

stuff I did last week...

we use $PV^n = C$ and

$$\frac{P_1 V_1}{T_1} = C$$

which is ① $P_1 V_1^n = P_2 V_2^n$

and

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{②}$$

we get similar terms together

$$\text{①} \quad \frac{P_1}{P_2} = \frac{V_2^n}{V_1^n}$$

$$\text{②} \quad \frac{P_1}{P_2} = \frac{V_2}{V_1} \times \frac{T_1}{T_2}$$

Put ① into ②, getting rid of P_1 and P_2 ...

$$\Rightarrow \frac{V_2^n}{V_1^n} = \frac{V_2}{V_1} \times \frac{T_1}{T_2} \quad \text{get } \frac{T_2}{T_1} \text{ on its own}$$

$$\frac{T_2}{T_1} = \frac{V_2}{V_1} \times \frac{V_1^n}{V_2^n}$$

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_1} \right) \times \left(\frac{V_1}{V_2} \right)^n$$

get V_1 on top

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{-1} \times \left(\frac{V_1}{V_2} \right)^n$$

multiplying bracket
so ADD indices

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{n-1}$$

Again we use $P_1 V_1^n = P_2 V_2^n$ and $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$

$$(1) \frac{P_1}{P_2} = \frac{V_2^n}{V_1^n}$$

$$(2) \frac{P_1}{P_2} = \frac{V_2}{V_1} \times \frac{T_1}{T_2}$$

This time, we want to get rid of V_1 and V_2 ... So rearrange (1)

$$(1) \frac{P_1^{1/n}}{P_2^{1/n}} = \frac{V_2}{V_1}$$

Now combine (1) and (2)

$$\frac{P_1}{P_2} = \frac{P_1^{1/n}}{P_2^{1/n}} \times \frac{T_1}{T_2}$$

rearrange for $\frac{T_2}{T_1}$

$$\frac{T_2}{T_1} = \frac{P_1^{1/n} P_2}{P_2^{1/n} P_1}$$

get all P_2 on top

$$\frac{T_2}{T_1} = \frac{P_2^{-1/n} P_2}{P_1^{-1/n} P_1}$$

Now sort out indices

$$\frac{T_2}{T_1} = \frac{P_2^{-1/n} P_2^1}{P_1^{-1/n} P_1^1}$$

we add indices again
but this time they
are fractions...

$$\frac{1}{1} + \frac{-1}{n} = \frac{n-1}{n}$$

so

$$\frac{T_2}{T_1} = \frac{P_2^{n-1/n}}{P_1^{n-1/n}}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$