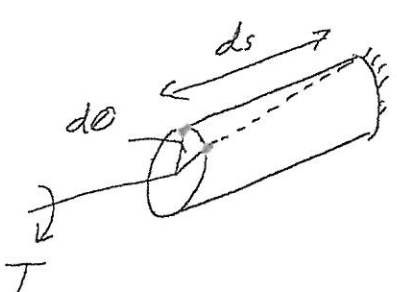


EXAMINATION SOLUTIONS: SESSION 2016/2017

School:	SCHOOL OF ENGINEERING		
Programme:	MECHANICAL & MANUFACTURING	Year:	3
Module/Unit:	ADVANCED STRESS ANALYSIS + FEA		
Author:	KELVIN LAKE	Page:	1 of

Question No	Question	Marks
1a;	 SINCE $\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{L}$ $\theta = \frac{TL}{GJ}$ $\therefore$ FROM DIAGRAM : $d\theta = \frac{Tds}{GJ}$	2

Continuation Sheet

Module/Unit:	ADVANCED STRESS ANALYSIS + FEA	Page:	2 of
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Question No	Question	Marks
(1a ii)	now $U = \frac{1}{2} T d\theta$ $\therefore$ FOR SECTION SHOWN $U = \frac{1}{2} T \frac{T ds}{GJ}$ $= \frac{T^2 ds}{2GJ}$ $\therefore$ TOTAL STRAIN ENERGY $U = \int_0^L \frac{T^2 ds}{2GJ}$ $= \frac{T^2 L}{2GJ}$	(4)
(ii)	RESILIENCE = STRAIN PER UNIT VOLUME IF $U = \frac{P^2 L}{2AE}$ $U = \frac{\sigma^2 A^2 L}{2AE}$ $\therefore U = \frac{\sigma^2 AL}{2E}$ $\therefore U/AL = \frac{\sigma^2}{2E}$ OR $U/V = \frac{\sigma^2}{2E}$	(4)

Continuation Sheet

Module/Unit:	ADVANCED STRESS ANALYSIS + FEA	Page:	3 of
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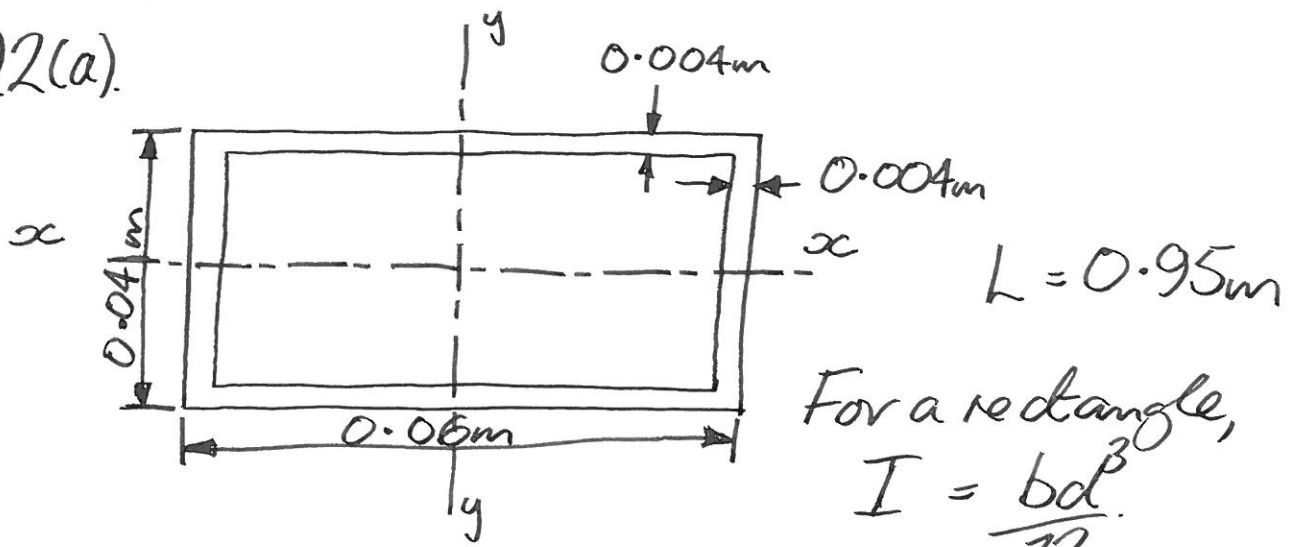
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Continuation Sheet

Module/Unit:	ADVANCED STRESS ANALYSIS + FEA	Page:	4 of
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Question No	Question	Marks
(biii)	$h = \frac{\sigma^2 L}{2Emg} - \frac{\sigma L}{E}$ $= \frac{(75 \times 10^6)^2 \times (4 \times 10^{-4} \times 4)}{2 \times 200 \times 10^9 \times 200 \times 9.81} - \frac{75 \times 10^6 \times 4}{200 \times 10^9}$ $= \frac{9 \times 10^{12}}{7.85 \times 10^{14}} - \frac{300 \times 10^6}{200 \times 10^9}$ $= 11.47 \times 10^{-3} - 1.5 \times 10^{-3}$ $= \underline{\underline{9.97 \text{ mm}}}$	(3)

Q2(a).



$$L = 0.95\text{m}$$

For a rectangle,

$$I = \frac{bd^3}{12}$$

$$\therefore I_{yy} > I_{xx}$$

Hence, strut will buckle about x-axis, i.e. lower value of second moment of area.

(4 marks).

(b). From Euler theory of bending with pinned ends;

$$P_e = \frac{\pi^2 EI}{L^2}$$
 where, $I = I_{xx}$.

$$\text{Now, } I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{0.06 \times 0.04^3}{12} - \frac{0.052 \times 0.032^3}{12}$$

$$\therefore I_{xx} = 0.178 \times 10^{-6} \text{m}^4$$

$$\therefore P_e = \frac{\pi^2 \times (200 \times 10^9 \times 0.178 \times 10^{-6})}{0.95^2}$$

So, Euler buckling load,

$$P_e = 389327.92 \text{ N}$$

$$(P_e = 389.33 \text{ kN})$$

(6 marks)

Q2(c). From Rankine-Gordon theory of bending;

$$P_R = \frac{\sigma_y \cdot A}{1 + a \cdot \left(\frac{L}{k}\right)^2}$$

where, A is c.s.a. of strut,
and, k is radius of gyration.

Now, $A = (0.06 \times 0.04) - (0.052 \times 0.032)$
 $= 0.736 \times 10^{-3} \text{ m}^2$

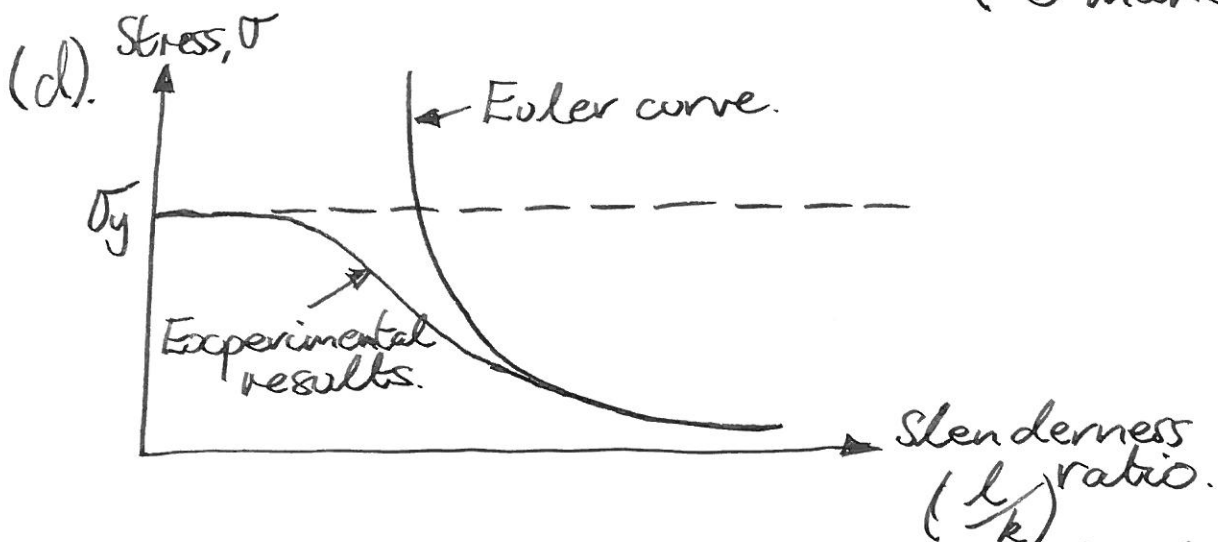
$$k = \sqrt{\frac{I}{A}} = \sqrt{\frac{0.178 \times 10^{-6}}{0.736 \times 10^{-3}}} = 0.0156 \text{ m}$$

$$\therefore P_R = \frac{325 \times 10^6 \times 0.736 \times 10^{-3}}{1 + \frac{1}{7500} \left(\frac{0.95}{0.0156}\right)^2}$$

$$P_R = 159726.7 \text{ N}$$

$$(P_R = 159.73 \text{ kN})$$

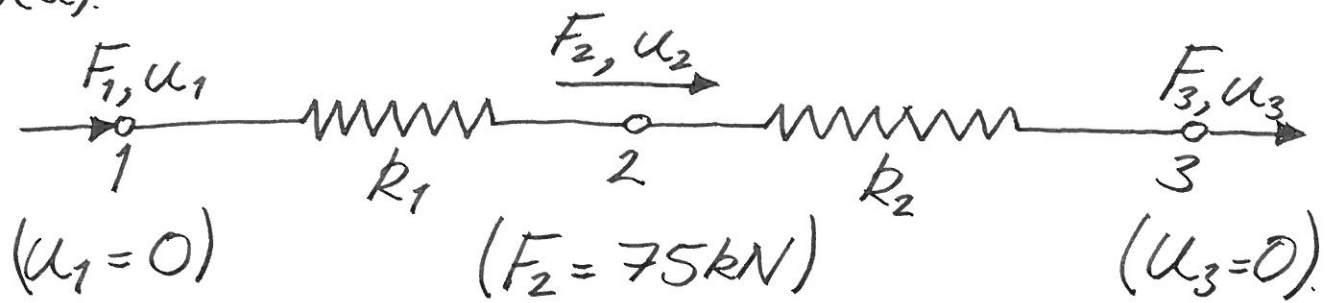
(6 marks).



Euler is only valid for large slenderness ratios (typically > 80 for structural steel). For lower values Rankine-Gordon theory should be used; as shown above, in this case is likely to be more accurate.

(4 marks).

Q3(a).



For a linear spring, $F = k \cdot u$

(4 marks).

(b)(i)

Bronze 'spring'; $\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$

Steel 'spring'; $\begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix}$

Expand matrices above so in equivalent form, and add because 'springs' in series;

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & k_1+k_2 & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \quad \text{i.e. } \{F\} = [K^e] \{u\}$$

where, $[K^e]$ is the element stiffness matrix.

Stiffness (k) of both bodies found from;

$$k = \frac{AE}{L}$$

Bronze: $k_1 = \frac{(400 \times 10^{-6}) \times (105 \times 10^9)}{0.2} = 210 \times 10^6 \text{ N/m}$
(210 kN/mm)

Q3(b)(i) Cont.

$$\text{Steel: } k_2 = \frac{(200 \times 10^6)(210 \times 10^9)}{0.125} = 336 \times 10^6 \text{ N/m} \quad (336 \text{ kN/mm})$$

Now,

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} = \begin{bmatrix} 210 & -210 & 0 \\ -210 & 546 & -336 \\ 0 & -336 & 336 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix}$$

$$\text{So, } F_1 = 210u_1 - 210u_2 \quad (1)$$

$$F_2 = -210u_1 + 546u_2 - 336u_3 \quad (2)$$

$$F_3 = -336u_2 + 336u_3 \quad (3)$$

Applying Boundary Conditions;

$$u_1 = 0 \quad \therefore \text{from (1), } F_1 = -210u_2 \quad (4)$$

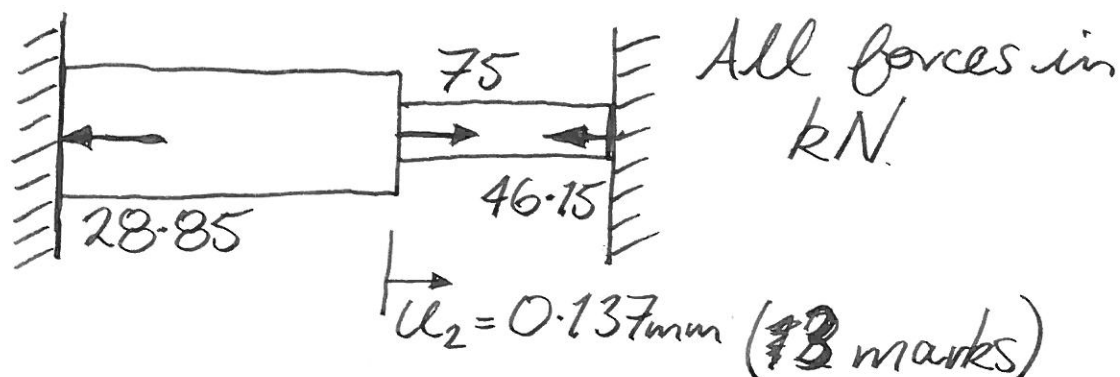
$$u_3 = 0 \quad \therefore \text{from (3), } F_3 = -336u_2 \quad (5)$$

$$F_2 = 75 \text{ kN} \quad \therefore \text{from (2), } 75 = 0 + 546u_2 - 0 \quad (6)$$

$$\text{From (6) above; } u_2 = \overset{\text{DIRECTION}}{0.137 \text{ mm}} \quad (12 \text{ marks})$$

$$(b)(ii) \text{ Hence, } F_1 = -210 \times 0.137 = -28.85 \text{ kN}$$

$$\text{and, } F_3 = -336 \times 0.137 = -46.15 \text{ kN}$$



Q3(b)(iii)

Forces at rigid end supports 'balance' applied force F_2 , i.e.

$$F_2 - F_1 - F_3 = 0.$$

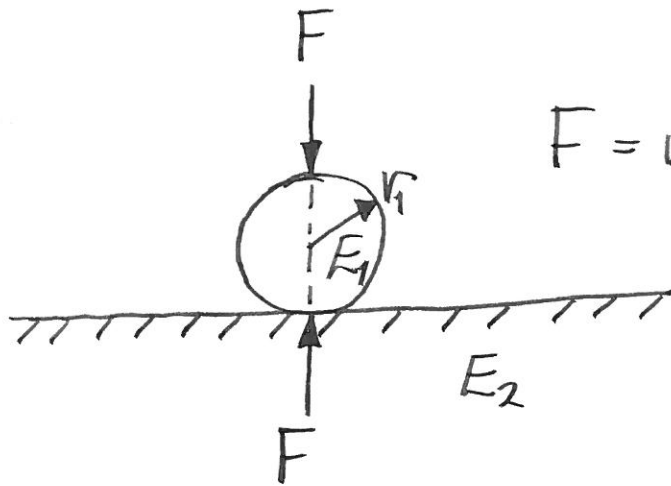
(as expected). (1 mark).

Q4(a).

- contacting bodies are isotropic, homogenous, and elastic
- contact areas are essentially flat & small relative to radii of curvature of undeflected bodies in immediate area of interface
- contacting bodies are perfectly smooth, therefore friction forces can be neglected.

(3 marks)

(b).



$$F = m \cdot g = 100 \times 9.81$$

$$\therefore F = 981 \text{ N}$$

$$E_1 = 72 \times 10^9 \text{ N/m}^2$$

$$E_2 = 210 \times 10^9 \text{ N/m}^2$$

$$r_1 = 0.0075 \text{ m}$$

Find, p_0 and δ .

$$p_0 = 1.5 \left(\frac{F}{\pi a^2} \right); a = 0.880 \sqrt[3]{F r_1 \Delta}; \Delta = \frac{1}{E_1} + \frac{1}{E_2}$$

$$\therefore, \Delta = \frac{1}{72 \times 10^9} + \frac{1}{210 \times 10^9} = 0.01865 \times 10^{-9}$$

$$a = 0.880 \sqrt[3]{981 \times 0.0075 \times 0.01865 \times 10^{-9}}$$

$$a = 0.4539 \times 10^{-3} \text{ m}$$

$$\therefore \text{Maximum pressure, } p_0 = 1.5 \left(\frac{981}{\pi \times (0.4539 \times 10^{-3})^2} \right)$$

$$p_0 = 2273.56 \text{ MN/m}^2$$

$$\therefore p_0 = 2273.56 \text{ MN/m}^2$$

Q4 (b) Cont.

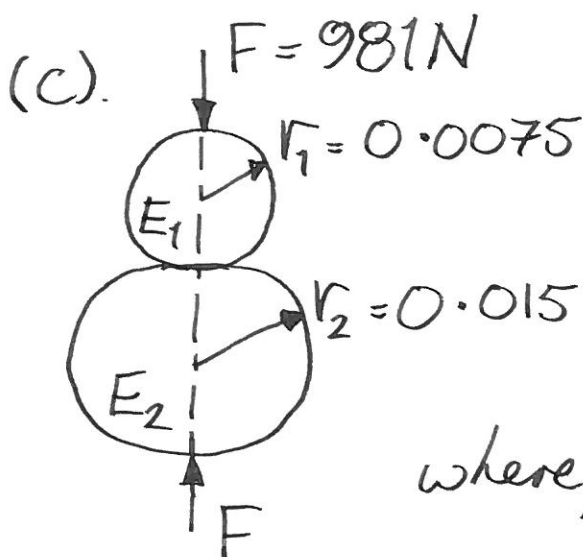
$$\delta = 0.775 \sqrt[3]{F^2 \frac{\Delta^2}{r_1}}$$

$$= 0.775 \sqrt[3]{(981)^2 \cdot \left(\frac{(0.01865 \times 10^{-9})^2}{0.0075} \right)}$$

$$\therefore \delta = 0.00002749 \text{ m}$$

(0.0275 mm)

(10 marks)



Now,

$$\delta = 0.775 \sqrt[3]{F^2 \Delta^2 m}$$

where, $m = \frac{1}{0.0075} + \frac{1}{0.015} = 200$

$$\text{So, } \delta = 0.775 \sqrt[3]{981^2 \times (0.01865 \times 10^{-9})^2 \times 200}$$

$$\therefore \delta = 0.00003147 \text{ m}$$

(0.0315 mm)

(4 marks)

(d). Increase in deflection for two spherical balls as radius of curvature of steel body decreases, resulting from a reduction in contact area and consequent increase in maximum pressure.

(3 marks)