

## 12 Pages

**Complete, fold and seal**  
(see note 2 below)

SURNAME: DYER

FORENAME(S): EMYR WYN

## Instructions to Candidates

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3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
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|   |                                  |                           |
|---|----------------------------------|---------------------------|
| 1 | Student Number                   | <del>1503008</del> 11     |
| 2 | Title of Examination Paper       | CONTROL & INSTRUMENTATION |
| 3 | Module Code                      | PSDE 5011                 |
| 4 | Date and Time of Examination     | 10/05/18<br>09:30         |
| 5 | Examination Venue                |                           |
| 6 | Seat Number                      |                           |
| 7 | Number of Answer Books handed in | 1                         |

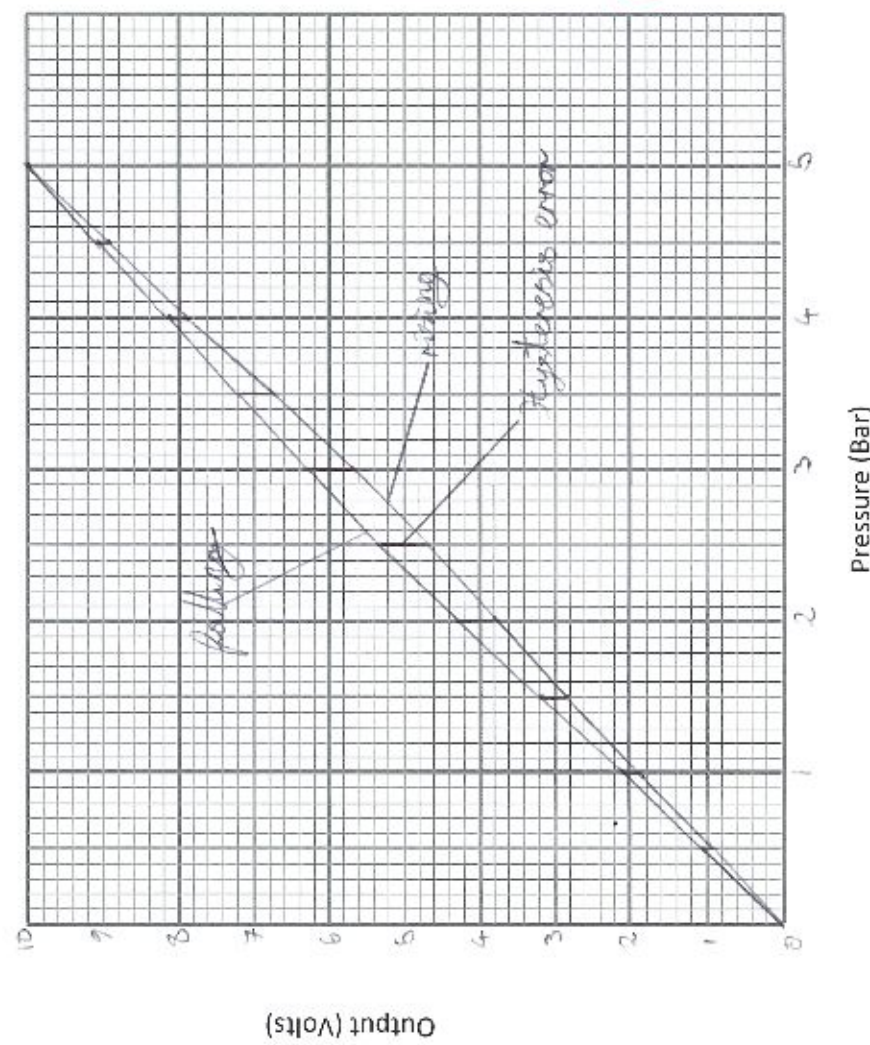
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22%

Table Q4

| Measured Pressure (Bar) | Output voltage (Volts) |         | Linear Value (Volts) | Non-linearity error (Volts) |         | Hysteresis Error (Volts) |
|-------------------------|------------------------|---------|----------------------|-----------------------------|---------|--------------------------|
|                         | rising                 | falling |                      | rising                      | falling |                          |
| 0                       | 0                      | 0       | 0                    | 0                           | 0       | 0                        |
| 0.5                     | 0.96                   | 1.03    | 0.995                | 0.035                       | -0.035  | 0.07                     |
| 1.0                     | 1.89                   | 2.12    | 2.005                | 0.115                       | -0.115  | 0.23                     |
| 1.5                     | 2.81                   | 3.17    | 2.990                | 0.180                       | -0.180  | 0.36                     |
| 2.0                     | 3.77                   | 4.28    | 4.025                | 0.255                       | -0.255  | 0.51                     |
| 2.5                     | 4.68                   | 5.33    | 5.005                | 0.325                       | -0.325  | 0.65                     |
| 3.0                     | 5.61                   | 6.25    | 5.93                 | 0.320                       | -0.320  | 0.64                     |
| 3.5                     | 6.73                   | 7.16    | 6.945                | 0.215                       | -0.215  | 0.43                     |
| 4.0                     | 7.88                   | 8.10    | 7.99                 | 0.110                       | -0.110  | 0.22                     |
| 4.5                     | 8.94                   | 9.05    | 8.995                | 0.055                       | -0.055  | 0.11                     |
| 5.0                     | 10.0                   | 10.0    | 10                   | 0                           | 0       | 0                        |

Graph for calibration results for QUESTION 4.



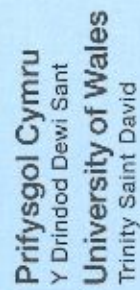
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ATTEMPT QUESTION 4.

(a) Static Sensitivity - Detection of Change

(ii) Non-linearity error - Has for the rising or falling value deviates from linear ideal

(iv) Hysteresis error - The difference between 2 values for non-linearity

100% = 22



**Complete, fold and seal**  
(see note 2 below)

SURNAME: Jones

FORENAME(S): Joseff Owaga

12 Pages

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|---|----------------------------------|-----------------------|--|
| 1 | Student Number                   | 1502999/1             |  |
| 2 | Title of Examination Paper       | Control + Instruments |  |
| 3 | Module Code                      | PSDE 5011             |  |
| 4 | Date and Time of Examination     | 10/5/18<br>9:30       |  |
| 5 | Examination Venue                | Sports Gym            |  |
| 6 | Seat Number                      |                       |  |
| 7 | Number of Answer Books handed in |                       |  |

(Gweler yr ochr arall am y fersiwn Cymraeg)

$$a) i) Z_1 = -2 \quad \checkmark$$

$$P_1 = -2 + j3 \quad \checkmark$$

$$P_2 = -2 - j3 \quad \checkmark$$

$$G(s) = \frac{(s - Z_1)(s - Z_2)}{(s - P_1)(s - P_2)}$$

$$G(s) = \frac{s(s - (-2))}{(s - (-2 + j3))(s - (-2 - j3))}$$

$$G(s) = \frac{s + 2}{(s + 2 - j3)(s + 2 + j3)}$$

$$= \frac{s + 2}{s^2 + 2s + j3s + 2s + 4 + 2j3 - j3s - 2j3 + 9}$$

$$= \frac{s + 2}{s^2 + 4s + 13}$$

b) ii) compare to standard second order

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\therefore \omega_n^2 = 13$$

$$\omega_n = \sqrt{13} = 3.606$$

$$ii) 2\zeta\omega_n = 4$$

$$\zeta = \frac{4}{2 \times 3.606} = 0.55$$

is it's underdamped

b)  $T(s) = \frac{G(s)}{1 + G(s)H(s)}$  ✓

$G(s) = \frac{s+2}{s^2+4s+13}$  ✓  $H(s) = K_1$  ✓

$T(s) = \frac{\frac{s+2}{s^2+4s+13}}{1 + \frac{s+2}{s^2+4s+13} \times K_1}$

multiply top and bottom by  $s^2+4s+13$

$T(s) = \frac{s+2}{s^2+4s+13 + K_1(s+2)}$

$T(s) = \frac{s+2}{s^2+4s+13 + K_1s + 2K_1}$

$= \frac{s+2}{s^2 + (4+K_1)s + (13+2K_1)}$  ✓

critical damping or real locus occurs at -5 ✓

$G(s) = \frac{s+2}{s^2-5s-5} \Rightarrow \frac{s+2}{s^2+10s+25}$

equating coefficients

$4 + K_1 = 10 \Rightarrow K_1 = 6$   
 $13 - 5K_1 = 25 \Rightarrow K_1 = 6$   
 $2K_1 = 12$   
 $K_1 = 6$  ✓

c)  $T(s) = \frac{G(s)}{1 + G(s)H(s)}$

$G(s) = \frac{s+2}{s^2+4s+13}$   $H(s) = 3.92$  ✓

d) The system should be stable because the zeros are stable.

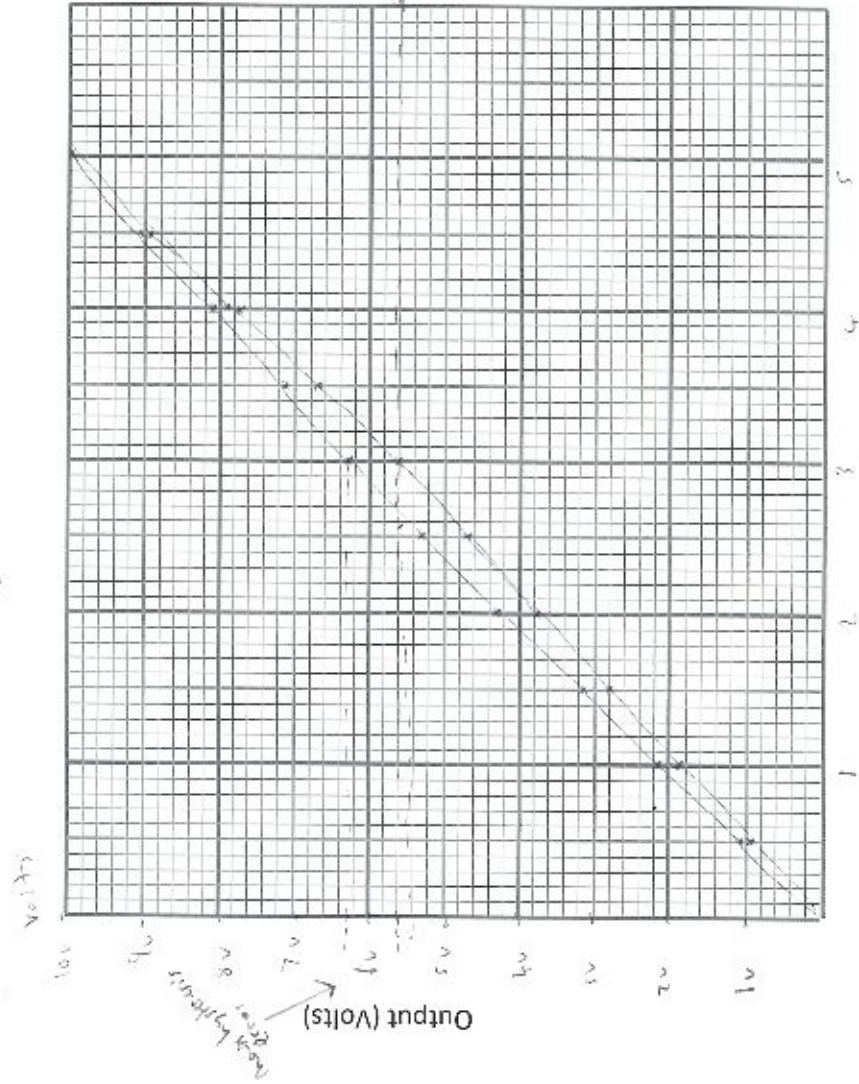
?

$\tau_{0.9} = 19\frac{1}{2}$   
 $20\frac{1}{2}$

Table Q4

| Measured Pressure (Bar) | Output voltage (Volts) |         | Linear Value (Volts) | Non-linearity error (Volts) |         | Hysteresis Error (Volts) |
|-------------------------|------------------------|---------|----------------------|-----------------------------|---------|--------------------------|
|                         | rising                 | falling |                      | rising                      | falling |                          |
| 0                       | 0                      | 0       | 0                    | 0                           | 0       | 0                        |
| 0.5                     | 0.96                   | 1.03    | 1                    | -0.04                       | +0.03   | 0.07                     |
| 1.0                     | 1.89                   | 2.12    | 2                    | -0.11                       | +0.12   | 0.23                     |
| 1.5                     | 2.81                   | 3.17    | 3                    | -0.19                       | +0.17   | 0.36                     |
| 2.0                     | 3.77                   | 4.28    | 4                    | -0.23                       | +0.28   | 0.51                     |
| 2.5                     | 4.68                   | 5.33    | 5                    | -0.32                       | +0.33   | 0.65                     |
| 3.0                     | 5.61                   | 6.25    | 6                    | -0.39                       | +0.25   | 0.74                     |
| 3.5                     | 6.73                   | 7.16    | 7                    | -0.27                       | +0.16   | 0.43                     |
| 4.0                     | 7.88                   | 8.10    | 8                    | -0.12                       | +0.1    | 0.22                     |
| 4.5                     | 8.94                   | 9.05    | 9                    | -0.06                       | +0.05   | 0.11                     |
| 5.0                     | 10.0                   | 10.0    | 10                   | 0                           | 0       | 0                        |

Graph for calibration results for QUESTION 4.



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Question 4

Question No.

a) (i) FSD

Full-scale deflection is the maximum output value minus the minimum. On table Q4, the FSD would be  $10.0 - 0 = 10V$  ✓

(ii) Static sensitivity

Static sensitivity is how the transducer reacts to the input. It is determined by dividing the FSD by the maximum allowed measurement. On table Q4, this would be  $\frac{10}{5} = 2V/B$  (volts per bar). ✓

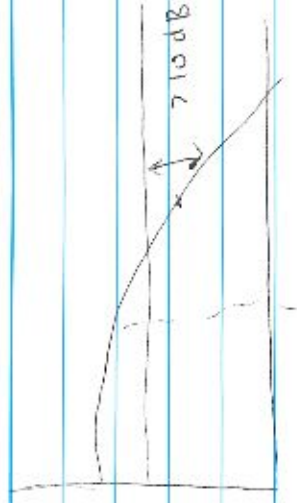
(iii) Non-linearity error

Non-linearity error occurs when the output voltage is different to the linear value and is the difference of the two. On Q4, the non-linearity error measured while the voltage is rising and falling and the values are obtained by noting the difference from the linear value. ✓

(iv) Hysteresis Error

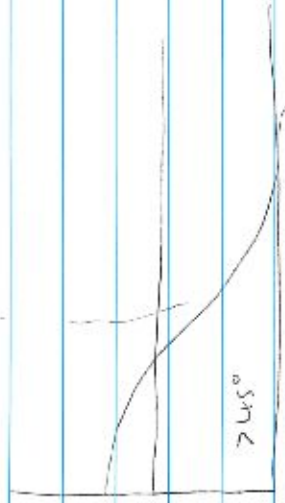
The hysteresis error is the difference between the non-linearity errors of both the rising voltage and the falling voltage. For example, in table Q4, at 4.5 linear volts there is a rising error of 0.23, and a falling error of 0.28,  $0.23 + 0.28 = 0.51V$ . ✓

| Question No. | Examiner Only                                                                                                                                                                                                                                                                                                                                                                                   | Question No. | Rhif y Cwestiwn                                        |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|--------------------------------------------------------|
|              | <p data-bbox="157 1083 199 1291">Question 2</p> <p data-bbox="241 742 346 1291">a) <math>T(s) = \frac{(s)}{1 + (s)(s+1)}</math></p> <p data-bbox="378 623 514 1068"> <math display="block">= \frac{\frac{1}{s(s+1)}}{\frac{s(s+1)}{s(s+1)} + 1}</math> </p> <p data-bbox="535 593 703 1261"> <math display="block">T(s) = \frac{1}{s(s+1) + 1}</math> </p> <p data-bbox="640 578 703 638">X</p> |              |                                                        |
|              | <p data-bbox="157 2804 199 2923">Question No.</p>                                                                                                                                                                                                                                                                                                                                               |              | <p data-bbox="1963 2804 2005 2953">Rhif y Cwestiwn</p> |



gain margin  
If the dB is over  
10, the system is stable

②

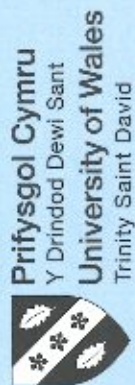


if the phase margin is  
over 45° the system is  
stable

①

c) Since the gain margin is only 4.27 <sup>and</sup> ~~the~~  
the phase margin is  $180 - 168 = 12^\circ$  the system  
is unstable.

⑤



## Examination Answer Books

12 Pages

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SURNAME:

DE DONCKER

FORENAME(S):

Scott

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|---|----------------------------------|-------------------------------|
| 1 | Student Number                   | 1203234/2                     |
| 2 | Title of Examination Paper       | Control and Instrumentation   |
| 3 | Module Code                      | PSDE 5011                     |
| 4 | Date and Time of Examination     | 10/5/18<br>9:30am             |
| 5 | Examination Venue                | Coley Sir Gar<br>Graig Campus |
| 6 | Seat Number                      |                               |
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Question 1

a.

$$U_A (\theta_L(s) - \theta_R(s)) = M_{cp} \frac{d\theta_R}{dt}(t) \quad \checkmark$$

$$U_A \theta_L(s) - U_A \theta_R(s) = M_{cp} \frac{d\theta_R}{dt}(t)$$

$$U_A \theta_L(s) = M_{cp} \frac{d\theta_R}{dt}(t) + U_A \theta_R(s) \quad \checkmark$$

Laplace transforms of both sides of =

$$U_A \theta_L(s) = M_{cp} \times s \theta_R(s) - \theta_R(0) + U_A \theta_R(s)$$

$$-\theta_R(0) = 0$$

$$\therefore U_A \theta_L(s) = M_{cp} \times s \theta_R(s) + U_A \theta_R(s) \quad \checkmark$$

Factorise in terms of  $\theta_R(s)$ 

$$\therefore U_A \theta_L(s) = \theta_R(s) (M_{cp} s + U_A) \quad \checkmark$$

$$H(s) = \frac{\theta_L}{\theta_R} = \frac{U_A}{M_{cp} s + U_A} \quad \times$$

$$H(s) = \frac{1}{s+1} \quad \text{and} \quad \tau = \frac{U_A}{M_{cp}} = \frac{783.13 \times 0.05}{0.1 \times 3250}$$

$$\text{Then } H(s) = \frac{1}{s+1} = \frac{1}{\frac{U_A}{M_{cp}} + 1} = \frac{1}{\frac{783.13 \times 0.05}{0.1 \times 3250} + 1}$$

$$H(s) = \frac{1}{0.125 + 1} \quad \times$$

Answer yn unig

Rhify Cwestiwn

~~$$\Theta_R(s) = \Theta_L(s) \times \frac{\Theta_L(s)}{s}$$~~

~~$$\Theta_R = \frac{1}{0.12s+1} \times \frac{\Theta_L}{s}$$~~

~~$$\Theta_R = \frac{1}{0.12s+1} \times \frac{\Theta_L}{s}$$~~

~~$$\Theta_R = \frac{1}{0.12s+1} \times \frac{\Theta_L}{s}$$~~

~~$$\Theta_R(s) = \int \Theta_R = \int \frac{\Theta_L}{0.12s^2+1} = \Theta_L \int \frac{1}{0.12s^2+1}$$~~

Using inverse Laplace for  $\Theta(t)$

~~$$\Theta_L(s) = \frac{\Theta_L}{s}$$~~

~~$$\Theta_R = H(s) \times \frac{\Theta_L}{s}$$~~

~~$$\Theta_R(s) = \frac{1}{0.12s+1} \times \frac{\Theta_L}{s} = \frac{\Theta_L}{s(0.12s+1)}$$~~

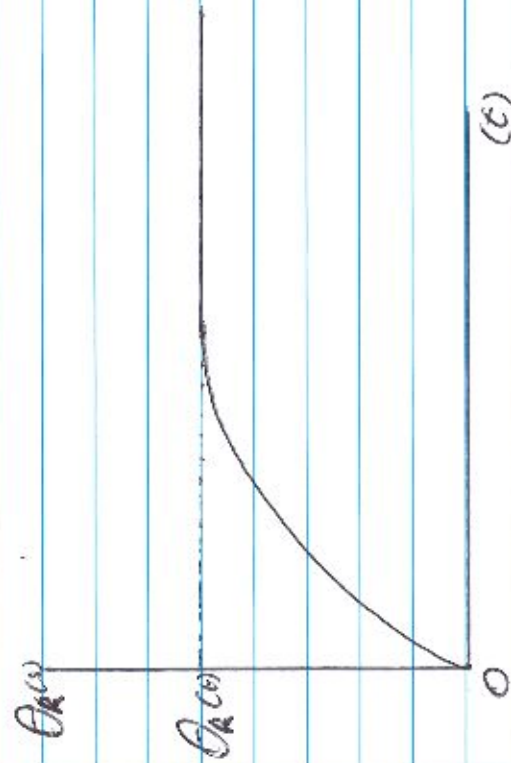
~~$$\Theta_R(s) = \int \Theta_R(s) = \int \frac{\Theta_L}{s(0.12s+1)} = \Theta_L \int \frac{1}{s(0.12s+1)}$$~~

Using inverse Laplace for  $\Theta(t)$

$$\Theta_R(t) = \Theta_L (1 - e^{-\frac{t}{\tau}})$$

$$\Theta_R(t) = \Theta_L (1 - e^{-\frac{0.44t}{0.12}})$$

iii.



$\Theta_R(t)$  shows the point when the dynamic response curve hits critical damping

$$\text{iv. } \Theta_R(t) = \Theta_L (1 - e^{-\frac{0.44t}{0.12}})$$

$$\therefore \Theta_R(25) = 75 (1 - e^{-(0.12 \times 25)})$$

$$\Theta_R(25) = 686.613^\circ \text{C}$$

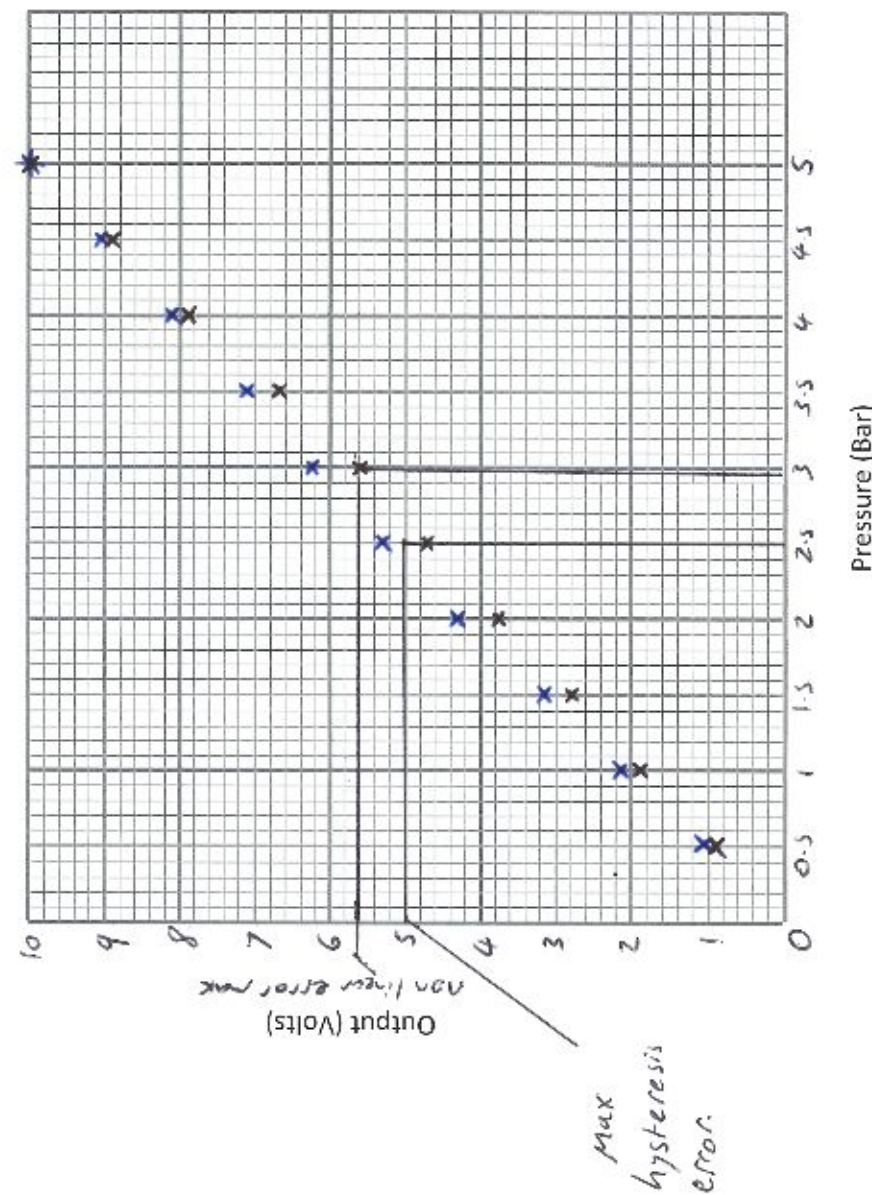
| Question No. | Examiner Only                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | Question No. | Examiner Only | Rhef y Cwestion | Arholwr yn unig |
|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|---------------|-----------------|-----------------|
| b.           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |              |               |                 |                 |
| i            | $T(s) = \frac{G(s)}{1 + H(s)G(s)}$ ✓<br><br>$H(s) = \frac{1}{8.3(0.12s+1)}$ and $G(s) = \frac{1}{120s+1}$<br><i>8.3 correct method</i><br><br>Then $T(s) = \frac{\left(\frac{1}{120s+1}\right) \times \left(\frac{1}{(0.12s+1)}\right)}{1 + \left(\frac{1}{(0.12s+1)}\right) \times \left(\frac{1}{(120s+1)}\right)}$ ✓ <i>correct method</i><br><br>multiply top and bottom by $(0.12s+1)(120s+1)$<br><br>$T(s) = \frac{0.12s+1}{(0.12s+1)(120s+1)+1}$<br><br>Multiply out the brackets<br><br>$T(s) = \frac{0.12s+1}{14.4s^2 + 0.12s + 120s + 1 + 1}$<br><br>$T(s) = \frac{0.12s+1}{14.4s^2 + 120.12s + 2}$<br><br>divide top and bottom by 14.4 to get $s^2$<br><br>$T(s) = \frac{0.008\bar{s} + 0.0694}{s^2 + 8.34s + 0.138}$ ✓ <i>4</i><br><br>$T(s) = \frac{0.0771}{s^2 + 8.34s + 0.138}$<br><br>Compare denominator with $s^2 + 2\zeta\omega_n s + \omega_n^2$ and equate. |              |               |                 |                 |
|              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |              |               |                 |                 |
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| Question 3.  |                                                                                                                                                                                                                                                                                                                                                                                                                                              | 6.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |               |
| a.           |                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |               |
| i.           | $z_1 = -2$ $p_1 = -2 + 3j$ $p_2 = -2 - 3j$ $G(s) = \frac{(s - z_1)}{(s - p_1)(s - p_2)}$ $G(s) = \frac{s - -2}{s - (-2 + 3j) \times s - (-2 - 3j)}$ $G(s) = \frac{2s}{(s + 2 - 3j) \times (s + 2 + 3j)}$ <p>Expand the brackets of denominator</p> $= \frac{2s}{s^2 + 2s + 6 - 9j^2 - 4 + 6j - 4 + 6j - 9j^2}$ $= \frac{2s}{s^2 + 2s + 13}$ $G(s) = \frac{2s}{s^2 + 4s + 13}$ <p>open loop T.F. <math>G(s)</math></p>                        | $K_1 = H(s)$ $T(s) = \frac{G(s)}{1 + H(s) \times G(s)}$ $T(s) = \frac{2s}{s^2 + 4s + 13} \times K_1$ <p>Multiply top and bottom by <math>s^2 + 4s + 13</math></p> $T(s) = \frac{2s}{s^2 + 4s + 13 + 2s \times K_1} = \frac{2s}{s^2 + 6s + 13 + K_1}$ <p>Natural Frequency = <math>\omega_n</math></p> <p>Consider <math>s^2 + 2\zeta\omega_n s + \omega_n^2</math></p> $\omega_n^2 = 13 \quad \therefore \omega_n = \sqrt{13} = 3.606 \text{ rad/s}$ $\zeta = \frac{4}{2 \times 3.606} = 0.55 \text{ underdamped}$ <p>form = 184</p> |               |
| ii.          | <p>Equating <math>G(s)</math> denominator with <math>s^2 + 2\zeta\omega_n s + \omega_n^2</math></p> $\omega_n^2 = 13 \quad \therefore \omega_n = \sqrt{13} = 3.606 \text{ rad/s}$ <p>for damping factor of <math>\zeta</math></p> $2\zeta\omega_n = 4 \quad \therefore \zeta = \frac{4}{2 \times \omega_n} = \frac{4}{2 \times 3.606} = 0.555$ <p><math>\zeta</math> is 0.555 which is less than 1, therefore the system is underdamped.</p> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |               |
| iii.         |                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |               |

Table Q4

| Measured Pressure (Bar) | Output voltage (Volts) |         | Linear Value (Volts) | Non-linearity error (Volts) |         | Hysteresis Error (Volts) |
|-------------------------|------------------------|---------|----------------------|-----------------------------|---------|--------------------------|
|                         | rising                 | falling |                      | rising                      | falling |                          |
| 0                       | 0                      | 0       | 0                    | 0                           | 0       | 0                        |
| 0.5                     | 0.96                   | 1.03    | 1                    | -0.04                       | 0.03    | 0.07                     |
| 1.0                     | 1.89                   | 2.12    | 2                    | -0.11                       | 0.12    | 0.23                     |
| 1.5                     | 2.81                   | 3.17    | 3                    | -0.14                       | 0.17    | 0.36                     |
| 2.0                     | 3.77                   | 4.28    | 4                    | -0.23                       | 0.28    | 0.51                     |
| 2.5                     | 4.68                   | 5.33    | 5                    | -0.32                       | 0.33    | 0.65                     |
| 3.0                     | 5.61                   | 6.25    | 6                    | -0.39                       | 0.25    | 0.64                     |
| 3.5                     | 6.73                   | 7.16    | 7                    | -0.27                       | 0.16    | 0.43                     |
| 4.0                     | 7.88                   | 8.10    | 8                    | -0.12                       | 0.10    | 0.22                     |
| 4.5                     | 8.94                   | 9.05    | 9                    | -0.06                       | 0.05    | 0.11                     |
| 5.0                     | 10.0                   | 10.0    | 10                   | 0                           | 0       | 0                        |

Graph for calibration results for QUESTION 4.



THIS PAGE MUST BE DETACHED FROM THE QUESTION PAPER  
AND INSERTED INTO YOUR ANSWER BOOKLET IT YOU  
ATTEMPT QUESTION 4.

## Question 4

iii. Non linearity voltage error - the deviation, either rising or falling, of the output voltage from the inputted linear voltage ✓

iv. Hysteresis error - the combination of both rising and falling non linearity errors ✓

b. \* See attached sheet \*

maximum non linearity error = 0.39V falling from 6V

maximum hysteresis error = 0.65V at 5V

total = 21



SURNAME: Beef

FORENAME(S): Joshua

3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.

1. Please give the information requested in the boxes below.
  2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
  3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
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  5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
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|   |                                  |                         |
|---|----------------------------------|-------------------------|
| 1 | Student Number                   | 120326212               |
| 2 | Title of Examination Paper       | Control + Inse          |
| 3 | Module Code                      | PDSE S011               |
| 4 | Date and Time of Examination     | 10/5/18<br>9:30 - 11:30 |
| 5 | Examination Venue                |                         |
| 6 | Seat Number                      |                         |
| 7 | Number of Answer Books handed in |                         |

[illegible]

Gweler yr ochr arall am y fersiwn Cymraeg)

|   |           |
|---|-----------|
| 2 | 72½ = 71% |
|---|-----------|

$$1a) i) UA(\theta_L(t) - \theta_R(t)) = M_C P \frac{d\theta_R(t)}{dt}$$

$$= UA\theta_L(t) - UA\theta_R(t) = M_C P \frac{d\theta_R(t)}{dt}$$

$$= UA\theta_L(t) = M_C P \frac{d\theta_R(t)}{dt} + UA\theta_R(t) \quad \checkmark$$

$$\theta_R(0) = 0 \quad \text{and} \quad \dot{\theta}_R = 0$$

same Laplace of both sides

$$= UA\theta_L(s) = M_C P (s\theta_R(s) - \theta_R(0)) + UA\theta_R(s)$$

$$= UA\theta_L(s) = M_C P s\theta_R(s) + UA\theta_R(s) \quad \checkmark$$

$$= UA\theta_L(s) = \theta_R(s) (M_C P s + UA) \quad \checkmark$$

$$\frac{\theta_R(s)}{\theta_L(s)} = \frac{UA}{M_C P s + UA} \quad \checkmark \quad \div \text{ by } UA$$

$$= \frac{1}{\frac{M_C P s}{UA} + 1}$$

$$H(s) = \frac{1}{\tau_s + 1}$$

$$\tau_s = \frac{M_C P s}{UA} \quad \text{secs} \quad \checkmark$$

$$= \frac{0.1 \times 3250}{785.15 \times 0.05}$$

$$= 0.83 \text{ secs} \quad \times$$

$$H(s) = \frac{1}{0.83s + 1} \quad \times \quad \checkmark$$

②

②

①

①

1/2

1/2



| Question No.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | Examiner Only                                                                                                                                                                       | Question No.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Examiner Only                                                                                                                                                                       |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $y_{ss} = \lim_{t \rightarrow \infty} \{y(t)\}$<br>$= \lim_{s \rightarrow 0} \{s Y(s)\}$<br>$Y(s) = R(s) \times T(s)$<br>$R(s) = \frac{75}{s}$<br>$T(s) = \frac{8.3 \times 10^{-3} s + 0.01}{s^2 + 1.21 s + 0.02}$<br>$Y(s) = \frac{75}{s} \left( \frac{8.3 \times 10^{-3} s + 0.01}{s^2 + 1.21 s + 0.02} \right)$<br>$y_{ss} = \lim_{s \rightarrow 0} \left\{ 8 \times \frac{75}{s} \left( \frac{8.3 \times 10^{-3} s + 0.01}{s^2 + 1.21 s + 0.02} \right) \right\}$<br>$= 75 \left( \frac{0.01}{0.02} \right)$<br>$= 37.5^\circ C$<br>$\text{Total} = 25\frac{1}{2}$ | <p>3a) <math>G(s) = \frac{(s-2)(s-2)}{(s-1)(s-1)}</math></p> <p><math>Z_1 = -2</math><br/> <math>Z_2 = 0</math><br/> <math>P_1 = -2 + j3</math><br/> <math>P_2 = -2 - j3</math></p> | <p>3a) <math>G(s) = \frac{(s-(-2))}{(s-(-2+j3))(s-(-2-j3))}</math></p> <p><math>= \frac{s+2}{(s+2+j3)(s+2-j3)}</math></p> <p><math>= \frac{s+2}{s^2+2s+4-j6+j6s+j^2 9}</math></p> <p><math>= \frac{s+2}{s^2+4s+13}</math></p> <p>ii) equate to <math>s^2+2\zeta\omega_n s + \omega_n^2</math></p> <p><math>\omega_n^2 = 13</math>   <math>\omega_n = \sqrt{13}</math>   <math>\omega_n = 3.61 \text{ rad/s}</math></p> <p><math>2\zeta\omega_n = 4</math>   <math>\zeta = \frac{4}{2(\omega_n)} = 0.55</math> ✓</p> <p><math>1 &gt; 0.55</math> so system is underdamped</p> <p>b) <math>G(s) = \frac{s+2}{s^2+4s+13}</math>   <math>H(s) = K_1</math></p> <p><math>T(s) = \frac{G(s)}{1+H(s)G(s)}</math></p> <p><math>= \frac{s+2}{1 + \frac{(s+2)(K_1)}{s^2+4s+13}}</math></p> <p><math>= \frac{s+2}{\frac{s^2+4s+13 + K_1(s+2)}{s^2+4s+13}}</math></p> <p><math>= \frac{s+2}{s^2+4s+13 + (s+2)K_1}</math></p> <p><math>= \frac{s+2}{s^2+4s+13 + K_1s + 2K_1}</math></p> | <p>3a) <math>G(s) = \frac{(s-2)(s-2)}{(s-1)(s-1)}</math></p> <p><math>Z_1 = -2</math><br/> <math>Z_2 = 0</math><br/> <math>P_1 = -2 + j3</math><br/> <math>P_2 = -2 - j3</math></p> |

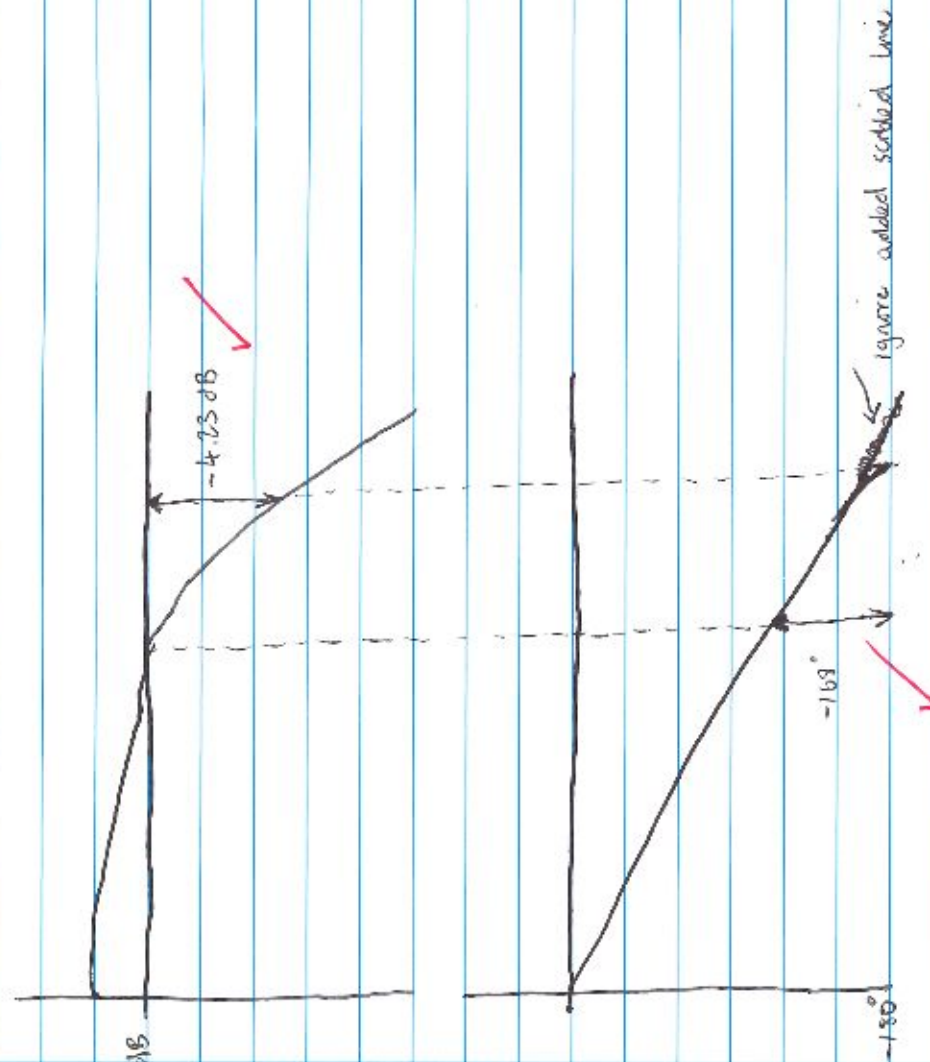
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Question No.

26)

0° & 0dB



$$10 \log 10 = 10$$

Examiner Only

Question No.

3

4

Examiner Only

Arholwr yn unig

Arholwr yn unig

Rhify Cwestiwn

Rhify Cwestiwn

## Examination Answer Books

12 Pages

### Instructions to Candidates

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SURNAME: HARRIES

FORENAME(S): DAVID SAMUEL

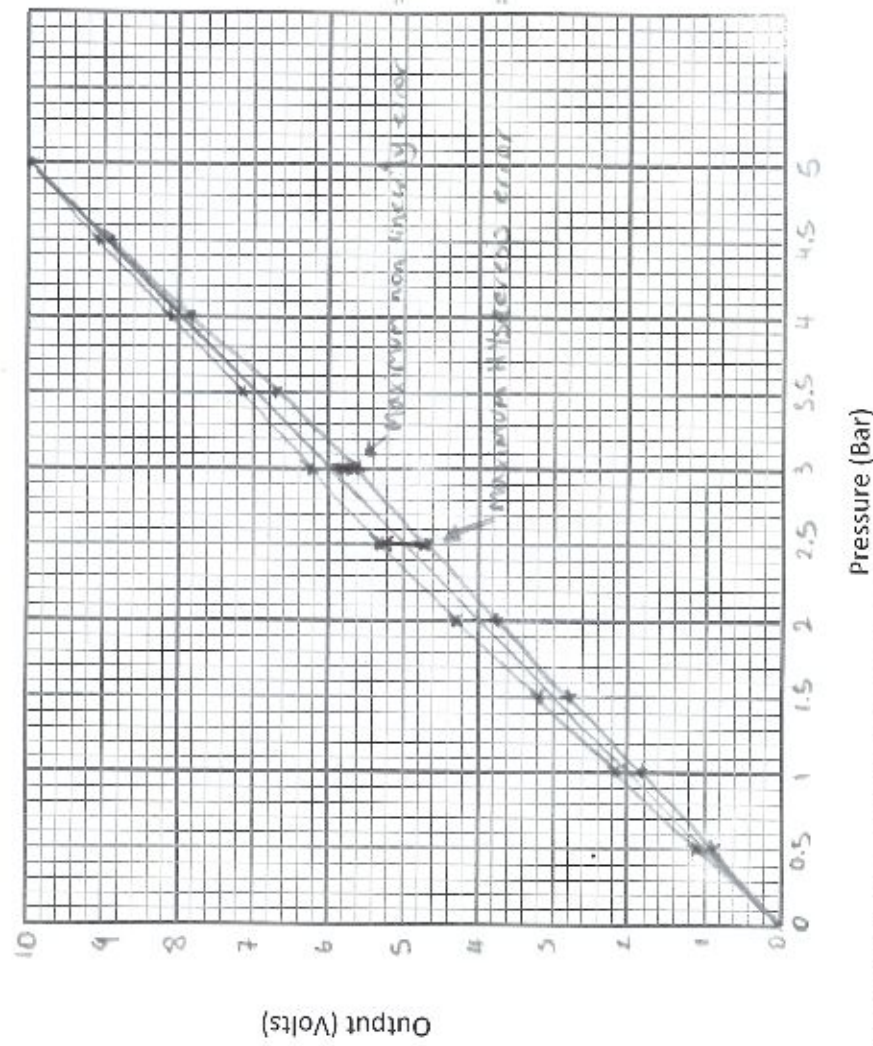
|   |                                  |                          |                                                      |                            |
|---|----------------------------------|--------------------------|------------------------------------------------------|----------------------------|
| 1 | Student Number                   | 1502947 / 1              |                                                      |                            |
| 2 | Title of Examination Paper       | CONTROL AND INSTRUMENTAL | List the numbers of the questions you have attempted | Q2 29<br>Q3 1<br>Q4 30 1/2 |
| 3 | Module Code                      | PSDE9011                 |                                                      |                            |
| 4 | Date and Time of Examination     | 10/5/18                  |                                                      |                            |
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| 6 | Seat Number                      |                          |                                                      |                            |
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Table Q4

| Measured Pressure (Bar) | Output voltage (Volts) |         | Linear Value (Volts) | Non-linearity error (Volts) |         | Hysteresis Error (Volts) |
|-------------------------|------------------------|---------|----------------------|-----------------------------|---------|--------------------------|
|                         | rising                 | falling |                      | rising                      | falling |                          |
| 0                       | 0                      | 0       | 0                    | 0                           | 0       | 0                        |
| 0.5                     | 0.96                   | 1.03    | 1                    | -0.04                       | 0.03    | 0.07                     |
| 1.0                     | 1.89                   | 2.12    | 2                    | -0.11                       | 0.12    | 0.23                     |
| 1.5                     | 2.81                   | 3.17    | 3                    | -0.19                       | 0.17    | 0.36                     |
| 2.0                     | 3.77                   | 4.28    | 4                    | -0.23                       | 0.28    | 0.51                     |
| 2.5                     | 4.68                   | 5.33    | 5                    | -0.32                       | 0.33    | 0.65                     |
| 3.0                     | 5.61                   | 6.25    | 6                    | -0.39                       | 0.25    | 0.64                     |
| 3.5                     | 6.73                   | 7.16    | 7                    | -0.27                       | 0.16    | 0.43                     |
| 4.0                     | 7.88                   | 8.10    | 8                    | -0.12                       | 0.10    | 0.22                     |
| 4.5                     | 8.94                   | 9.05    | 9                    | -0.06                       | 0.05    | 0.11                     |
| 5.0                     | 10.0                   | 10.0    | 10                   | 0                           | 0       | 0                        |

Graph for calibration results for QUESTION 4.

1 small square = 0.2 V



Pressure (Bar)

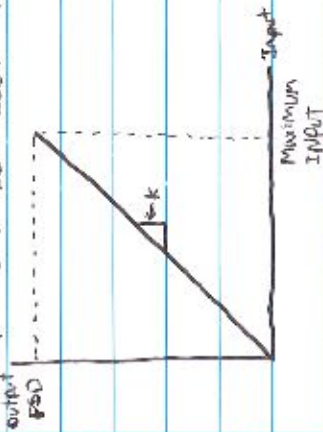
THIS PAGE MUST BE DETACHED FROM THE QUESTION PAPER  
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4a) FSD: Full scale Deflection is the highest output value for a transducer.

i) Static Sensitivity = Output Range of Transducer

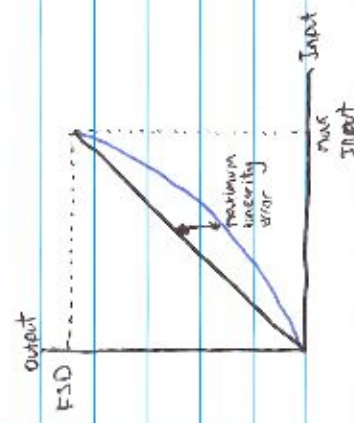
Input Range of Measurand

Static sensitivity can be defined as the rate at which the output of a transducer changes in relation to the input at a static state. This can be seen in the graph below:



The gradient of the linear relationship is the static sensitivity.

ii) Non-linearity error. An ideal transducer would have a linear relationship between its output and the measurand's input. However, in practicality transducers do not have a linear relationship, and instead have linearity error. This is illustrated below:



The black line represents an unrealistic linear transducer and the blue line represents a realistic transducer's linearity error.

Max linearity error = Transducer output - linear value

Linear value = static sensitivity  $\times$  T + V<sub>0</sub>

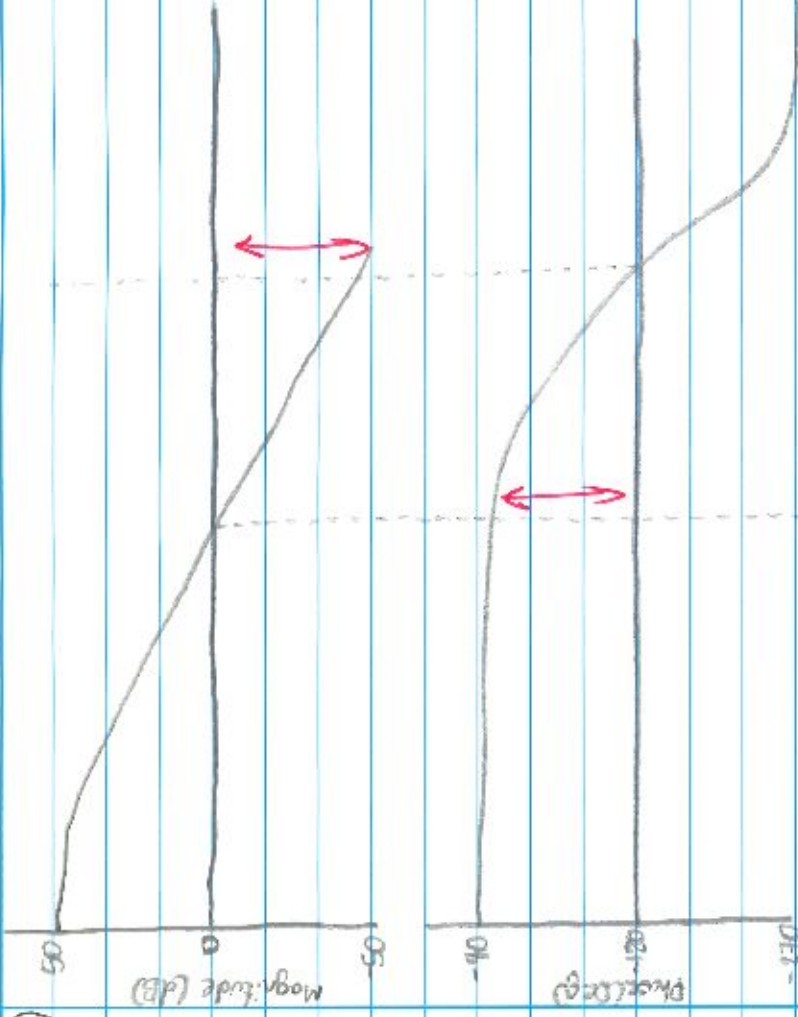
Non linearity error can be caused by badly calibrated transducers, poor signal conditioning and hysteresis.

iv) Hysteresis error is introduced when a value has been previously measured.

This happens when the input measurand changes. This can be seen illustrated on the next page.

| Question No.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | Examiner Only | Question No.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Examiner Only                                          |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------|
| <p>Output</p> <p>As the input increases and decreases, the output from the transducer deviates from an accurate linear value and starts providing different values for the same previously measured input value.</p> <p>b) Maximum non-linearity error = <math>-0.34V</math><br/> <math>\% \text{ maximum non-linearity error} = \frac{\text{max error} \times 100}{\text{FSO}} = \frac{0.34 \times 100}{10} = 3.4\%</math></p> <p>Maximum Hysteresis error = <math>0.65V</math><br/> <math>\% \text{ maximum Hysteresis error} = \frac{0.65 \times 100}{10} = 6.5\%</math></p> <p><u>TOTAL = 30.5</u></p> | <p>(1/2)</p>  | <p>Q2 a) <math>Y(s) = G(s) E(s)</math><br/> <math>E(s) = R(s) - H(s) Y(s)</math><br/> <math>Y(s) = G(s) R(s) - G(s) H(s) Y(s)</math><br/> <math>Y(s) + G(s) H(s) Y(s) = G(s) R(s)</math><br/> <math>Y(s) (1 + G(s) H(s)) = G(s) R(s)</math><br/> <math>Y(s) = \frac{G(s)}{1 + G(s) H(s)}</math></p> <p>Without Velocity Feedback:<br/> <math>G(s) = \frac{9}{s(s+4)}</math> <math>H(s) = 1</math> <math>\therefore Y(s) = \frac{9}{R(s) (s^2 + 4s)}</math><br/>         Multiply by <math>s(s+4)</math>:<br/> <math>Y(s) = \frac{9}{R(s) (s^2 + 4s + 9)}</math><br/> <math>2\zeta\omega_n = 4</math> <math>\therefore \zeta = \frac{4}{2\omega_n} = \frac{4}{2 \times 3} = 0.6</math> damping factor.<br/> <math>\omega_n = \sqrt{9} = 3</math> <math>\therefore \zeta &lt; 1</math> and <math>\zeta &gt; 0</math> therefore underdamped.</p> <p>With velocity feedback:<br/> <math>G(s) = \frac{9}{s(s+4)}</math> <math>H(s) = \frac{3}{s}</math> <math>\therefore Y(s) = \frac{9}{R(s) (s^2 + 7s + 9)}</math><br/> <math>2\zeta\omega_n = 7</math> <math>\therefore \zeta = \frac{7}{2\omega_n} = \frac{7}{2 \times 3} = 1.16</math> therefore overdamped.<br/> <math>\omega_n = 3</math></p> <p>overdamped step response:  <br/>         underdamped step response: </p> | <p>(3)</p> <p>(3)</p> <p>(6)</p> <p>(3)</p> <p>(1)</p> |

b)



Gain margin is when a comparison of a system is done between gain and phase when the gain is at 0 dB. A system can be considered stable as long as the phase is above  $-180^\circ$ . A system with a phase shift greater than  $-180^\circ$  is unstable. The gain margin is to ensure a system is not close to being unstable. Therefore anything with a phase shift greater than  $-135^\circ$  is close to becoming unstable and has passed the gain margin.

Likewise, the phase margin can be used to determine the stability of a system by looking at the gain at a  $-180^\circ$  phase shift. If the gain is negative the system is stable and if the gain is positive the system is unstable.  $-3 \text{ dB}$  is in danger of becoming unstable.

At 0 dB, the phase shift is  $-168^\circ$ . Therefore the system is stable. However it is dangerously close to being unstable. At  $-180^\circ$ , the gain is  $-4.23 \text{ dB}$ . It is negative, therefore again the system is stable. However it is again getting close to instability. Both gain and phase margins show a stable system.

3a)  $\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$   $H(s) = 1$

$\frac{Y(s)}{R(s)} = \frac{K \omega_n}{s^2 + 2\zeta \omega_n s + \omega_n^2}$  Natural frequency  $\omega_n = \sqrt{4}$  from  $2\zeta \omega_n$  damping factor  $= 2$

i)  $(s+3)(s-3) = s^2 - 3s + 3s - 9$

$\frac{Y(s)}{R(s)} = \frac{1}{s^2 - 9}$   $\omega_n = \sqrt{9} = 3 \text{ rad/s}$   $\zeta = 0$

c)  $\Delta = \ln \left( \frac{\% \text{ steady state error}}{\pi} \right) \sqrt{\frac{1 - \zeta^2}{\zeta^2}}$   $\% OS = 100 e^{\frac{-\zeta \pi}{\sqrt{1 - \zeta^2}}}$

X

①

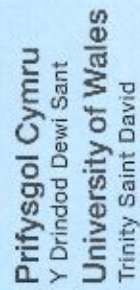
④

③

②

③

Score = 29



## 12 Pages

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8. **Every book must be handed in. In no circumstances may be taken out of the examination venue.**

**Complete, fold and seal**  
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SURNAME: Smith

FORENAME(S): Nathan, John

|   |                                  |                             |
|---|----------------------------------|-----------------------------|
| 1 | Student Number                   | 1103912/2                   |
| 2 | Title of Examination Paper       | Control and Instrumentation |
| 3 | Module Code                      | PSDE 5011                   |
| 4 | Date and Time of Examination     | 10/5/18, 9:30               |
| 5 | Examination Venue                | Gym                         |
| 6 | Seat Number                      |                             |
| 7 | Number of Answer Books handed in |                             |

[illegible]

(Gweler yr ochr arall am y fersiwn Cymraeg)

$$\frac{\theta_L \times \theta_R}{s} = \frac{\theta_L \times \theta_R}{s}$$

Examiner Only

Question No.

$$100e^{-\frac{2\pi}{(1-s)^2}}$$

① a) i)  $UA(\theta_L(t) - \theta_R(t)) = MCP \frac{d\theta_R(t)}{dt}$

$$\frac{1}{MCPs+1}$$

$$UA \times \theta_L(t) - UA \theta_R(t) = MCP \frac{d\theta_R(t)}{dt}$$

$$UA \theta_L(t) = MCP \frac{d\theta_R(t)}{dt} + UA \theta_R(t)$$

② ✓

Take Laplace transform of both.

$$UA \theta_L(s) = MCP (s \theta_R(s) - \theta_R(0)) + UA \theta_R(s)$$

$$\theta_R(0) = 0$$

$\frac{1}{2}$

$$s \text{ So } UA \theta_L(s) = MCP s \theta_R(s) + UA \theta_R(s)$$

③ ✓

$$UA \theta_L(s) = \theta_R(s) (MCPs + UA)$$

$$+1(s) = \frac{\theta_R(s)}{\theta_L(s)} = \frac{UA}{MCPs + UA}$$

By OA to give  $\frac{1}{MCPs + 1} + 1 = \frac{1}{Ts + 1}$

$$T = \frac{MCP}{UA} = \frac{0.1 \times 3250}{78313 \times 0.05} = 8.300$$

④ ✓

$$H(s) = \frac{1}{8.3s + 1}$$

⑤ ✓

Answer yn unig

Rhif y Cwestiwn

a) ii)  $OLC = HS \times \frac{OL}{S}$  ①

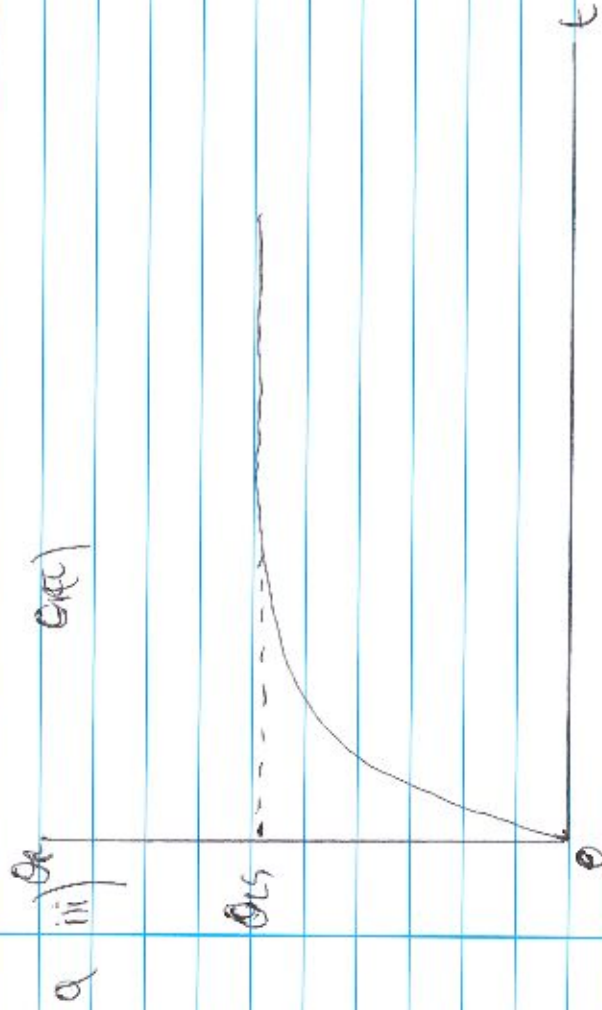
$OL(S) = (1 - e^{-t/\tau})$

$OL(t) = \frac{1}{8.35+1} \times \frac{OL}{S}$

$\frac{OL}{S(8.35+1)}$

$OL(S) \left( \frac{OL}{S(8.35+1)} \right) = \left[ OL \left( \frac{1}{S(8.35+1)} \right) \right]$

$OL(S) = OL \left( 1 - e^{-t/\tau} \right) = OL \left( 1 - e^{-t/(8.35)} \right)$  ①



$OL$  = Temperature of the medium being measured.

$OL_s$  = Temperature of the probe (std)

The dynamic response is in a linear shape showing a positive steady rise of the medium being measured against time.

$OL \times (1 - e^{-t/\tau})$

$OL \times (1 - e^{-25/8.3})$

$75 \times (1 - e^{-25/8.3}) = \underline{71.3^\circ C}$  ③

b) i)  $T(S) = \frac{G(S)}{(1 \times G(S) \times H(S))}$  ✓

$G(S) = \frac{1}{(120s+1)}$  ✓  $H(S) = \frac{1}{(8.35s+1)}$  ✓

$T(S) = \frac{1}{\frac{1}{(120s+1)} \times \frac{1}{(8.35s+1)} \times \left( \frac{1}{8.35s+1} \right)}$

Simplify  $= \frac{(8.35s+1)}{(120s+1)(8.35s+1) \times 1}$  ①

$\frac{(8.35s+1)}{996s^2 + 120s + 8.35s + 1 + 1}$

$= \frac{(8.35s+1)}{996s^2 + 128.35s + 2}$

divide by 996 to get  $S^2 + 23.027s + 0.002$

$= \frac{0.00835s + 0.001}{S^2 + 0.12835s + 0.002}$  ✓

NEXT PAGE! →

| Question No.                                                                                                                                                                                                                                                                                                                                                                                   | Examiner Only         | Question No.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Examiner Only         |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------|
| <p>equale with formulae <math>s^2 + 2\zeta\omega_n s + \omega_n^2</math></p> <p><math>= \omega_n^2 = 0.002</math> <math>\omega_n = \sqrt{0.002} = 0.045</math> ✓</p> <p><math>2\zeta\omega_n = 0.1289</math> <math>\zeta = \frac{0.1289}{2 \times 0.045} = 1.432</math> ✓</p> <p>As the value is 1.432, this is over the value of 1 for critical damping meaning the system is overdamped.</p> | <p>③ ✓</p>            | <p>② a) without <math>= T(s) = \frac{G(s)}{1 + G(s) + 1(s)}</math></p> <p><math>G(s) = \frac{9}{s(s+4)}</math> <math>+1(s) = 1</math></p> <p><math>T(s) = \frac{9}{s(s+4) + 1 \times 9}</math></p> <p><math>\frac{9}{s^2 + 4s + 9}</math> ✓</p> <p>equale with <math>s^2 + 2\zeta\omega_n s + \omega_n^2</math> ✓</p> <p><math>\omega_n^2 = 9</math> <math>\omega_n = \sqrt{9} = 3</math> ✓</p> <p><math>2\zeta\omega_n = 4</math> <math>\zeta = \frac{4}{2 \times 3} = 0.6</math> ✓</p> <p>0.6 = underdamped without velocity feedback. ✓</p> <p>with velocity feedback <math>T(s) = \frac{G(s)}{1 + G(s) + 1(s)}</math></p> <p><math>G(s) = \frac{9}{s(s+4)}</math> <math>+1(s) = \frac{s}{3}</math></p> <p>so <math>T(s) = \frac{9}{s(s+4) + \frac{s}{3}}</math> ✓</p> | <p>Examiner Only</p>  |
| <p>b) ii) <math>y_{ss} = (s) y(s)</math> <math>y_s = 0.4 \times (T(s))</math></p> <p><math>T(s) = \frac{7s}{s}</math> ✓</p> <p><math>y(s) = \left( \frac{7s}{s} \times \frac{0.0083s + 0.001}{s^2 + 0.1289s + 0.002} \right)</math></p> <p><math>y_{ss} = \frac{75 \times 0.001}{0.002} = 37.5^\circ\text{C}</math> ✓</p> <p><math>100m = 25\frac{1}{2}</math></p>                             | <p>① ✓</p> <p>③ ✓</p> | <p>Answer yn unig</p> <p>Rhify Cwestiwn</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | <p>Rhify Cwestiwn</p> |

$$= \frac{9}{s(s+4) + \frac{9s}{3}}$$

$$= \frac{9}{s^2 + 4s + \frac{9s}{3}} = \frac{9}{s^2 + 4.3s + 3}$$

equale with outside feedback and for unity feedback

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad Gs = \frac{9}{s^2 + 4.3s} \quad Hs = 1$$

$$= \frac{9}{s^2 + 4.3s} = \frac{9}{s^2 + 4.3s + 1 \times 9}$$

$$= \frac{9}{s^2 + 4.3s + 9} \quad \text{Equate with } s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$= \omega_n^2 = 9 \quad \omega_n = \sqrt{9} = 3$$

$$2\zeta\omega_n = 4.3 \quad \zeta = \frac{4.3}{2 \times 3} = 0.716$$

The System is still underdamped at  $\zeta = 0.716$ , but the damping has improved with the input of the velocity feedback on the System. as  $\zeta$  is nearer to 1.

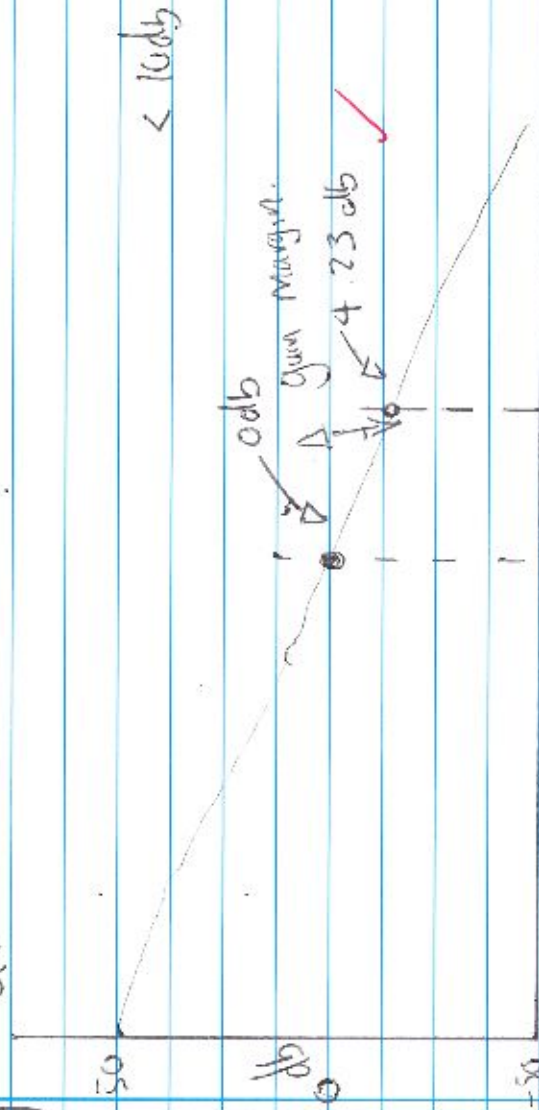
correct  
wrong  
point  
marked

10db

74.5°

GAIN

b)



&lt; 10db

Gain Margin

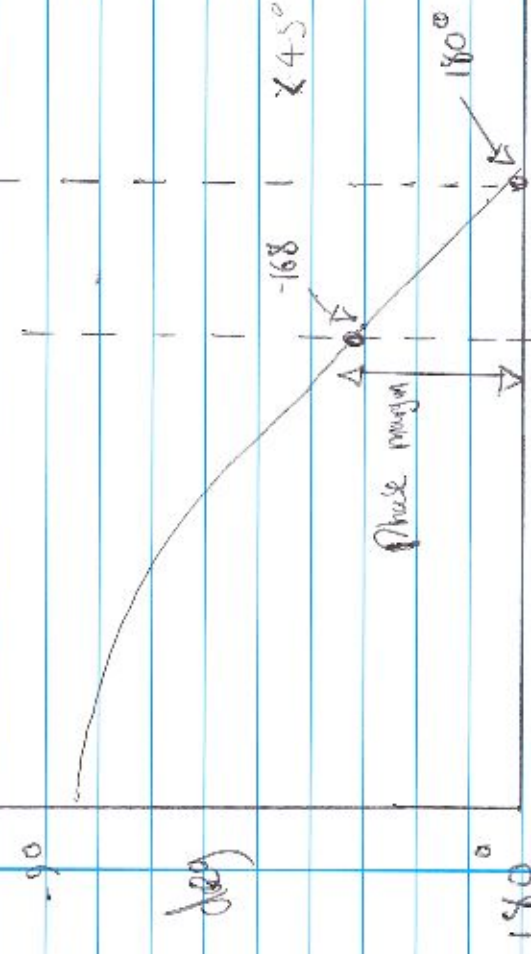
4.23 db

5

Phase

deg

3



-168

Phase margin

180°

3

Phase margin will =  $180 - 168 = 12^\circ$ , as the magnitude of the gain = 0 dB the phase margin below is measured from the straight line down which gave  $-168^\circ$ , this is taken away from  $180^\circ$  to give the phase margin. The phase margin needs to be under  $4.5^\circ$  to be unstable.

gain margin is taken at the point the line from the phase diagram reaches  $180^\circ$ , this is linked up with the top gain graph to measure the 'db' magnitude. It read 4.23 dB which is under 10 dB making it unstable.

TOTAL = 30.5

C

$$T_o = \frac{0.66}{1 + 0.66 \text{ HCS}}$$

- ④ a) FSD = full scale deflection, this is the difference between the range of the transducer. So the range is the maximum and minimum values the transducer can measure. So with the g table Q4 the FSD would be the difference between 0-5 Bar for the measured pressure or 0-10V for the linear value. ✓

- Static Sensitivity is the <sup>range</sup> output of the transducer input of the transducer. ✓

So 5/0 or 10/0? X

- Non linearity error is the difference from for rising and falling values against the linear voltage against the output voltage. The non linearity error will give plots to plot a graph to work out hysteresis error. In table Q4 the non linearity error is the difference for the rising of 1 volt linear volt, minus the output voltage rising of 0.96V to give -0.04, and falling would be ~~1.03~~ - 1 = 0.03 (falling). ✓

- Hysteresis error is the falling non linearity error minus the rising non linearity error. This gives a calculated value for the hysteresis error from the two non-linearity errors.

% FSD is done by  $\frac{\text{max non-linearity}}{\text{max hysteresis}} \div 10 \times 100$

max non linearity = 0.03 X,  $0.03 \div 10 \times 100 = 3.3\%$  FSD X  
 max hysteresis error = 0.05,  $0.05 \div 10 \times 100 = 5.5\%$  FSD ✓  
 Graph inserted behind !! X

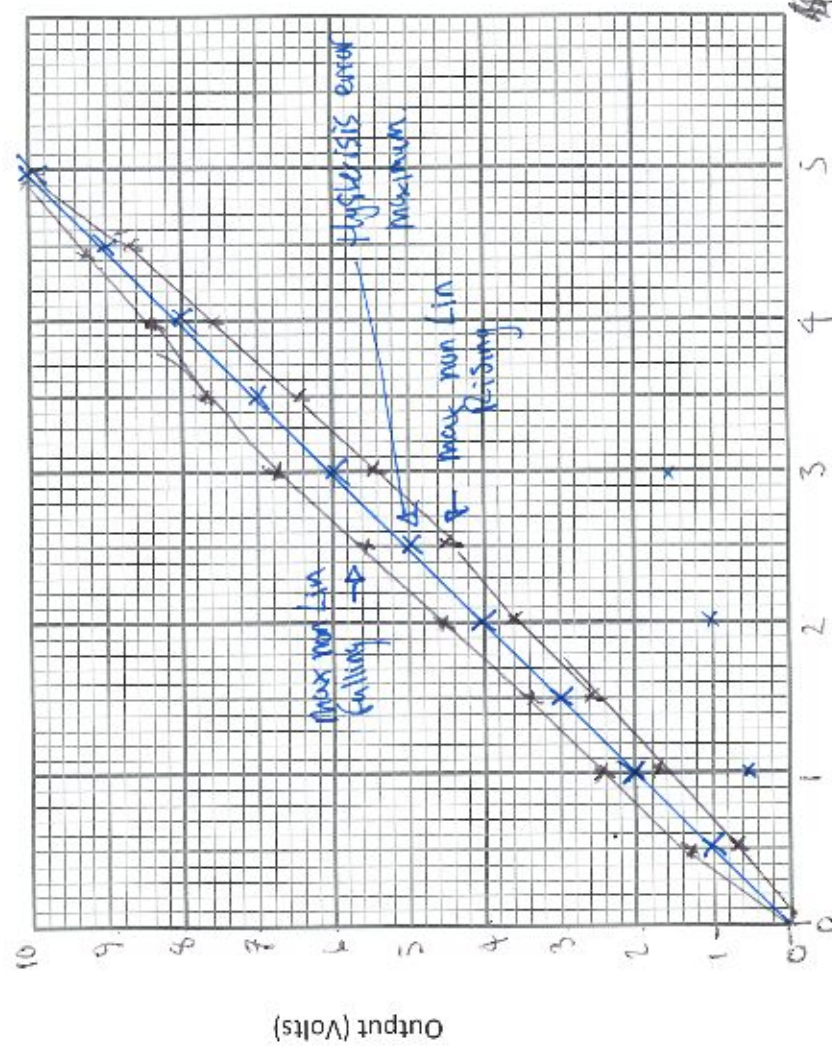
% FSD = max  $\div 10 \times 100$

100% = 31

Table Q4

| Measured Pressure (Bar) | Output voltage (Volts) |         | Linear Value (Volts) | Non-linearity error (Volts) |         | Hysteresis Error (Volts) |
|-------------------------|------------------------|---------|----------------------|-----------------------------|---------|--------------------------|
|                         | rising                 | falling |                      | rising                      | falling |                          |
| 0                       | 0                      | 0       | 0                    | 0                           | 0       | 0                        |
| 0.5                     | 0.96                   | 1.03    | 1                    | -0.04                       | 0.03    | 0.07                     |
| 1.0                     | 1.89                   | 2.12    | 2                    | -0.11                       | 0.12    | 0.23                     |
| 1.5                     | 2.81                   | 3.17    | 3                    | -0.19                       | 0.17    | 0.36                     |
| 2.0                     | 3.77                   | 4.28    | 4                    | -0.23                       | 0.28    | 0.51                     |
| 2.5                     | 4.68                   | 5.33    | 5                    | -0.32                       | 0.33    | 0.65                     |
| 3.0                     | 5.61                   | 6.25    | 6                    | -0.39                       | 0.25    | 0.64                     |
| 3.5                     | 6.73                   | 7.16    | 7                    | -0.27                       | 0.16    | 0.43                     |
| 4.0                     | 7.88                   | 8.10    | 8                    | -0.12                       | 0.1     | 0.22                     |
| 4.5                     | 8.94                   | 9.05    | 9                    | -0.06                       | 0.05    | 0.11                     |
| 5.0                     | 10.0                   | 10.0    | 10                   | 0                           | 0       | 0                        |

Graph for calibration results for QUESTION 4.



THIS PAGE MUST BE DETACHED FROM THE QUESTION PAPER  
AND INSERTED INTO YOUR ANSWER BOOKLET IT YOU  
ATTEMPT QUESTION 4.