



Complete, fold and seal
(see note 2 below)

12 Pages

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1. Please give the information requested in the boxes below.
2. Enter your name in the space provided to the right, fold over and seal down (using the label provided) to hide your name.
3. The answers must be legibly written. The Examiners can either ignore illegible script or have it typed and the expense of typing charged to the candidate.
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5. Begin each answer on a fresh page. Do all rough work in the answer book and cross it through.
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1	Student Number	2111222
2	Title of Examination Paper	Further engineering mathematics
3	Module Code	SGEES001
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5	Examination Venue	Coleg Sir Gar
6	Seat Number	2
7	Number of Answer Books handed in	1

[illegible]

67%

(Gweler yr ochr arall am y fersiwn Cymraeg)

①
a)

$$4 \frac{dy}{dx} = 8x^2 + 2$$

$$\int 4 \frac{dy}{dx} dx = \int (8x^2 + 2) dx$$

$$4y = \frac{8x^3}{3} + 2x + C$$

$$y = \frac{2}{3}x^3 + \frac{1}{2}x + C \quad \checkmark \quad 6$$

$$0 = \frac{2}{3}(2)^3 + \frac{1}{2}(2) + C \quad \begin{array}{l} y=2 \text{ when } x=0 \\ \text{Not } y=0 \text{ when } x=2 \end{array}$$

$$0 = \frac{16}{3} + 1 + C$$

$$-\frac{19}{3} = C$$

Method correct

$$-6.33 = C$$

$$y = \frac{2}{3}x^3 + \frac{1}{2}x - 6.33$$

2
8/10 Part a

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4(\frac{2}{3})(-6.33)}}{2(\frac{2}{3})}$$

$$\frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4(\frac{2}{3})(-6.33)}}{2(\frac{2}{3})}$$

$$\frac{dy}{dx} = \frac{8}{3}x^2 + \frac{1}{2}$$

$$-6 \pm \sqrt{0^2 - 4(\frac{8}{3})(\frac{1}{2})}$$

$$b) \quad x^2 \frac{dy}{dx} + xy = x^3$$

$$\frac{dy}{dx} + \frac{xy}{x^2} = \frac{x^3}{x^2}$$

$$\frac{dy}{dx} + \frac{1}{x} y = x \quad \checkmark \quad 2$$

$$\int \frac{1}{x} dx = \ln x + c \quad \text{Constant but not needed}$$

$$y e^{\ln x + c} = \int e^{\ln x + c} x dx$$

$$y x e^c = \int x \times x dx$$

$$y x e^c = \int x^2 dx \quad \checkmark \quad 2$$

$$y x = \frac{x^3}{3}$$

$$y = \frac{x^3}{3x}$$

$$2 = \frac{x^3}{3x}$$

$$y x e^c = \frac{x^3}{3} + c$$

$$y'x + C = x^3 + Cx$$

$$y'x = x^3 + Cx - C$$

$$y = x^2 + C - \frac{C}{x}$$

$$2 = (0)^2 + C - \frac{C}{0}$$

$$2 = C - 0$$

$$2 = C$$

Ans 6/10

$$y = x^2 + C$$

$$y = x^2 + 2$$

Q1 16

②

$$2 \frac{\partial^2 y}{\partial x^2} + 7 \frac{\partial y}{\partial x} + 6y = 0$$

compare to auxillary

$$2m^2 + 7m + 6 = 0 \quad \checkmark \quad \begin{matrix} 2 & 6 \\ 2 & 3 \\ 1 & 6 \end{matrix}$$

$$(2m + 3)(m + 2) = 0$$

$$2m + 3 = 0$$

$$m = -1.5 \quad \times$$

Sign error

$$m + 2 = 0$$

$$m = -2 \quad \checkmark$$

real & different roots

$$y_c = Ae^{m_1 x} + Be^{m_2 x}$$

$$y_c = Ae^{-1.5x} + Be^{-2x} \quad \text{General solution}$$

$$y_p = Ae^{-1.5x} + Be^{-2x}$$

$$y_p = Ae^{-1.5(0)} + Be^{-2(0)}$$

$$3 = A + B$$

$$\frac{dy_p}{dx} = -A1.5e^{-1.5x} + B(-2)e^{-2(x)}$$

$$A = 3 - B \quad (1)$$

$$B = 3 - A \quad (2) \quad \checkmark$$

$$\frac{dy_p}{dx} = A1.5 - 2B$$

sub (3) into (1)

$$3 = -A1.5 - 2B$$

$$\frac{3 + 2B}{1.5} = 3 - B$$

$$\frac{3 + 2B}{1.5} = A \quad (3)$$

$$3 + 2B = 4.5 - 1.5B$$

$$\frac{3 - 1.5A}{-2} = B \quad (4)$$

$$3 + 3.5B = 4.5$$

$$3.5B = 1.5$$

$$B = \frac{3}{7}$$

method correct

sign error in start of work

check

sub (4) into (2)

$$A = 3 - B \quad \leftarrow \text{wrong}$$

$$= 3 - \frac{3}{7}$$

$$\frac{3 - 1.5A}{-2} = 3 - A$$

$$3 - 1.5A = -\frac{6}{2} + 2A \quad \leftarrow \text{error}$$

$$3 = -1.5 + 3.5A$$

$$4.5 = 3.5A$$

$$\frac{9}{7} = A$$

$$3 - 1.5A = -6 + 2A$$

$$3 = -6 + 3.5A$$

$$9 = 3.5A$$

$$\frac{18}{7} = A$$

recheck

$$A = 3 - B$$

$$= 3 - \frac{3}{7}$$

$$= \frac{18}{7} \quad \checkmark$$

$$y = y_c + y_p$$

$$\frac{18}{7} + \frac{3}{7} = 3$$

$$y = Ae^{kx}$$

$$y = Ae^{1.50x} + Be^{-2x} + 3$$

Q2

16

③

Hrs	Freq	Hrs $\times$ freq	$f(x - \bar{x})^2$
7	5	35	35.1125
8	21	168	57.1725
9	25	225	10.5625
10	20	200	2.45
11	16	176	29.16
12	8	96	44.18
13	5	65	56.1125
Total	100	965	234.75 $\checkmark$ S

$$\frac{965}{100} = 9.65 \text{ Hrs on average } \checkmark$$

$$Z = \frac{x - \mu}{\sigma} \quad Z = \frac{x - 9.65}{\sigma}$$

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{965}{100} = 9.65$$

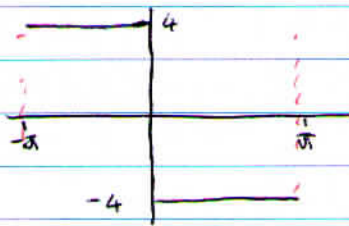
$$\sigma = \sqrt{\frac{\sum f(x - \bar{x})^2}{\sum f}} = \sqrt{\frac{965 \times 234.75}{100}}$$

$$= \sqrt{2.3475} = 1.5322 \text{ s.d } \checkmark$$

Q3 10

(4)

$$f(x) = \begin{cases} 4 & (-\pi < x < 0) \\ -4 & (0 < x < \pi) \end{cases}$$



$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_0 = \frac{1}{2\pi} \left(\int_{-\pi}^0 f(x) dx + \int_0^{\pi} f(x) dx \right)$$

$$a_0 = \frac{1}{2\pi} \left([4x]_{-\pi}^0 + [-4x]_0^{\pi} \right)$$

$$= \frac{1}{2\pi} (+4\pi - 4\pi)$$

$$= \frac{1}{2\pi} (+8\pi) = 4$$

$a_0 = 0$ ✓
 $\therefore$ odd series S

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin(nx) dx$$

$$= \frac{2}{\pi} \int_0^{\pi} -4 \sin(nx) dx$$

$$= \frac{2}{\pi} \left[\frac{4}{n} \cos(nx) \right]_0^{\pi}$$

$$= \frac{2}{\pi} \left[\frac{4}{n} \cos(n\pi) - \frac{4}{n} \right]$$

$$= \frac{8}{n\pi} (\cos n\pi - 1)$$

down to 4n

$$b_{g1} = \frac{2}{\pi} \left(\frac{4}{1} \cos(1\pi) - \frac{4}{1} \right) \quad \cos \pi = -1$$

$$= \frac{2}{\pi} (3.994 - 4)$$

$$= -3.82 \times 10^{-3} = -0.003820 \quad \times$$

$$b_2 = \frac{2}{\pi} \left(\frac{4}{2} \cos(2\pi) - \frac{4}{2} \right)$$

$$\cos 2\pi = 1$$

∴ b_2 and even terms = 0

$$= \frac{2}{\pi} (-0.012)$$

$$= -7.639 \times 10^{-3} = -0.007639 \quad \times$$

$$b_3 = \frac{2}{\pi} \left(\frac{4}{3} \cos(3\pi) - \frac{4}{3} \right)$$

$$= \frac{2}{\pi} (-0.018)$$

$$= -0.01146 \quad \times$$

$$b_4 = \frac{2}{\pi} \left(\frac{4}{4} \cos(4\pi) - \frac{4}{4} \right)$$

$$= \frac{2}{\pi} (-0.024)$$

$$= -0.01525 \quad \times$$

$$f(x) = 0 + 0.003820 \sin(x) - 0.007639 \sin(2x)$$

$$- 0.01146 \sin(3x) - 0.01525 \sin(4x) \dots \times \dots$$

①4.

15

⑤

a)

I) $2 + t^2 + 3e^{3t}$

↓

$$\frac{2}{s} + \frac{2}{s^3} + \frac{3!}{s-3}$$

✓

II) $e^{-2t} \sin(4t)$

↓

$$\frac{1}{s+2} \left(\frac{4}{s^2 + 4^2} \right)$$

✗

$$= \frac{1}{s+2} \left(\frac{4}{s^2 + 16} \right)$$

$$= \frac{4}{(s+2)(s^2 + 16)}$$

$$= \frac{4}{s^3 + 16s + 2s^2 + 32}$$

= 8

$$= \frac{4}{s^3 + 2s^2 + 16 + 32}$$

✗

III) $\frac{2}{s-3}$

↓

$$2e^{3t}$$

✓

IV) $\frac{1}{s^2 + 8s + 20}$

20

1	20
2	10
4	5

$$(s+4)^2 + 4$$

$$\frac{-8 \pm \sqrt{48^2 - 4(1)(20)}}{2}$$

$$= \frac{-8 \pm \sqrt{-16}}{2}$$

$$\frac{1}{s(s+8)+20}$$

$$s^2 + 8s + c$$

8

$$(s+4)^2 + 4$$

42

$$s^2 + 2(4)s + 4^2$$

$$(s+a)^2$$

$$= s^2 + 2as + a^2$$

$$s^2 + 2(4)s + 4^2$$

$$s^2 + 8s + 16$$

+4

$$s^2 + 8s + 20$$

$$(s^2 + 4)^2 + 4$$

$$b) \frac{dy}{dt} + 12y = 8$$

$$s +$$

$$\int \frac{dy}{dt} + 12y dt = \int 8 dt$$

$$1 + 12y$$

$$\int \frac{dy}{dt} dt + 12y = \int 8 dt$$

$$t + 12y = 8t$$

$$\frac{1}{s^2} + 12\bar{y} = \frac{8}{s^2}$$

$$12\bar{y} = \frac{7}{s^2}$$

$$\frac{1}{s^2 + 8s + 20} = \frac{1}{(s+4)^2 + 4} \checkmark$$

$$\frac{b}{(s+a)^2 + b^2}$$

$$\frac{2}{(s+4)^2 + 4} \checkmark$$

$$\frac{1}{(s+4)^2 + 4}$$

$$b^2 \quad b = 2$$

$$a = \frac{3}{6}$$