

Linear Programming Example

A store has requested a clothing manufacturer to produce trousers and sports jackets.

For materials, the manufacturer has 750 m² of cotton textile and 1,000 m² of polyester. Every pair of trousers (1 unit) needs 1 m² of cotton and 2 m² of polyester. Every jacket needs 1.5 m² of cotton and 1 m² of polyester.

The price of the trousers is fixed at 50 and the jacket, 40.

What is the number of trousers and jackets that the manufacturer must give to the stores so that these items obtain a maximum sale?

1. Choose the unknowns.

x = number of trousers

y = number of jackets

2. Write the objective function.

$$\text{Cost} = 50x + 40y$$

3. Write the constraints as a system of inequalities.

To write the constraints, use a table:

	trousers	jackets	available
cotton	1	1,5	750
polyester	2	1	1,000

$x + 1.5y \leq 750$ multiply by 2 to keep the numbers even:

$$2x + 3y \leq 1500$$

$$2x + y \leq 1000$$

As the number of trousers and jackets are natural numbers, there are two more constraints:

$$x \geq 0$$

$$y \geq 0$$

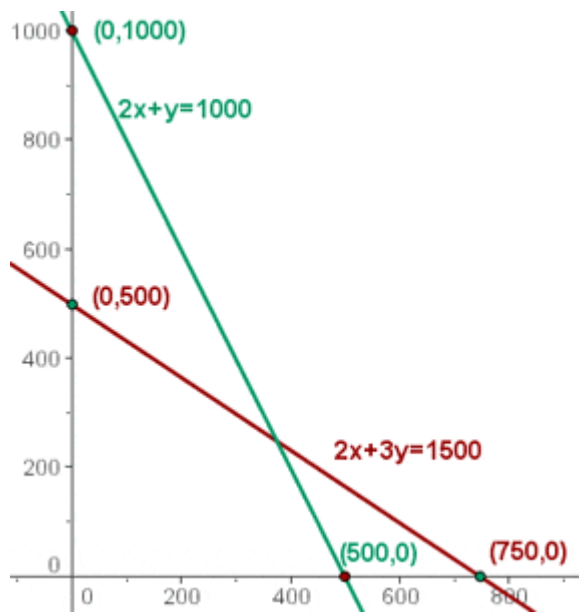
4. Find the set of feasible solutions that graphically represent the constraints.

Represent the constraints graphically.

As $x \geq 0$ and $y \geq 0$, work in the first quadrant.

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Represent the straight lines from their points of intersection with the axes.



Solve the inequation graphically: $2x + 3y \leq 1500$, and take a point on the plane, for example $(0,0)$.

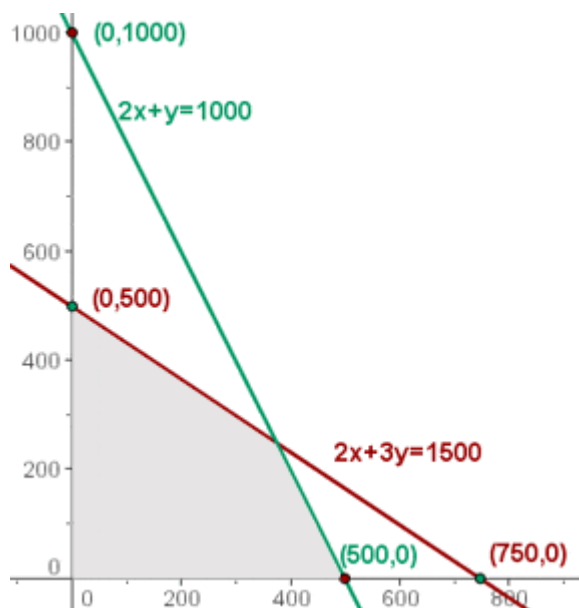
$$2 * 0 + 3 * 0 \leq 1,500$$

Since $0 \leq 1,500$ then the point $(0,0)$ is in the half plane where the inequality is satisfied.

Similarly, solve $2x + y \leq 1,000$.

$$2 * 0 + 0 \leq 1,000$$

The area of intersection of the solutions of the inequalities would be the solution to the system of inequalities, which is the set of **feasible solutions**.



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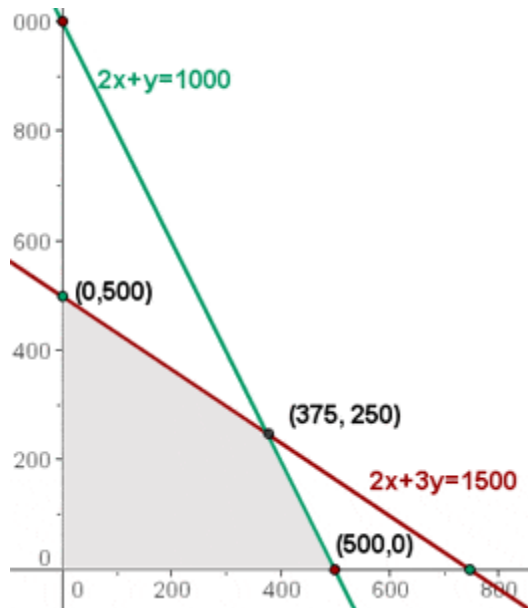
5. Calculate the coordinates of the vertices from the compound of **feasible solutions**.

The optimal solution, if unique, is in a vertex. These are the solutions to the systems:

$$2x + 3y = 1,500; x = 0 \quad (0, 500)$$

$$2x + y = 1,000; y = 0 \quad (500, 0)$$

$$2x + 3y = 1,500; 2x + y = 1,000 \quad (375, 250)$$



6. Calculate the value of the objective function at each of the vertices to determine which of them has the maximum or minimum values. It must be taken into account the possible non-existence of a solution if the compound is not bounded.

In the **objective function**, place each of the vertices that were determined in the previous step.

$$f(x, y) = 50x + 40y$$

$$f(0, 500) = 50 \cdot 0 + 40 \cdot 500 = 20,000$$

$$f(500, 0) = 50 \cdot 500 + 40 \cdot 0 = 25,000$$

$$f(375, 250) = 50 \cdot 375 + 40 \cdot 250 = 28,750$$