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Shaped Charge Design

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Abstract

This report provides a literature review of shaped charge history and development, a review of underlying principles used by the explicit solver Abaqus, two-dimensional modelling and analysis of two charge designs utilising different materials for the shaped charge cone liner, as well as a three-dimensional model of the final design selected from one of the two-dimensional design models. It provides a review and comparison of pressures, velocities and thermal temperatures experienced by the liners throughout the simulations, as well as different approaches to modelling and simulation techniques, such as mesh sizing and refinement, adaptive mesh techniques, interactions between part surfaces, Smoothed Particle Hydrodynamics (SPH) and validation of concept.

Keywords:

Abaqus, Shaped charge, Ballistics, Impact Analysis, Finite Element Analysis, Jones-Wilkins-Lee, Johnson-Cook, Lagrangian Mechanics, Eulerian Mechanics, Smooth Particle Hydrodynamics, Mie-Gruneisen, Equation of State.

Table of Contents

Abstract	ii
List of Figures	iv
List of Tables	v
List of Equations	v
List of Abbreviations	vi
1 Introduction to Shaped Charges	1
1.1 A Brief History of Shaped Charges	2
1.2 Development of the Shaped Charge	2
2 Underlying Principles Integrated by Abaqus	4
2.1 Jones-Wilkins-Lee	4
2.2 Johnson-Cook	5
2.2.1 Johnson-Cook Strength Model	5
2.2.2 Johnson-Cook Failure Model	6
2.3 Mie-Gruneisen	6
2.3.1 The Hugoniot State	7
2.3.2 Energy Partitioning/The General Mie-Gruneisen Equation	8
2.4 Lagrangian Mechanics	8
2.5 Eulerian Mechanics	10
2.6 Coupled Eulerian-Lagrangian	12
2.7 Arbitrary Lagrangian-Eulerian	12
2.8 Smoothed Particle Hydrodynamics	13
3 Case Study – Shaped Charge	14
3.1 Material Properties	14
3.2 Setting up Abaqus to Solve an Explosive Problem	16
3.3 Jet Formation Comparison	17
3.4 Pressure Analysis	20
3.5 Velocity of Detonation	22
3.6 Thermal Analysis of Liner	24
3.6.1 Thermal Diffusivity	25
3.7 Three-Dimensional Simulation	27
3.7.1 Smoothed Particle Hydrodynamic Adaptation	32
4 Conclusion	35
5 References	36

List of Figures

Figure 1 - " <i>Schematic diagram of a shaped charge</i> " adapted from [2]	1
Figure 2 - Simulation Failure Error Message	17
Figure 3 - Incorrect Meshing and Excessive Element Distortion	17
Figure 4 - Mixed Mesh Simulation Failure	18
Figure 5 - Liner Part Isolation	19
Figure 6 - Excessively Distorted Element Isolation.....	19
Figure 7 - Successful Shaped Charge Jet Formation	20
Figure 8 - Pressure Analysis Initial Formation of Jet	20
Figure 9 - Pressure Analysis Peak Overpressure.....	21
Figure 10 - Pressure Analysis Liner Isolation	21
Figure 11 - Pressure Analysis Graphs.....	22
Figure 12 - Velocity Analysis of Liner	23
Figure 13 - Velocity Analysis Graphs	23
Figure 14 - Thermal Analysis of Liner.....	24
Figure 15 - Thermal Analysis Peak Temperature Graphs	24
Figure 16 - Thermal Analysis Temperature Spread Graphs.....	25
Figure 17 - Three-Dimensional Analysis Initial Point of Failure	28
Figure 18 - Distortion Control of the Three-Dimensional Solve	29
Figure 19 - Three-Dimensional Jet Formation	30
Figure 20 - Temperature Profile of Liner of Three-Dimensional Model.....	30
Figure 21 - Three-Dimensional Temperature Graph for Magnesium Liner	31
Figure 22 - SPH Conversion Settings for Three-Dimensional Model.....	32
Figure 23 - Final point of Lagrangian Meshed Three-Dimensional Model Prior to SPH Conversion	33
Figure 24 - Initial Conversion of Liner to SPH	33
Figure 25 - Formation of Jet from SPH.....	34

List of Tables

Table 1 - Properties of Metals Used for Shaped Charge	15
Table 2 - Properties of HE Used for Shaped Charge	15
Table 3 - Thermal Material Properties of Liner Materials.....	26

List of Equations

Equation 1 - JWL equation	4
Equation 2 - JC Strength Model	5
Equation 3 - JC Failure Model	6
Equation 4 - Linear Velocity Relationship Between Shock Waves and Material Particles.....	7
Equation 5 - Hugoniot Pressure Equation	7
Equation 6 - Final Pressure Equation	8
Equation 7 - Lagrangian Function Equation	8
Equation 8 - Using the Lagrangian Function to Reach Equilibrium	9
Equation 9 - Lagrangian Global Equilibrium Equation	9
Equation 10 - The Material Derivative	10
Equation 11 - The Conservation of Mass	11
Equation 12 - The Conservation of Momentum	11
Equation 13 - The Conservation of Energy	11
Equation 14 - Thermal Diffusivity.....	26

List of Abbreviations

ALE	Arbitrary Lagrangian-Eulerian
CAE	Computer Aided Engineering
CEL	Coupled Eulerian-Lagrangian
EFP	Explosively Formed Penetrators
EOS	Equation Of State
ERA	Explosive Reactive Armour
FEA	Finite Element Analysis
HE	High Explosive
JC	Johnson-Cook
JWL	Jones-Wilkins-Lee
SPH	Smoothed Particle Hydrodynamics

1 Introduction to Shaped Charges

Shaped charges are specialist devices designed to drive energy into a narrow, focal point, to drive metal particles at high velocities out from this point, in order to penetrate thick steel, concrete or armour. It does this by utilising a conical or hemispherical metal liner surrounded by high explosives to concentrate force into a small area [1].

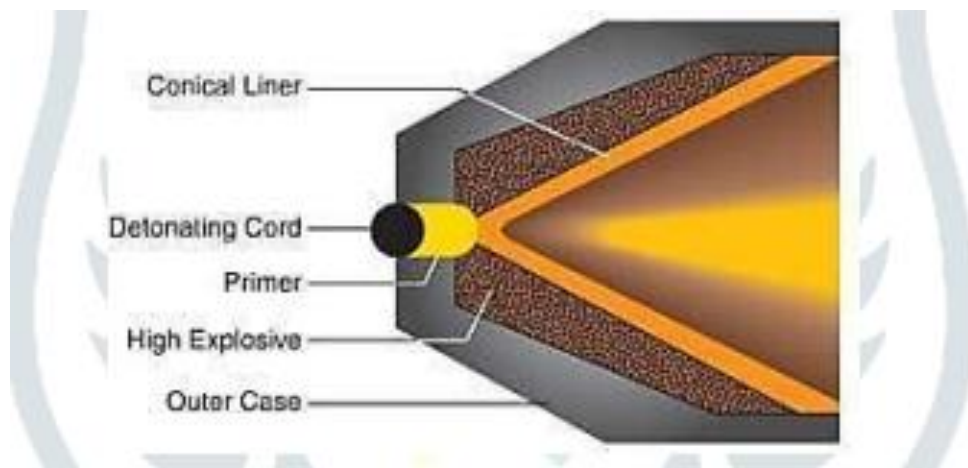


Figure 1 - "Schematic diagram of a shaped charge" adapted from [2]

Figure 1 gives a graphical representation of the inner workings of a shaped charge, where when the high explosive is detonated, the pressure produced by the explosion drives the liner to collapse inwards upon itself, causing a peak overpressure on the central axis of the shaped charge body. This peak overpressure then drives the compressed liner material forwards, like a dart, forming a high velocity jet. The Jet stretches due to a gradient of velocity between the tip and the tail. This is further influenced by a hardened outer case, which, due to a mass differential between the liner and the case, forces the detonation to drive away from the case in the desired vector/direction [2].

Shaped charges can also be referred to as "hollow charge" or "Explosively Formed Penetrators" (EFPs) [3].

1.1 A Brief History of Shaped Charges

The earliest mention of shaped charges was that of the journals of Franz Xaver von Baader, a German mining engineer who advocated a conical or mushroom shaped empty cavity in the forward end of a blasting charge, as a way to increase the explosive effect as well as to also save powder [4].

Although this was mentioned, the earliest formal research on shaped charge design was by that of American chemist Charles Munroe in 1888, and as a result was coined 'The Munroe Effect' across the USA, for his work on unlined shaped charges [3]. Although this was only later acknowledged by Europe as the foundation, it was formerly labelled across Europe and the United Kingdom as 'The Neumann Effect' after chemist Max Neumann and explosives expert Egon Neumann, who researched the effects on steel of a conical indentation on a block of TNT [5,6,7]. This work led to the research of shaped charges in military application, as it was adopted to be used in the development of tactical trench demolition, with its first use in the battlefield in 1940 by the German forces to successfully invade the previously thought 'impregnable' Belgium fortress of Eben-Emael [6,8]. This victory gained increased interest from both British and American military research and development sectors and resulted in the first U.K. commissioned 'hollow charge' application, the no.68 grenade, an anti-tank rifle grenade capable of penetrating thick armour [6].

1.2 Development of the Shaped Charge

The success at Eben-Emael created a surge of nations rapidly weaponizing lined-cavity shaped charges into portable, infantry wielded anti-tank weapons such as the previously mentioned British no.68 anti-tank rifle grenade, the American bazooka, the German panzerfaust and later the soviet RPG [9].

As a result, main battle tanks adopted Explosive Reactive Armour (ERA), which detonates outwards to disrupt incoming piercing jets, to combat these portable anti-tank devices, which led weapons developers to invent what is known as the tandem-charge warhead – a warhead which utilises two sequential shaped

charges [10]. Modern day systems continue to refine this concept, such as the Saab HEAT 758 [11].

Other developments include the use of EFPs, where instead of a conical liner, a hemispherical dish-shaped liner is used instead, producing a single, aerodynamically stable kinetic slug. Drawbacks of this development are it has reduced raw armour penetration; however it benefits from being able to travel longer distances through the air before striking its target, which is known as standoff.

2 Underlying Principles Integrated by Abaqus

This report discusses a solution which has been completed using the Dassault Systemes program Abaqus, a finite element analysis (FEA) solver software which utilises a number of underlying physics principles to solve complex problems using computer aided engineering (CAE). For this section, the physics principles used by the solver as well as available alternatives are discussed.

2.1 Jones-Wilkins-Lee

The Jones-Wilkins-Lee (JWL) equation of state (EOS) is used in hydrosimulations as a descriptor of thermodynamics for both detonations products and unreacted high explosives and can be referred to as the JWL equation.

$$P = A \left(1 - \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left(1 - \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \frac{\omega E}{V}$$

Equation 1 - JWL equation

Where:

P – Pressure of the detonation products

V – Relative volume (which is the ratio of initial density to current density)

E – Internal energy per unit of initial volume

A, B, R_1, R_2 – Empirical constants

ω – Grüneisen coefficient

These parameters are derived from cylinder expansion tests, where a metal tube is filled with explosives, detonated and the expansion is measured.

In numerical modelling, the JWL equation allows accurate prediction of shock wave propagation through material, including metal acceleration and blast pressure.

[12,13]

2.2 Johnson-Cook

Johnson-Cook (JC) modelling describes materials plastic behaviour as well as progressive failure of metals which undergo severe deformation and can be split into two independent parts: the JC strength model and the JC failure model.

2.2.1 Johnson-Cook Strength Model

The JC strength model multiplies three distinct physical phenomena to calculate dynamic yield stress (σ_y): strain hardening, strain rate sensitivity and thermal softening.

$$\sigma_y = [A + B(\varepsilon_p)^n] \left[1 + C \ln \left(\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) \right] [1 - (T^*)^m]$$

Equation 2 - JC Strength Model

This equation can be broken down into the follow three brackets:

- Strain Hardening Bracket $[A + B(\varepsilon_p)^n]$ – Defines static material behaviour at reference temperature and reference strain rate, where:
 - A – Initial yield stress of the material
 - B – Hardening Constant
 - ε_p – Equivalent plastic strain
 - n – Hardening exponent
- Strain Rate Bracket $\left[1 + C \ln \left(\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) \right]$ – Scales yield strength based on the speed of deformation, where:
 - C – Strain rate sensitivity coefficient
 - $\dot{\varepsilon}_p$ – Current plastic strain rate
 - $\dot{\varepsilon}_0$ – Reference strain rate
 - ε^* - In some sources, $\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0}$ is referred to as ε^* and represents the dimensionless plastic strain rate

- Thermal Softening Bracket $[1 - (T^*)^m]$ – material strength is reduced as either adiabatic heating or ambient temperature rises to the melting point, where:

- m – Thermal softening exponent
- T^* - Homogenised temperature

2.2.2 Johnson-Cook Failure Model

The failure model calculates the expected onset of damage as per the cumulative plastic strain.

$$\varepsilon_f = [d_1 + d_2 \exp(d_3 \eta)] \left[1 + d_4 \ln \left(\frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) \right] [1 + d_5 T^*]$$

Equation 3 - JC Failure Model

Where:

η – represents $\frac{\sigma_m}{\sigma_{eq}}$, known as the stress triaxiality factor, where σ_m is hydrostatic stress and σ_{eq} is Von Mises equivalent stress

d_1, d_2 and d_3 – Triaxiality damage constants

d_4 – Strain rate damage constant

d_5 – Thermal softening damage constant

Damage is continuously evaluated during calculation across integration points to understand damage accumulation.

[14,15]

2.3 Mie-Gruneisen

Mie-Gruneisen EOS is used to model pressure responses of solid materials under extreme shock loading conditions through thermodynamic relations. This is done by lining any high velocity impact pressure directly to changes in material density and internal energy. The underlying principles can be separated into two parts: the Hugoniot state and energy partitioning.

2.3.1 The Hugoniot State

When a material is compressed by a shock wave, its thermodynamic properties move along a path known as the hugoniot curve, which represents an estimate for all possible shock states of that material.

The kinematics of the shock wave to pressure relation can be represented by the Rankine-Hugoniot jump equations combined with the empirical linear relationship between the shock wave velocity and material particle velocity:

$$U_s = C_0 + sU_p$$

Equation 4 - Linear Velocity Relationship Between Shock Waves and Material Particles

Where:

U_s – Shock wave velocity

C_0 – The bulk speed of sound in the material at zero pressure

s – A dimensionless empirical material constant

U_p – Material particle velocity

This linear-velocity relation can be substituted into the conservation of mass and momentum equation to calculate a reference hugoniot pressure as a function of nominal volumetric strain:

$$P_H = \frac{\rho_0 c_0^2 \mu (1 + \mu)}{[1 - (s - 1)\mu]^2}$$

Equation 5 - Hugoniot Pressure Equation

Where:

P_H – Hugoniot pressure

ρ_0 – initial material density

μ – Nominal compressive strain, defined by density change where: $\mu = \frac{\rho}{\rho_0} - 1$

2.3.2 Energy Partitioning/The General Mie-Gruneisen Equation

This can be used to split the total pressure into two distinct components: a reference hugoniot pressure somewhere along the hugoniot curve at a given volume, and any additional thermal pressure generated by atomic vibrations as internal energy changes (known as the gruneisen effect). This can be combined with the hugoniot pressure equation to calculate final pressure and expressed as:

$$P = \frac{\rho_0 c_0^2 \mu \left[1 + \left(1 - \frac{\Gamma_0}{2} \right) \mu \right]}{[1 - s\mu]^2} + \Gamma_0 \rho_0 E_m$$

Equation 6 - Final Pressure Equation

Where:

P – Final pressure

Γ_0 – The dimensionless gruneisen gamma coefficient at reference state

E_m – The current internal energy per unit mass

This allows an understanding of the relationship of how the pressure of a shockwave can affect the energy within a material, as well as the final pressure output from the impact.

[16,17]

2.4 Lagrangian Mechanics

Lagrangian mechanics is the formulation of classical mechanics which focuses on energy instead of force vectors, which is to say Newtonian mechanics. What this means is rather than forces, the manipulation of an object is defined by two specific forms of energy, kinetic and potential energy. The difference in this is known as the Lagrangian function and can be expressed as follows:

$$L = T - V$$

Equation 7 - Lagrangian Function Equation

Where L is the Lagrangian function, T is kinetic energy and V is potential energy.

In FEA software, in particular Abaqus, there are two ways in which a material moves through space: Eulerian, which will be discussed in the next section, and Lagrangian, where the mesh is designated to ‘stick’ to the material, therefore if the material stretches, so does the element.

It is well understood that nature takes a path that minimises the amount of action, this is known as Hamiltons principle, the same can be said for Lagrangian mechanics, where in FEA it transforms the into the principle of virtual work and uses the Lagrangian function to reach equilibrium.

$$\delta \int_{t_1}^{t_2} (T - V) dt = 0$$

Equation 8 - Using the Lagrangian Function to Reach Equilibrium

For a Lagrangian mesh in Abaqus, the global equilibrium equation at any time can be understood as:

$$M\ddot{u} + I - P = 0$$

Equation 9 - Lagrangian Global Equilibrium Equation

Where:

M – Mass matrix

$\ddot{u}$ – Nodal accelerations

I – Internal force vector (i.e. the stresses within the elements)

P – External force vector (i.e. applied loads)

Abaqus handles large deformations using two specific Lagrangian frameworks: total and updated.

Total Lagrangian describes that all variables (stress, strain) are referred back to the original configuration, where time (t) = 0.

Updated Lagrangian describes that all variables are referred to the current configuration (the state at the end of the last increment).

There is a point in which standard Lagrangian mesh fails due to extreme distortion, in which one of two other techniques should be incorporated: Coupled Eulerian-Lagrangian (CEL) or Arbitrary Lagrangian-Eulerian (ALE).

[18,19]

2.5 Eulerian Mechanics

Eulerian Mechanics are a formulation of continuum mechanics, where the instead of focusing on mesh distortion, an FEA solver such as Abaqus focuses on the environment in which the simulated problem is situated. It is used to handle fluids, gases and solids which undergo extreme deformation that would lead to a mesh failure in a standard Lagrangian mesh, which makes it excellent at solving ballistic problems, especially for explosives.

In order to do this, it uses a material derivative to bridge the gap between tracking a moving particle, the Lagrangian perspective, and observing a fixed point, the Eulerian perspective. The total rate of change of any physical property over time can be described by this material derivative:

$$\frac{D\phi}{Dt} = \frac{\partial\phi}{\partial t} + \mathbf{v} \cdot \nabla\phi$$

Equation 10 - The Material Derivative

Where:

$\frac{D\phi}{Dt}$ – The total rate of change (Lagrangian)

$\frac{\partial\phi}{\partial t}$ – the local rate of change at a fixed spatial position

$\mathbf{v} \cdot \nabla\phi$ – the convective term used to represent transport of material moving through spatial gradients

How this works in Abaqus and other FEA solvers is by splitting each time step into two distinct phases:

1. Lagrangian phase – the solver temporarily allows the mesh to move and deform with the material, applying standard Lagrangian calculations to compute internal forces, accelerations and updated element geometries.
2. Eulerian phase – the solver maps the deformed mesh back to the material in its original, un-deformed spatial state. During the remap the solver calculates how much mass, momentum and energy has moved across the element boundaries and updates the material state accordingly

The calculations for each of these elements are governed by three Eulerian conservation laws:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Equation 11 - The Conservation of Mass

$$\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \otimes \mathbf{v}) = \nabla \cdot \boldsymbol{\sigma} + \mathbf{b}$$

Equation 12 - The Conservation of Momentum

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho e \mathbf{v}) = \boldsymbol{\sigma} : \mathbf{D} + \mathbf{b} \cdot \mathbf{v} - \nabla \cdot \mathbf{q}$$

Equation 13 - The Conservation of Energy

Where:

ρ is the material density

$\mathbf{v}$ is the velocity vector

$\boldsymbol{\sigma}$ is the Cauchy stress tensor

$\mathbf{b}$ is the body force vector

e is the internal energy

$\mathbf{D}$ is the rate of deformation tensor

$\mathbf{q}$ is the heat flux

[19]

2.6 Coupled Eulerian-Lagrangian

CEL is an advanced formulations which combines both Lagrangian and Eulerian reference frames into a single model whilst at the same time maintaining that both domains are separated in space. It eliminates the core weaknesses of using either formulation by themselves by partitioning physical domains into distinct regions.

It does this by enforcing interactions at the boundary where the Lagrangian mesh meets the Eulerian material, utilising a penalty method whereas if the Eulerian material attempts to cross or penetrate the Lagrangian element surface, the penalty algorithm applies a virtual spring force normal to the interface to push them apart, thus conserving momentum.

This technique is primarily used for applications where a fluid might meet a solid, or to understand the interaction between two solids moving at high speeds where mesh elements might overlap or compose together in a simulation, which would not be possible in a real-world application.

[20]

2.7 Arbitrary Lagrangian-Eulerian

ALE is a hybrid formulation which combines both Lagrangian and Eulerian into a single framework. It differs from CEL as it allows a single domain to transition dynamically between both Lagrangian and Eulerian behaviours

It does this by allowing the mesh to move arbitrarily based on user defined optimisation rules, maintaining high element quality. What this means is that at the domain boundaries, the mesh will behave in a typical Lagrangian manner, maintaining well defined geometric edges and contact interactions. At the same time, within the domain interior the mesh will behave in a modified Eulerian manner, allowing the internal elements to reposition as the mesh displaces or shears, to prevent grid tangling and element inversion. The user definition part of this is the refinement of exactly when to apply this technique, at specific timesteps

or moments of distortion, as the computing requirement is resource intensive therefore it can add both time and resource requirement to the solve.

[21]

2.8 Smoothed Particle Hydrodynamics

Smoothed Particle Hydrodynamics (SPH) is a mesh-free, particle-based continuum, in which elements are converted into a particle cloud with indiscriminate mesh boundaries and interactions. It is useful for eliminating mesh distortion issues which lead to structural issues when solving high speed impact problems.

In Abaqus and other FEA programs, it can either be set from step zero where a model converts and behaves entirely as a large mass of particles, with no boundary interactions other than force velocity. This is best for describing how a model may act as a fluid however drawbacks would be any assembly is completely disregarded.

The other method is to use it as a conversion from Lagrangian at (or just before) the point in which the mesh would normally fail due to excessive distortion, which can be defined by either a time step or maximum plastic failure strain. This allows maximum mesh interaction prior to conversion, to produce more accurate results.

Applications of SPH in industry are for ballistics, fragmentation, hydrodynamics of fluids, and aerospace modelling of soft tissue impact such as bird strikes.

[22,23]

3 Case Study – Shaped Charge

As part of this report, it was required to model and solve a shaped charge problem, with two two-dimensional simulations that included a change in at least one aspect of the model, as well as a three-dimensional model to understand the complexity and limitations of modelling in three dimensions, as well as alternative functions to aid the simulation in providing results which may allow a different level of accuracy or output of understanding.

3.1 Material Properties

The shaped charge was comprised of 3 components:

- Case
- High Explosive (HE)
- Liner

This required deciding which materials would be better suited to each part, with two different materials being chosen for the liner in order to compare results.

For the HE, RDX was chosen as it is a widely understood explosive material which has been around for over half a century, therefore researching and collecting information on this was simple.

For the liner, it was felt that a comparison of a reactive material along with a stable material would provide the best results for analysis, therefore magnesium and aluminium were chosen.

For the case, it was important not only to choose something simple, but also a material which would be significantly more robust than the liner material, so to emphasise on the driving force of the shaped charge and provide a clear jet formation. Mild steel was chosen for this component.

The properties of each component are separate, split into metals and explosives in each of the two following tables:

Material →		Mild Steel [24,25,26,27,28]	Magnesium AZ31 [29,30,31,32]	Aluminium 5083- H116 [33,34,35,36,37]
Property ↓				
Thermal Conductivity ($W/m \cdot k$)		51	96	117
Specific Heat Capacity ($J/kg \cdot k$)		475	1000	900
Johnson Cook Damage	D1	0.02	-0.35	0.0261
	D2	1.5	0.6025	0.263
	D3	0.2	-0.4537	-0.349
	D4	0.002	0.206	0.147
	D5	0	7.2	16.8
	Melting Temperature (K)	1798	891	873.15
	Transition Temperature (K)	298	293	293
	Reference Strain Rate	1	1	1
Johnson Cook Plastic	A	363000000	141600000	167000000
	B	792712200	271700000	596000000
	n	0.5756	0.2105	0.551
	m	1.64566	1.83	0.859
	C	0.03	0.013	0.001
	Epsilon dot zero	0.6	1	1
Density (Kg/M^3)		7850	1770	2660
Shear Modulus		78000000000	17000000000	71000000000
Mie-Gruneissen Equation of State	c0	4569	4516	5240
	S	1.49	1.256	1.4
	Gamma zero	2.17	1.43	1.97

Table 1 - Properties of Metals Used for Shaped Charge

Material →		RDX [38,39,40,41,42]
Property ↓		
Thermal Conductivity ($W/m \cdot k$)		2060
Specific Heat Capacity ($J/kg \cdot k$)		1005
Density (Kg/M³)		1820
Mie-Gruneissen Equation of State	Detonation Wave Speed (m/s)	8540
	A	611300000000
	B	107000000000
	Omega	0.32
	R1	4.4
	R2	1.2
	Detonation Energy Density	5846000
	Pre-detonation Bulk Modulus	19600000000

Table 2 - Properties of HE Used for Shaped Charge

These properties were hand picked out not only to ensure the deformation due to pressure and velocity could be analysed, but also the temperature as a result of reaction, as magnesium reacts by burning vigorously at high temperatures when ignited, and is used in specialised shaped charges that are used to damage structural concrete [1,43,44].

3.2 Setting up Abaqus to Solve an Explosive Problem

Once the material properties had been obtained, the model was set up in abacus, first by modelling the parts. For this, a simple cup shaped case and liner with a 45-degree cone angle was used, with only half the two-dimensional model drawn, as a boundary condition was later set, in order to reduce computing requirements.

Each part was assigned a material property, with the liner being assigned the properties of both aluminium and magnesium, where one of the properties was suppressed for each simulation.

Following that, the parts were assembled into a finished model, and a step was assigned to the model as dynamic, temperature displacement, explicit, to ensure thermal properties were captured, as well as to set the run time of the simulation. Interactions between parts as well as boundaries were set, ensuring that the centreline formed a solid boundary.

As this is an explosive problem, no load was required to be set.

The meshing of the parts formed one of the more complex area's where a global mesh was applied to the case, but a graduated mesh was applied to both the HE and the liner, with the most refined point being central to the shaped charge and graduating out to match the mesh sizing at the point where the HE and liner interacted with the case.

The elements for the mesh were assigned as explicit, coupled temperature-displacement to ensure any energy transfer was monitored, at a linear geometric order and tetrahedrally.

Reactive state-of-change interactions were investigated, as high temperature detonations could lead to the transition from solid to gases, however it was

discovered that this would require additional user sub-routines which was not part of the software provided.

3.3 Jet Formation Comparison

Some of the initial simulations failed due to excessive mesh distortion as seen in figure 2:

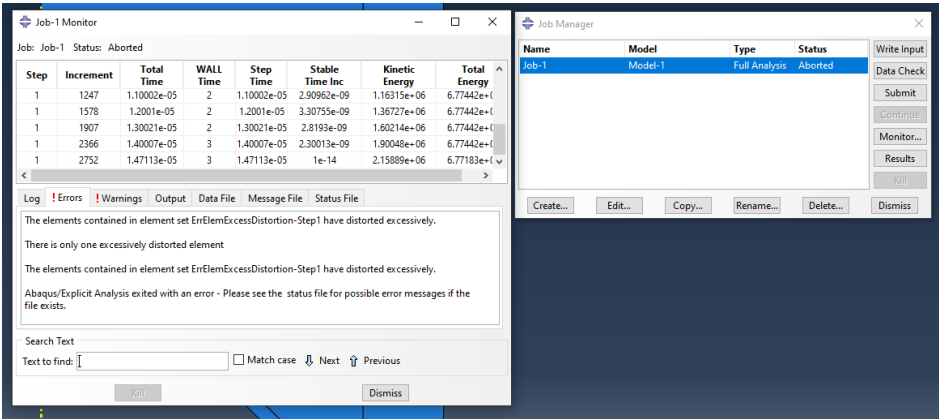


Figure 2 - Simulation Failure Error Message

The simulation was analysed and it was realised that part of the gradient on the mesh of the HE had been set incorrectly. This and the excessively distorted element at the top of the HE can be seen below:

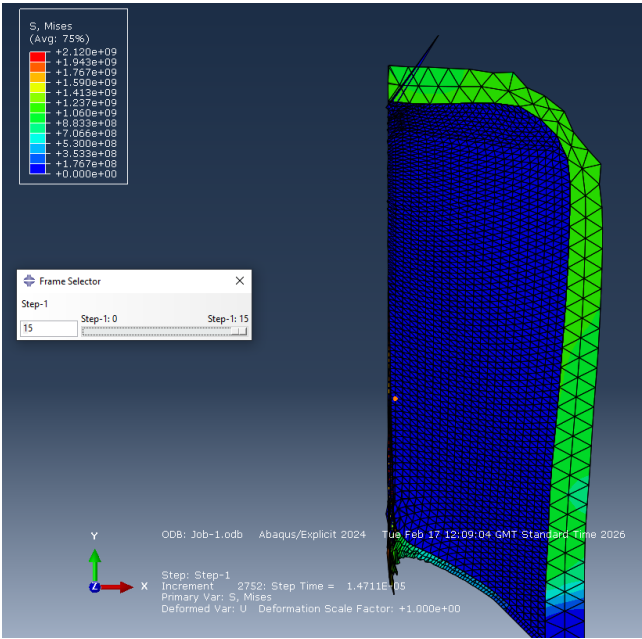


Figure 3 - Incorrect Meshing and Excessive Element Distortion

Once this had been corrected, the simulation was run again for both liner materials. This time it failed a short time later, due again to excessive element distortion. It was realised at this point that the mesh for the liner had been set to hexahedral, rather than tetrahedral. This mismatch of elements caused the excessive distortion as seen below:

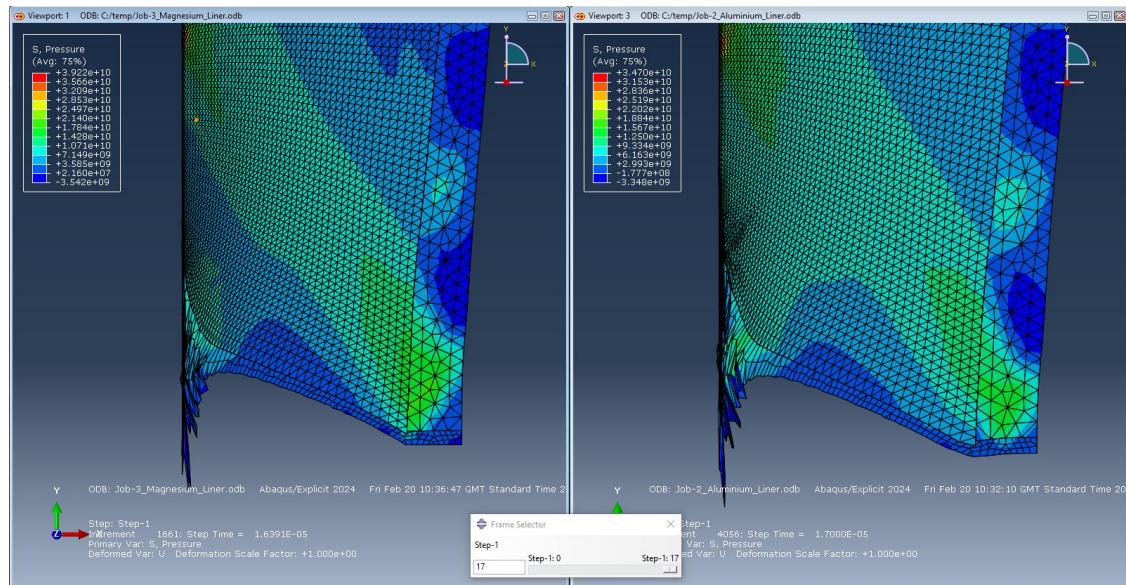


Figure 4 - Mixed Mesh Simulation Failure

The results of this were analysed and the software used to pinpoint the exact elements which had excessively distorted, first by isolating the view to the liner only:

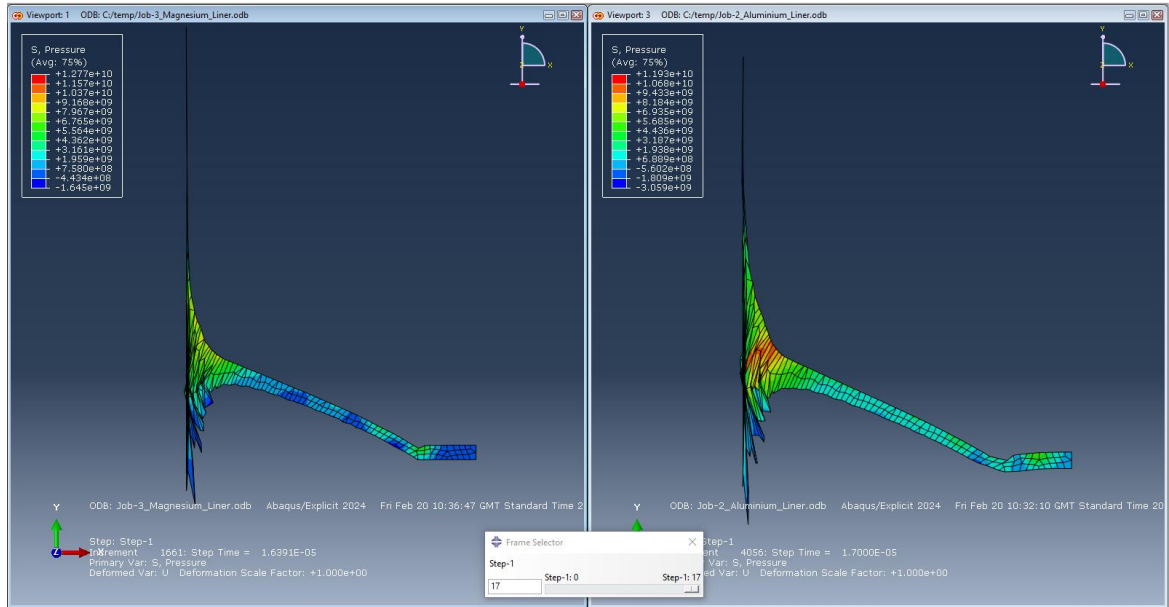


Figure 5 - Liner Part Isolation

Then by isolating the offending elements:

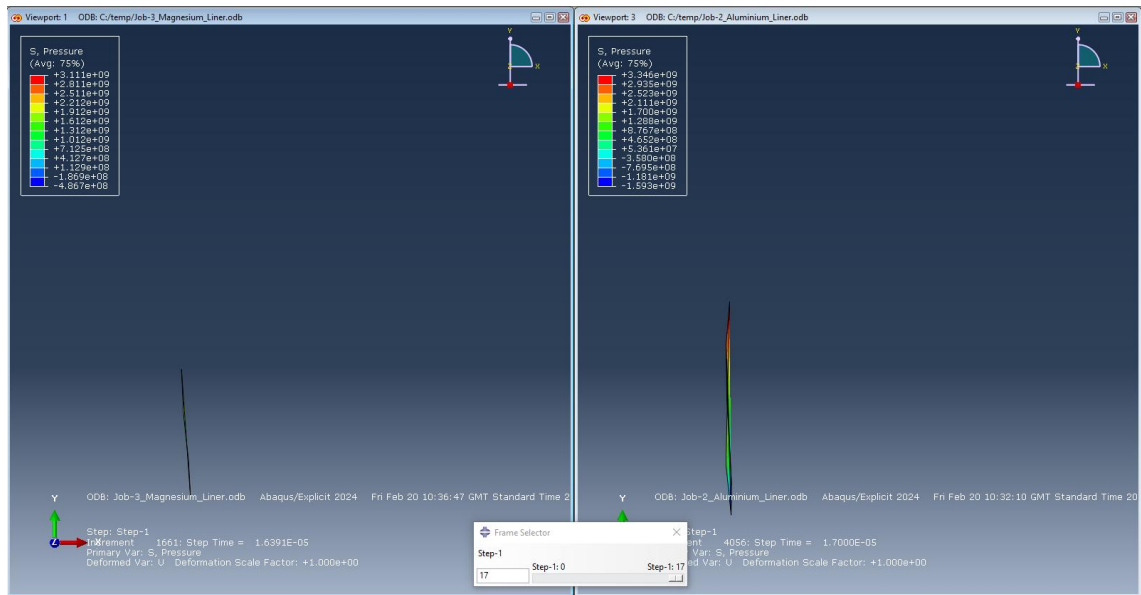


Figure 6 - Excessively Distorted Element Isolation

Following a rectification in the mesh elements, where the size and shape between the interacting faces of the parts were matched, a successful jet formed for both simulations:

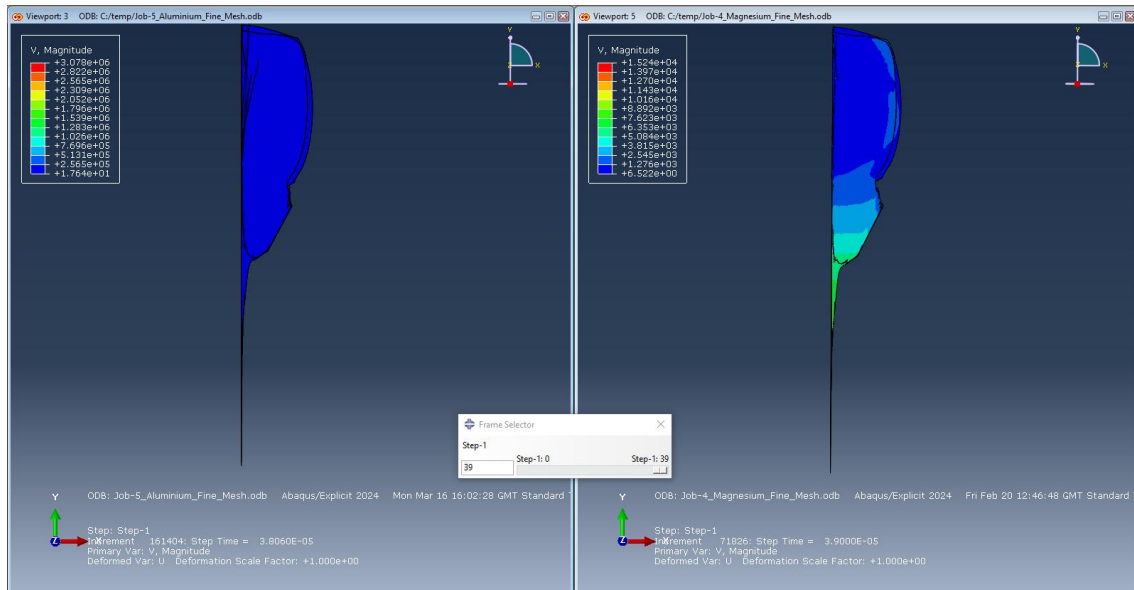


Figure 7 - Successful Shaped Charge Jet Formation

From these results, only a minor amount of information could be ascertained, therefore it was necessary to conduct further analysis.

3.4 Pressure Analysis

Following the successful formation of the jet, a series of analytics were undertaken on the results. First, pressure analysis was conducted.

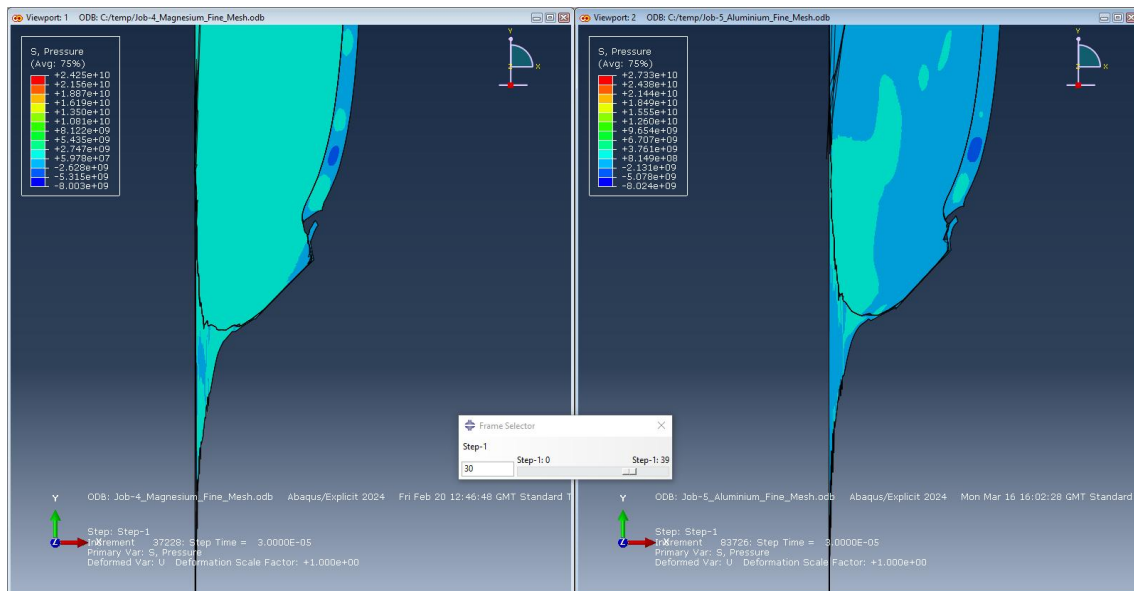


Figure 8 - Pressure Analysis Initial Formation of Jet

Upon initial formation of the jet, the pressure can be seen to be higher throughout the body of the material of the magnesium liner.

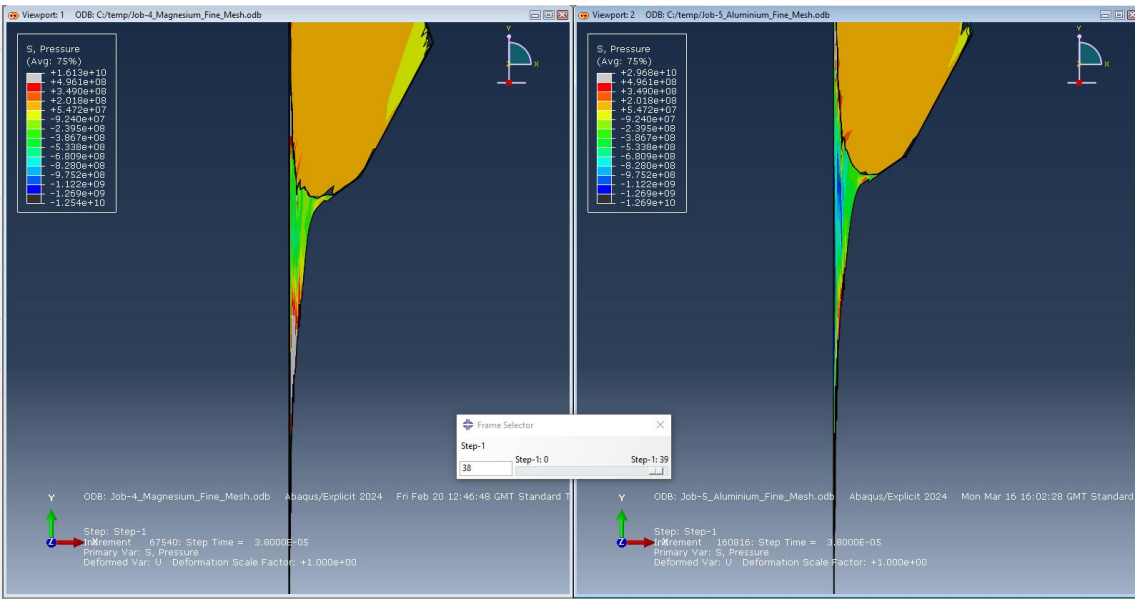


Figure 9 - Pressure Analysis Peak Overpressure

However, once the Jet had fully formed, the peak overpressure in the tip of the aluminium jet can be seen here as greater than that of the magnesium, whilst the magnesium had higher pressure distributed overall, as well as a slighter thicker jet.

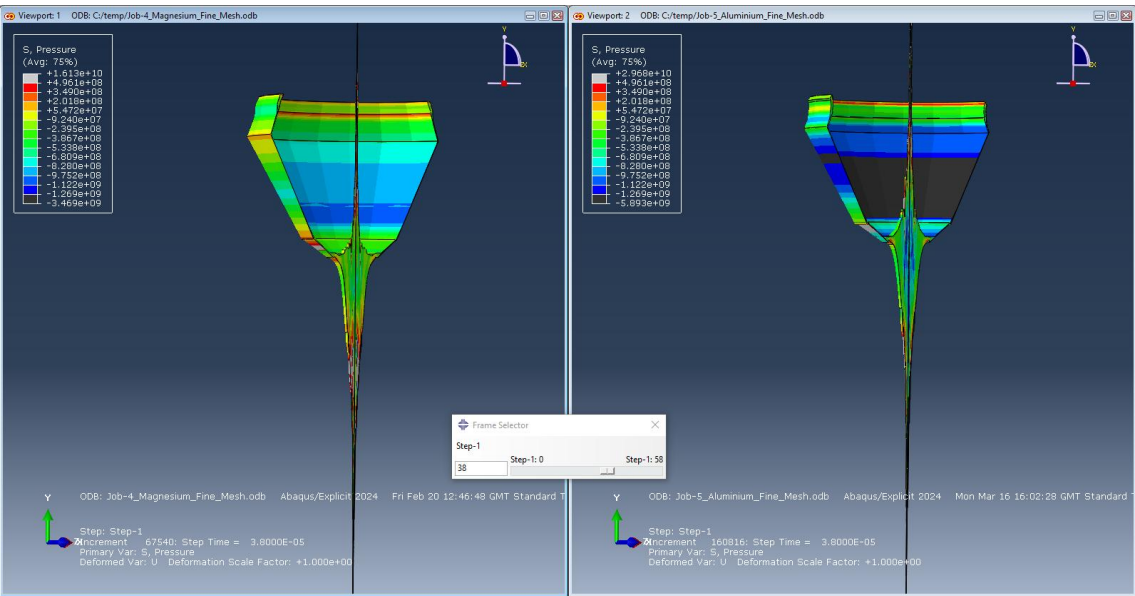


Figure 10 - Pressure Analysis Liner Isolation

In figure 10, the liner was isolated as well as the visuals extruded about 180-degrees to provide the best visual representation. This confirms that the magnesium jet had formed thicker, with the pressure spread more evenly throughout.

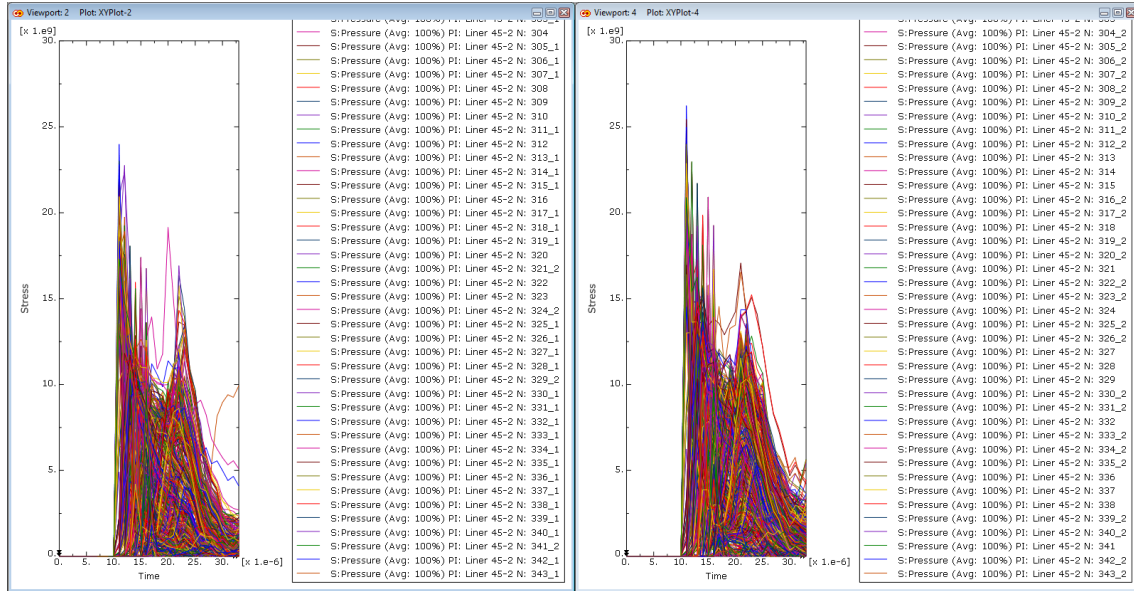


Figure 11 - Pressure Analysis Graphs

Nodal analysis was undertaken on a selection of nodes from the liners of each. This further confirms that whilst there was a larger spread of high pressure across the magnesium liner, the aluminium liner exhibited a higher peak overpressure through some of its elements.

3.5 Velocity of Detonation

The velocity of the detonation and liner jet was the next thing to be analysed. From our previous results, it was predicted that the aluminium would exhibit higher peak velocities, whilst the magnesium would exhibit a higher velocity spread.

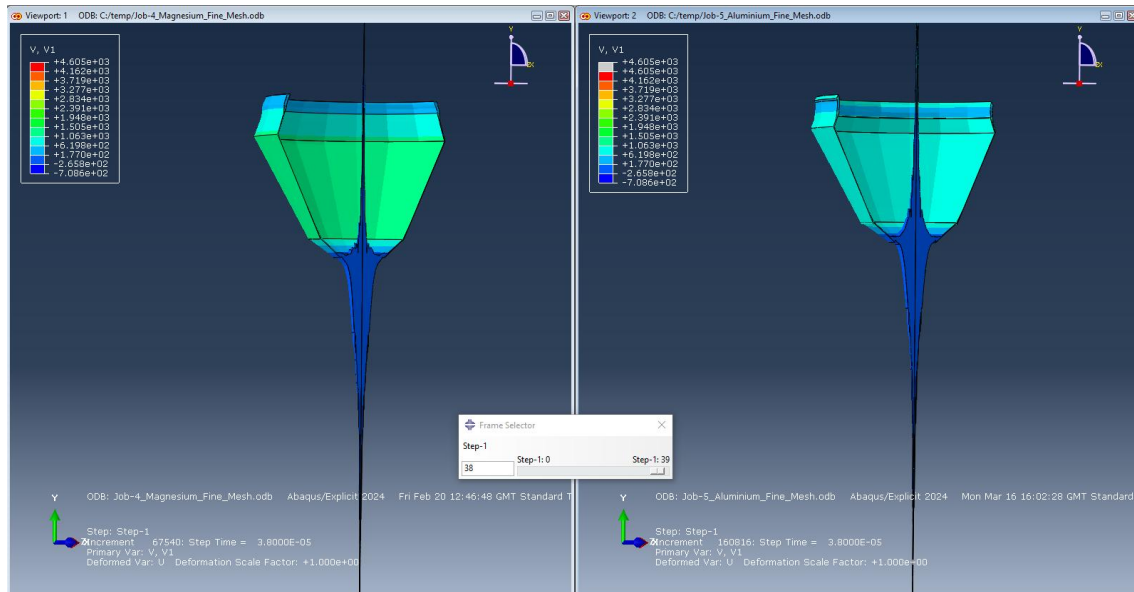


Figure 12 - Velocity Analysis of Liner

Figure 12 confirms that at least for the magnesium liner, the velocity spread was higher than in the aluminium. It was difficult to identify any peak velocities from this visual analysis alone.

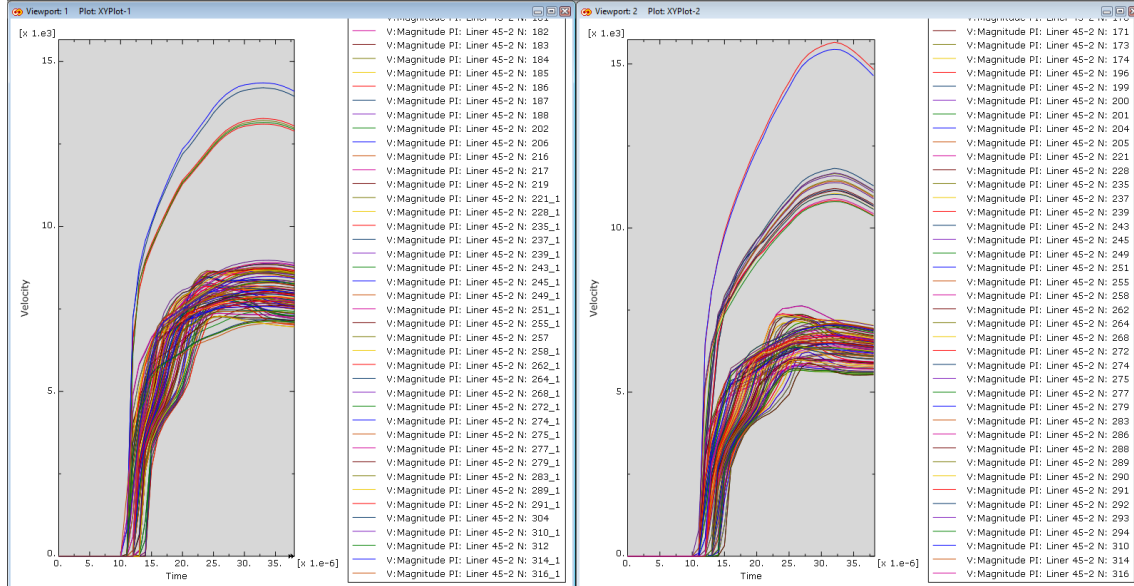


Figure 13 - Velocity Analysis Graphs

The velocity graphs however validated that the peak velocities experienced by the aluminium were higher than that of the magnesium. It also further confirmed the higher velocity spread in the magnesium.

3.6 Thermal Analysis of Liner

The final area of analysis was the thermal analysis of the two liners. From the earlier research on reactivity of the metals used, it was expected that the magnesium would exhibit a higher temperature output than the aluminium.

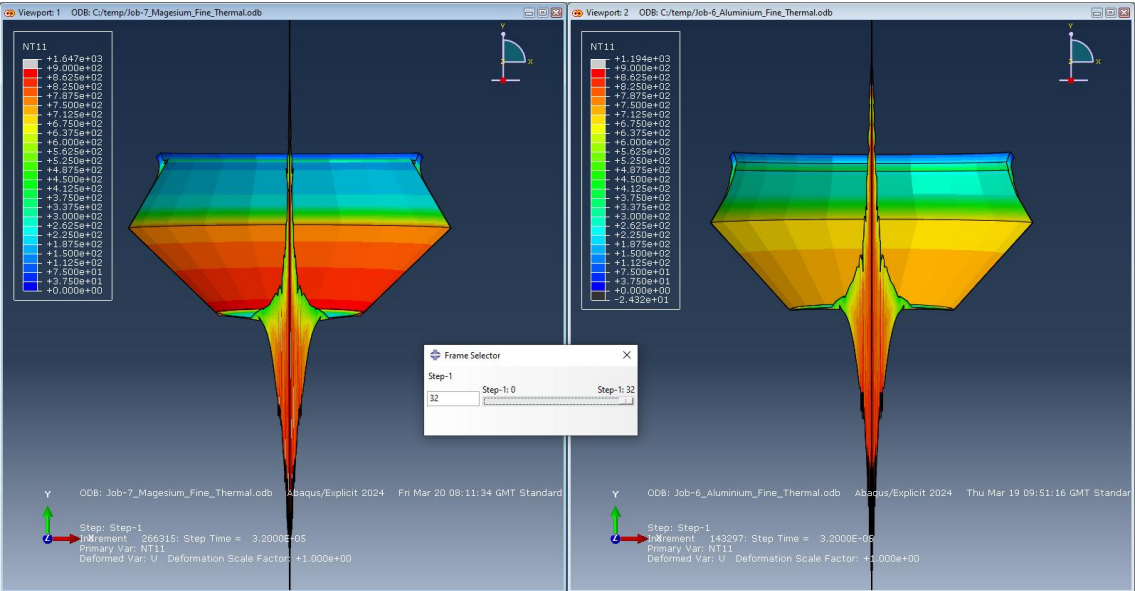


Figure 14 - Thermal Analysis of Liner

Figure 14 demonstrates that the temperature across the magnesium exceeded that of the aluminium across the liner, as expected.

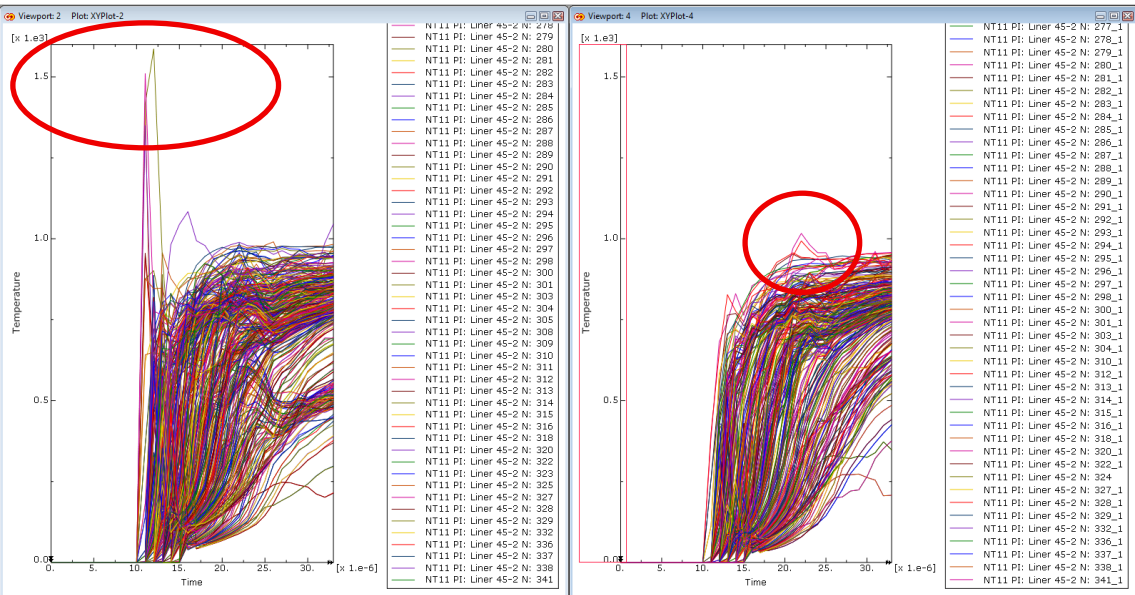


Figure 15 - Thermal Analysis Peak Temperature Graphs

The graph shows that the magnesium liner reached a considerably higher peak temperature compared to that of the aluminium, potentially signifying that of an autoignition of the material.

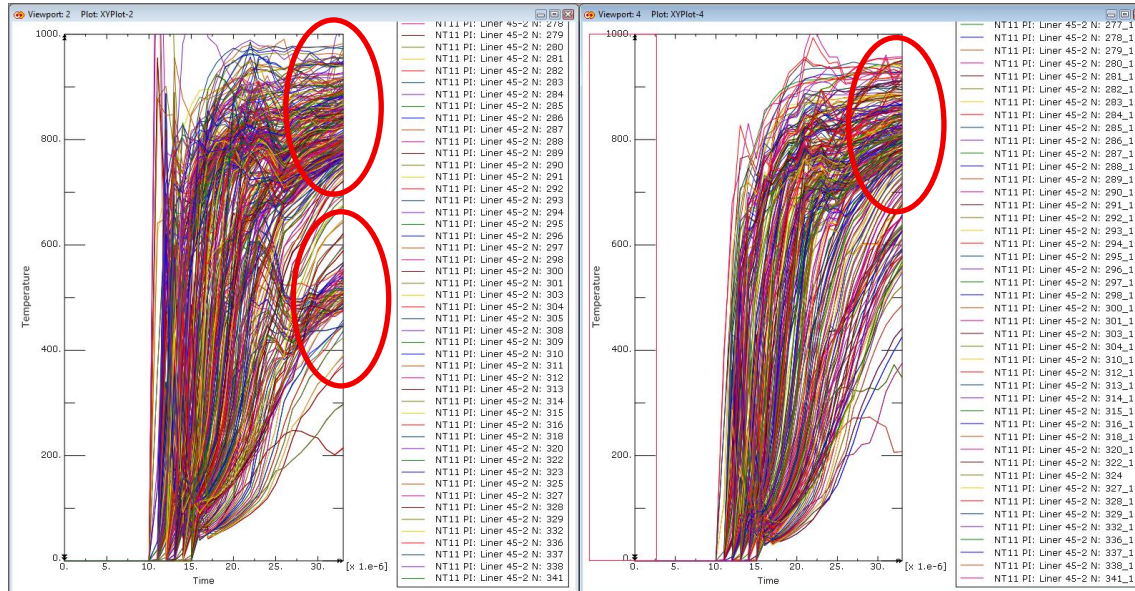


Figure 16 - Thermal Analysis Temperature Spread Graphs

Zooming in on the results, the temperature experienced across the liners can be seen, with the aluminium being largely uniform with an area of high concentration, whilst the magnesium has two focal points, which would suggest a higher thermal diffusivity.

3.6.1 Thermal Diffusivity

It is important to understand thermal diffusivity at this point, to validate whether the results shown in the graph align with mathematical conclusions drawn from calculating thermal diffusivity and conductivity.

Thermal diffusivity is a material specific property which measures the rate at which heat propagates through a material during transient temperature changes, quantifying how fast a substance reacts and changes to its internal temperature to match, rather than how much total heat energy it can move.

It can be described mathematically by its core formula [45]:

$$\alpha = \frac{k}{\rho C}$$

Equation 14 - Thermal Diffusivity

Where:

α – Thermal Diffusivity (m^2/s)

k – Thermal Conductivity ($W/m \cdot k$)

ρ – Density (kg/m^3)

C – Specific Heat Capacity ($J/kg \cdot k$)

Following this, the thermal values taken from the earlier research can be split out from the prior table.

Material →	Magnesium AZ31	Aluminium 5083-H116
Property ↓		
Thermal Conductivity ($W/m \cdot k$)	96	117
Specific Heat Capacity ($J/kg \cdot k$)	1000	900
Density (Kg/M^3)	1770	2660

Table 3 - Thermal Material Properties of Liner Materials

These figures can be used to calculate the thermal diffusivity of each material.

For magnesium:

$$\alpha = \frac{96}{1770 \cdot 1000} = 5.42 \times 10^{-5} m^2/s \text{ or } 54.2 mm^2/s$$

For aluminium:

$$\alpha = \frac{117}{2660 \cdot 900} = 4.89 \times 10^{-5} m^2/s \text{ or } 48.9 mm^2/s$$

These Calculations confirm that the aluminium has a lower thermal diffusivity than the magnesium therefore the heat travels faster through the magnesium, resulting in waves of heat travelling through the material. Normally, aluminium would have a higher diffusivity than magnesium however the aluminium selected for this solution is that of an alloy, dramatically lowering the baseline thermal conductivity. This does however validate the results found by the thermal analysis of the simulation.

3.7 Three-Dimensional Simulation

A three-dimensional model was constructed from the model, using the same figures as before and applying magnesium as the property for the liner. This was extruded about 180-degrees, and a boundary was set along the cutaway, to be able to give a better analysis.

This differed from the prior 180-degree visual as the simulation considers the mesh entirely differently when calculating the results and completing the solve.

Initially, this caused an issue with the simulation, where it would crash early in the solve due to excessive element distortion, as per figure 17 below:

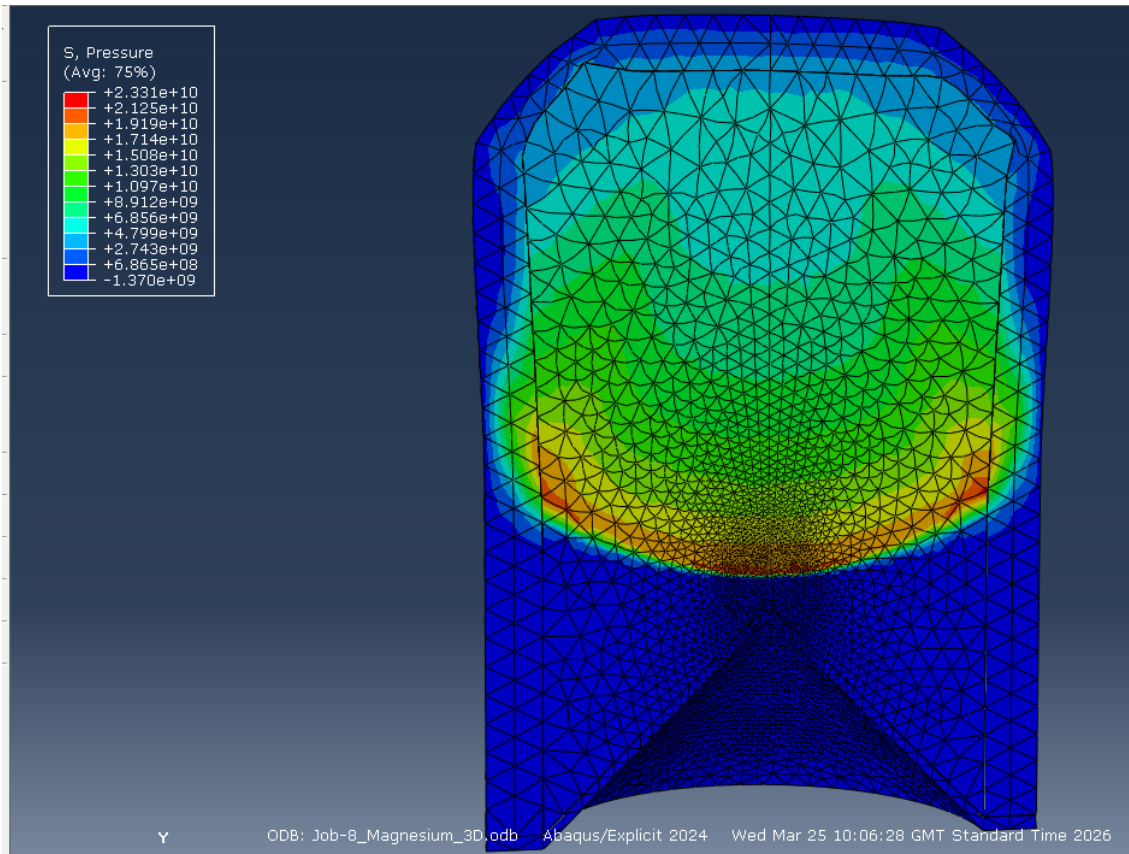


Figure 17 - Three-Dimensional Analysis Initial Point of Failure

This was resolved by adjusting the settings. Initially ALE mesh adaptation was attempted, however this did not resolve the issue.

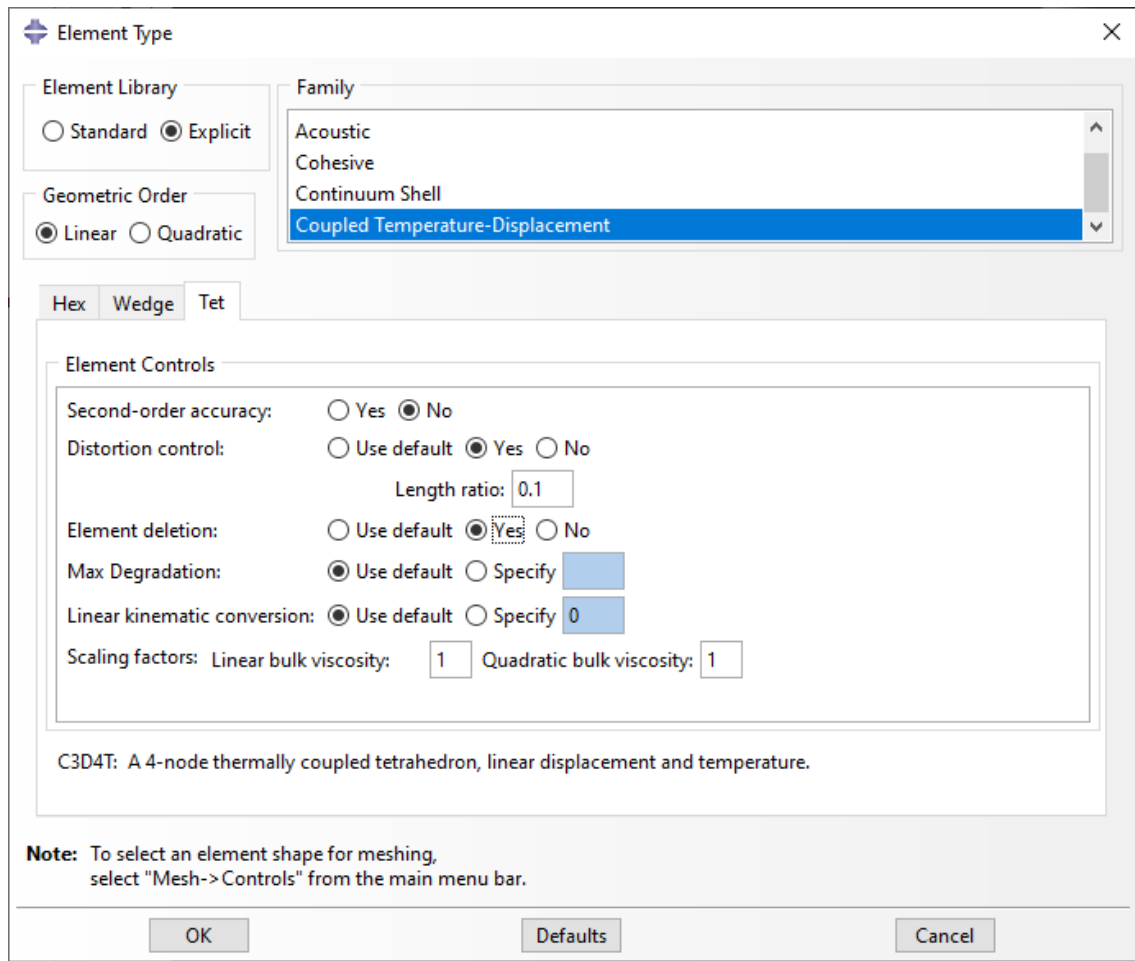


Figure 18 - Distortion Control of the Three-Dimensional Solve

Distortion control was applied to the model, where if an element distorted to more than 0.1 of its length ratio, then it would delete the element to prevent excessive distortion.

This resulted in a successful solve:

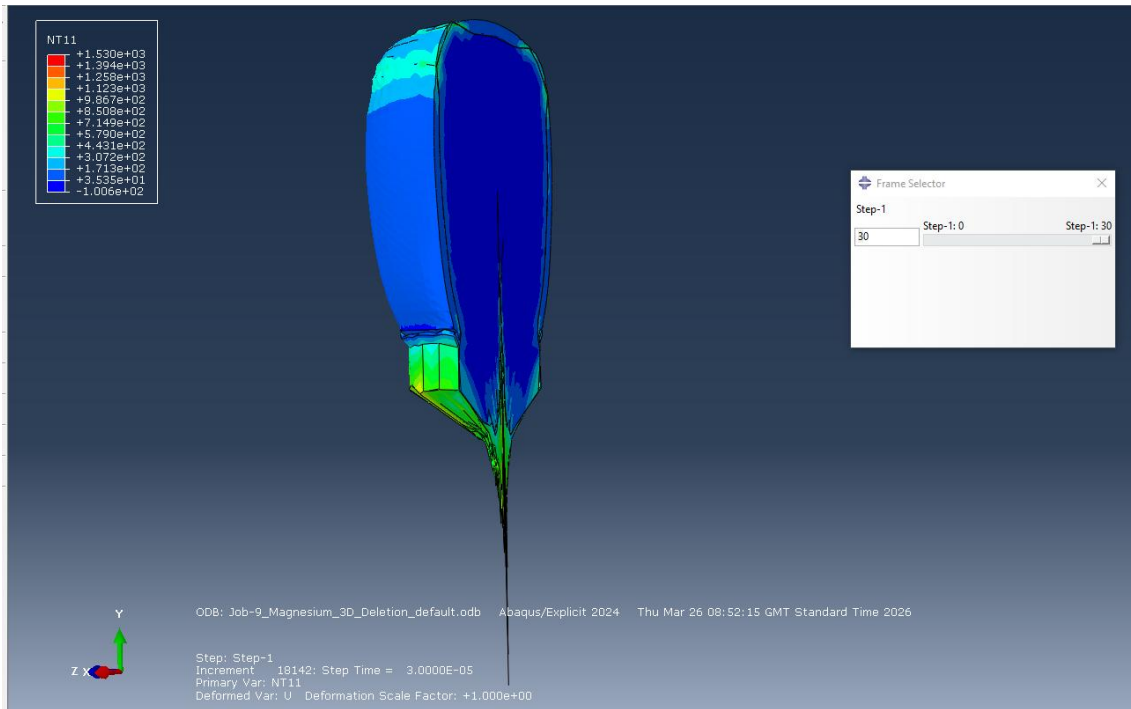


Figure 19 - Three-Dimensional Jet Formation

The temperature profile of the liner was then analysed.

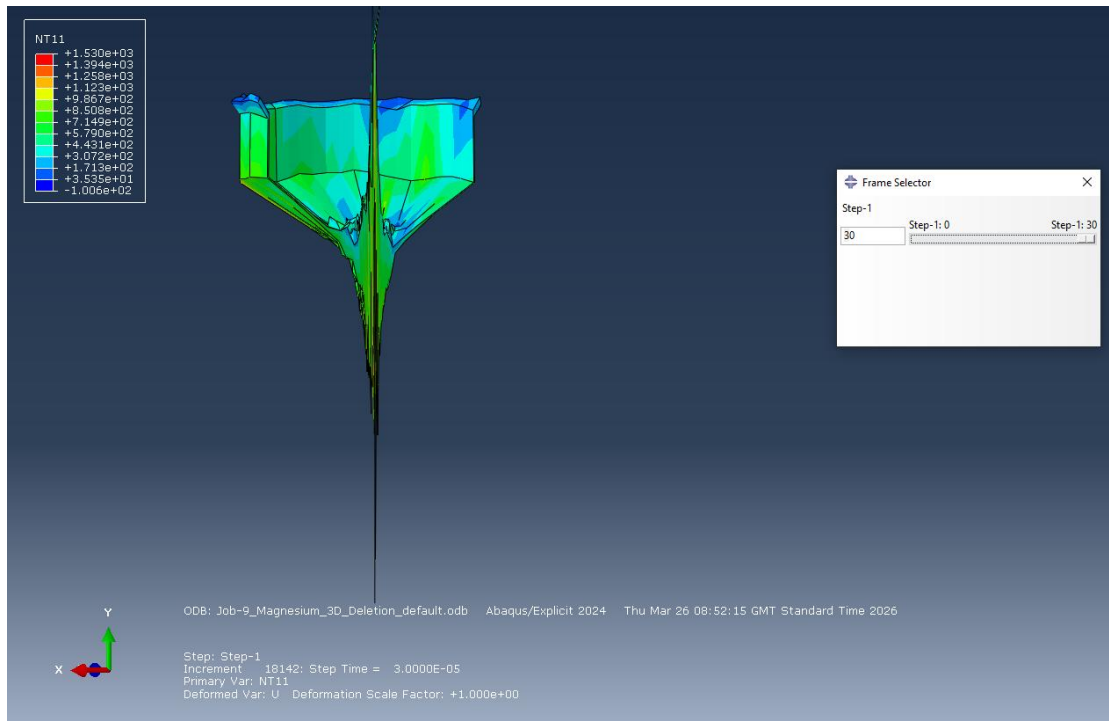


Figure 20 - Temperature Profile of Liner of Three-Dimensional Model

This was important as it demonstrated the difference between the two-dimensional and three-dimensional models, where in the two-dimensional model the temperature spread was banded and uniformly distributed across the liner, whilst the three-dimensional model adapts for the extra dimension.

The temperature profile graph also reflected this:

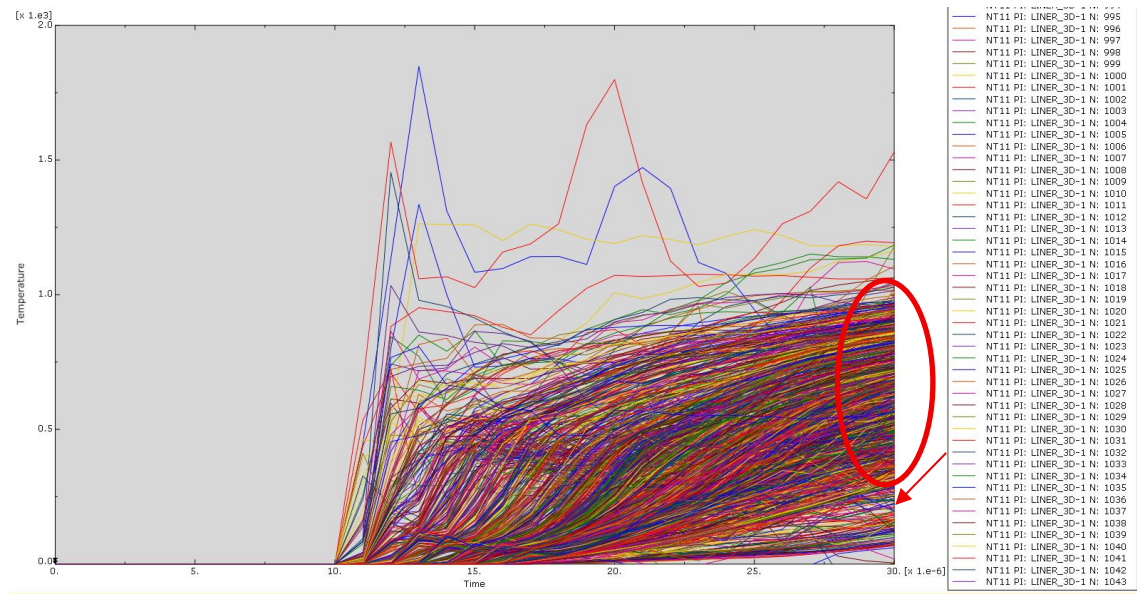


Figure 21 - Three-Dimensional Temperature Graph for Magnesium Liner

This highlights the importance of using three-dimensional modelling for more accurate results, as the element temperature is more evenly distributed and less localised, although a small separation gap can still be seen.

3.7.1 Smoothed Particle Hydrodynamic Adaptation

The final analysis conducted on the case study was the SPH adaptation of the three-dimensional model.

Some of the setup was required to be changed, as SPH does not adapt for thermal because as previously mentioned, it turned the elements to act independently as particles and only interact kinematically, therefore the step was changed to only dynamic, explicit.

A further adjustment was made to ensure that the elements converted to SPH particles at a specific timeframe:

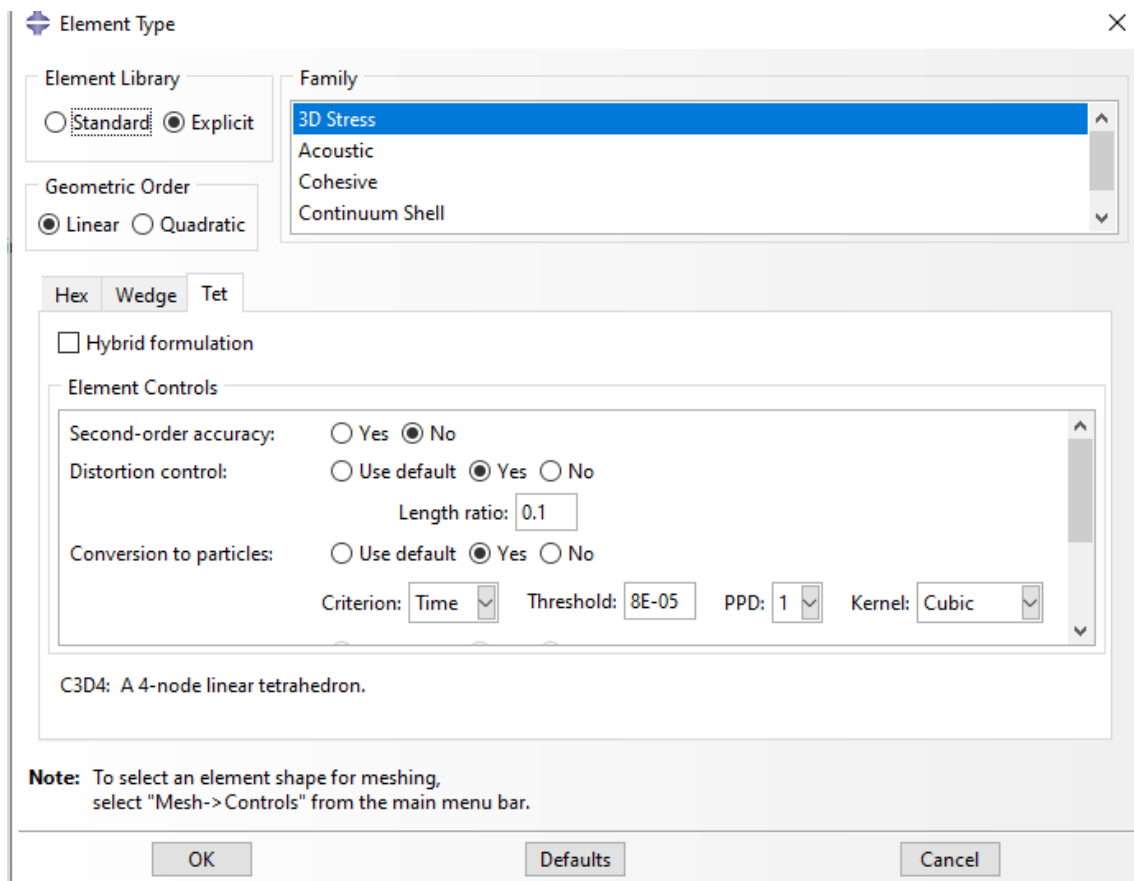


Figure 22 - SPH Conversion Settings for Three-Dimensional Model

This was run and the results can be seen in figures 23-25.

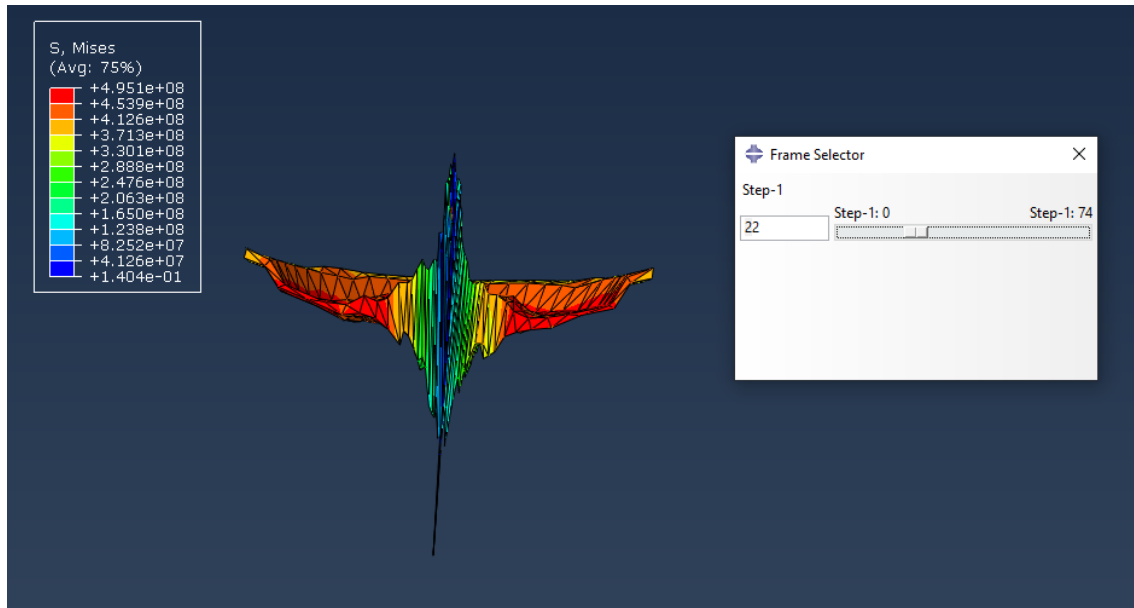


Figure 23 - Final point of Lagrangian Meshed Three-Dimensional Model Prior to SPH Conversion

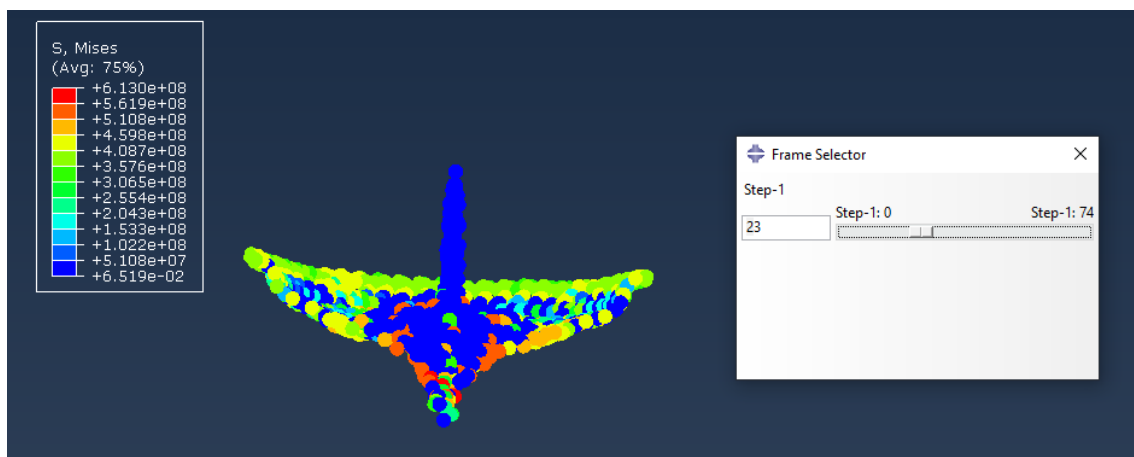


Figure 24 - Initial Conversion of Liner to SPH

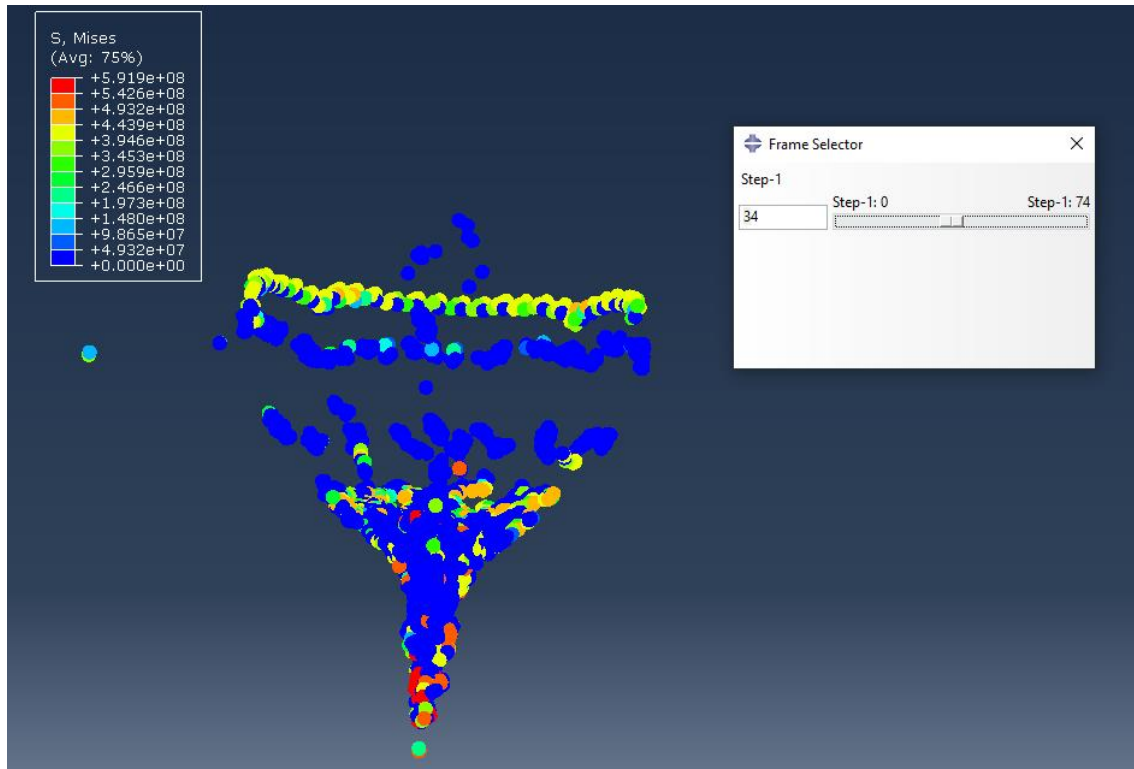


Figure 25 - Formation of Jet from SPH

This shows how SPH can be used to adjust three-dimensional models to understand the hydrodynamics of complex kinematic problems, however the accuracy is greatly decreased as the interactions such as high-strain thermal coupling are disregarded.

4 Conclusion

For this report shaped charge design was reviewed, with a literature review of the history and development of shaped charges, as well as shaped charge modelling conducted in the explicit solver software Abaqus. Underlying principles were reviewed and described and a comprehensive analysis of the results of both the two-dimensional and three-dimensional models conducted, using various techniques.

The analysis of these models demonstrated how the principles of the software could be used to generate a solution which could be used in design, as well how some of the underlying principles and advanced techniques can be applied to shaped charge modelling to provide a greater result. The analysis of three-dimensional modelling provided valuable insight to the limitations of two-dimensional modelling, although was far more difficult to model and required fault finding techniques in order to ensure the simulation executed correctly, as well as requiring significantly more computing time and resource. The advanced technique of SPH provided some useful insights but overall provided significantly less detail.

interpreting temperature analysis and validating it using the underlying physics principles calculations was important as it could be used in the future of design of shaped charges to understand materials which provide a higher heat output for specialised explosives devices in the field. However, one limitation of this was the inability to add any reactive state change modelling to the simulation, which after some research it was understood that could be done but only if user sub-routines were a part of the software, which for the version supplied to conduct this was not included. Other limitations included licence and resource limitations, where the element nodes were limited to 250,000. In order to produce a greater level of detail then a larger number of element nodes as well as the introduction of user subroutines could generate a much more complex yet detailed result.

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