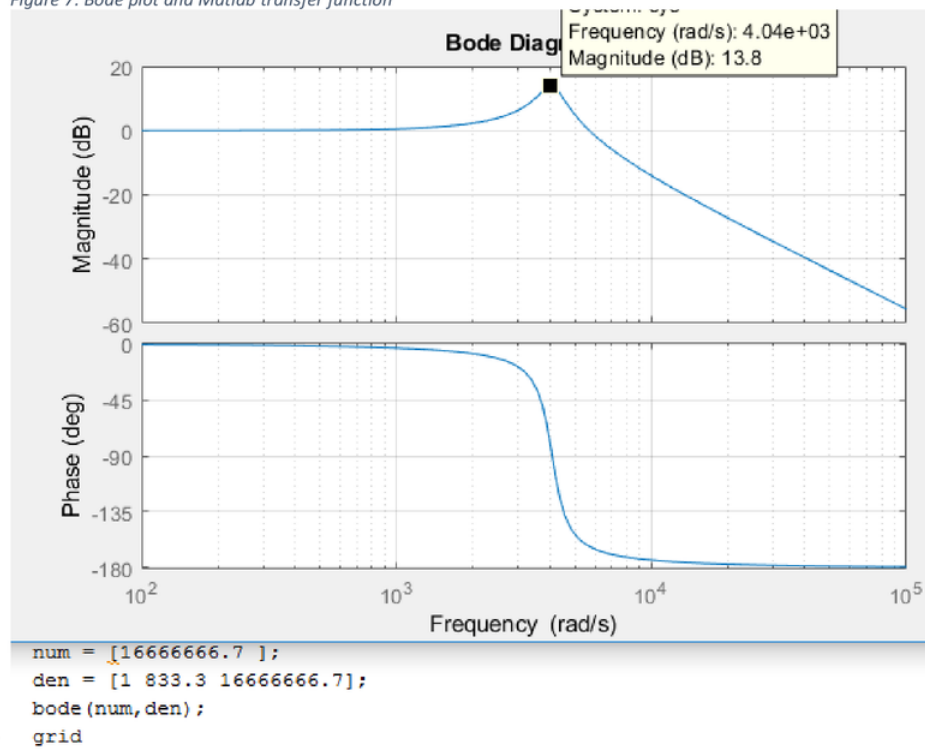


Figure 6: Critical step response

Figure 7: Bode plot and Matlab transfer function



For under damping, the value for R is significantly lowered in order to pull the damping factor below 1. This means that the maximum frequency  $\omega_m$  can be calculated. The value for R was changed to

$$50. \omega_m = \sqrt{\frac{1}{LC} - \frac{1}{2} \left( \frac{R}{L} \right)^2} \rightarrow \sqrt{\frac{1}{0.06 \times 1 \mu F} - \frac{1}{2} \left( \frac{50}{0.06} \right)^2} = 4039.73 \text{ rad/s}$$

Using Matlab, a bode plot was carried out using the transfer function with a resistance of 50Ω. It can be seen that the peak maximum frequency is  $4.04 \times 10^3 \text{ rad/s}$ , this ties in with the calculated value of 4039 rad/s.

The reason the resistor value had to be dropped to 50 was because if the value 489 was used, it would have produced a negative square root which cannot be done. This is because the value for  $\omega_m$  tends towards  $\omega_n$  as R is varied. The next bode plot in figure 8 displays all four resistance values, 50, 200, 489 and 800Ω.

With no damping, the value for R is non-existent and produces a graph where  $\omega_n$  and  $\omega_m$  are the same, creating a huge peak shown in the following bode plot.

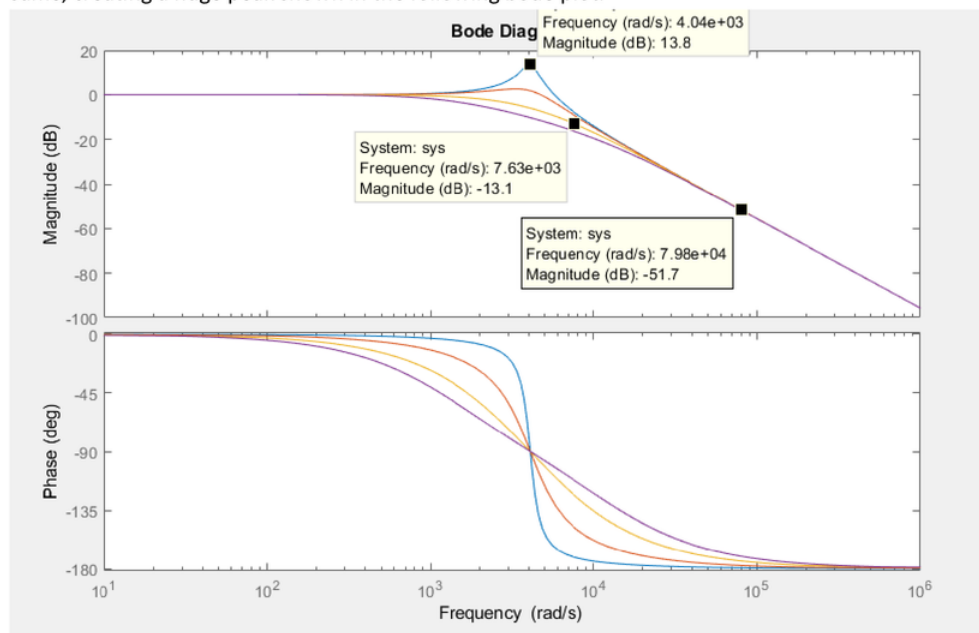


Figure 8: Bode plot containing four resistance values

The phase shift of the filter can always be seen to be the same, no matter the value of resistor. Proved by the bode/phase diagram featuring the different values of R, the angle changes but the phase remains the same.

The Quality factor of the original circuit is worked out to be  $Q = \frac{1}{R} \sqrt{\frac{L}{C}} \rightarrow \frac{1}{200} \sqrt{\frac{1\mu F}{0.06}} = 1.22$ , a larger value of Q determines how high and sharp the peak is. If a value of 1 is used for the resistor, ie, no damping:  $Q = \frac{1}{R} \sqrt{\frac{L}{C}} \rightarrow \frac{1}{1} \sqrt{\frac{1\mu F}{0.06}} = 244.94$ , This is shown in the above bode plot as a very high peak.

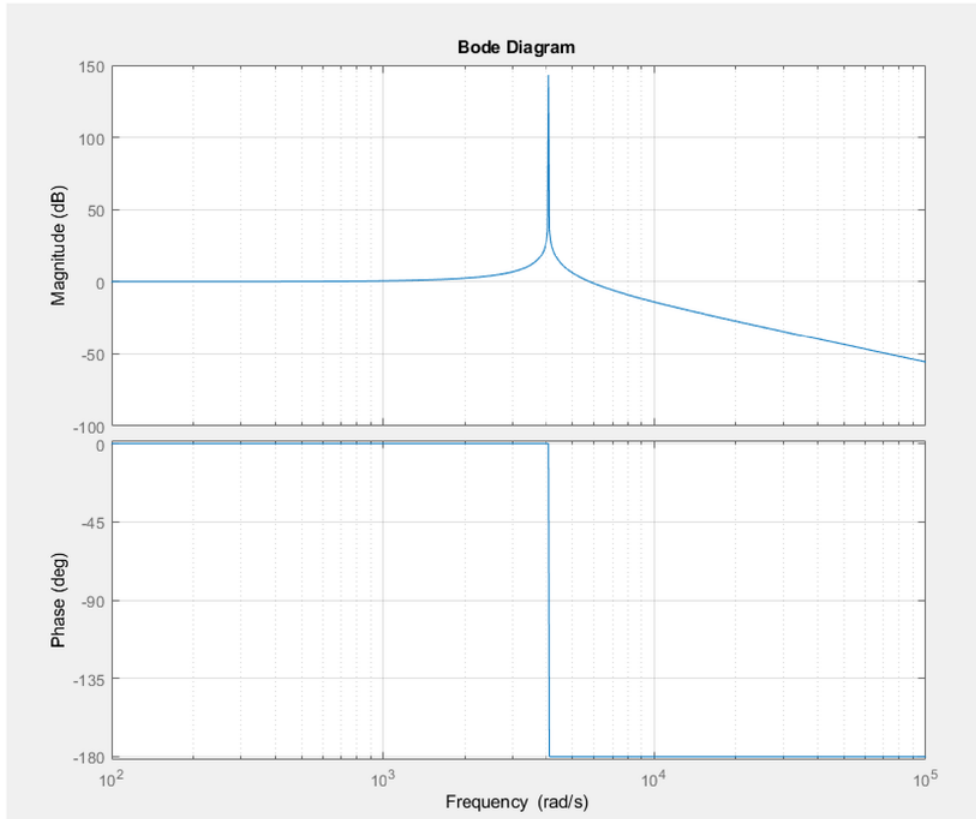


Figure 9: No damping

### Parallel resonant circuit

Using the potential divider rule;

$Y(s) = \frac{Z(s)}{R+Z(s)} X(s)$  with the impedance of the inductor and capacitor being represented as  $Z$ , to provide the following Laplace transform.

$$H(s) = \frac{s \frac{1}{RC}}{s^2 + s \frac{1}{RC} + \frac{1}{LC}}$$

$$R = 450\Omega$$

$$L = 1H$$

$$C = 25\mu F$$

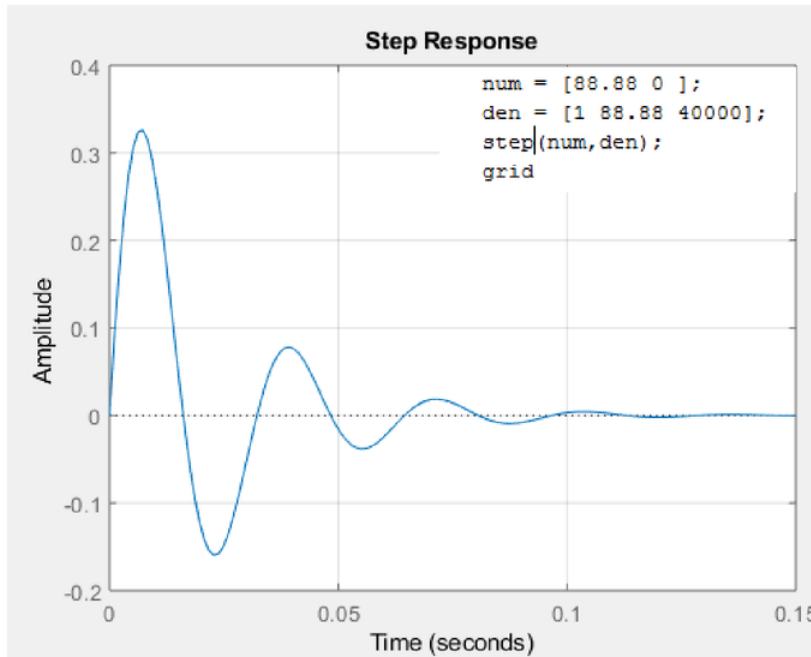
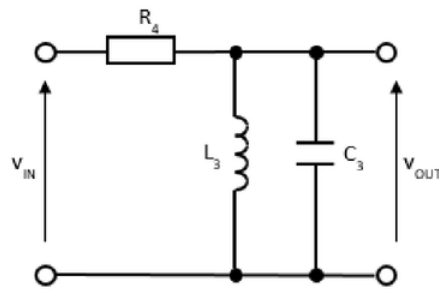


Figure 10: RLC step response and Matlab transfer function.

Figure 10 shows the RLC step response with a resistor value of 450 Ohms, the graph suggests that the system is currently underdamped with that resistance value.

The bandwidth of the parallel resonant circuit will now be calculated to determine the cut off points of this band pass filter.

$$\omega_{c1} = -\frac{1}{2 \times (450)(25\mu F)} + \sqrt{\left(\frac{1}{2 \times (450)(25\mu F)}\right)^2 + \frac{1}{1 \times (25\mu F)}} = 160.43Hz$$

$$\omega_{c2} = \frac{1}{2 \times (450)(25\mu F)} + \sqrt{\left(\frac{1}{2 \times (450)(25\mu F)}\right)^2 + \frac{1}{1 \times (25\mu F)}} = 249.323Hz$$

So therefore, the bandwidth is worked out to be  $\omega_{c2} - \omega_{c1} = 88.8Hz$

A Matlab bode plot will now be performed.

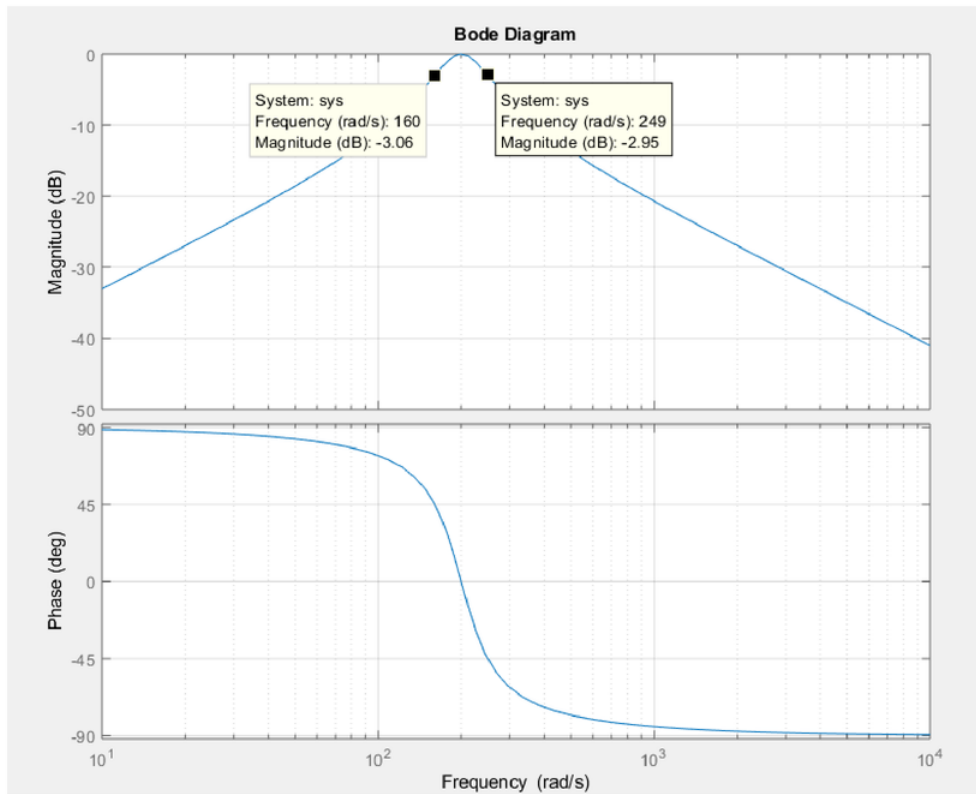


Figure 11: Matlab bode plot showing -3db cut off points

Figure 11 shows the bode plot with the bandwidth cut off frequencies highlighted. This ties into the calculations as the  $\omega_{c1}$  value is shown to be 160Hz at -3dB compared to the calculated value of 160.43Hz, the same goes for  $\omega_{c2}$ .

The Q factor can also be worked out as:  $Q = \frac{\omega_o}{\beta}$  and  $\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{1(25\mu F)}} = 200$

Therefore  $Q = \frac{\omega_o}{\beta} = \frac{200}{88.8} = 2.25$

As Bandwidth =  $\frac{1}{RC}$  the size of the resistor has a large effect when changed. To make the bandwidth smaller, a higher resistance is used, for example:  $\beta = \frac{1}{700(25\mu F)} = 57.14Hz$

To compare, a smaller resistance produces  $\beta = \frac{1}{300(25\mu F)} = 133.4Hz$

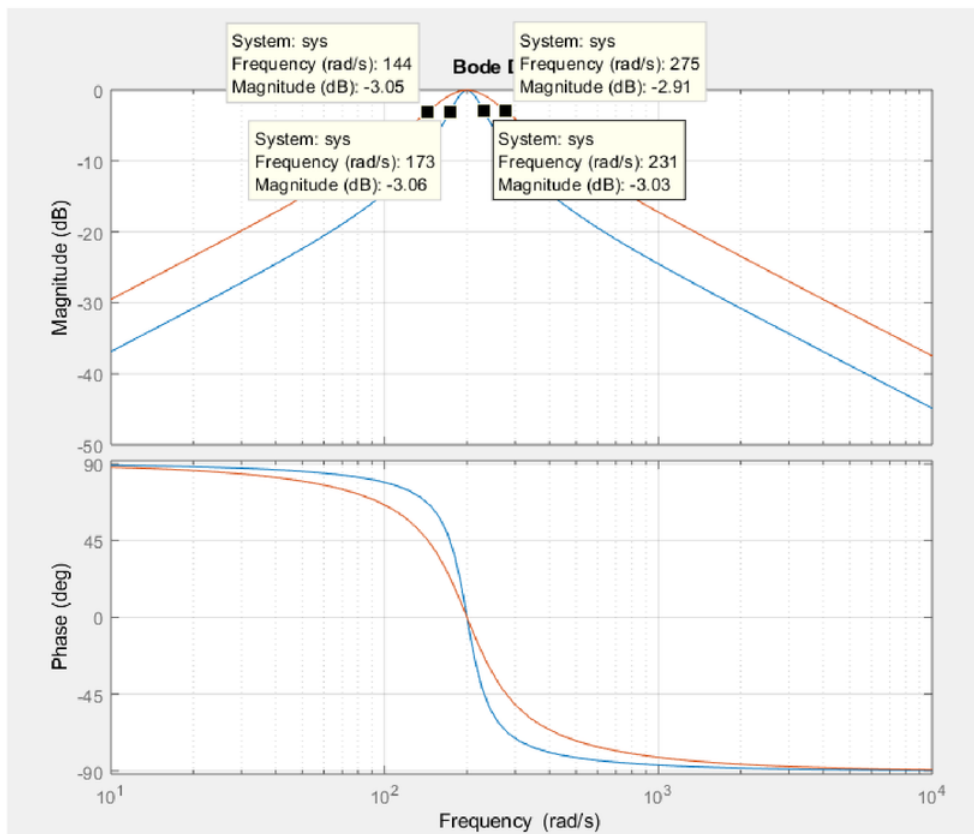
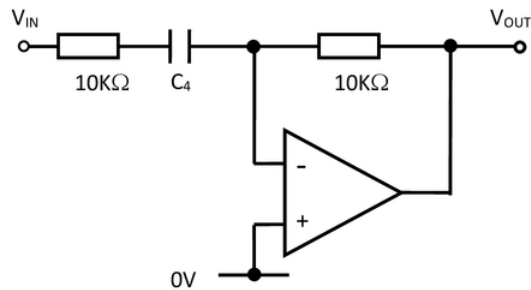


Figure 12: Bandwidth changes with resistance

Figure 12 shows the orange response with a bandwidth of  $275 - 144 = 133\text{Hz}$  with a resistance of 300 Ohms, the blue graph shows the 700 Ohm resistance with a bandwidth of  $231 - 173 = 58\text{Hz}$ . This ties in with the calculations proving that the higher the resistance, the smaller the bandwidth.

## Analysis of First Order Electronic Systems

### High pass filter



For this exercise a value will be chosen for  $C_4$  and calculations conducted to produce results which can be compared to a Matlab response.

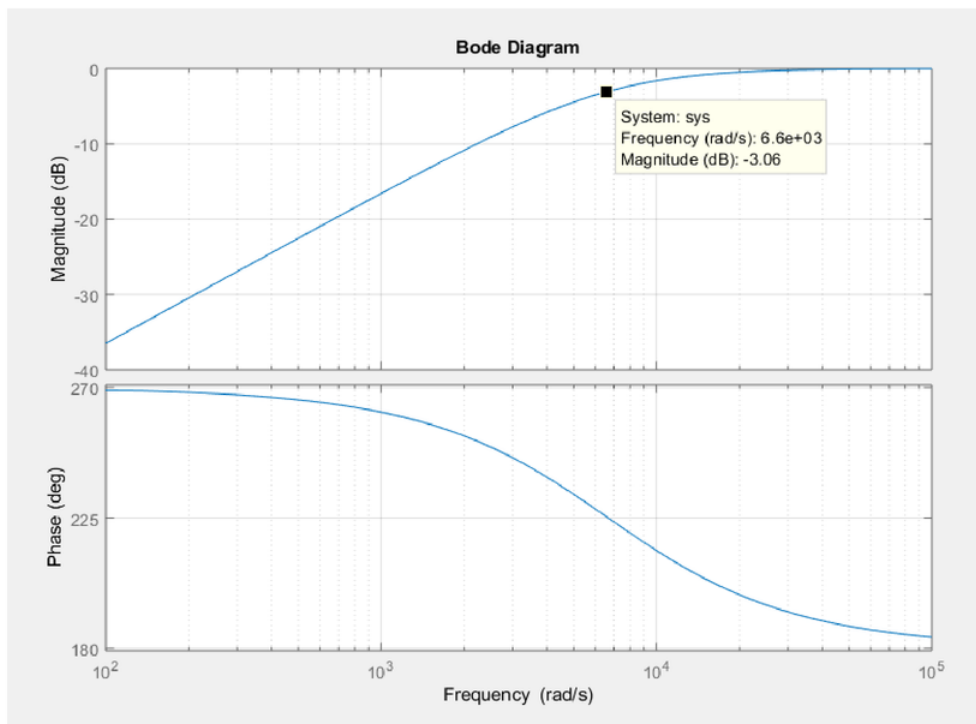


Figure 13: Bode plot for first order high pass filter

```
1 - num = [-1 0];
2 - den = [1 6666.6];
3 - bode(num,den);
4 - grid
```

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The resultant graph in figure 13 displays the bode plot of the simulation in Matlab. For this,  $C_4$  is set to 15nF.

$$K = \frac{10k}{10k} = 1 \quad \omega_c = \frac{1}{10k \times 15nF} = 6666.6rad/s$$

The transfer function for the first order high pass is:

$$H(s) = -K \left\{ \frac{s}{s + \omega_c} \right\} \rightarrow -1 \left\{ \frac{s}{s + 6666.6} \right\} \quad \text{The value of } K = 1 \text{ in decibels is } 20\log 1 = 0 \text{ therefore, the cut off at 3dB below this should be -3dB.}$$

These results were input into Matlab as shown in figure 13, the graph shows a trend typical of that of a high pass filter. The cut off point detailed in the graph is at approximately -3dB with the maximum decibels at 0 and the frequency of 6600rad/s. These results relate to what is expected in theory and calculations. As the filter is only a first order, the roll off does not have a particularly steep gradient and is rather gentle, this filter may not be as effective as other types due to this.

## Low pass filter

The first order low pass filter will be evaluated using 500nF for the value of  $C_5$  capacitor.

$$K = \frac{5.6k}{1k} = 5.6 \quad \omega_c = \frac{1}{5k \times 500nF} = 357.143 \text{ rad/s}$$

The transfer function for the first order high pass is:

$$H(s) = -K \left\{ \frac{\omega_c}{s + \omega_c} \right\} \rightarrow -5.6 \left\{ \frac{357.143}{s + 357.143} \right\}$$

The value of  $K = 5.6$  in decibels is  $20 \log 5.6 = 14.96$

therefore, the cut off at 3dB below this should be 11.96dB.

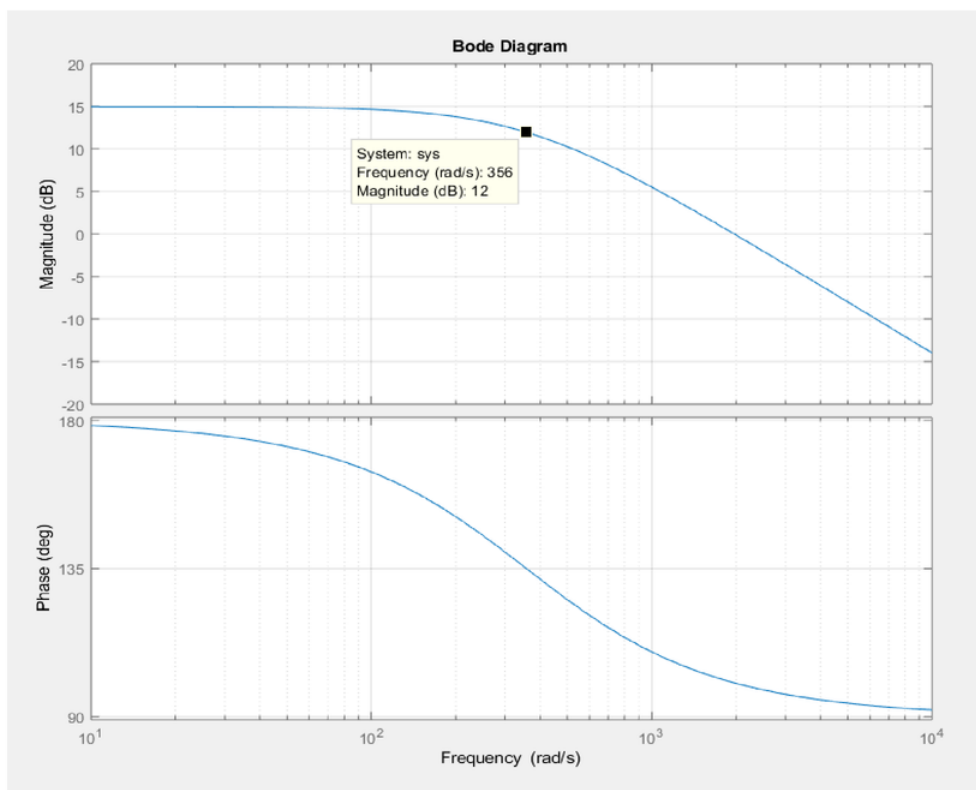
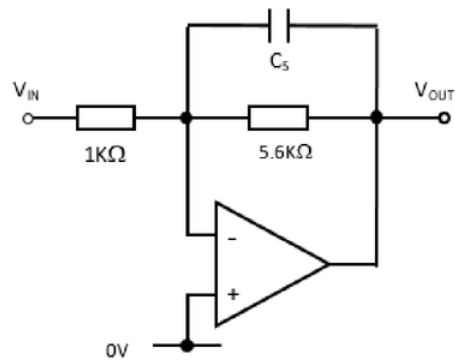


Figure 14: Low pass filter bode plot

```
1 - num = [-2000];
2 - den = [1 357.143];
3 - bode(num,den);
4 - grid
```

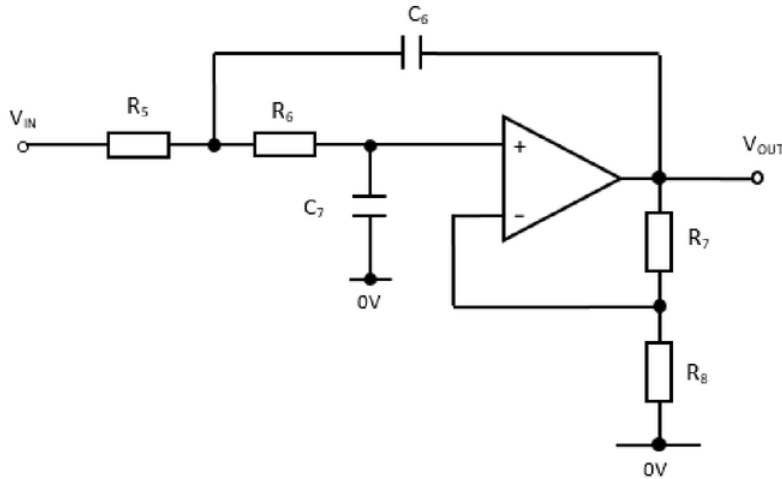
Figure 14 shows the bode plot for the transfer function input into Matlab, the curve clearly that of a low pass filter. The maximum magnitude is roughly 15dB, tying in with the calculated value of 14.96

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for the K value in decibels. From this, the cut off frequency is at the calculated 12dB and at 356rad/s. This matches the calculations, as from here the gentle roll off typical of a first order curve is present.

## Analysis of Second Order Electronic Systems

### Sallen-key low pass filter



Above is the Sallen-key design for the second order low pass filter, values for the capacitors and resistors will be calculated in order to evaluate the system. For a cut-off frequency of 357.143rad/s to be selected and a capacitance of 500nF to allow comparison against the first order circuit responses.

Let  $R_5 = R_6$  and so as  $\omega c = \frac{1}{RC}$ ,  $R = \frac{1}{357.143 \times 500nF} = 5600\Omega$ . To work out  $R_7$ , let  $R_8 = 2k\Omega$  with a K value of 1.586 to coincide with the Butterworth filter design. Therefore,  $R_7 = (1.586 - 1) \times 2000 = 1172\Omega$ .

This means that,  $Q = \frac{1}{3-K} = 0.707$        $\frac{\omega c}{Q} \rightarrow \frac{357.143}{0.707} = 505.15$

The transfer function is as follows:  $H(s) = \frac{K \omega c^2}{s^2 + s(\frac{\omega c}{Q}) + \omega c^2}$

With input values this changes to:  $H(s) = \frac{1.586 \times (357.143)^2}{s^2 + 505.15s + 357.143^2} \rightarrow H(s) = \frac{202296.08}{s^2 + 505.15s + 127551.12}$

Figure 15 shows the response curve and Matlab bode plot simulation. It can be seen that the cut off frequency remains to be 357rad/s at 1dB, 3dB less than the maximum 4dB magnitude. The shape of the response is very steep, typical of the second order filters, however the roll off remains gradual in keeping with the Butterworth filter design.

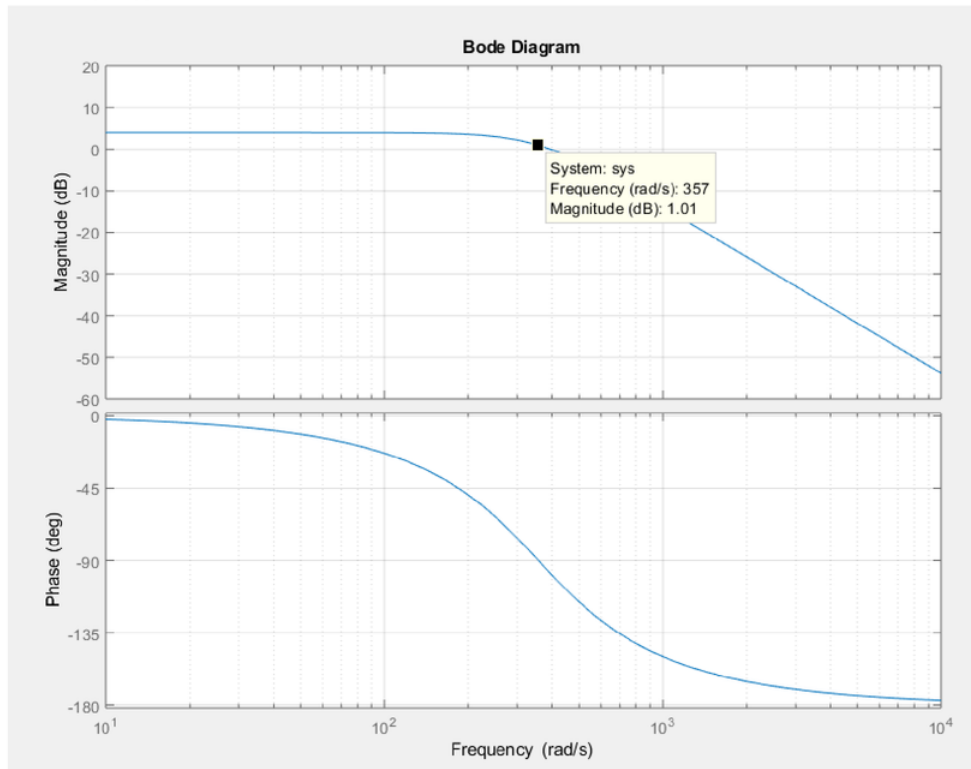


Figure 15: Second order low pass filter bode plot

```

1 - num = [202296.08 ];
2 - den = [1 505.15 127551.12];
3 - bode(num,den);
4 - grid

```

In figure 15, the bode plot is shown across the frequency range along with the numbers input into Matlab in order to achieve this.

Figure 16 shows the comparison of this second order low pass filter (blue trend) against the first order low pass filter from task 3 (orange trend). The cut off frequency for both remains to be 357rad/s, however this is the main similarity. The most notable difference is the roll off rate from the cut off frequency. The first order system does not decline as much as the second order. Theory states that the first order system should roll off at a rate of 20dB per decade, whereas the second order is 40dB per decade. The second order system at  $10^3$  is at -10dB, this declines to -50dB at  $10^4$  as can be seen on the graph in figure 16. The first order system drops from 5dB at  $10^3$  to -15db at  $10^4$ , a 20dB drop per decade. Both these rates of decline are typical of the second and first order theory.

Another notable feature is the difference in phase shift between the two, the first order system shifts phase of 90 degrees across the frequency range, whereas the second order system shifts 180 degrees out of phase. A phase shift of 90 degrees per order is apparent.

The difference in maximum magnitude for these filters is down to the value for K, as for the second order, if  $K = 20\log(1.586)$  a value of 4dB is given, compared to  $K = 20\log(5.56) = 11.96\text{dB}$  for the first order bode plot. To make these the same, the K value could be modified to produce the same value which would subsequently alter the value for Q also.

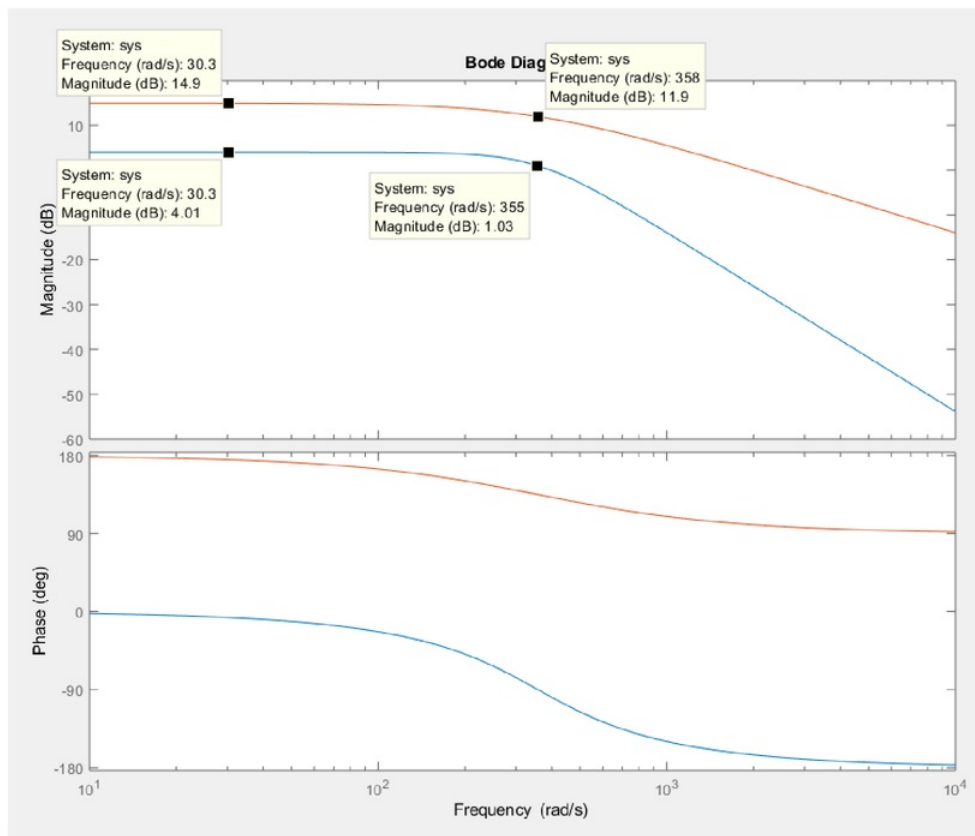
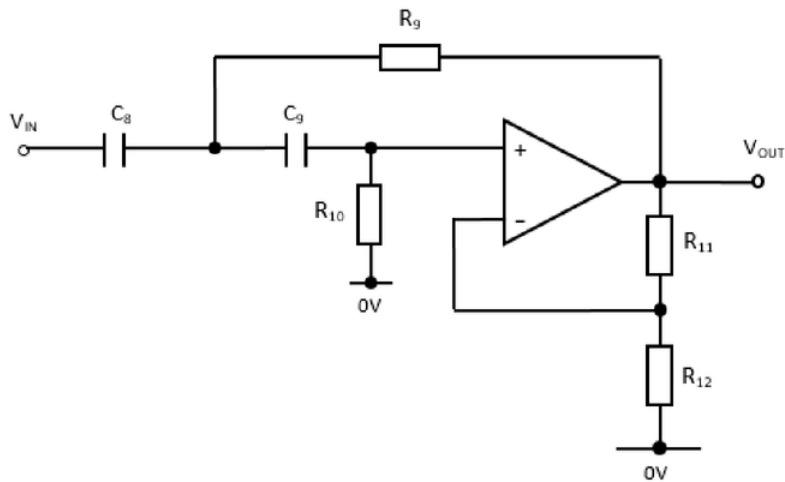


Figure 16: Low pass second order filter comparison

## Sallen-key high pass filter



Above is the Sallen-key design for the second order high pass filter, values for the capacitors and resistors will be calculated in order to evaluate the system. For a cut-off frequency of 6666.6rad/s to be selected and a capacitance of 50nF to allow comparison against the first order circuit responses.

Let  $R_9 = R_{10}$  and so as  $\omega c = \frac{1}{RC}$ ,  $R = \frac{1}{6666.6 \times 50nF} = 3000\Omega$ . To work out  $R_7$ , let  $K = 1.586$  to coincide with the Butterworth filter design.

This means that,  $Q = \frac{1}{3-K} = 0.707$   $\frac{\omega c}{Q} \rightarrow \frac{6666.6}{0.707} = 9429.4$

The transfer function is as follows:  $H(s) = \frac{K s^2}{s^2 + s(\frac{\omega c}{Q}) + \omega c^2}$

With input values this changes to:  $H(s) = \frac{1.586 s^2}{s^2 + 9429.4s + 44443555.56}$

Figure 17 shows the response curve and Matlab bode plot simulation. It can be seen that the cut off frequency remains to be 6666.6rad/s at 1dB, 3dB less than the maximum 4dB magnitude. As this is a second order filter, the rate of cut off is steeper than that of the first order.

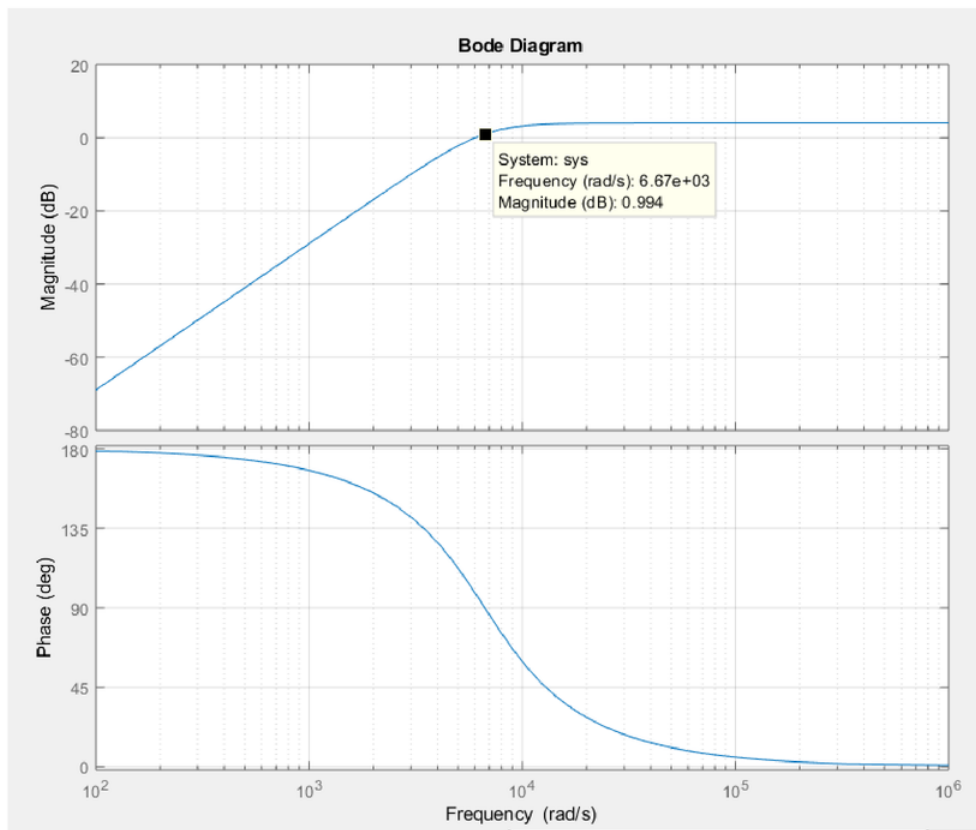


Figure 17: Second order high pass filter bode plot

```

1 - num = [1.586 0 0];
2 - den = [1 9429.4 44443555.56];
3 - bode(num,den);
4 - grid

```

Figure 18 shows the graph from the second order high pass filter (blue trend), with that of the first order high pass filter from task 3 (orange trend). The cut off frequency for both is 6666.6rad/s, however, the rate of decline after the cut off frequency is rather different. For the second order filter, the drop off rate is 40dB per decade, as can be seen in figure 18. At a frequency of  $10^3$  the magnitude is -30dB, whereas at  $10^2$  the magnitude has dropped by 40dB to -70dB. The first order curve drops 20dB from -15dB to -35dB across the same decade.

The difference in phase shift can also be seen as 180 degrees for the second order and 90 degrees for the first order.

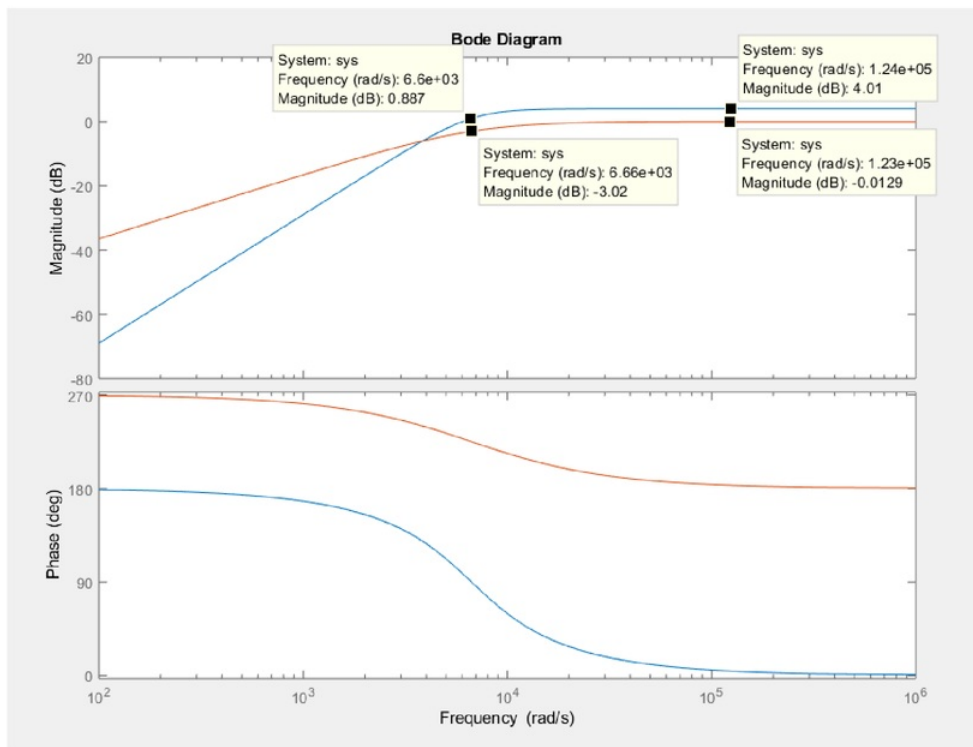


Figure 18: High pass filter second order comparison

## Conclusions and Recommendations

When analysing the first order low and high pass filters, it became apparent that altering the time constant would ultimately adjust the value for the cut off frequency. The component which had the largest effect on both was the resistor, the larger the resistor value, the smaller the time constant.

With regards to the second order circuits, the resistor had the greatest effect on damping and bandwidth. A larger resistor would produce a smaller bandwidth when it came to the parallel resonant circuit than a smaller one, but too much would run the risk of over damping the circuit. The low pass second order filter would achieve a high peak frequency if R is negligible with no damping where  $\omega_m$  tends towards  $\omega_n$  as R is varied because with no damping, they are effectively the same, as shown in figure 8.

First order electronic systems can be tuned to the desired cut off frequency through varying either the resistors or capacitors. The value for K determines the maximum magnitude which shall be passed. The decline rate of 20dB per decade is apparent with the first order system.

The second order systems have twice the decline rate per decade, at 40db compared to 20 db of a first order system. These would be more effective at cancelling out unwanted frequencies than those of lower orders.

# PSDE5009 Assignment 1

## GRADEMARK REPORT

FINAL GRADE

GENERAL COMMENTS

**Instructor**

70/100

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## GRADING FORM: CSG-RUBIC

**DANIEL INCE****70****RESEARCH-INFORMED LITERATURE**

N/A

**KNOWLEDGE AND UNDERSTANDING**

Sections 2, 3: Explanations for Electric Circuits – .Generally well explained from first principles referring to the potential divider rule. Laplace models require 's' in the components on each diagram. (7/10) Sections 4, 5: Explanations for Electronic Systems – Briefly stated with clear diagrams but further marks were available for additional explanations to help your reader understand where your equations and some values came from. (6/10)

**13****ANALYSIS**

Sections 2, 3: Analysis, Modelling and Testing of Electric Circuits - . Very good analysis of circuits and results, further marks were available for

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analysis of phase changes. (15/20) Sections 4, 5: Analysis, Modelling and Testing of Electronic Systems - Very good evidence of analysis with some annotations on graphic results. However, further marks were available for additional observations - Comparisons made in section 5 would have been better using the same gain and the full Matlab program given. Also, graphs could have more labels. (15/20) Section 6: Conclusions and recommendations – Very good conclusions drawn but no recommendations given. (6/10)

#### **PRACTICAL APPLICATION AND DEP.**



Sections 2,3,4,5: Use of Matlab – Good use of this software in evidence though the full programs should have been given. (7/10) Sections 2,3,4,5: Accuracy and Presentation Quality of Results – Excellent (10/10)

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#### **SKILLS FOR PROFESSIONAL PRAC.**



Section 1: Introduction: None offered. (0/5) All sections: Overall Report Quality - Well presented and written. However, further marks were available for numbering your sections. (4/5)

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