

Assignment: Deflection of beams

Course: BEng Ordnance, Munitions & Explosives- Safety

Module: ACDA6009 Structural mechanics

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1. Abstract

This assignment will be looking at the results gained from a laboratory experiment where incrementally increasing weight is applied to three beams of different material and their deflection measured. The three materials that were used for the experiment were aluminium, brass and mild steel. Analyses of these results was carried out looking at what effects the different materials had on the beams deflection. It will also check the validity of the results gained by comparing them to calculated results. The results gained from the experiment and the ones calculated were taken and input into tables as can be seen later in the assignment, also showing these results in graphical form. With all the information gained and analysed it proved that the reason for the change in deflection was down to the material properties of each of the beams. The comparison that was carried out showed that the mild steel had the least deflection followed by the brass with aluminium having the most deflection for each of the given weights added. This was true for both the experimental and calculated results giving confidence in the accuracy of the experimental results gained. Research was then carried out into the material properties of each of the three different beams, this was done for further comparison and to further prove if the conclusion that was come to was correct. Which it proved the beam with the most to least deflection from the experimental results gained coincided with the trend that should be expected from the research carried out.

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3. Summary

To best do this assignment it has been split into sections, it will firstly introduce the background to the assignment. It will then look at relevant background information on how the laboratory experiment was set up and the calculated results gained. The assignment will then move on to look at the results that have been gained and comparing the experimental and calculated ones as well as the behaviour of the varied materials. These results gained will then be discussed comparing calculated deflections with measured deflections. It will also discuss and compare how the different materials have performed for both type of results gained. It will conclude by giving an overview of the whole assignment explaining how real-world testing compares with idealistic theoretical results. Finally, it will then include all relevant sources used in the reference section and finish with an appendix of all other relevant information for the assignment.

4. Introduction

The aim of this assignment as given in the brief is to carry out an investigation into the deflection of cantilever beams made from different materials, this will be done by carrying out an experimental laboratory test as well as theoretical calculations. There will be different incrementally increasing loads applied to the different beams and their deflection taken at each. This applied force from the weight due to gravity to the beam is one of the factors that effects a beams diffraction. Another factor is the distance from the support where the load is applied, this was kept the same for each of the experiments and calculations. The second moment of area for the beam also has an effect and is a measurement of the beams shapes ability to resist change in this case bending caused by the applied load. [1] The type of material also influences the amount of deflection that can occur which is what will be investigated for this assignment.

This data that was required to be input to conduct the experiment was kept the same through to ensure a fair comparison. The weight that was added to the beams was the same for each of the materials and done in increasing increments. The set up that was used for this experiment had the beam clamped at one end with the other end free to move. Research was carried out looking at how the deflection calculated results would be gained, a formula was required to be found to use for this type of experimental set up. This information was gained from laboratory guidance provided by the manufacturer of the laboratory equipment for the experiment deflection of beams and cantilevers ES4 by TecQuipment [2].

5. Background Information

The aim of this assignment is to carry out a comparison between theoretical calculated results for beam deflection at different loads with the results gained from a laboratory exercise. This assignment tested three different types of materials brass, aluminium and mild steel. The calculated and experimental results can be seen later in the assignment with additional information shown in the appendix.

To find the deflection value for the beam at varying weights it was required firstly that the second moment of area be calculated, this was done by taking the value for b and d. These values were the same for each of the different materials also for the calculated and experimental results for a fair comparison of results gained. These values were gained by measuring the areas shown below in figure 1.

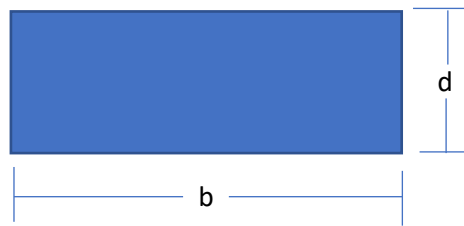


Figure 1- Image of b and d measurement

The values for b and d for the beam were measured to give b as 0.0095 meters and d as 0.0031 meters. Given this information the second moment of area (I) was now able to be calculated, this was done by using the formula shown below in equation 1.

$$I = \frac{bd^3}{12}$$

Equation 1- Formula for calculating I [2]

$$I = \frac{0.0095 * 0.0031^3}{12}$$

Equation 2- Calculation for finding I (second moment of area)

The values for b and d were input into the formula as can be seen in equation 2 which gave a value for the second moment of area of $2.36 \times 10^{-11} \text{ m}^4$ to two decimal place. The length of the bar that was used for the experiment was 0.24 meters long. The various Youngs modulus for the three different materials under test is shown below in table 1.

Material	Youngs Modulus
Brass	105 GPa
Mild steel	200 GPa
Aluminium	69 GPa

Table 1- Table of young's modulus for Brass, Mild steel and Aluminium [2]

The image below in figure 2 shows the incrementally increasing weight that were added to the beam.



Figure 2- Image of varying weights for experiment

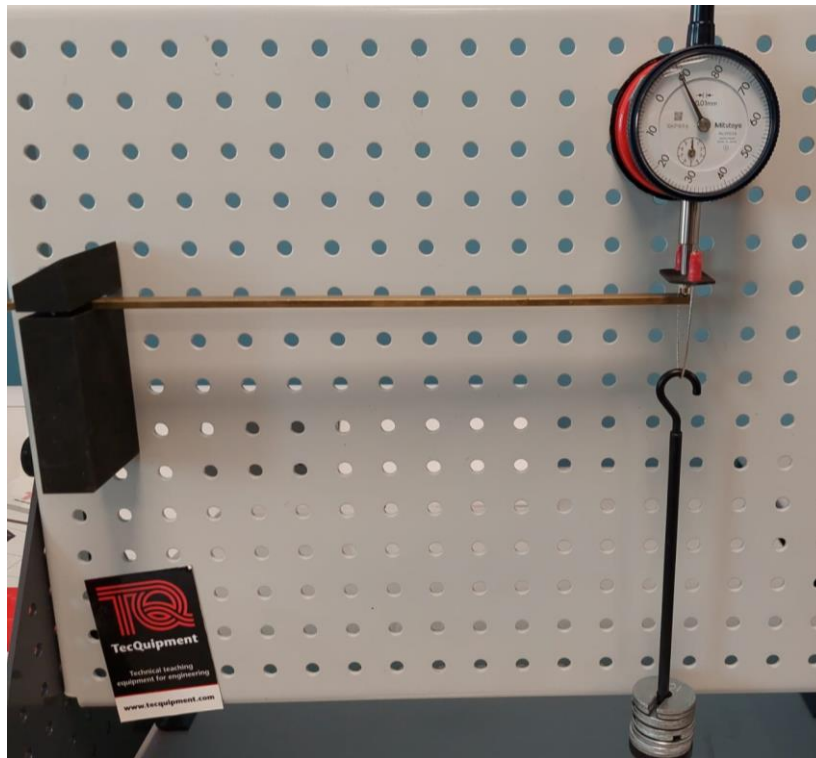


Figure 3- Image of experimental set up

The experimental laboratory set up that was used is shown above in the image in figure 3, this shows how the beam was clamped, loaded and measurements taken. As is shown the beam was clamped at one end with the dial test indicator at the other end 0.24 meters away. The beam then had four incrementally increasing loads applied of fifty grams at the same end as the dial test indicator and measurements taken of the deflection for each of these loads. Once all the required readings for the deflection were gained the load could then be removed from the end of the beam.

The beam was then replaced for one of the other materials under test, the same process was followed for each beam until each of the different material had been tested. Once all the different beams had been tested under the required loads the calculated results for γ the beams deflection for varying weights was now carried out. This was done by firstly converting the weight into SI units of kilo grams, then to force due to gravity by multiplying each of the weights by 9.81. The deflection could now be calculated using the formula shown below in equation 3, this was all done using Microsoft Excel for ease of calculations and then each calculation checked with a calculator.

$$\gamma = \frac{WL^3}{KEI}$$

Equation 3- Formula for calculating γ (deflection) [2]

Where:

W= Load (N)

L= Length (m)

K= Constant for beams support condition (3 for this experiment) [2]

E= Youngs modulus (Pa)

I= Second moment of area (m^4)

A comparison of all the results gained was now required to be carried out, this was done by firstly inputting the experimental results into excel with the calculated results. A graph was then plotted of deflection against force for a visual representation of the data and the difference in the results gained. The difference in the measurements for the laboratory and experimental results for each material at each load was calculated for comparison using excel.

6. Results

Table 2 below shows the calculated and laboratory results for the beams deflection under varying loads for the three different materials. The information for the calculated results shown were gained using the formulas and information given in the background information section of the assignment.

		Lab measurements results (m)			Calculated measurements results (m)		
Weight (g)	Force (N)	Brass	Mild Steel	Aluminium	Brass	Mild Steel	Aluminium
0	0	0	0	0	0.00	0.00	0.00
50	0.4905	0.81	0.45	1.31	0.91	0.48	1.39
100	0.981	1.8	0.88	2.72	1.83	0.96	2.78
150	1.4715	2.71	1.46	4.38	2.74	1.44	4.17
200	1.962	3.67	1.87	5.5	3.65	1.92	5.56

Table 2- Table of laboratory and calculated results

The graph below in figure 5 shows the forces due to gravity in Newtons that was applied against the resultant deflection in millimetres. It shows this information for each of the three different materials under test for both the laboratory experimental results gained and the calculated results that were gained.

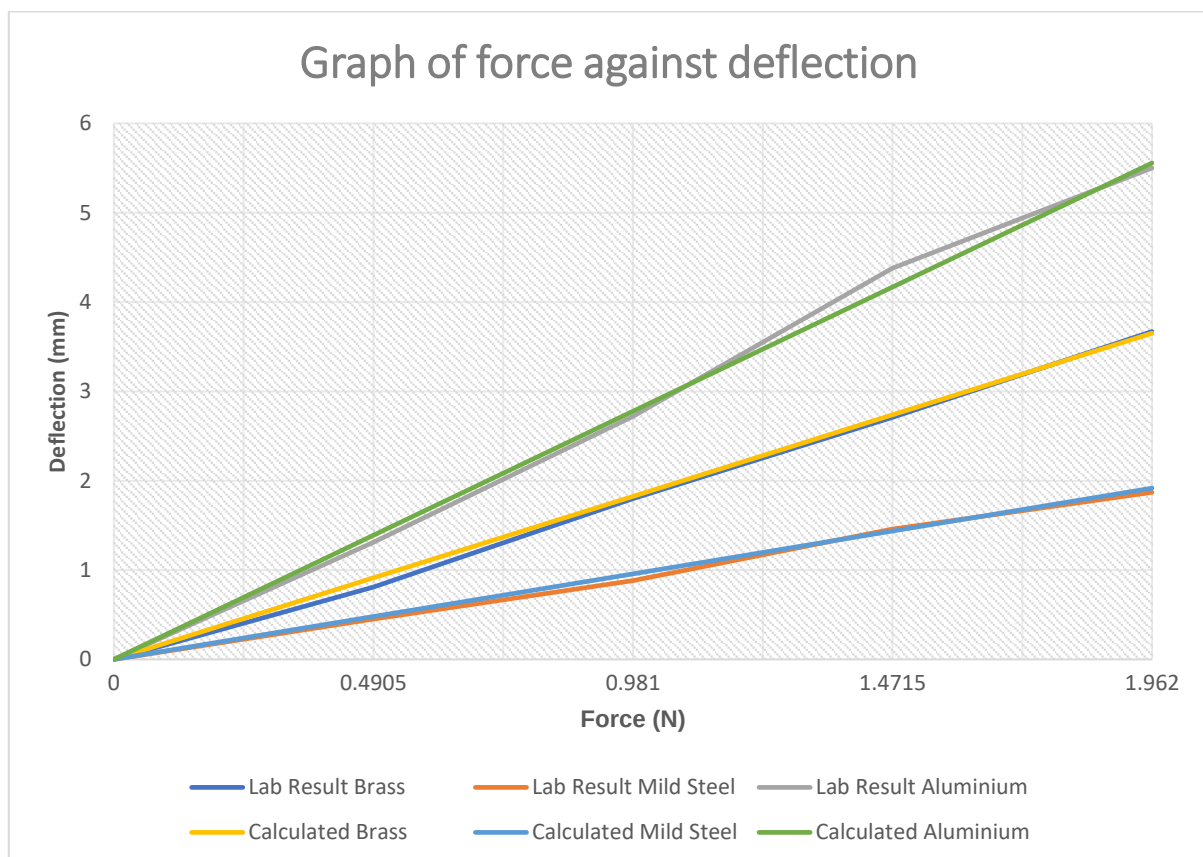


Figure 4- Graph of force against deflection for calculated and lab results

Table 4 below shows the difference between the laboratory and calculated results that were gained for each of the different materials at different weights.

Experimental laboratory results against calculated results				
		Difference in measurements (m)		
Weight (g)	Force (N)	Brass	Mild Steel	Aluminium
0	0	0.00	0.00	0.00
50	0.4905	0.10	0.03	0.08
100	0.981	0.03	0.08	0.06
150	1.4715	0.03	0.02	0.21
200	1.962	0.02	0.05	0.06

Table 3- Difference between experimental laboratory results and calculated results

7. Discussion

This section will discuss the difference in the results that have been gained from the laboratory experiment and the ones calculated, also looking at the difference in deflection between the different materials. As can be seen in the method section for both the calculated results and the experimental results the beams length, where the load was applied, the incremental increases in weight, the shape and therefore the moment of inertia were all kept the same. This was done to ensure that the conducted test was a fair comparison of the results with only one variable in place. This means that the clear difference that can be seen in the results is due to each of the materials different properties.

The findings showed that aluminium had the highest deflection for each weight followed by brass and the material with the lowest deflection was mild steel. This can be clearly seen in the table 2 and visually in figure 5. Further evidence of this can be seen in the appendix in tables 5 and 6 also visually in the graphs in figures 6 and 7, where it shows the experimental and laboratory results in table and graphical form separately. Both the calculated results as well as the experimental results showed the same trend which further solidified and validated the results and the conclusion that was gained.

The trend in the performance of the different materials was expected due to the differing material properties of each of the beams. Their shear modulus and young modulus gives a good indication of whether one material will have more deflection than another. The values for the different beams materials young modulus are shown earlier in the assignment in table 1, below in table 4 it shows the shear modulus for each of the materials. As can be seen in both these tables the material with the higher shear modulus and young's modulus had the lowest deflection.

Material	Shear modulus (GPa)	Reference
Aluminium	25.0	[3]
Brass	35.0 - 44.0	[4]
Mild Steel	70.0 - 80.0	[5]

Table 4- Table of material properties

The varied materials of the beams can be split into two main groups the first being ferrous metals which is where the mild steel sits, this is due to it containing iron. The second group is the nonferrous metals which is where the brass and the aluminium sit. Each of these materials has its own unique stress to strain graph

which gives them their own unique elastic zone, yield strength, plastic zone, ultimate tensile strength and fracture point.

The elastic properties region on a stress to strain graph is where the material can undergo changes but will return to its original shape without any deformation or change occurring. Passing the yield strength is the point at which the elastic point of the material has been reached and any further deformation will not allow the material to return to its original form, the area passed this point is called the plastic zone. If the material undergoes further strain it will then hit its ultimate tensile strength, this is the most amount of stress the material can take. If it is then pushed further with the strain increased it will eventually cause a fracture to the material.

Ductile and brittle materials behave differently on a stress to strain graph which explains their behaviour during the exercise. A material with high ductility undergoes higher plastic deformation at lower strain than that of a lower ductile material. This can be seen with the aluminium being highly ductility and having the most deformation and mild steel having the least deformation and being the most brittle. This can be seen visually in the graph below in figure 5 showing a brittle material and a ductile material on a stress to strain graph. This graph was gained from Ryan P. Wilkerson from University of California [6].

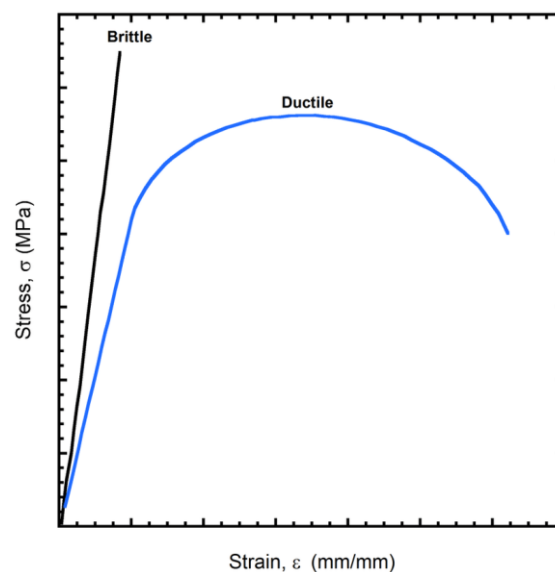


Figure 5- Brittle vs Ductile Stress strain graph [6]

The calculated results for the deflections of each of the beams was gained using Macaulay's method utilizing Microsoft Excel for ease as described in the method section. Macaulay's method is used where a force is applied discretely or discontinuous to a Euler-Bernoulli beam, these situations make it difficult to predict where the most deflection will occur using standard methods. Macaulay's method is also known as the double integral method, this is due to it taking the bending moment equation and integrating it to find a beams deflection [7].

The difference in the readings between the calculated and experimental results were similar, as can be seen in figure 5 and table 3. This information shows that except for one of the readings they were all at

least 0.1 meters difference which is very slight. The one anomaly in the data is for aluminium at a force of 1.4715 Newtons where there was a difference in the two methods of 0.21 meters. This is likely due to an error in the lab results as can be seen visually in figure 5 with a spike only occurring at that one point. All the other data act in a linear fashion when the force increases the deflection increase at a similar rate with it. The beginning of the data with the anomaly acts the same with one spike and then it heads back towards the linear increasing trend like the calculated results.

8. Conclusion & recommendations

The work above was carried out as a preliminary investigation into the effect using different beam material has on deflection. This assignment as described previously used a cantilever beam that was clamped rigidly at one end with the other end being free to move and a weight applied to it being varied. An experimental laboratory excessive was conducted with the results validated with calculated results. With all the information gained and analysed this proved that the reason for the change in the deflection was due to the different material properties of each of the beams. The results showed aluminium to have the most deflection followed by brass with mild steel having the least. The information gained from the theoretical idealistic calculations confirmed the accuracy of the experimental results due to their close relation to one another. These results followed the same trend with also the research carried out also confirming this trend.

As discussed in the discussion section there was one anomaly in the results gained, given this information the test should have been done again to see if the same thing occurred or if it was an abnormal error. Given more time each of the materials would have been loaded three times and an average of these results taken for further accuracy. To increase the accuracy further the experiment could be conducted in a temperature-controlled room so that it is consistent. The dial test indicator that was used for measuring could have also been calibrated to a higher standard and an uncertainty of measurement for the equipment gained for better understanding of the experimental results.

The other reason that there could be slight difference in results is due to the young modulus that is used being for idealistic theoretical material. The manufacturing process can never create beams that have the exact same material properties as the theoretical ones that have been gained. There could also be an error in the equipment used due to the beams used for the experiment had been previously used an unknown number of times with unknown loads applied. These loads could have exceeded the beams yield strength and caused deformation to occur in the plastic region as explained above. The reality of experimental test work against calculated work is that they never exactly match. This highlights the importance of carrying out real world testing as opposed to pure theoretical work.

9. References

9.1. Reference List

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9.2. Bibliography

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10. Appendix

Below are additional tables and graphs of the experimental results and calculated results that were gained.

Laboratory Results				
		Measurements (mm)		
Weight (g)	Force (N)	Brass	Mild Steel	Aluminium
0	0	0	0	0
50	0.4905	0.81	0.45	1.31
100	0.981	1.8	0.88	2.72
150	1.4715	2.71	1.46	4.38
200	1.962	3.67	1.87	5.5

Table 5- Table of Laboratory results

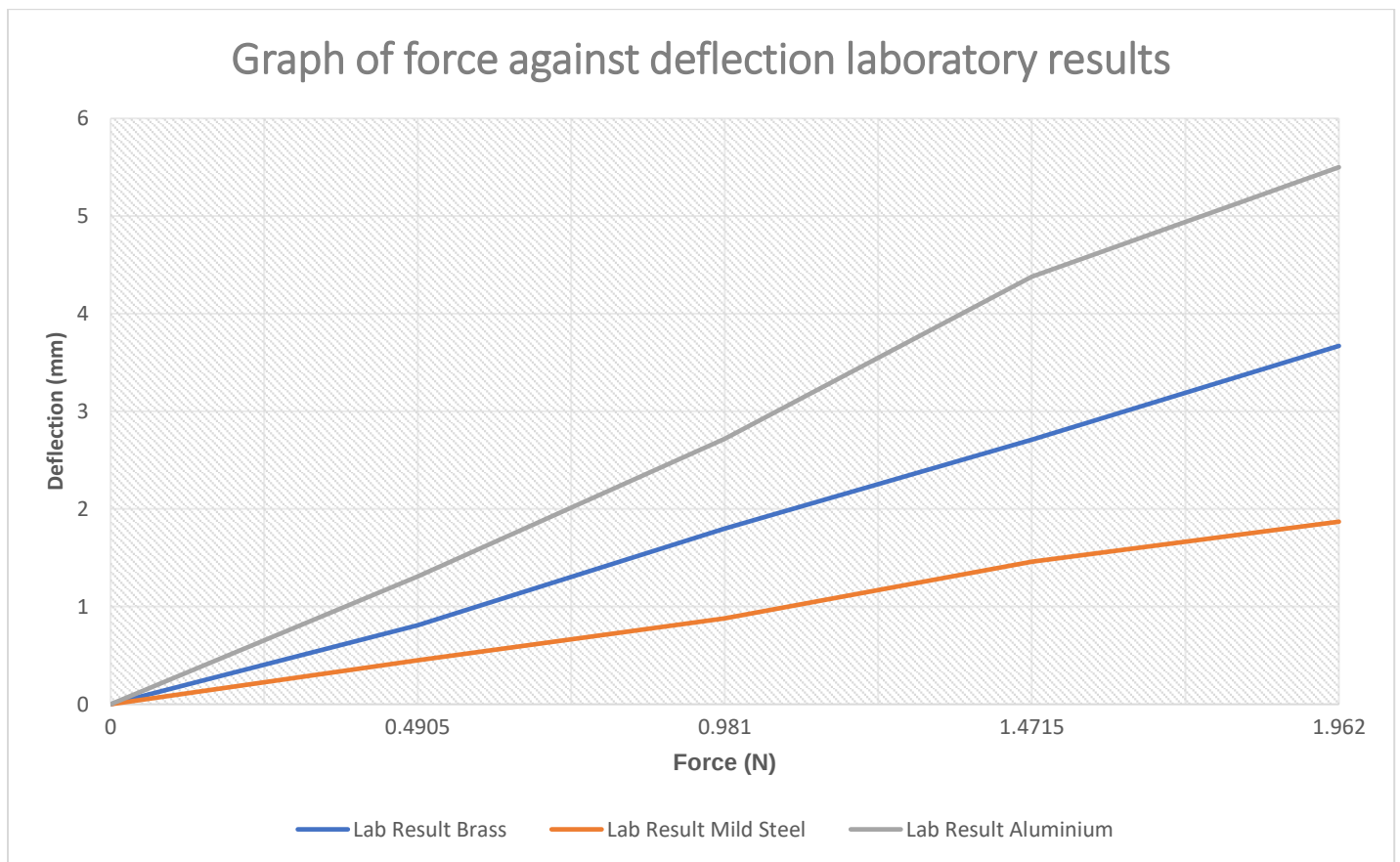


Figure 6- Graph of force against deflection for Laboratory results

Calculated results				
		Measurements (mm)		
Weight (g)	Force (N)	Brass	Mild Steel	Aluminium
0	0	0.00	0.00	0.00
50	0.4905	0.91	0.48	1.39
100	0.981	1.83	0.96	2.78
150	1.4715	2.74	1.44	4.17
200	1.962	3.65	1.92	5.56

Table 6- Table of Lab results

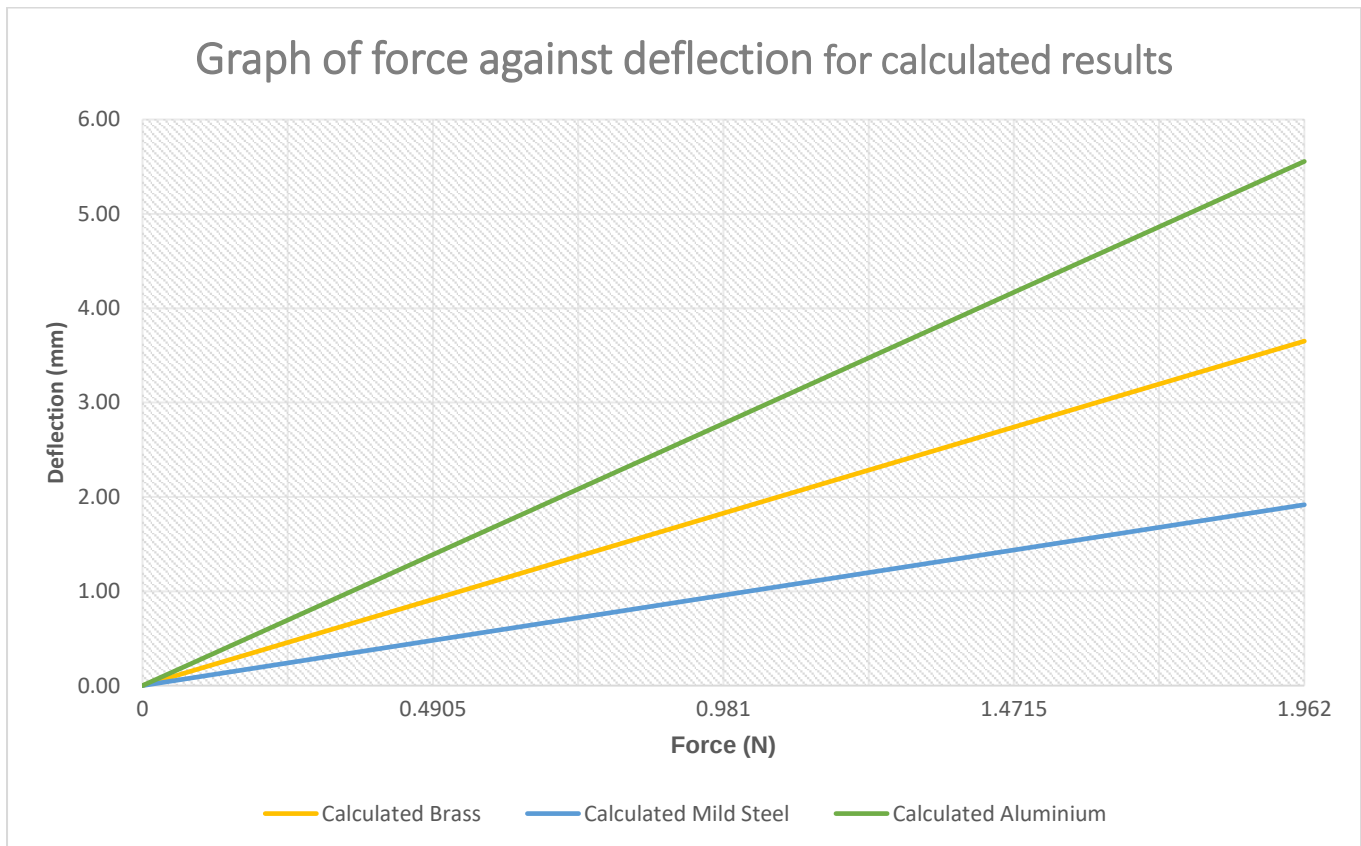


Figure 7- Graph of force against deflection for calculated results